Advanced Design of Steel Structures

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Lecture - 41
Curved Section

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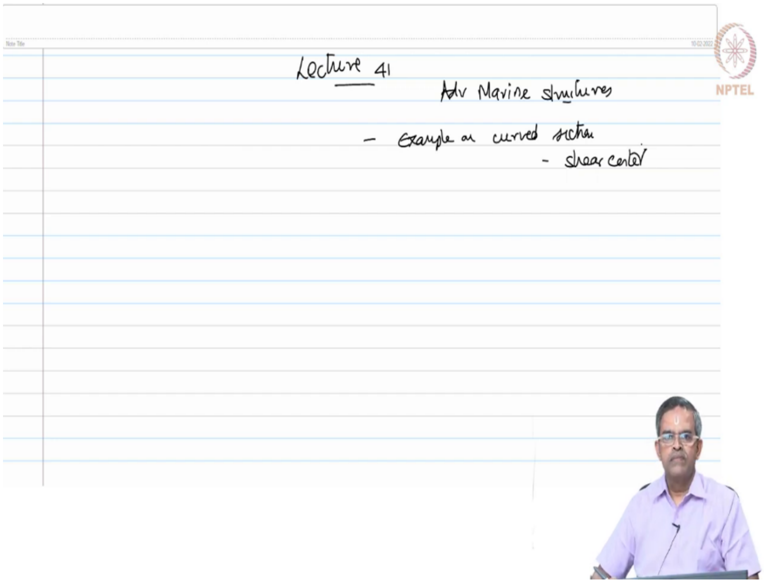

Friends, welcome to the lecture 41 of Advanced Marine Structures. In this lecture, we are going to learn example on Curved Sections to Locate the Shear Center. So, we already studied locating shear center from the first principles simple exercise provided a section has got at least one axis of symmetry. If axis of symmetry does not exist, then the section will be subject to unsymmetric bending. Then, we have got to combine both the problems of unsymmetric welding, locate the u u v v axis, then find the shear center. I will come to that problem slightly later. We will do that.

So, in the previous examples, we discussed about rectilinear sections, like I section, channels, t, etcetera. In this example, it is quite interesting we will take a curved section because we are going to slowly understand the analysis and design of curved beams and crane hooks. So, we need to at least examine the curved section for shear center calculations. Let us do that. We will take I will call this example 1 for this lecture. We will talk about curved sections.

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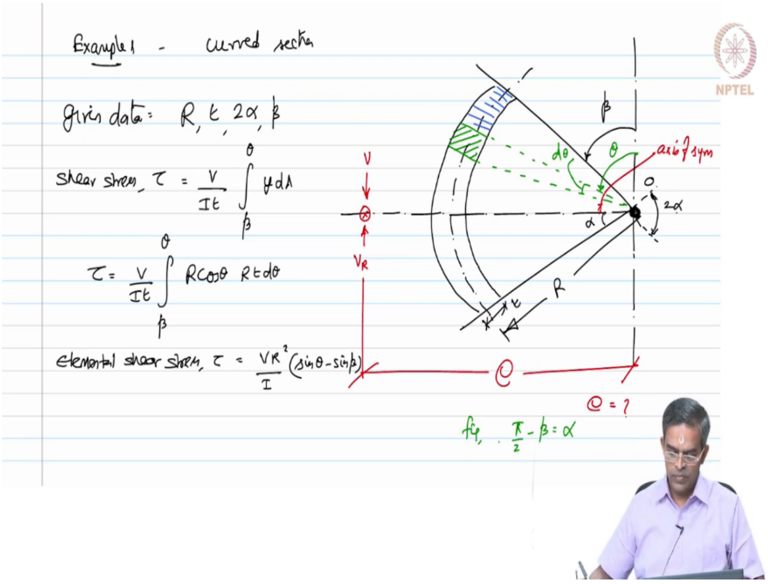

So, now, let us say, I have an axis of symmetry of the section. We will mark a curved section symmetric about this axis. So, let us say it has got a center line, and that center line eta at a distance R, from here and I call this point as O, ok, this point as O. Let us say it subtends an angle 2 α symmetric about the center, so this angle is α. Let us believe the section has got uniform thickness t and the curvature start of this section from the axis of rotation is taken as β.

