Advanced Design of Steel Structures
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Lecture - 37
Unsymmetric bending - example problems

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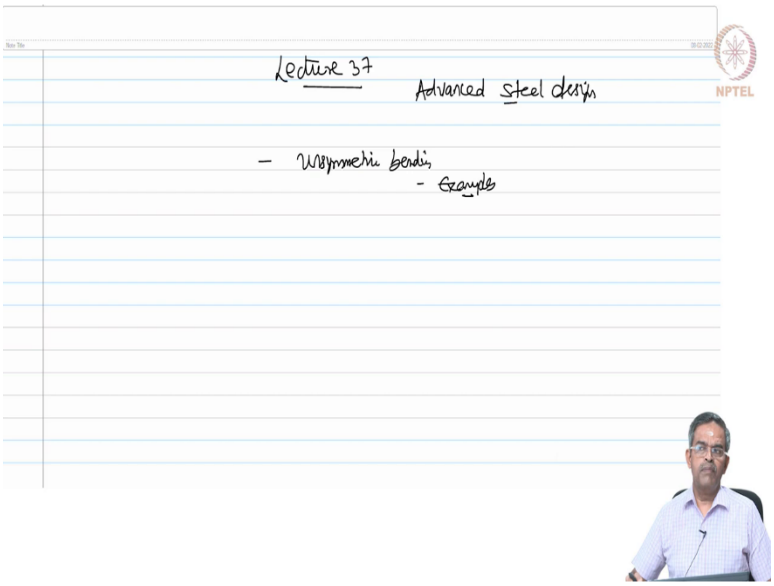

Friends, welcome to the 37th lecture on Advanced Steel Design course, in this lecture we are going to learn Unsymmetric bending with example problems. So, we learnt in the last lecture how to identify the u-u and v-v axis which are principal axis of inertia, their inclination with respect to y z axis of the cross section and the meaning of the angle α and how to compute this α angle if I know the principal moments of inertia, we have learnt these equations in the last lecture.

So, the question asked was; if a section or a member is subject to unsymmetric bending, how do we find the stresses at any point in the cross section?

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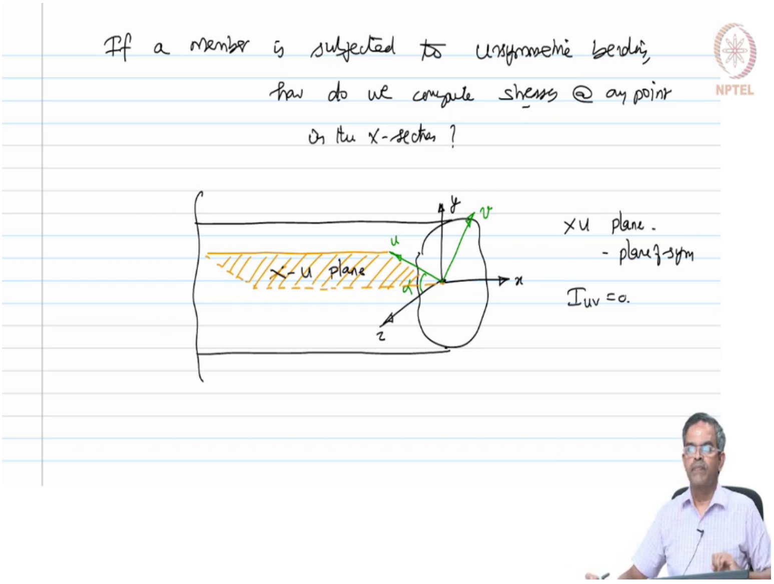

If a member is subjected to unsymmetric bending, how do we compute stresses at any point in the cross section because we said that the classical bending equation cannot be used to find out the stresses. So, how do we get this? So, let us explain this with a figure let us say I have a cross section which is arbitrary whose area is no, let us say this is my y axis and this remains as z axis and x is normal.

Let us mark it here let us mark it here as to make it easy, I will mark z here I will mark a three-dimensional view I will put x here, if this is x axis; obviously, this becomes my member. length of the member let us say. Now, I draw a plane or I draw the u-u, v-v axis. So, I draw the u-u axis at an angle α from z axis and also draw the v-v axis here this is u axis. Let me draw the shaded portion representing a specific plane. So, this hatched portion represents a specific plane which is x-u plane.

So, I will say this is write it here this is x-u plane this is x-u plane, this is the plane of symmetry is the plane of symmetry, hence I_{uv} should be 0 about that. So, once we establish this relationship, we can use conventional bending equation to get the stresses let us see how.

