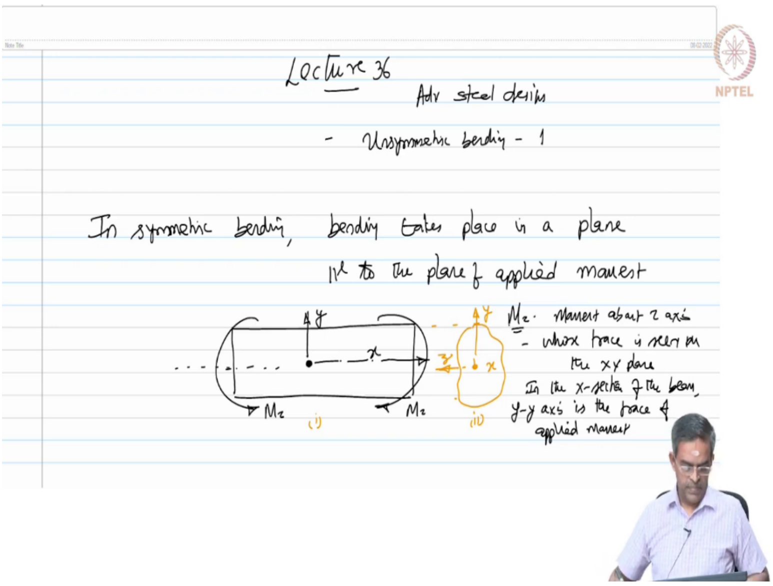
Advanced Design of Steel Structures
Dr. Srinivasan Chandrasekaran
Department of Ocean Engineering
Indian Institute of Technology, Madras

Lecture - 36
Unsymmetric bending - 1

(Refer Slide Time: 00:18)



Friends, welcome to the 36th lecture on Advanced Steel Design, where we are going to learn something about Unsymmetric Bending. So, we calls unsymmetric bending, lecture 1. In the previous lectures, we have learnt the importance of plastic design form resistant design, material properties at elevated temperature for steel, newly proposed materials as functionally graded materials and their advantages.

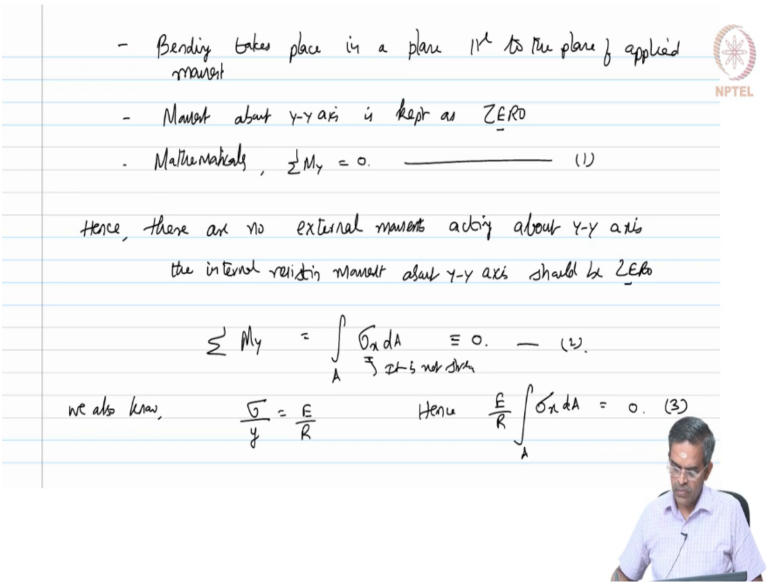
We also did some examples on plastic design and analysis, then for form dominant design we understood that stability which is one of the modes of failure is important to assess and we developed stability functions for various application conditions of axial compressive load, axial tensile load and etcetera and we did some problems and used MATLAB intensively to find out the critical buckling load so that I can assess stability of this systems.

Now, unsymmetric bending is another area of challenge in advanced design procedures. Generally, friends, in symmetric bending, bending takes place in the plane parallel to the plane of applied movement. Let us try to explain this with some graphical illustration. So, let us take a beam. So, let us say this is my x-axis of the beam which is along the span of the beam and let us call this as y and you know the z-axis is normal to this laptop screen.

So, with that axis I apply a moment about that axis I call that as M_z. So, M_z is the moment about z-axis whose trace is seen on the XY plane, right. So, now, if I cut a section, if I cut a section and if the section cross-section is known to me where this becomes my x-axis now and I call this as y and this as z-axis, correct? Imagine this figure 1 and figure 2. Figure 1 positive z is normal to the laptop screen, look it from the side.

So, you will see y in the vertical direction and z towards left side, am I right. So, I think I will remove this write it better here and remove this maybe write it here. So, now, I can say a about in the cross-section of the beam y-y axis is the trace of applied moment; z-z axis is the axis about which moment is applied. We call that is y as M_z.

(Refer Slide Time: 05:21)



Having said this, we can now write a condition saying bending takes place in a plane parallel to the plane of applied moment.