So, now; obviously, being this the axis of symmetry shear center will lie on this axis. Let us mark the shear center somewhere here. If the force and the reaction pass through the shear center the section will not be subjected in distinct moment. We call this distance of the shear center from the axis of curvature as e. So, the question is now I want to find e. The given data for the section are the following, radius of the section, thickness of the section, the included angle and the subtended angle. Let me write it clear; are known thing.

So, let us now cut the section which is subtending an angle dθ atθ from here. Now, let us say this is my hatched portion. Now, the shear stress tau is given /the classical equation V /It integral y dA. Now, my question of integration is going to be, for the element is going to be varying from β toθ. That is my control, is it not? That is my dA. Look at here, this is my dA, right. That is what I am looking at.

So, let me do that which will be equal to V /It, integral β toθ, y will be R cos θ and R tθ dθ, t dθ, R dθ t will be the area of the hatched portion which is green in color. This is y bar. So, let us we can do this. So, the elemental shear stress tau is equal to VR square /I, sinθ - sin β, am I right. That is my elemental shear stress. Also, friends, we have an interesting relationship here. From this figure we can sayπ/2 - β is α. Can I say that? From this figure, from the geometry this is true.

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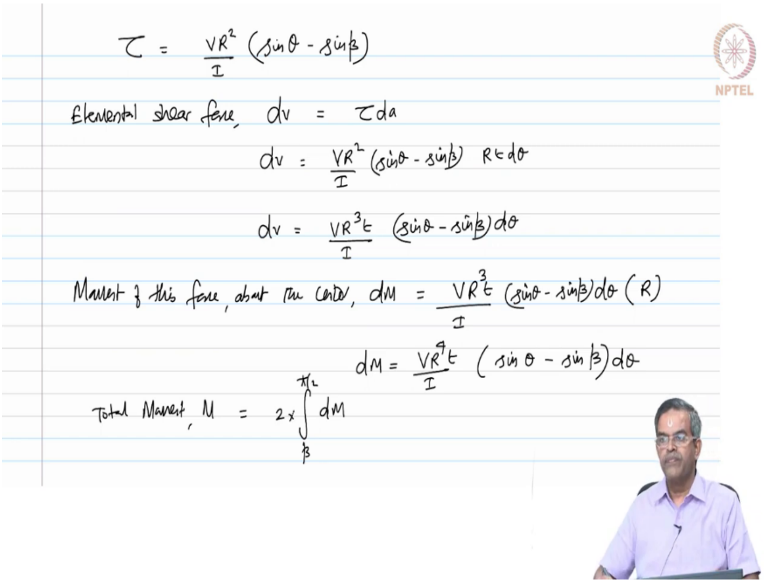

Having said this, so we said that tau is VR square /I sinθ - sin β. Now, I can find the elemental shear force, which is dv, which will be actually tau da. So, now, I can find dv as VR square /I, sinθ - sin β into R t dθ. So, now, this becomes VR cube t /I, sinθ - sin β of dθ. This is my dv. Now, the moment of this shear force about the center is given by dM. dM will be straight away into R, is it not. The distance is R radius.

So, I can say VR cube t /I, sinθ - sin β dθ multiplied by R. So, this will become VR 4 t /I, sinθ - sin β into dθ. That is my dM. So, once we obtain the elemental moment, we can find the total moment, yeah which will be integral of dM varying from β toπ/2, 2 times of that.