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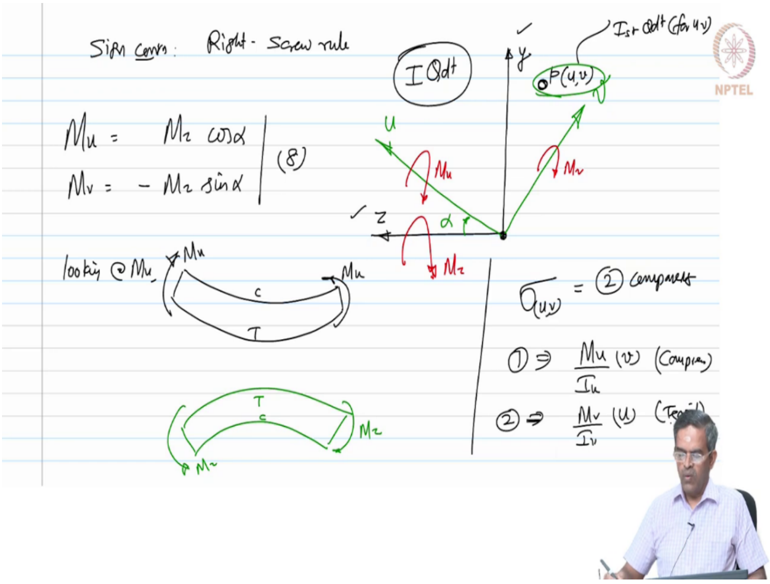

Let us say give me y axis and we mark this as my z axis, an axis here I mark this as my u-axis, this as my v-axis and we know that this angle is α measured from the positive z to positive u this is. So, this is my first quadrant remember that positive y positive z. So, it is the first quadrant.

So, I take any point p here which is marked in terms of u and v. Remember; for u and v this the first quadrant. Now let us put a moment which is M_z acting about this axis, we call this moment as M_u and this moment as M_v. Now I can write a relationship using the sign convention right screw rule. So, we can say M_u is M_z cos α and M_v is M_z sin α, but minus we call this equation number 8 for continuity what we had from the previous lecture.

$$M_u = M_z \cos cos\, \alpha$$

$$M_v = - M_z \sin sin\, \alpha$$

$$\sigma_{(u,v)} = \frac{M_u}{I_u}(\mathrm{v})\ (\text{Compression})$$

$$\sigma_{(u,v)} = \frac{M_v}{I_v}(\mathrm{u})\ (\text{Tension})$$

So, interestingly friends if you look at M_u if you look at M_u it produces bending this form. So, tension at the bottom compression at the top this is M_u, can you visualize that? But on the other hand, if you see M_z, M_z is reverse the tension at the top compression at the bottom. Let us take any point u-v we write the equation. So, the stress at the point u v is given by two combinations 2 components one component will come from M_u by I_u into v, the other component will come from M_v by I_v into u.

Looking at these figures the component which is caused by u will be compressive, at the same location the component caused by M_v will be tensor, Let us take an extreme top fibre that is what the situation is.

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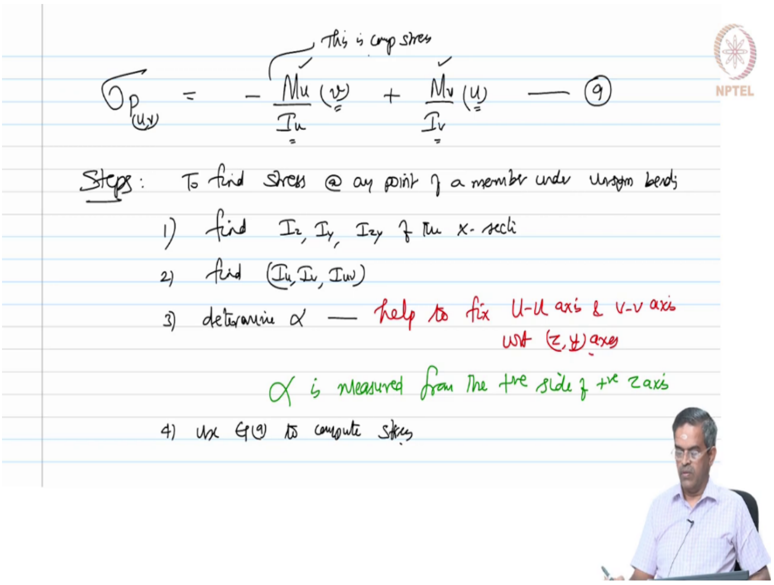

Now, we can say

$$\sigma_{P(u,v)} = -\frac{M_u}{I_u}(v) + \frac{M_v}{I_v}(u)$$

but; however, this is minus because this is a compressive stress given by equation 9. So, friends, equation 9 is similar to a classical bending equation provided I have replaced the moments properly resolved to u and v axis.

And the components of measurement and the moment of inertias or with respect to the principal axis. So, I am transforming the problem from an unsymmetric bending to uni-planar bending where the moments of inertia are now transformed to the principal axis of moments of inertia and I am trying to find out the moments about this axis and using a classical bending theory. Let us see, what are the steps involved in finding the stress? what are the steps involved to find the stress at any point of a member under unsymmetric bending.

First step find I_x, I_y, I_z, I_{zy} of the cross section if you know this find I_u, I_v, and I_{uv}. So, then determine α that is this will help you to fix the u-u axis and v-v axis with respect to z y axis. Remember, α is measured from the positive psi of positive z axis, then use equation 9 to compute this stress, these are the steps involved. To demonstrate this, we will take up a simple problem and gradually transform the problem into a very interesting theory, we will do that.