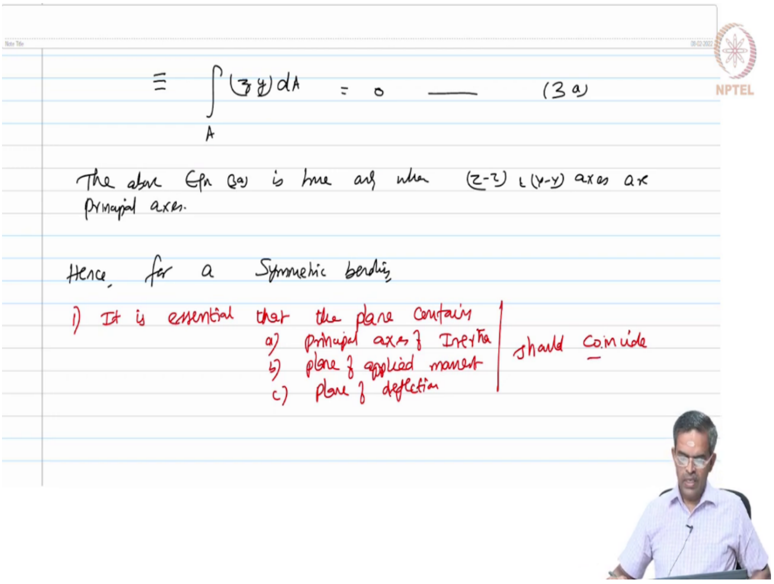
So, therefore, friends we know that moment about y-y axis is kept as zero. Mathematically algebraic sum of moments about y is 0, we calls equation number 1. Therefore, there are no external moments acting about y-y axis, right. Since there are no external moments acting about y-axis the internal resisting moment about y-y axis should be 0.

So, can I say mathematically M_y should be integral of over area A sigma x dA, 0 equation 2. So, now, we also know

$$\frac{\sigma}{y} = \frac{E}{R}$$

, hence $\frac{E}{R}$ into sigma x d A of the area a should be equal to 0. So, now, sigma x is the product of moment of inertia.

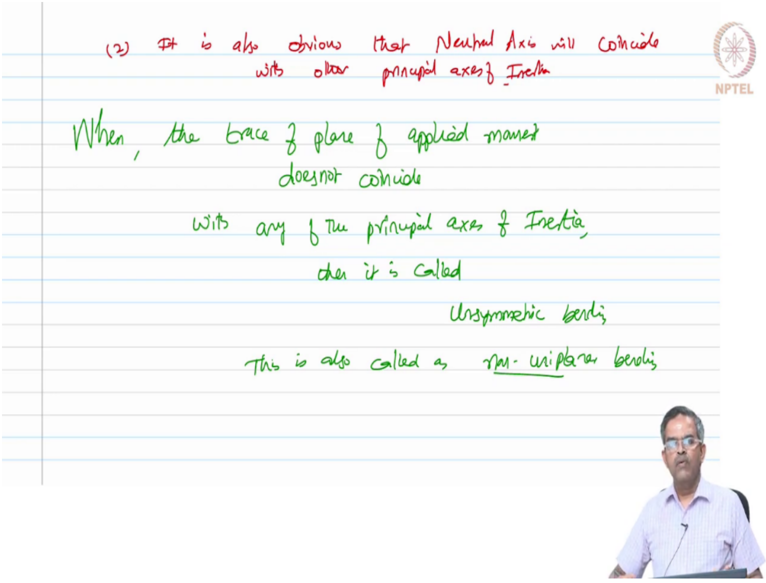
(Refer Slide Time: 08:18)



So, let us do that way saying it should be same as over the area A I should say z y dA should be termed as 0, we call this equation 3. Do not mistake it as stress it is not stress to ensure this condition, we should imply that they should be satisfied. So, we can say z y dA should be termed as 0. We call this 3(a) the above equation is only true when Z – Z and Y – Y axis are principal axis.

The above equation 3(a) is true only when Z – Z and Y – Y axes or principal axes. Hence for the symmetric bending there are certain conditions. 1 - it is essential that the plane containing a – principal axes of inertia; b – plane of applied moment; c – plane of deflection should coincide, then only a symmetric bending can happen.