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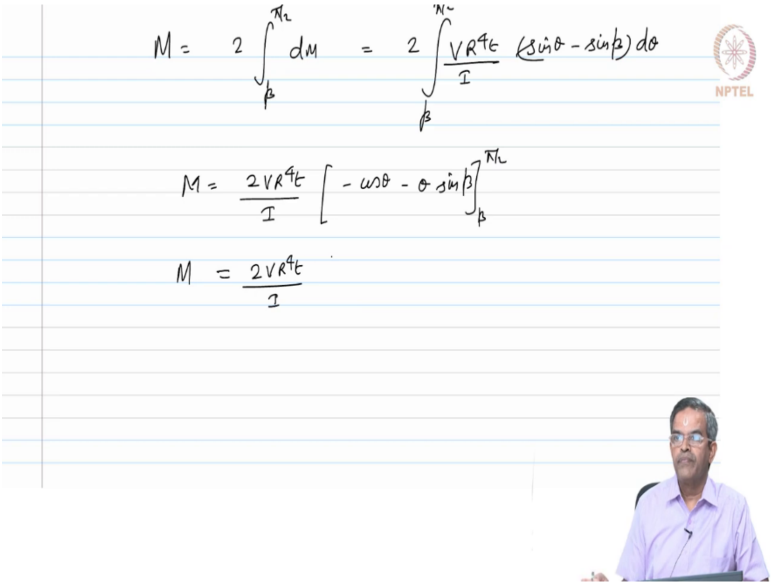

So, the total moment M is twice of integral β $2\pi/2$ dM which will be twice of integral β $2\pi/2$ VR 4 t /I, sinθ - sin β dθ. So, which will give me M as 2 VR 4 t /I of - cosθ -θ sin β, apply the limits from β to$\pi/2$. So, that will give me M as 2 VR 4 t /I.

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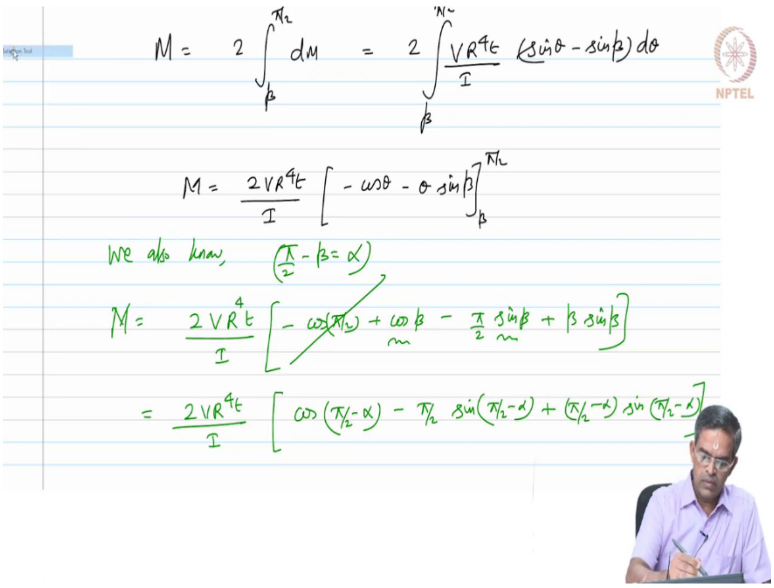

Before that we also know$\pi/2$ - β is α. Can you see this here? Let me use that relationship here. So, now, saying that M will now become 2 VR 4 t /I - cos$\pi/2$ + cos β -$\pi/2$ sin β + β sin β. So, we can say this is 0. And we have cos β sin β, right. Now, I can simplify this equation as 2 VR 4 t /I. I am replacing β as$\pi/2$ - α.

So, can I write this as cos$\pi/2$ - α - π /2 sin$\pi/2$ - α +$\pi/2$ - α of sin$\pi/2$ - α. I am bringing all in terms of α. Let me copy this equation to the next screen.