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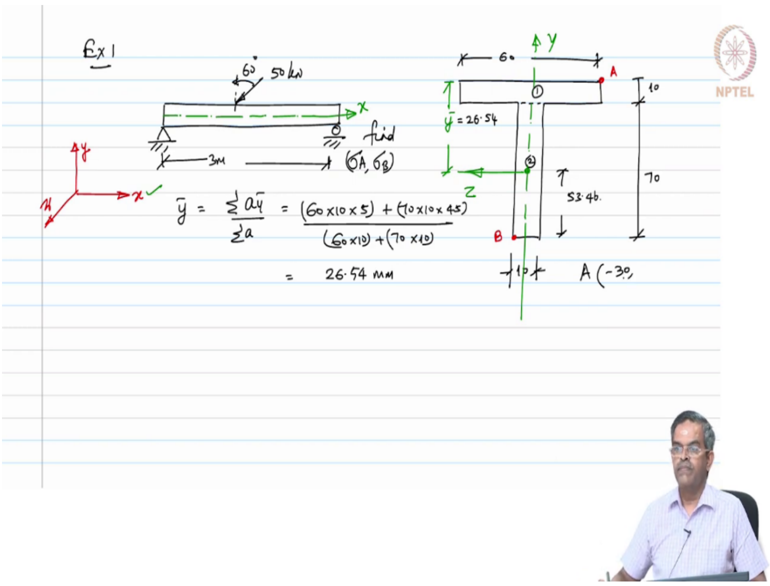

So, let us say example 1 we will take a T section this dimension is 60. And this dimension is 10,70 and this thickness is 10. We will divide this into 2 components we will call see component number 1 and component number 2. So, now, this is the beam which is simply supported for a span of 3 meters subjected to an inclined load of 50 kilo newtons, the inclination is 60 degrees to the vertical. So, let us draw the coordinate axis for this, this is x, let me do it here this is x axis, this is y axis and this is z axis.

Let us mark two points one point is A other point is B. So, the question is, find the stresses at A and B?. So, we need to locate the neutral axis. So, now, interestingly this has one axe of symmetry let us locate the z axis here and this is going to be my y axis is a cross section, because x is along the length of the member see. Let me call this distance as y bar and from first principles I can easily find y bar let us do that.

So, we know,

$$\bar{y} = \frac{\Sigma a\bar{y}}{\Sigma a} = \frac{(60\times10\times5)+(70\times10\times45)}{(60\times10)+(70\times10)}$$

$$\bar{y} = 26.54mm$$

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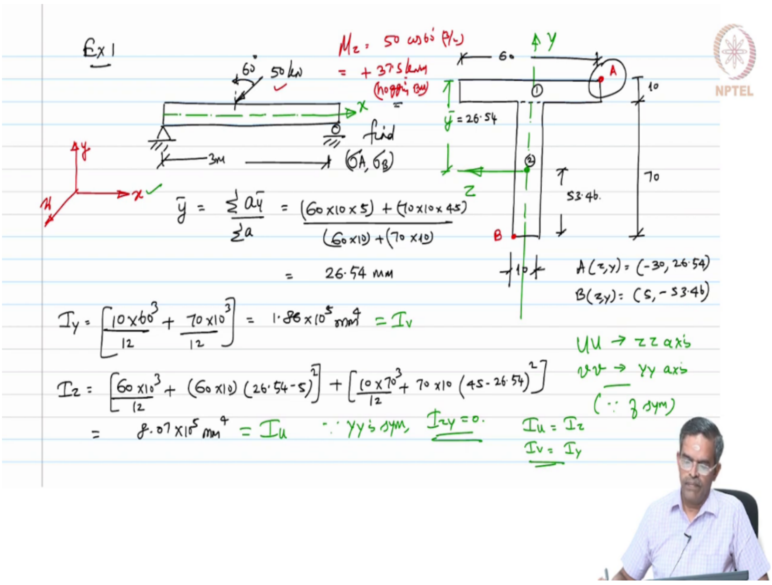

Let us say

$$A(z, y) = (-\ 30, 26.54)$$

$$B(z, y) = (5, -\ 53.46)$$

Having said this let us compute I_y it has got axis of symmetry. So, we can use simple theorem to compute this let us do this going to be

$$I_y = \left[\frac{10\times60^3}{12} + \frac{70\times10^3}{12}\right] = 1.86\times10^5\ mm^4$$

$$I_y = I_v$$

Let us find out I z I will use parallel axis theorem. So, I will say

$$I_z = \left[\frac{60\times10^3}{12} + (60\times10)(26.54 - 5)^2\right] + \left[\frac{10\times70^3}{12} + (70\times10)(45 - 26.54)^2\right]$$

$$I_z = 8.07\times10^5 mm^4$$

$$I_z = I_u$$

Now I can straight away say since the axis symmetry is matching, I can straight away say u-u axis will be same as z-z axis, v-v axis will be same as y-y axis because of symmetry therefore, I can say I_u is same as I_z and I_v is same as I_y. So, I can now say this is I_u and this is I_v, I_y is I_v and I_u is I_z. Since, y-y axis is symmetric axis I can straight away say I_{zy} will be 0 which will give me the angle of α. I_z will be 0, I do not have to calculate that.

