Advanced Design of Steel Structures
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Lecture - 38
Shear center - 1

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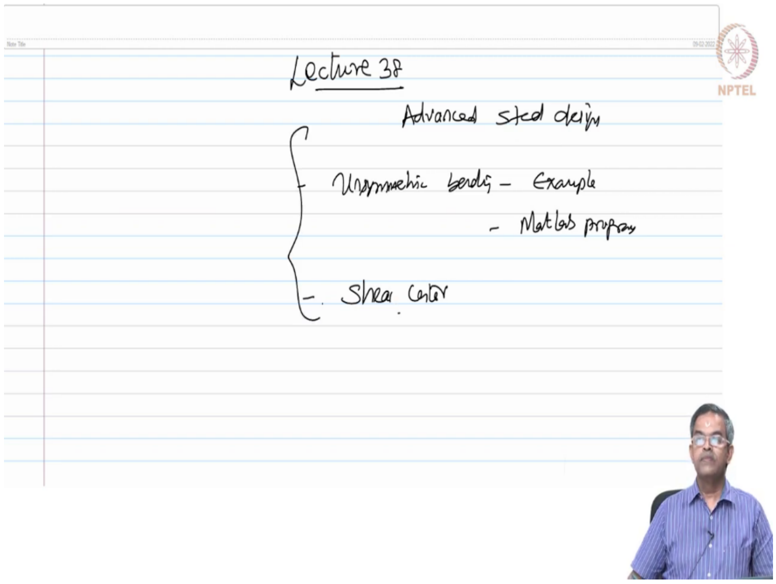

Friends, welcome to the 38 lecture on Advanced Steel Design. In this we are going to learn more example on unsymmetric bending. We are also going to use the MATLAB program for solving this problem. We will also start learning more on Shear centre. So, friends we already said in the last lecture, that unsymmetric bending is caused under special circumstances. And we know how we have modified the classical equation of bending appropriately to find out the stresses at any point in a cross section of a member subjected to unsymmetric bending.

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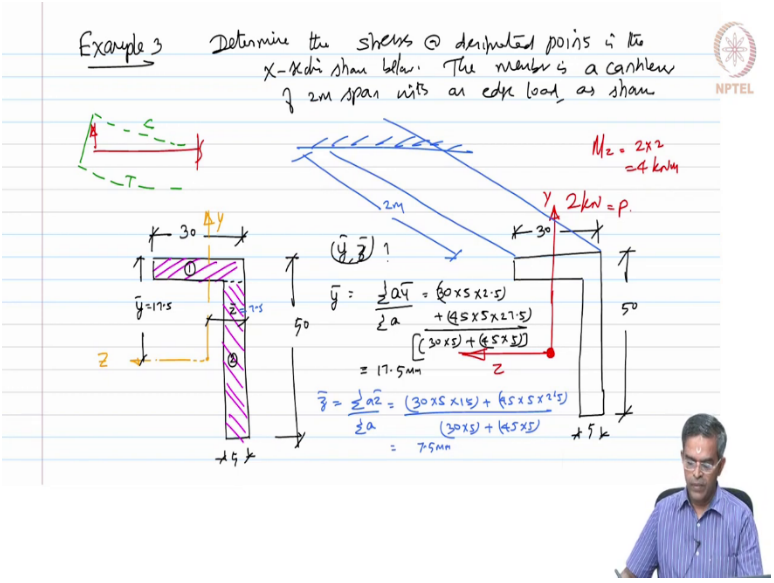

We will now take up one more example. I will call this as example 3, because we have already done couple of examples in the last lecture, I am continuing that; obviously, the example says determine the stresses at the designated points in the cross section shown below.

The member is a cantilever of 2-meter span with an edge load as shown. So, let us say; this is my cross section, this is 30 mm and this is 50 mm and it has got an uniform thickness of 5 mm. And let us say this is a cantilever subjected to an upward loop of 2 km and the cantilever span is 2 meters.

Let us say this is my z-y axis. The load is applied this way and normal to the screen is my x axis. Let us say I have a centroidal axis for this. Let me draw the figure again here separately to mark the geometric properties of this cross section. So, this is 15, this is of thickness 5 uniform and this dimension is 30. We call this as 1 and this as 2. So, there are two things here this is the first element then the second element.

So, let us say it has got a centroid here. This is going to be my z axis this is going to be my y axis and I would like to compute these values. I call this as y bar and I call this as z bar. So, we need to find $\bar{y}$ and $\bar{y}$ we need to find. First principles very simple. let us do that. So,

$$\bar{y} = \frac{\Sigma a\bar{y}}{\Sigma a} = \frac{(30\times5\times2.5)+(45\times5\times27.5)}{[(30\times5)+(45\times5)]} = 17.5mm$$

$$\bar{z} = \frac{\Sigma a\bar{z}}{\Sigma a} = \frac{(30\times5\times15)+(45\times5\times2.5)}{[(30\times5)+(45\times5)]} = 7.5mm$$

First principle is very simple I think there will be no difficulty in doing this. Now, I want to compute I_z and I_y, because we know to compute the principal moments of inertia. I need to compute these values, because the equation is based on this.