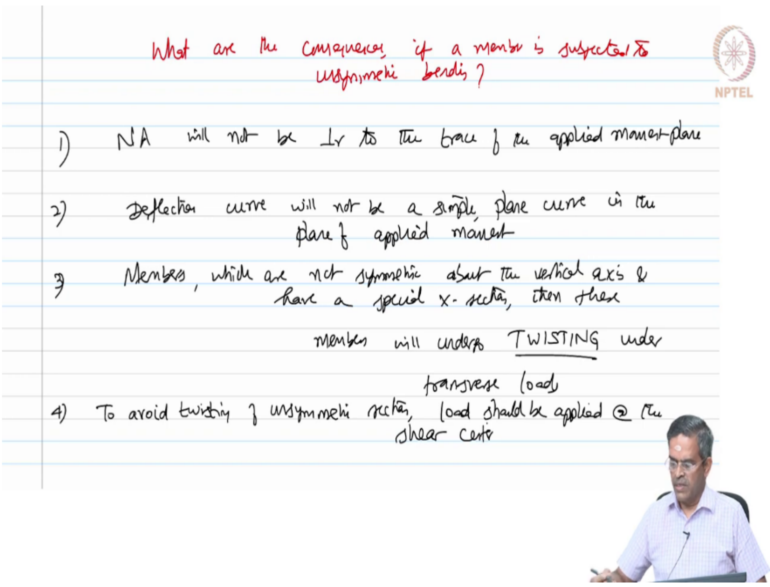
(Refer Slide Time: 10:51)



The second condition for the symmetric bending to happen is, it is also obvious that the neutral axis will coincide with other principal axes of inertia. So, therefore, friends, when the trace of plane of applied moment does not coincide with any of the principal axes of inertia, then it is called unsymmetric bending.

This is also called by another name which is non uniplanar bending. This is another name of unsymmetric bending.

(Refer Slide Time: 12:55)



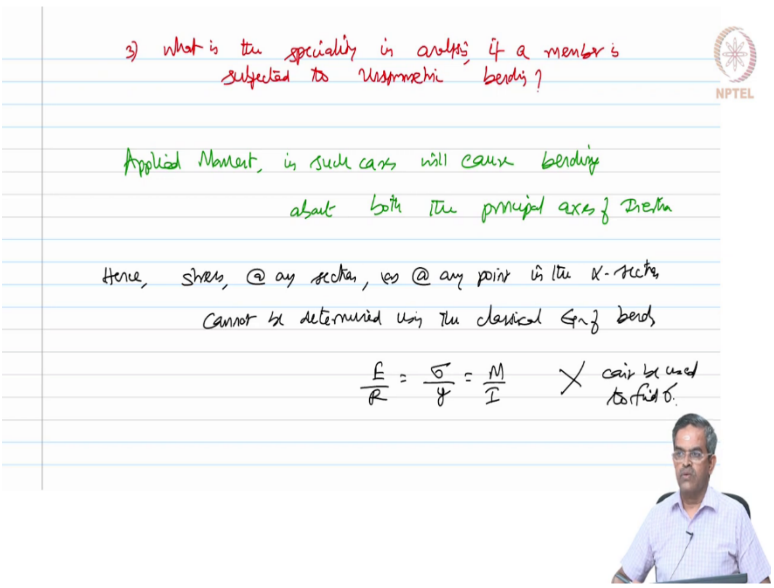
Now, the question comes if a member or a cross-section is subject to unsymmetric bending what will be the consequence?

So, we will ask a question what are the consequences if a member is subjected to unsymmetric bending. 1st, the neutral axis will not be perpendicular to the trace of the applied moment plane it makes an inclination. 2 – The deflection curve is also not a plane curve or we should say will not be a simple plain curve in the plane of applied moment.

3 – members which are not symmetric because you may get members which are not symmetric in cross-section. So, members which are not symmetric about the vertical axis and have a special cross-section, then these members will undergo twisting under transverse loads, that is a very serious consequence we have when the members are subjected to unsymmetric bending.

The 4th could be to avoid twisting of unsymmetric section load should be applied at the shear center.

(Refer Slide Time: 16:33)

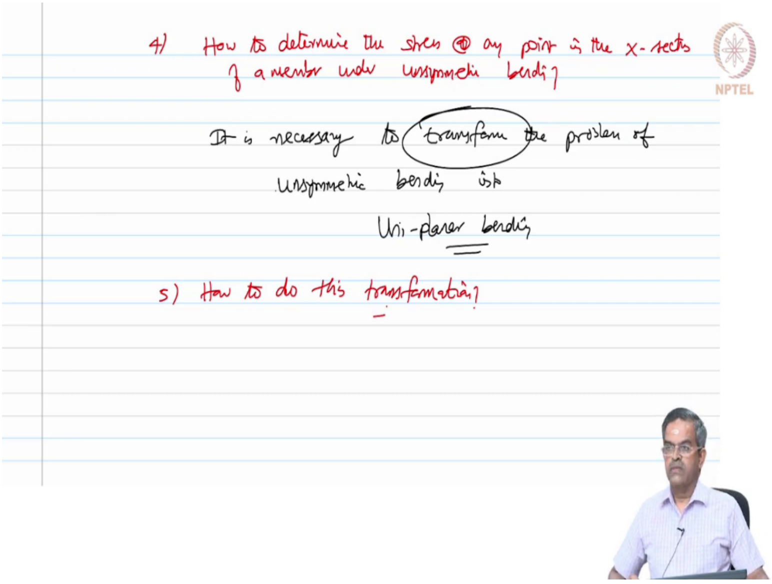


Now, the question comes what is a specialty in analysis if a member is subjected to unsymmetric bending. The answer is the applied moment, in such cases will cause bending about both the principal axes of inertia. Therefore, stress at any section or at any point in the cross section cannot be determined using the classical theory of bending that

$$\frac{E}{R} = \frac{\sigma}{y} = \frac{M}{I}$$

This equation cannot be used to determine the stress. So, it is very complicated problem now.