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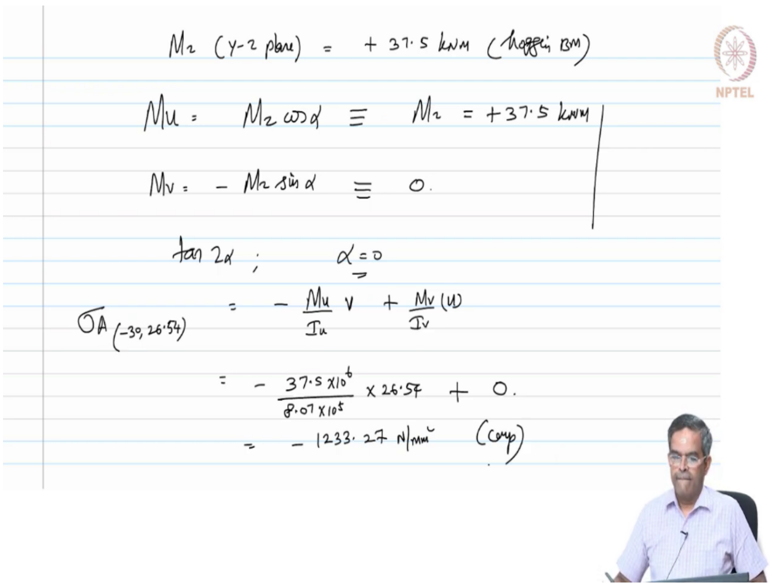

Now, let us find out M_z. So, M_z will be in the y- z plane see the original equation. So, it is going to be

$$M_z(y - z\,plane) = 50 \cos cos\, 60 \left(\frac{3}{2}\right) = +37.5 \text{ kNm (Hogging BM)}$$

we can write it here itself M_z. M_z is going to be +37.5 kNm which is a hogging moment I will say it is positive.

The load is downward hogging moment tension at the bottom. Now I can straight away say M_z is plus 37.5 kNm hogging bending moment. So, I can now say

$$M_u = M_z \cos cos\, \alpha = M_z = + 37.5 kNm$$

$$M_v = - M_z \sin sin\, \alpha = 0$$

$$\alpha = 0;$$

M_v is minus $M_z \sin sin\, \alpha$ we have this equation with us. So, α 0 so, this is going to be 0. So, we can also write tan 2 α which will also become; obviously, α is 0.

So, let us compute σ_A. So, if you want to find σ_A at

$$\sigma_{A(-30,26.54)} = - \frac{M_u}{I_u}(v) + \frac{M_v}{I_v}(u)$$

$$\sigma_{A(-30,26.54)} = \frac{-37.5\times10^6}{8.07\times10^5} \times 26.54 + 0$$

$$\sigma_{A(-30,26.54)} = - 1233.27 \frac{N}{mm^2} \text{(Compression)}$$

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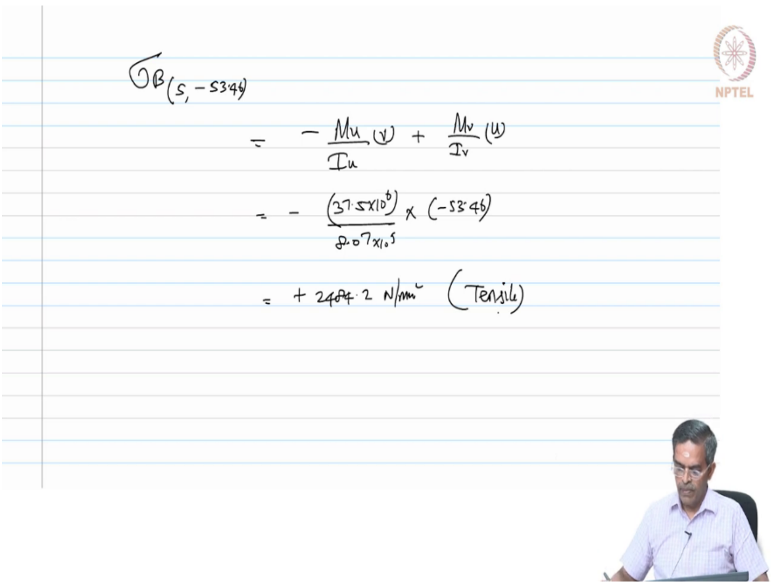

Let us find out σ_B at the point (5,-53.46),

$$\sigma_{B(5,-53.46)} = -\frac{M_u}{I_u}(v) + \frac{M_v}{I_v}(u)$$

$$\sigma_{B(5,-53.46)} = \frac{-37.5\times10^6}{8.07\times10^5}\times(-\ 53.46)+0$$

$$\sigma_{B(5,-53.46)} = +\ 2484.2\ \frac{N}{mm^2}(\text{Tensile})$$

Friends, also see this figure the point A is lying on the extreme top fibre. So, point A will be subjected to compression because the hogging bending mode and the bottom fibre will be in tension. So, we have got this is tensile and this is compressed.