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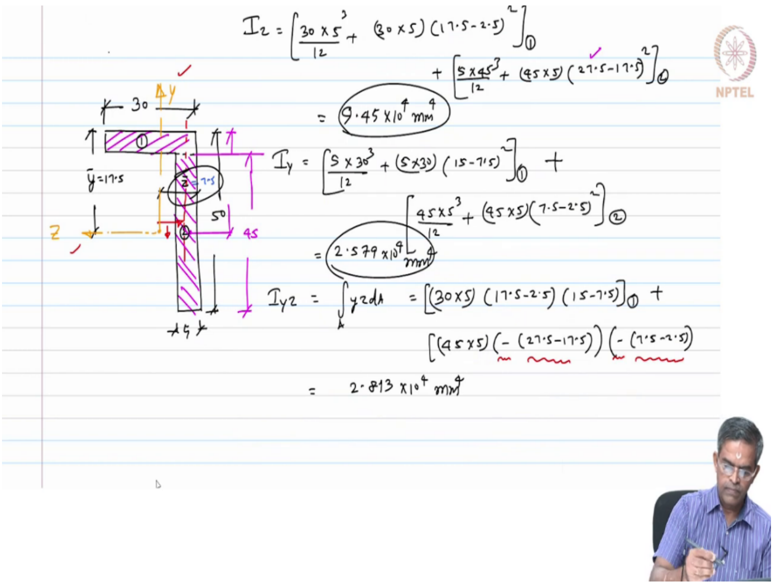

So, now let us compute I_z. Looking at the reference to this figure I can use parallel axis theorem. So, let me copy this figure for our convenience. Let me copy this figure and put it here and this dimension is 50 let us mark it here, because we need it. This is 50 let us mark it here. So, now, let me compute I_z, which is 30 into 5 cube by 12 plus parallel axis theorem, 30 into 5 into 17.5 minus 2.5 the whole square.

This is for piece number 1 plus 5 into 45 cube by 12 plus 45 into 5 into 27.5 minus 17.5. I think the cg of this piece if you carefully look at this the cg of this piece will be from here, this value is 45. So, 22.5 plus 5, 27.5 minus 27.5 the whole square this is for piece number 2. So, this value is 9.45, 10 power 4 mm 4. Let us also find out I_y again using parallel axis theorem.

$$I_z = \left[\frac{30\times5^3}{12} + (30\times5)(17.5 - 2.5)^2\right]_1 + \left[\frac{5\times45^3}{12} + (45\times5)(27.5 - 17.5)^2\right]_2 = 9.45\times10^4 mm^4$$

$$I_y = \left[\frac{5\times30^3}{12} + (5\times30)(15 - 7.5)^2\right]_1 + \left[\frac{45\times5^3}{12} + (45\times5)(7.5 - 2.5)^2\right]_2 = 2.579\times10^4 mm^4$$

$$I_{yz} = \int yz\, da = [(30\times5)(17.5 - 2.5)(15 - 7.5)]_1 + [(45\times5)(-(27.5 - 17.5))(-(7.5 - 2.5))]_2$$

$$I_{yz} = 2.813\times10^4 mm^4$$

So, which is very simple, let us do this 5, 30 cube by 12 plus 5 into 30 into 15 minus 7.5, because this value is 7.5. This is for piece number 1 plus piece number 2, 45 into 4 cube by 12 plus 45 into 5 into 7.5 minus 2.5 the whole square for 2 which is also equal to now 2.579 into 10 power 4 mm power 4.

Let us compute also I_{yz}, which will be yzdA for the entire area. Let us do it here. Which will be equal to 30 into 5 that is for the first piece. So, which is going to be 17.5 minus 2.5, because this is positive and z bar is also positive, which will be 15 minus 7.5. This is for piece number 1. Similarly we can do for piece number 2, 45 into 5. And we are talking about minus going to be minus of 27.5 minus 17.5, because this is going to be below the cg, into minus of 7.5 minus 2.5.

I think we know very clearly how are we getting this minus. For example, the cg of this piece is towards the negative direction of z. Therefore, there is a minus here. Similarly, the cg of this is towards the negative direction. For example, towards negative direction of y therefore, there is minus. So, if I compute this I get I_{yz} as 2.813 into 10 power 4 mm to the power 4.

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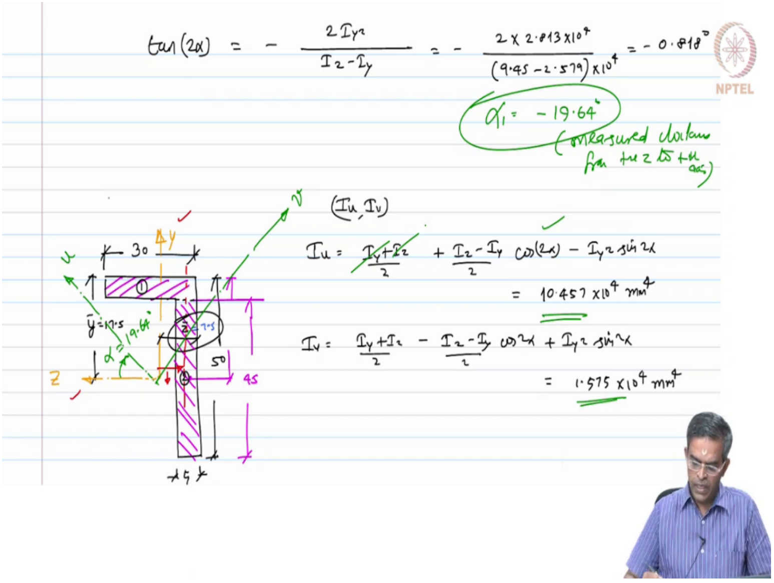

$$\tan tan\ (2\alpha) =- \frac{2I_{yz}}{I_z - I_y} =- \frac{2\times2.813\times10^4}{(9.45-2.579)\times10^4} =- 0.818^o$$

$$\alpha =- 19.64^o$$

I get this value as minus 0.818 degrees. If you work out I get α 1 as minus 19.64 degrees.