(Refer Slide Time: 18:41)

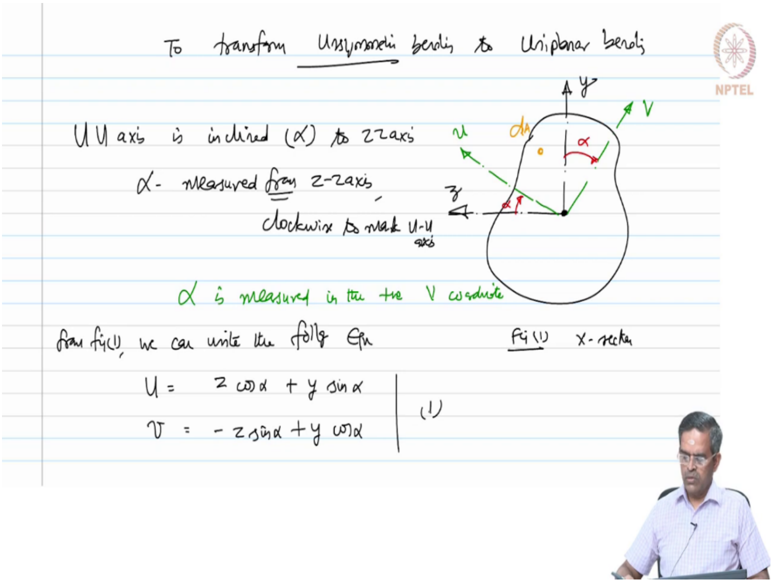


Now, what do I do to determine the stress when a member is subjected to unsymmetric bending?

So, the question is then how to determine the stress at any point in the cross-section of a member under unsymmetric bending. To do that, it is necessary to transform the problem of unsymmetric bending into uni-planar bending. So, we need to do a transformation of the problem.

Now, the question comes how will you transform the problem into uni-planar bending? that is a interesting question because we need to find the stresses at the cross sections.

(Refer Slide Time: 20:16)



Let us look into that. So, the objective is now to transform unsymmetric bending problem to uni-planar bending that is objective now. Let us take a cross-section. Let us mark these two axes as z and y; x is normal to the screen. Let us also mark two more additional axis which is called the v and u axis.

$$u = z\cos cos\,\alpha + y\sin sin\,\alpha$$

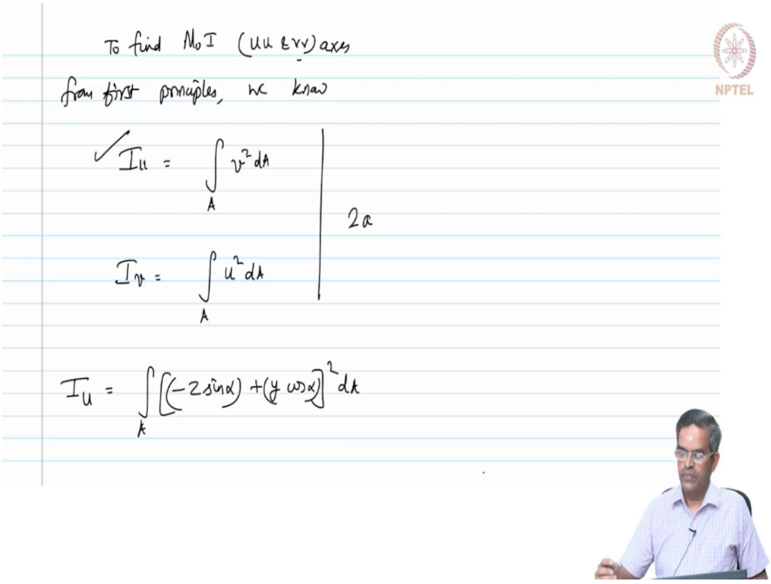
$$v =- z\sin sin\,\alpha + y\cos cos\,\alpha$$

Let us measure the angle of u-axis from z-axis or v-axis from y-axis as α. Let us take a point whose area is dA, it has got specific coordinates u and v or x, y depending upon our own choice of coordinates. Let us refer this as figure one a typical cross section of a member.

One can very well see here that uu axis is inclined α to zz axis α is measured from z-z axis clockwise to mark u-u axis, that is a condition here. So, to be very specific α is measured in the positive V coordinate. So, we have positive V, we measure from that. So, from this figure I can write a basic relationship which is very easy write down that. So, from figure 1, we can write the following equation.

What are they? $u = z\cos cos\,\alpha + y\sin sin\,\alpha$. We call this equation number 1; it is a basic equation written with reference to figure number. having said this, I want to find the moment of inertia.