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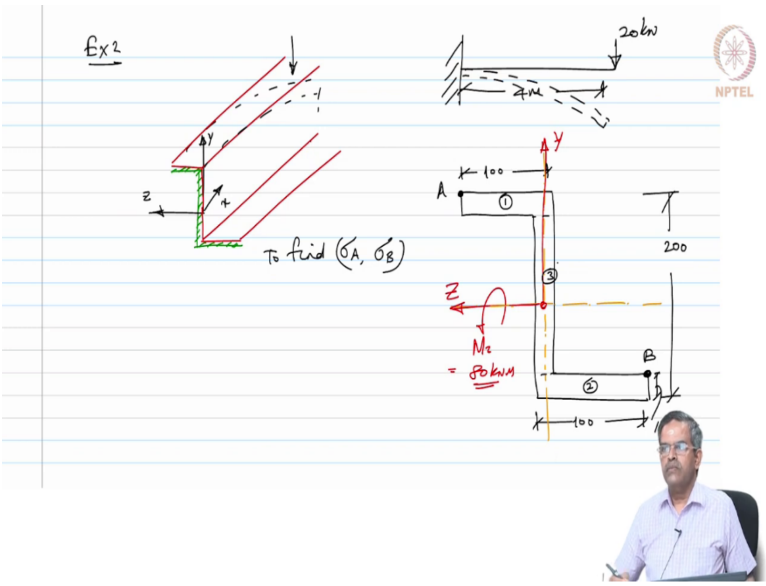

Let us do one more problem let us do one more problem which is example 2 here we take a cantilever beam subjected to a load of 20 kN for a span of 4 meters. So, the beam is going to bend this way. So, let us mark the axis, let us say this is my y-axis, this is my z-axis and this becomes my x-axis let me draw the section.

So, this is my section I am looking for a z section. So, the z section is marked here say this is 100 mm and the whole depth is 200 mm and the thickness is 15 mm. So, we all know we got two axis of symmetry taking this also as 100, now we will mark this axis as my Y-axis and this axis as my Z-axis an axis somewhere here and I will apply M_z here where this M_z will be equal to 80 kNm.

So, here actually the load is applied and the section is going to bend this way and so on. So, we want to find two points this is A and this is B. So, to find stress as A and B that is the question.

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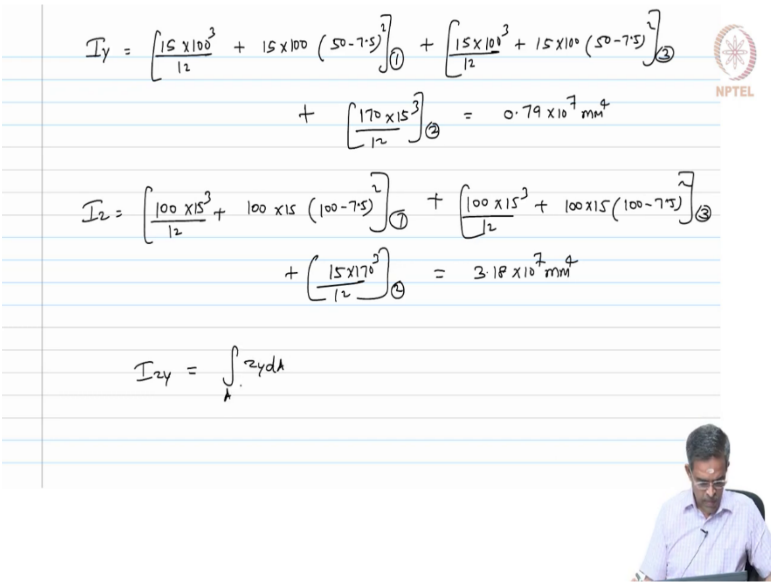

Let us find I_y for this problem which is a straightforward exercise because it has got two axis of symmetry.

$$I_y = \left[\frac{15\times100^3}{12} + (15\times100)(50 - 7.5)^2\right] + \left[\frac{15\times100^3}{12} + (15\times100)(50 - 7.5)^2\right] + \left[\frac{170\times15^3}{12}\right]$$

$$I_y = 0.79\times10^7 mm^4$$

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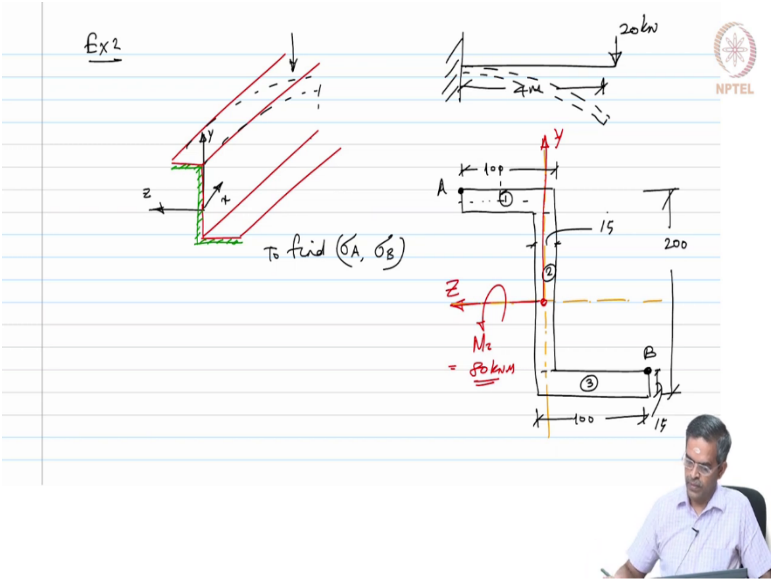

One can find I z z is an horizontal axis. So, I can say

$$I_z = \left[\frac{100\times15^3}{12} + (100\times15)(100 - 7.5)^2\right] + \left[\frac{100\times15^3}{12} + (100\times15)(100 - 7.5)^2\right] + \left[\frac{15\times170^3}{12}\right]$$

$$I_z = 3.18\times10^7 mm^4$$

Since, z-y are axis of symmetry we know I_{zy} will be integral z by dA of the whole area.