$$I_u = \frac{I_z + I_y}{2} + \frac{I_z - I_y}{2} \cos cos\ 2\alpha\ - I_{yz} \sin sin\ 2\alpha\ = 10.457\times10^4 mm^4$$

$$I_v = \frac{I_z + I_y}{2} - \frac{I_z - I_y}{2} \cos cos\ 2\alpha\ + I_{yz} \sin sin\ 2\alpha\ = 1.575\times10^4 mm^4$$

So, measured clockwise from positive z to positive u that is what we do. Let us mark that. So, let me copy this figure. Let me copy this figure, let me put it here. Let me mark the u u axis 19 degrees so, this is my u this could be my v and this angle is α which is 19.64 degrees.

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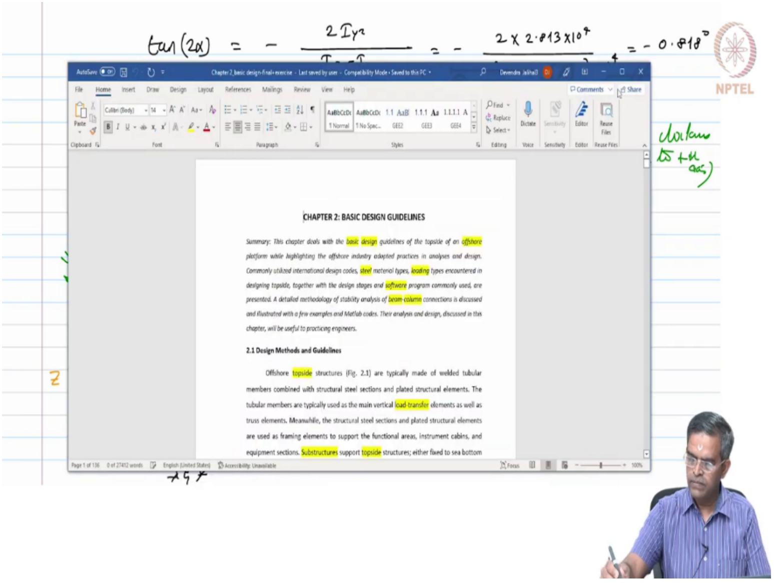

Now, once I have this with me. I can now find I_u and I_v. I_u is given by the equation $\frac{I_z+I_y}{2} + \frac{I_z-I_y}{2}\cos cos\ 2\alpha - I_{yz} \sin sin\ 2\alpha$. I substitute please check you will get this value as 10.457 into 10 power 4 mm 4. We can also find I_v from this equation $\frac{I_z+I_y}{2} - \frac{I_z-I_y}{2}\cos cos\ 2\alpha + I_{yz} \sin sin\ 2\alpha$, which is actually 1.575 into 10 power 4 mm 4. I do not think we have difficulty in finding out these values by substitution, because α value is here. We have 2 α and we have $I_z I_y$ everything in the previous step. So, we can easily find this.

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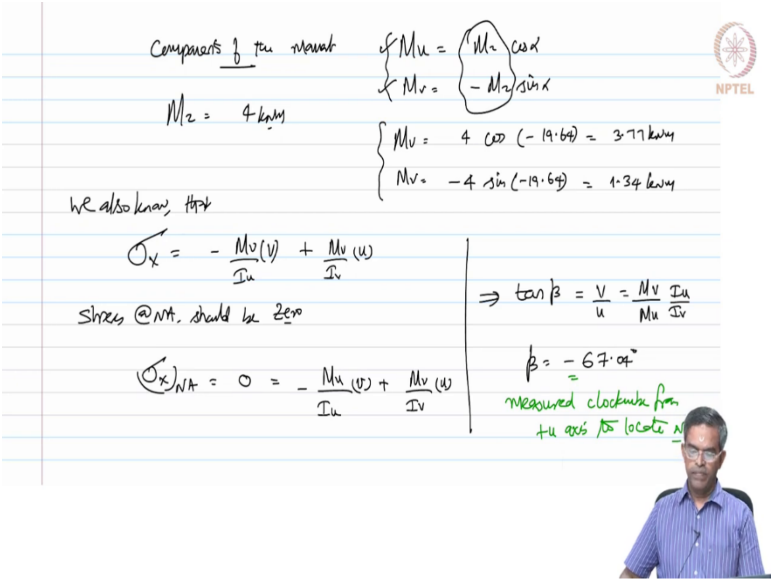

Now, having said this let us compute the components of the moment. So, now, we want to compute the components of the moment. We know the M_z value from the equation you know M_z value will be 2 into 2, which is 4 kNm. Which will cause bending this way. So, tension at the bottom compresses the top.