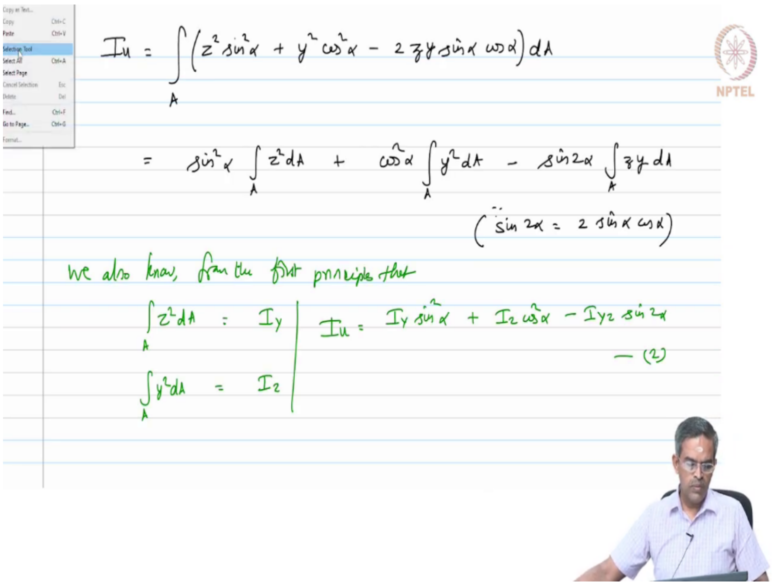
(Refer Slide Time: 24:27)



Let us say to find the moment of inertia about uu and vv axes remember they are inclined to zz and yy. So, it is not that easy to use a principle parallel axes or perpendicular axes theorem to find out that. They are inclined by an angle α. So, it is not that easy to find out I_{uu} and I_{vv}. So, let us use first principles. So, from first principles, we know I_u is integral v^2 dA for the whole area A. I_v is integral u^2 dA for the whole area a. We call this equation as 2(a).

Now, we have expression for u and v? Do we have expressions for u and v, let us substitute it here and see what happens. So, now, I v becomes integral over area A u is z of cos α, v is minus z z sin α. So, minus z sin α plus y cos α the whole square dA. So, I am getting I u, I am getting I u. So, v is here I am using this relationship. Let us expand this.

(Refer Slide Time: 26:36)



So, let us say I u is equal to integral z square sin square α plus y square cos square α minus 2zy sin α cos α of dA. So, which will be sin square α of z square dA whole area A plus cos square α of y square dA whole area A minus sin 2 α integral zy A dA.

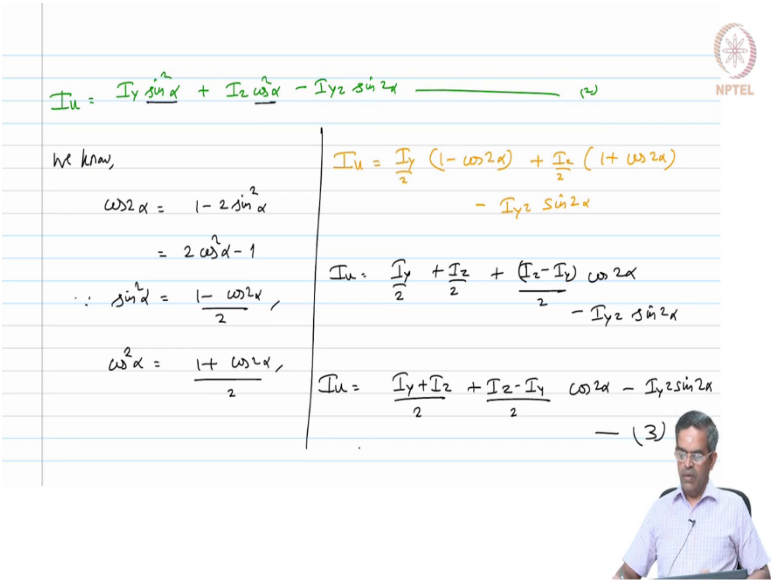
How do we write this? We know sin 2α is 2 sin α cos α. We also know from the first principles that z square dA for the whole area A can be said as I y integral y square dA for the whole area A can be said as I z when we know this then I can write I u from the above equation as,

$$I_u = I_y sin^2\alpha + I_z cos^2\alpha - I_{yz} \sin sin\, 2\alpha$$

We will call this equation number 2.

Let us simplify this. Let us copy this equation here. I will use a tool. I will copy this equation.