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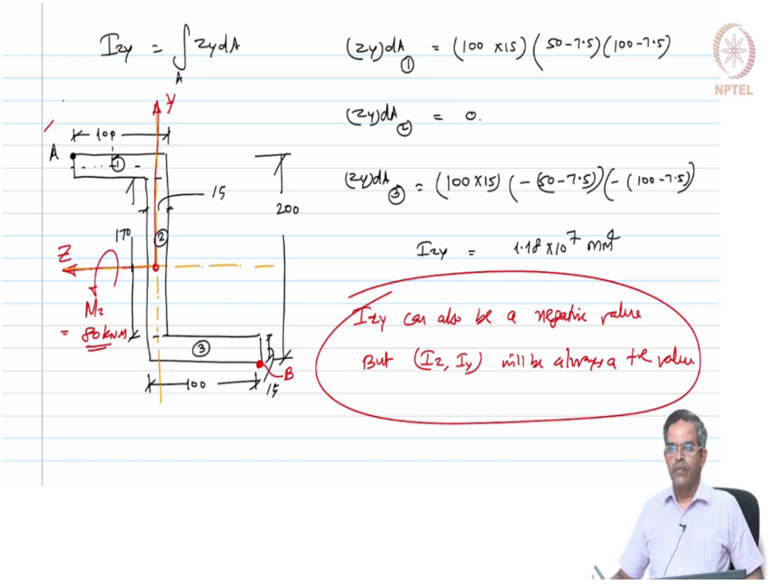

$$(zy)dA_1 = (100\times15)(50 - 7.5)(100 - 7.5)$$

$$(zy)dA_2 = 0$$

$$(zy)dA_3 = (100\times15)(-(50 - 7.5))(-(100 - 7.5))$$

$$I_{zy} = 1.18\times10^7 mm^4$$

So, add them together I_{zy} will become $1.18\times10^7 mm^4$. Friends, very important about the z sections, though it is not act of symmetry I_z is not 0. Also, there is a note I_{zy} can also be a negative value, but I_z I_y will be always a positive value. Having said this, we are looking for let us we will change the point B somewhere here we will mark this as point B and A we want extreme fibres.

So, therefore, since I z y is not 0 we want to find the angle α. Let us do that.

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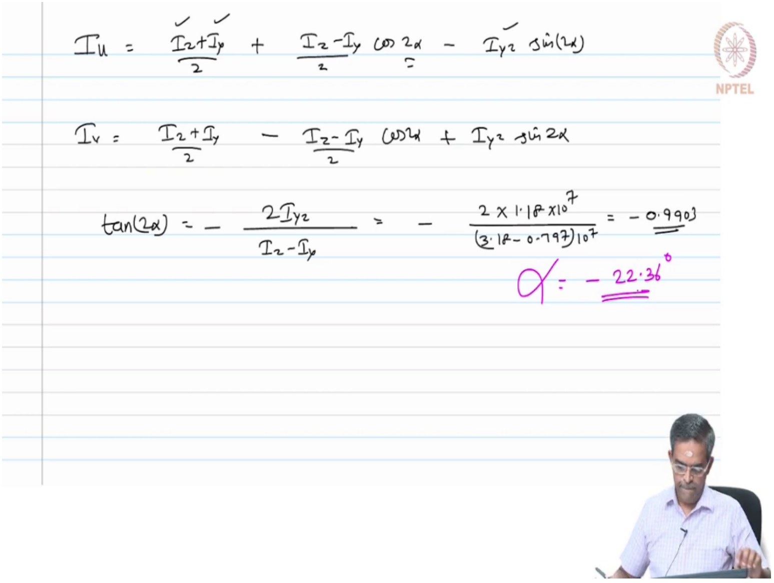

$$I_u = \frac{I_z + I_y}{2} + \frac{I_z - I_y}{2} \cos cos\, 2\alpha \; - I_{yz} \sin sin\, 2\alpha$$

$$I_v = \frac{I_z + I_y}{2} - \frac{I_z - I_y}{2} \cos cos\, 2\alpha \; + I_{yz} \sin sin\, 2\alpha$$

$$\tan tan\, (2\alpha) = - \frac{2I_{yz}}{I_z - I_y}$$

$$\tan tan\, (2\alpha) = - \frac{2 \times 1.18 \times 10^7}{(3.18 - 0.797) \times 10^7} = - \, 0.9903$$

$$\alpha = - \, 22.36^o$$

So, what does it mean, when I say α is negative what does it mean? Measured clockwise from positive z axis towards positive view that is what it is.

So, I should say. So, I will mark this figure I will take this figure again.