So, now M_z is 4 kilo newton meter. Now, we can say

$$M_u = M_z \cos cos\, \alpha$$

$$M_v =- M_z \sin sin\, \alpha$$

$$M_z = 4kNm$$

$$M_u = 4 \cos cos\, (- 19.64) = 3.77kNm$$

$$M_v =- 4 \sin sin\, (- 19.64) = 1.34kNm$$

$$\sigma_{(u,v)} = \frac{M_u}{I_u}(\text{v})\ \text{(Compression)}$$

$$\sigma_{(u,v)} = \frac{M_v}{I_v}(\text{u})\ \text{(Tension)}$$

$$\tan tan\, \beta = \frac{v}{u} = \frac{M_v}{M_u}\frac{I_u}{I_v}$$

$$\beta =- 67.04^{o}$$

Fiends please note we are not substituting any value for these whether it is hogging or sagging.

Because these equations are generated from a generic idea we can only interpret whether stresses will be tensile or compressive based upon the nature of bending. Which we will do later, it is an interpretation only. Now, we also know to compute the stresses. Stress at any point x is given by minus M_u by I_u into v. I am using the same classical bending theory equation, but I have resolved M and I n distance of fibres depending upon the principal axis. I have modified them, plus M_v by I_v into u.

So, we say stress at the neutral axis should be 0. So, therefore, can I say stress as neutral axis, which is equal to M_u I_u into v plus M_v I_v into u, which implies tan beta, where beta is the angle of inclination of the neutral axis from the u u axis. Which can be simply given by v by u which is M_v by M_u into I_u by I_v. Now, we have all the values.

We have Mu, we have M_u, we have M_v, we have I_u and I_v. So, can we find beta? Beta is coming to be minus 67.04 degrees, which is now measured clockwise, because it is negative from positive u axis to locate the neutral axis. So, let us locate the neutral axis? Let us take this figure, let us copy this figure again.

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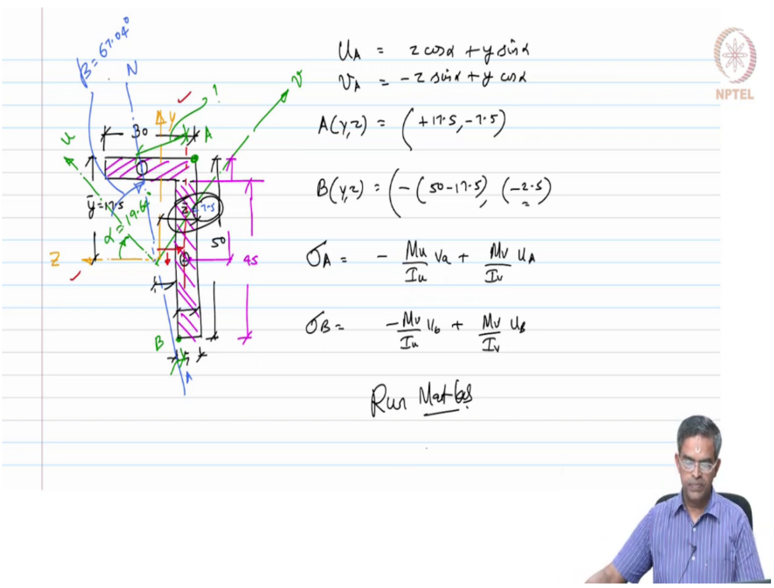

Let us copy this figure again, let us draw it here. Let me locate the neutral axis in Blue colour so, at 67 degrees. So, I should say very close here. This is my neutral axis. Measure 67 degrees further from u-u axis. So, we have mark the neutral axis. now we also know, if you want to locate a point if you want to locate a point A and the point B, I need to measure the distance perpendicular from the neutral axis.

Let us say the neutral axis is not passing through this line. So, let us say I want to measure this distance to find the stresses is not it, but that is very tedious, because it is very difficult to find out this component. So, instead we have resolved it very simply we can find the u_A and v_A component of the point A, u_A we said it is z cos α plus y sin α we derived it already.

Similarly, this is minus z sin α plus y cos α. Let us first fix the y z points for the corner A to do that we can easily look at the figure. The y z point will be plus 17.5 and minus 7.5 is it for the point A. Similarly for the point B, if you want to find y z it should be minus of 15 minus 17.5.

$$u_A = z \cos cos\, \alpha + y \sin sin\, \alpha$$

$$v_A =- z \sin sin\, \alpha + y \cos cos\, \alpha$$

$$A(y, z) = (+\ 17.5, -\ 7.5)$$

$$B(y, z) = (-\ (50\ -\ 17.5), (-\ 2.5))$$

$$\sigma_A =- \frac{M_u}{I_u}\left(v_A\right) + \frac{M_v}{I_v}\left(u_A\right)$$

$$\sigma_B =- \frac{M_u}{I_u}\left(v_B\right) + \frac{M_v}{I_v}\left(u_B\right)$$

This is for y and z straight away it is minus 2.5. We are looking for actually the value of this, because we know this distance is 7.5 and this is 5. So, I can get this value as minus 2.5. Since, I know the coordinates y z for the points A and B. I can easily use the above equation, which is going to be; if you want to find stress at A, I will say minus Mu by I_u into v_A plus M_v I_v into u_A whereas, α is known to me, I can find u_A and v_A.