(Refer Slide Time: 29:27)



Let me copy it here. So, I am going to simplify this equation. Let me remove this number.

$$I_u = I_y sin^2\alpha + I_z cos^2\alpha - I_{yz} \sin sin\ 2\alpha$$

So, I am going to replace these two values with these new expressions.

$$\cos cos\ 2\ \alpha = 1 - 2sin^2\alpha$$

$$\cos cos\ 2\ \alpha = 2cos^2\alpha - 1$$

$$sin^2\alpha = \frac{1 - \cos cos\ 2\alpha}{2}$$

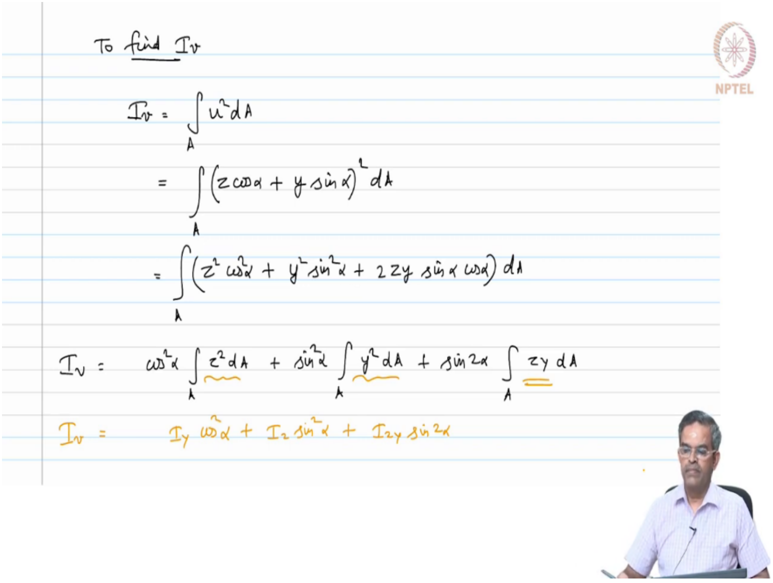
$$cos^2\alpha = \frac{1 + \cos cos\ 2\alpha}{2}$$

$$I_u = \frac{I_y}{2}(1 - \cos cos\ 2\alpha) + \frac{I_z}{2}(1 + \cos cos\ 2\alpha) - I_{yz} \sin sin\ 2\alpha$$

$$I_u = \frac{I_y + I_z}{2} + \frac{I_z - I_y}{2} \cos cos\ 2\alpha - I_{yz} \sin sin\ 2\alpha$$

call this equation number 3. So, let us take equation number 3 as one of the important reference derivatives.

(Refer Slide Time: 32:19)



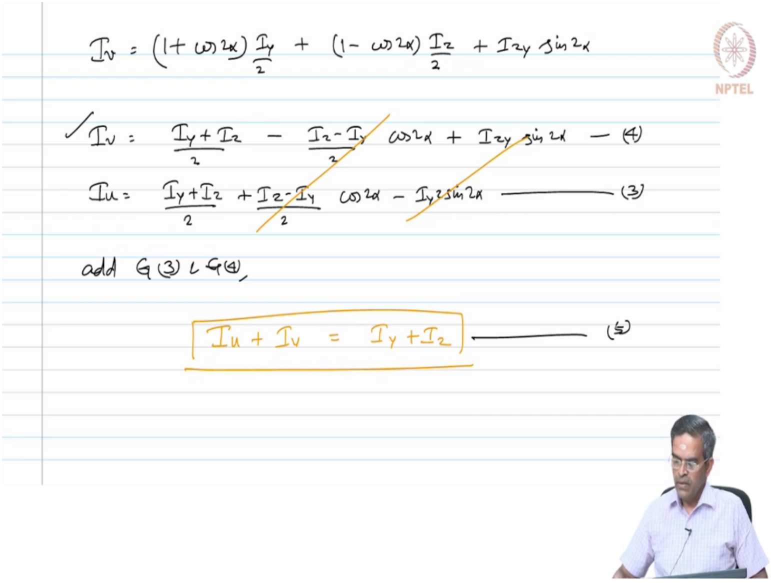
Now, let us move on to I_v. Now, let us find I_v. We know I_v is integral u^2 dA for the entire A; we already know u we already know u as this value, $u = z\cos cos\,\alpha + y\sin sin\,\alpha$. Let us substitute it here which is going to be equal to integral for the whole area $z\cos cos\,\alpha + y\sin sin\,\alpha$ the whole square dA. Let us expand this which will be z square cos square α y square sin square α plus 2z y sin α cos α of dA integrated whole area A.

Now, I can write I_v as cos square α integral z square dA for the whole area A plus sin square α integral y square dA for the whole area plus sin 2 α integral whole area z y dA. I think there is no confusion in writing this expression. Now, I can write this as z square dA is I_y cos square α y square dA is I z sin square α plus this is going to be I_{yz} sin 2 α. This is I_v, Now, let us take this I_v and expand it.

$$I_v = I_y cos^2\alpha + I_z sin^2\alpha + I_{zy}\sin sin\,2\alpha$$

We have a substitution.