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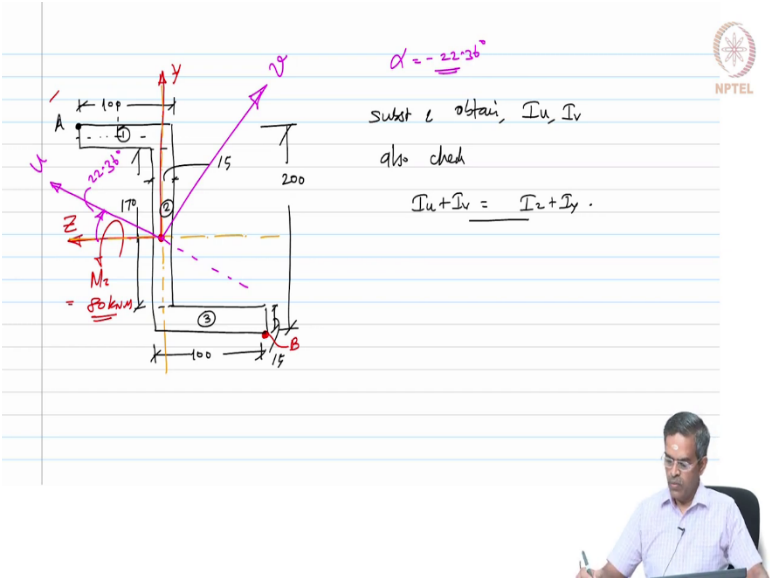

Let me mark the u-u and v-v axis and we know this angle is 22.36 degrees. So, we have marked this angle which we obtained it is always measured from positive z towards positive u that is what you have got. So, now, we know α is minus 22.36 degrees let us substitute that and get I_u and I_v. So, substitute and obtain I_u, I_v also. Check, $I_u + I_v = I_z + I_y$. you should check this.

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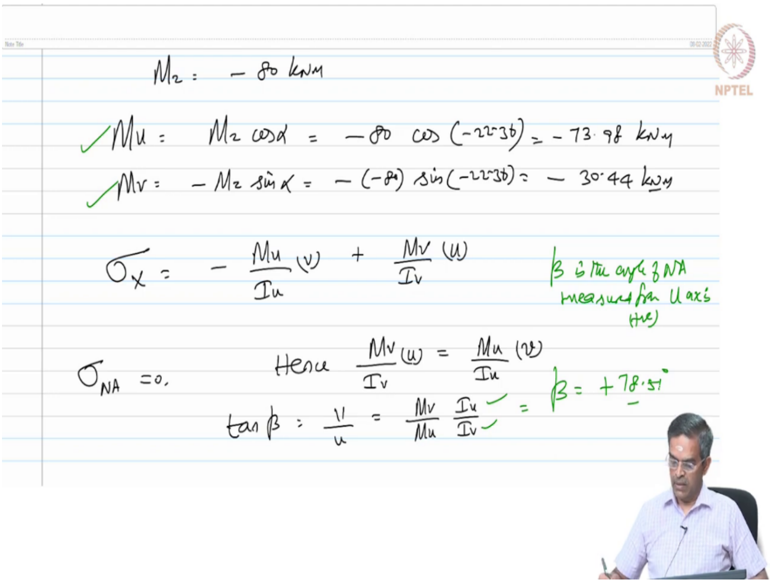

So, now we have M_z as minus 80 kilo newton meter see here. So, this is a force and M_z is creating tension at the top.

$$M_z = -\ 80 kNm$$

$$M_u = M_z \cos cos\ \alpha\ = -\ 80 \times \cos cos\ (-\ 22.36)\ =\ -\ 73.98 kNm$$

$$M_v = -\ M_z \sin sin\ \alpha\ = -\ (-\ 80) \times \sin sin\ (-\ 22.36)\ =\ -\ 30.44 kNm$$

$$\sigma_x = -\ \frac{M_u}{I_u}(v) + \frac{M_v}{I_v}(u)$$

$$\sigma_{NA} = 0;$$

$$\frac{M_v}{I_v}(u) = \frac{M_u}{I_u}(v)$$

So, straight away I can say $\tan tan\ \beta$ is an angle which is

$$\tan tan\ \beta\ = \frac{v}{u} = \frac{M_v}{M_u}\frac{I_u}{I_v}$$

Now, what is beta? Beta is the angle of neutral axis measured from the u-u axis positive.

$$\beta = +\ 78.51^o$$

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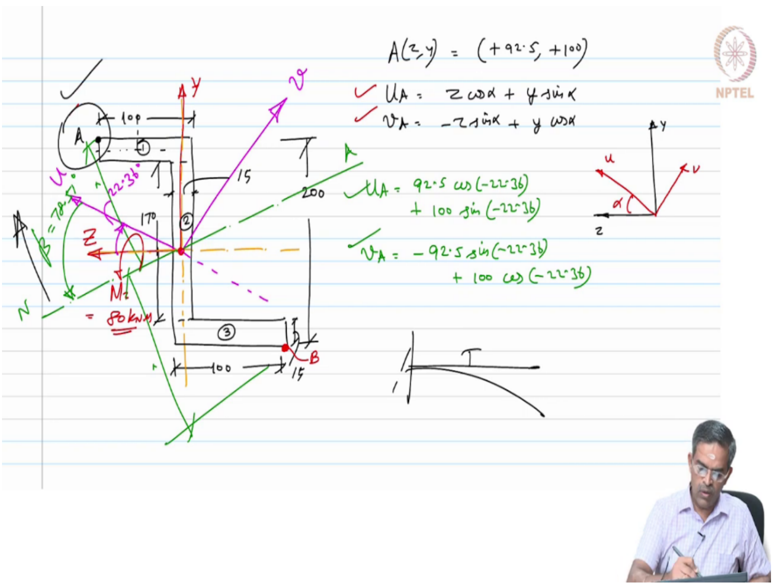

Let us mark this I will go to this figure, pick up this figure again let me mark the beta angle which will be. So, this be neutral axis and this is my angle beta which is 78.51 degrees. So, I must know this distance and I must know this distance, to find the stresses. So, that is very complicated. So, what I am going to do I am not going to compute these distances I want to find out these distances in terms of u and v that is what we are trying to do.