Similarly, I can find stress at B minus M_u by I_u into va plus M_v I_v into u_B. We can easily do this, Now, I want to run this using MATLAB. Let us run MATLAB to compute this. I will hold the screen we will run the MATLAB now.

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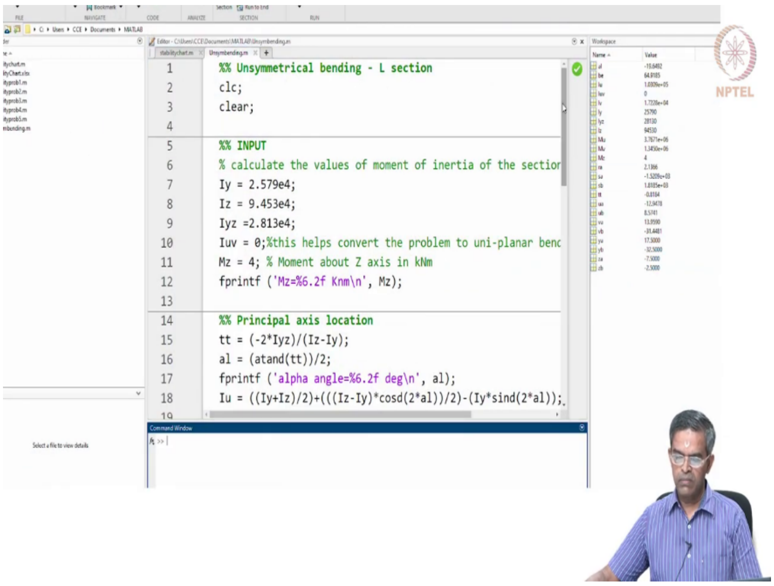

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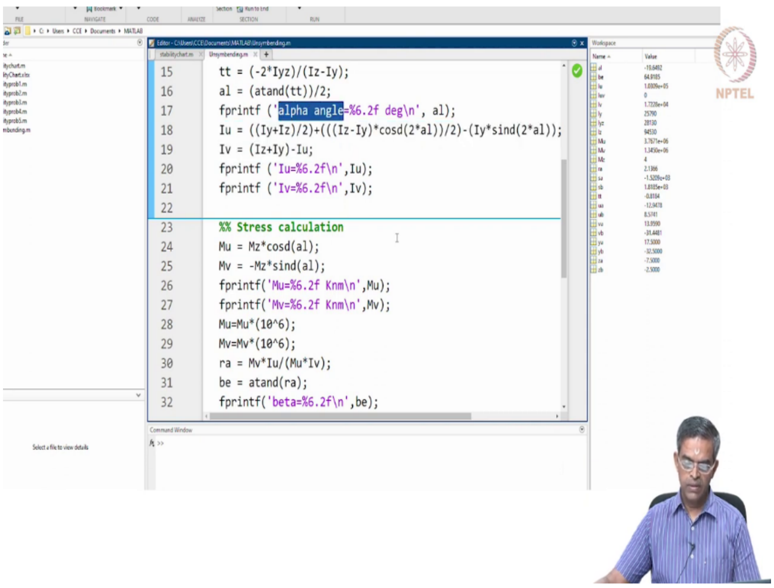

So, friends what you see on the screen is the MATLAB program to compute unsymmetrical building l section that is what we are doing. So, let us enter the values of y y as 2.579 I_z and I_{yz}, which are all taken from the lecture and M_z is 4. We already have in kilo newton meter.

So, now, it is calculating I_u and I_v automatically. Then it also calculates the α angle. It calculates the α angle.

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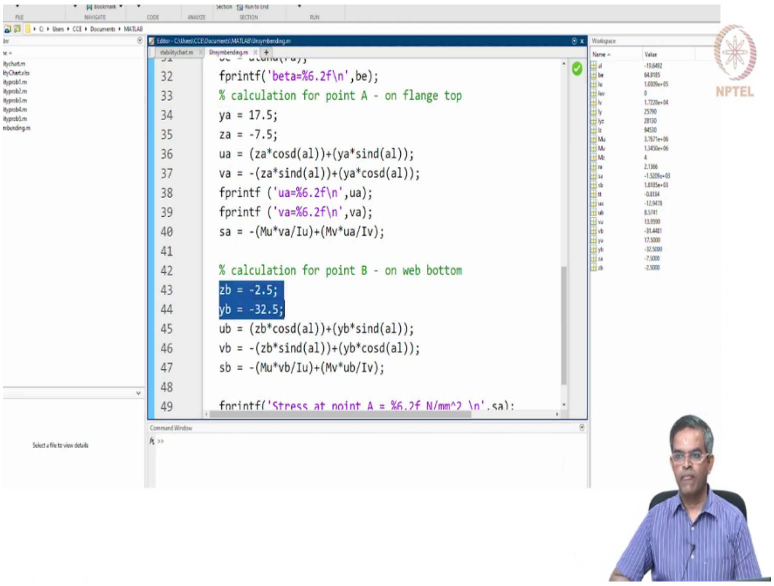