(Refer Slide Time: 34:41)



$$I_v = (1 + \cos cos\, 2\alpha)\frac{I_y}{2} + (1 - \cos cos\, 2\alpha)\frac{I_z}{2} + I_{zy} \sin sin\, 2\alpha$$

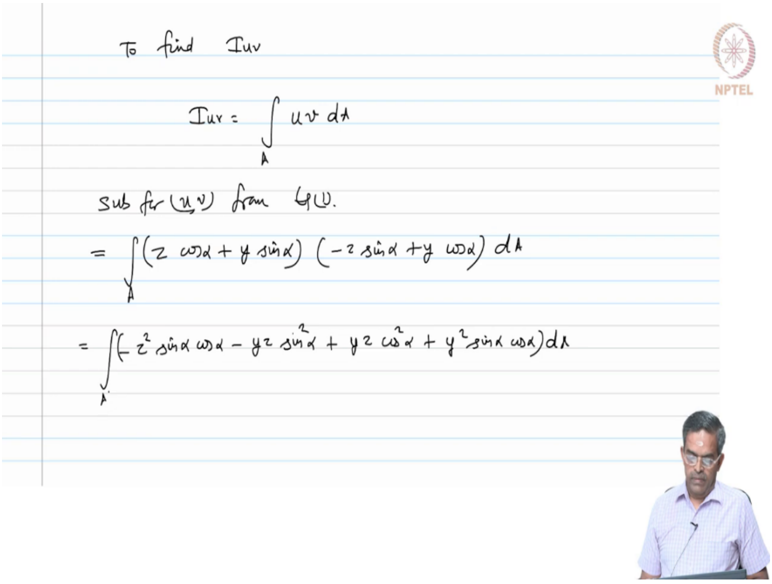
$$I_v = \frac{I_y+I_z}{2} - \frac{I_z-I_y}{2}\cos cos\, 2\alpha + I_{ZY} \sin sin\, 2\alpha$$

$$I_u = \frac{I_y+I_z}{2} + \frac{I_z-I_y}{2}\cos cos\, 2\alpha - I_{yz} \sin sin\, 2\alpha$$

$$I_u + I_v = I_y + I_z$$

I get this classical relationship, I call this equation number 5. It is a classical relationship.

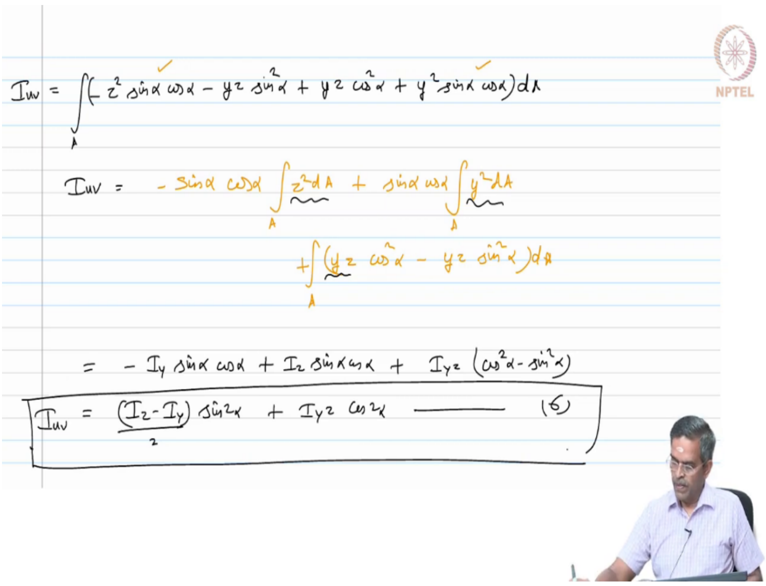
(Refer Slide Time: 37:31)



Let us also find out the cross product which is I_{uv}. We know I_{uv} is given by integral u v dA over the area A. So, we already have the values for u and v; let us see here in equation. We have equation 1, we have u and v. So, let me write it here. So, substitute for u and v from equation 1.

So, let us do that which will be equal to integral z cos α plus y sin α into minus z sin α plus y cos α of dA for the whole area which now become integral minus z square sin α cos α minus y z sin square α plus y z cos square α plus y square sin α cos α. This is for the whole area.

(Refer Slide Time: 39:18)