So, now let us talk about the point A, point A has got z and y as plus 92.5 and plus 100.

A(z,y)=(+92.5,+100)

$$u_A = z \cos cos\, \alpha + y \sin sin\, \alpha$$

$$v_A = - z \sin sin\, \alpha + y \cos cos\, \alpha$$

$$u_A = 92.5 \cos cos\, (-22.36) + 100 \sin sin\, (-22.36)$$

$$v_A = - 92.5 \sin sin\, (-22.36) + 100 \cos cos\, (-22.36)$$

We already said that look at this axis if this is my z and y if these are my u and v and this is my α this equation is clear.

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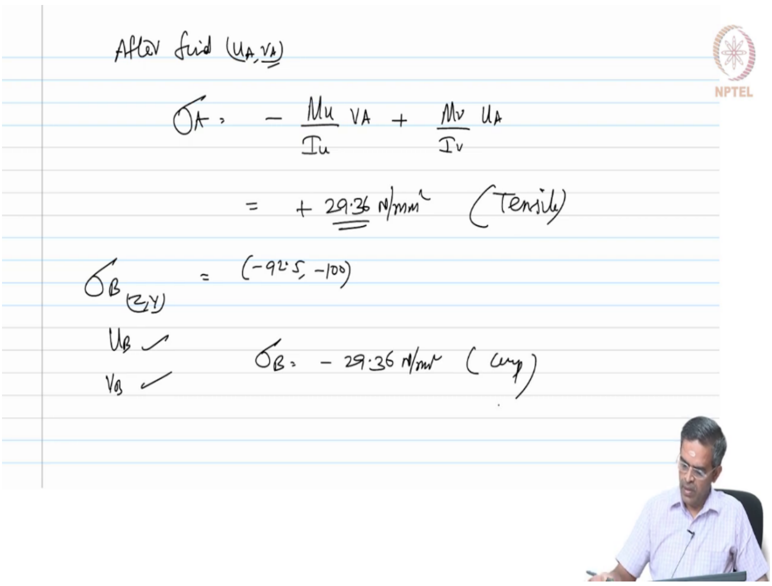

$$\sigma_A = - \frac{M_u}{I_u}\left(v_A\right) + \frac{M_v}{I_v}\left(u_A\right) = + 29.36 \frac{N}{mm^2} (Tensile)$$

$$\sigma_{B(z,y)} = (-92.5, -100)$$

$$\sigma_{B(-92.5,-100)=-29.36}\frac{N}{mm^2}\ (Compression)$$

And this indicates its compressive, look at the figure the point A is above the neutral axis and I have a cantilever. So, I have tension at the top and I get this as tensile and compression at the bottom get compression.

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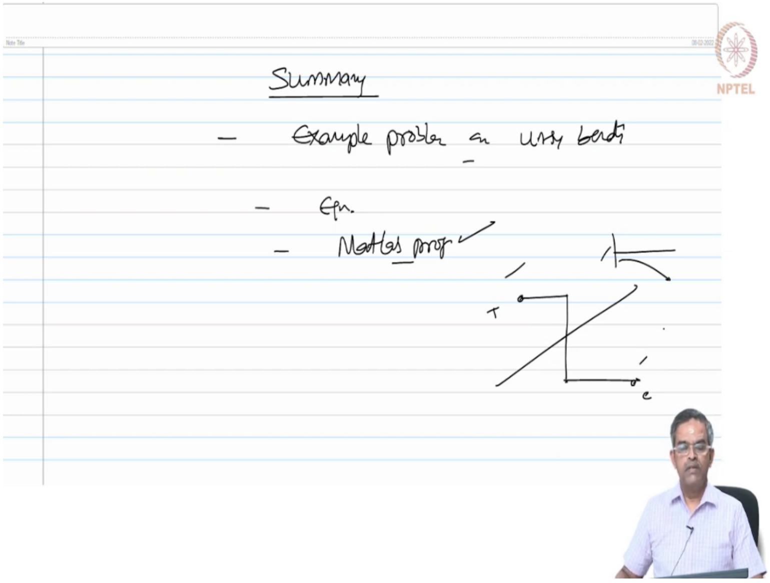

So, friends in this lecture we learnt how to solve 2 problems, how to solve example problems on unsymmetric bending. We know how to use the equations; in the next lecture we will use MATLAB program to solve this and see how we are able to use MATLAB effectively for solving such problems.

So, in a z section interestingly when the neutral axis here, this point is tensile, this point is compressive for a cantilever and we got them exactly verified from the stat. So, friends, this lecture helps you to compute the stresses of a section subjected to unsymmetric bending by converting it into uniplanar bending and then using the classical equation for finding out the stresses with modifications in both M and I values. Use this in more examples are available in reference textbooks.

Please, see them use the MATLAB programs effectively and try to solve as many problems as possible. So, that you get good hold on understanding of the problem.

Thank you very much and have a good day bye.