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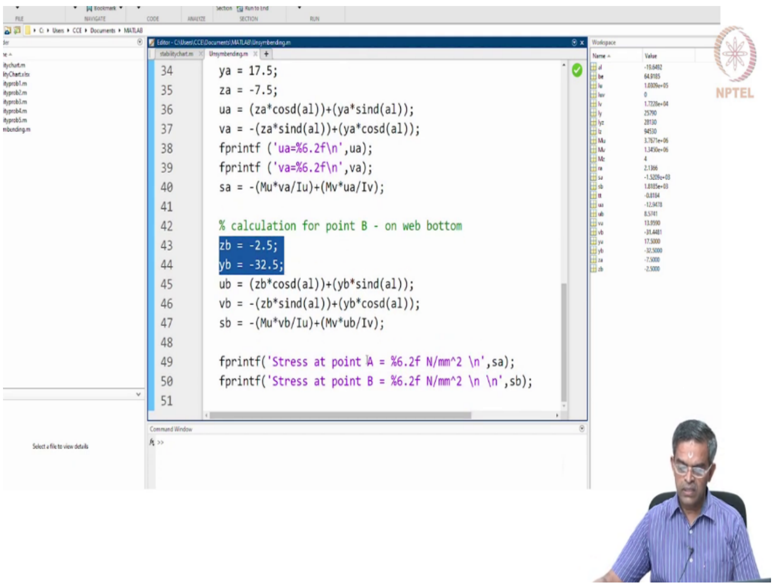

Then it computes M_u and M_v then we have to enter the coordinates for the point A and coordinates for the point B. Please see that. Once we enter the program runs. I will get the stress at point A and point B. Let us run this program. So, the program is run.

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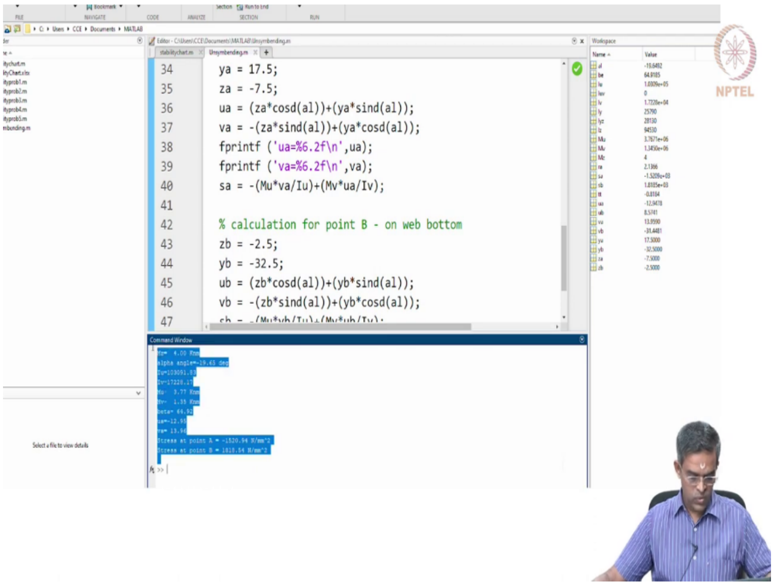

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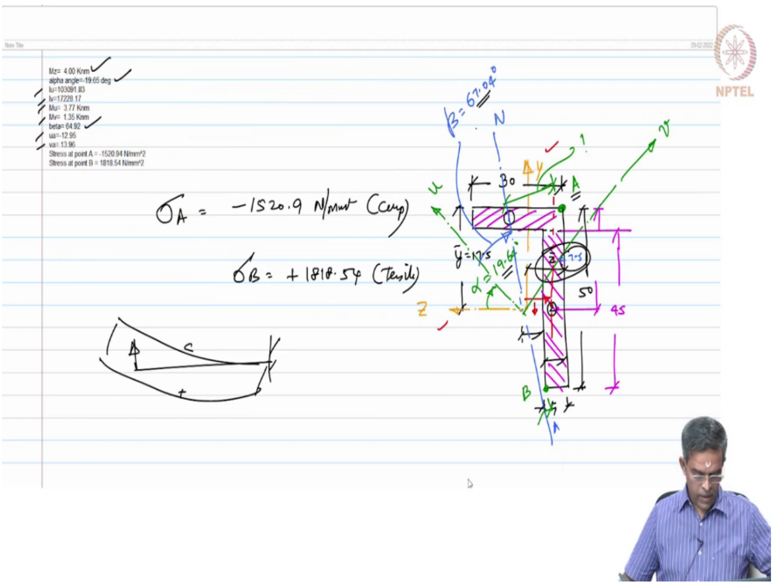

I have the values; I am copying all these values into my screen here and putting them here. All these values are copied here. Let me also copy this figure and put it here. Let me move this figure here friends. So, let us check the values. We said M_z for me problem is 4, yeah I got 4. A angle we got is 19.64 program is giving 19.65. I_u, I_v we have. M_u, M_v we have. Beta angle we computed 64.92 degree, it is 67. So, this is the small difference, because of the round off error in I_u and I_v.

So, beta we have. So, we have already values of u_A and v_A, u_B and v_B. Now, the stress at the point A is minus 1520.9 and stress as point B is plus 1818.54. Friends, we already said it is a cantilever with an upward. So, the beam is bending like this. This is compression and tension. So, the top point is receiving compression and the bottom point is receiving tension.

So, friends we have used the MATLAB program effectively to solve this problem and we have the answers as we see on the screen. So, and this is what we have interpreted we have these values with us. So, this is what you can run the program and try to show.