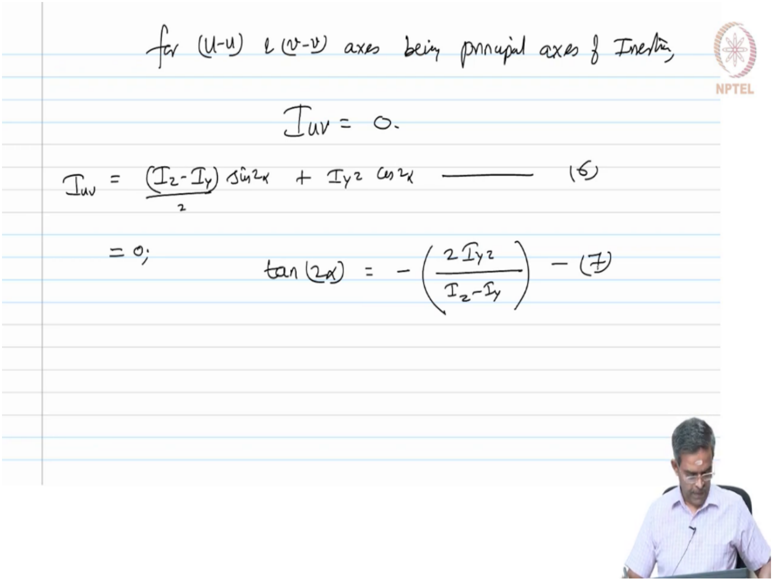
Let me copy this equation take it to the next screen for our reference which is now this is I_{uv}. So, I_{uv} is now. So, we have two terms sin α cos α twice. So, what I will do that is I can write sin α cos α integral z square dA, let us have a minus sign plus sin α cos α integral y square dA plus y z dA. we will do it. This way plus y z cos square α minus y z sin square α dA for the whole area A.

So, now I can expand this further. So, we know this is minus of I_y sin α cos α and this is I_z. So, plus I_z of sin α cos α plus you know y is A dA is I_{yz}, cos square α minus sin square α, am I right? Which now amounts to I_z minus I_y of sin 2 α by 2 plus I_{yz} cos 2 α, equation 6. This is my I_{uv} it is a classical equation.

$$I_{uv} = \frac{(I_z - I_y)}{2} \sin sin\, 2\alpha + I_{yz} \cos cos\, 2\alpha$$

Friends, we must subsequently substitute I_{uv} to be 0. So, now subsequently we should substitute I_{uv} as 0. Why do we have to do that? In simple terms, I_{uv} need to be indicated as an understanding of explaining non-symmetric bending.

(Refer Slide Time: 42:50)



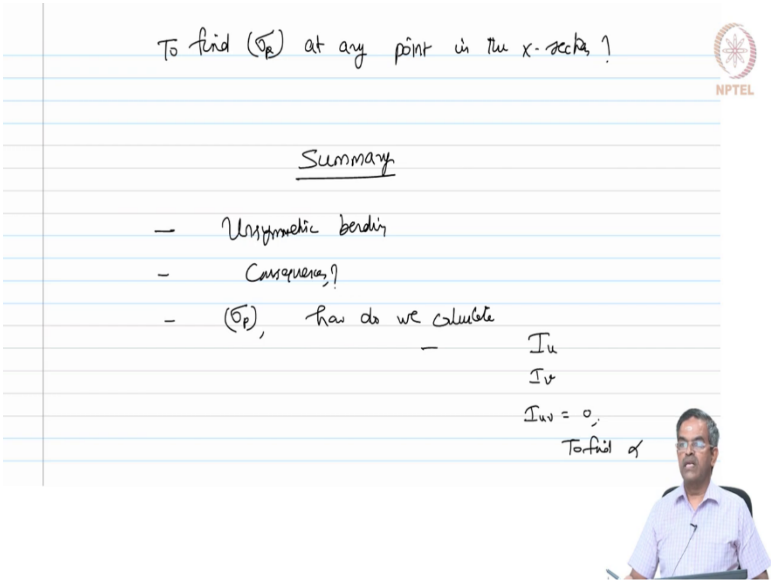
$$I_{uv} = 0;$$

$$I_{uv} = \frac{(I_z - I_y)}{2} \sin sin\, 2\alpha + I_{yz} \cos cos\, 2\alpha$$

$$\tan tan\, (2\alpha) = -\left(\frac{2I_{yz}}{I_z - I_y}\right)$$

So, now friends, u and v being principal axes of inertia I_{uv} should be 0. So, what is I_{uv}? I am just copying this equation. So, if you equate this to 0, I will get $\tan tan\, (2\alpha) = -\left(\frac{2I_{yz}}{I_z - I_y}\right)$. We call this equation number 7. So, friends we are now interested to find out stress at any point in a given cross section.

(Refer Slide Time: 44:38)



So, our objective is to find stress at any point in the cross-section. That is objective. So, we will look into this in detail in the next lecture.

Let us now write the summary what we learnt in this lecture. We learnt what is meant by unsymmetric bending or non-uniplanar bending. We have also learnt what are the consequences if a member is subjected to unsymmetric bending. If I want to find the stress at any point P in a cross section or a member under unsymmetric bending, what do I do what do we or how do we calculate. So, we said that we need to convert the unsymmetric bending problem into a uniplanar bending problem.

To do that we need to find the principal moments of inertia and we equated the product to 0 being principal axis and to find the inclination of α. Remember α is an angle is an angle of u-u axis with z-z axis measuring from the positive z to positive u or positive v to positive y, in a specific manner. So, if I know this using the equation, we will be able to find the stresses. That is what we discussed briefly and we will explain the equation how to find stresses in case of an unsymmetric bending in the next lecture.

Thank you very much and have a good day.