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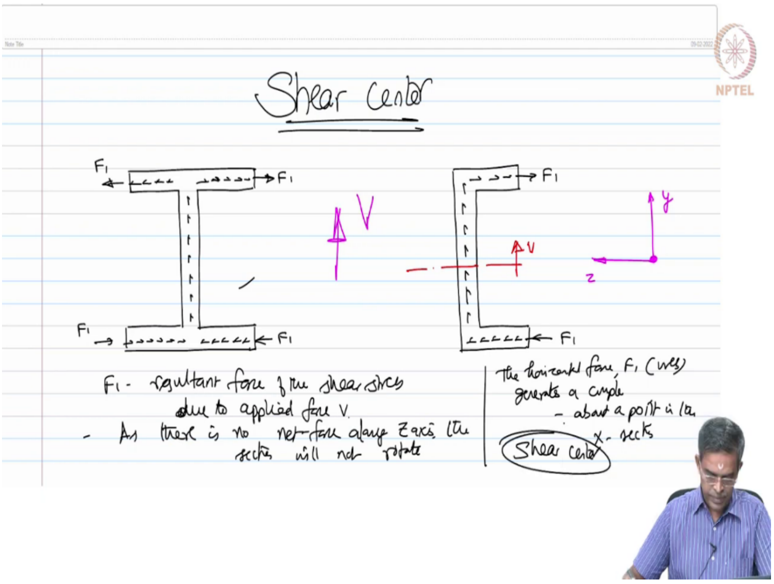

Now, let us start discussing something on shear center. So, we will does something about shear center to understand shear center. Let us first see the internal force distribution in a given section. Let us take for example, an I section. And a channel section whatever may be the dimension.

So, for both these, we say this is my z axis this becomes my y axis and of course, x is normal to the screen. If I apply a force V upward, let us see the force distribution of this this is how the internal forces will get distributed, which I call the net force as F1 and F1 F1 and F1. On the other hand if you look at the channel section and say this is my centroidal axis and apply a force m bar V here. The force distribution in this case is what I am marking on the screen now.

So, we call this as F1 and this as F1. So, friends in both these figures we can say F1 is the resultant force of the shear stress due to the applied force v. There will be no net force along z axis the section will not rotate in this case. As there is no net force along z axis the section will not rotate. We also know that the vertical force is taken care of by the web.

The vertical force is taken care of by the web, but the flange forces F1 generates a couple. The horizontal force F1, which is acting in the web, generates a couple correct. This couple is generated about a point in the cross section that point is called as shear center.

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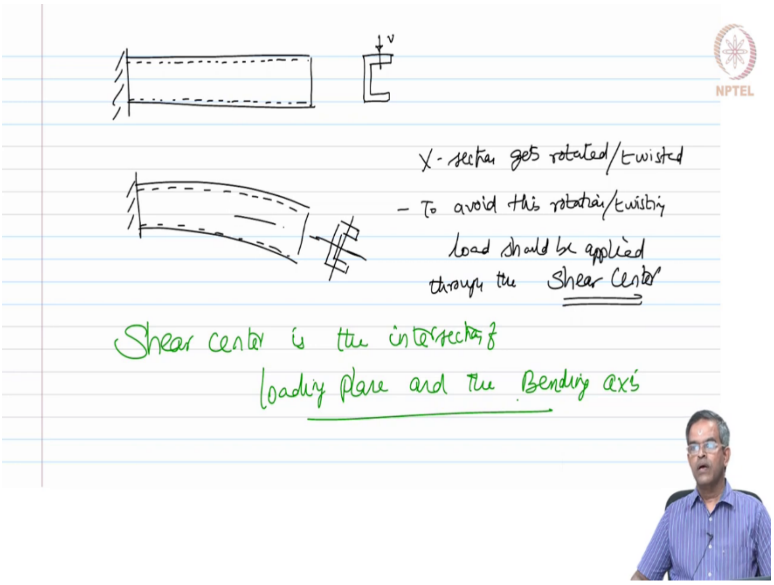

Now, let us take a cantilever beam. Let me draw the flange thickness. Let us say the section is a channel subjected to some load v. The cantilever will start deflecting under this load, where the cross section will also have this effect. So, what we say here is? Cross section gets rotated or we can say gets twisted. Now, to avoid this load should be applied through the shear center. Therefore, friends shear center is the intersection of loading plane and the bending axis.

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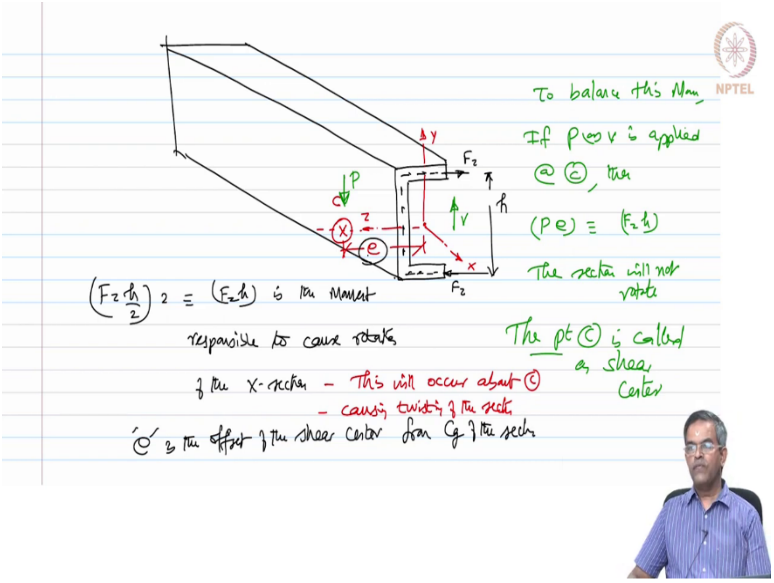

Now, interestingly let us enlarge this figure and see what happens to my channel section? So, I have a channel section drawn here again. Let us say this is my length of the member, it is got 3 axis now let me mark those axis. This is my vertical axis y my horizontal axis z this is my another axis, which is x. Let us say it is a thinned section the depth of the section is measured from center to center of the web call that as h.

We call this force, the internal force, the horizontal internal force Fz and Fz. Let us say this is subjected to a vertical force b. Now, looking at this figure we can say fz into h by 2 of 2, which is nothing but Fz into h is the moment responsible to cause rotation of the cross section. This will happen about which point, let us extend this. This lapped amount of point c, lapped amount of point c, at a distance e from the centroid so, this will happen or this will occur about c causing twisting of the section.

Now to balance this if P or v is applied at c then imagine that p or v is applied here then we can say P into e will balance this moment. So, section will not rotate. So, in the whole context of discussion friends the point c is called as shear center. So, it is ideal that the load should pass through the shear center to avoid twisting of the cross section. Having said this let us also say in the same thing whereas, e is the offset of the shear center from the Cg of the section e is offset.

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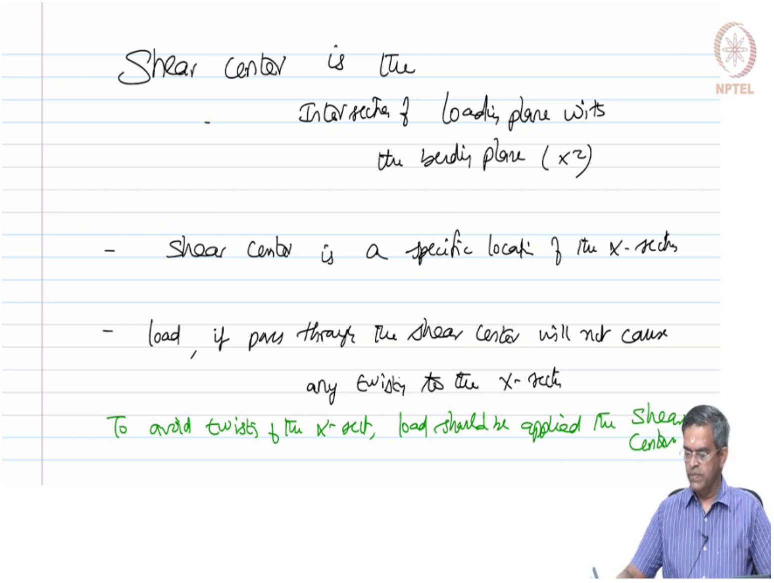

Therefore friends, shear center is the intersection of loading plane with the bending plane in my case this is x z. Further shear center is a specific location of the cross section. Load if pass through the shear center will not cause any twisting moment to the section. So, therefore, to avoid twisting of the cross section load should be applied or made to pass through the shear center.

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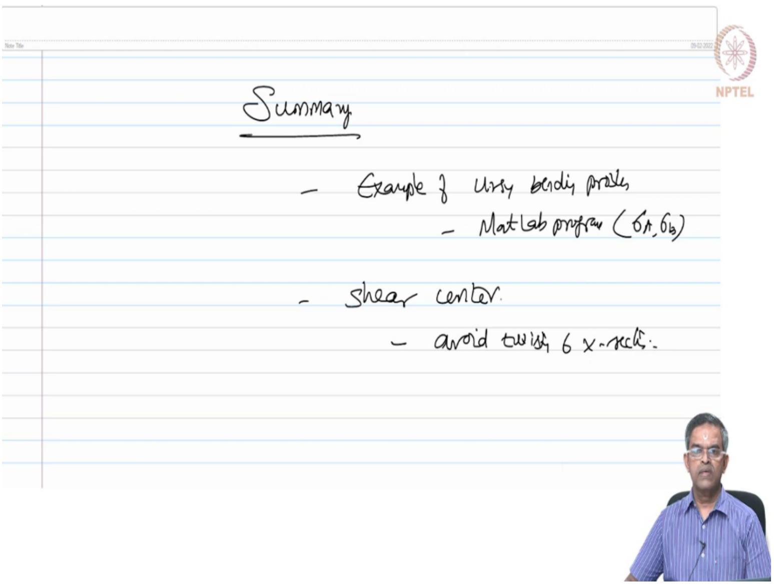

friends, in this lecture we learnt one more example of unsymmetric bending problem we have used effectively the MATLAB program to find the stresses at the designated points. We have also started learning about the importance of identifying a shear center and how if load is applied to the shear center can avoid twisting of the cross section. We will discuss more in detail about the shear center and more numerical examples of various shape of cross sections and find shear center using the conventional equations in the coming lectures.

Thank you very much and have a good day, bye.