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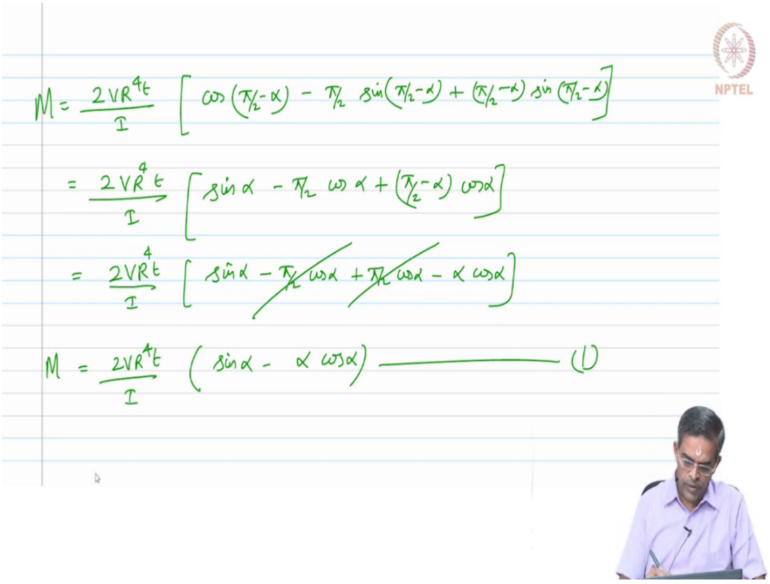

So, M was equal to this which now becomes 2 VR 4 t /I, sin α -π/2 cos α +π/2 - α cos α. Which is further simplified as 2 VR 4 t /I, sin α -π/2 cos α +π/2 cos α - α cos α. So, this goes away; which will now land up as 2 VR 4 t /I sin α - α cos α. We call this equation number 1. It is an interesting equation which is a classical theory available in the literature of a curved sections. Now, let us take moment about O. This is the figure we have. Let us copy this figure.

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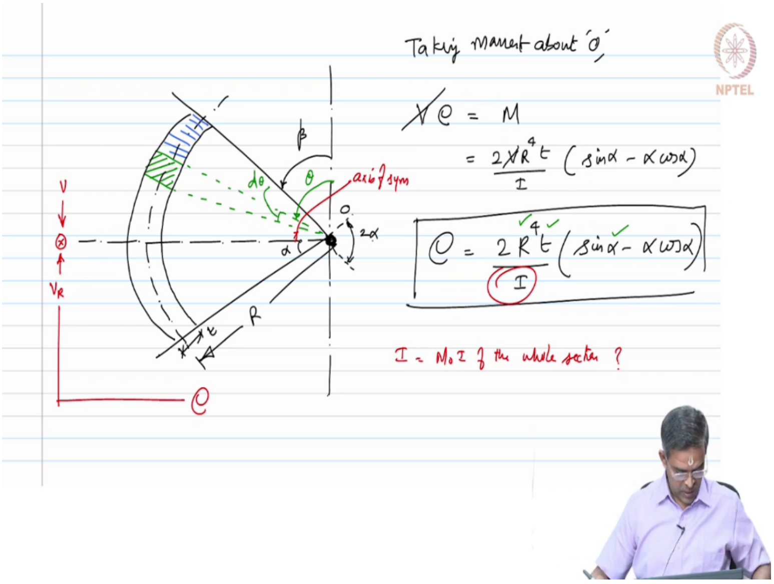

We will take moment about O. So, V into e is equal to that moment. So, which I say is equal to 2 VR 4 t /I sin α - α cos α. That is what the moment we have here,. I have equated this. So, now, I can easily get e as sorry, 2 R 4 t /I sin α - α cos α that is my e. If we know the value of e, I can easily find the shear center, locate the center. So, look at this equation, in this equation R is a geometric property, t is a section property, α is the angle of this curved section subtended the center to α rather.

But the question is how about I. I is a moment of inertia of the whole section. Is it not? That is what we understand. It is a segmented section; it is not a full curve. So, that we can find I readily; the segmented section. So, the problem is how to find this. Let us do that.

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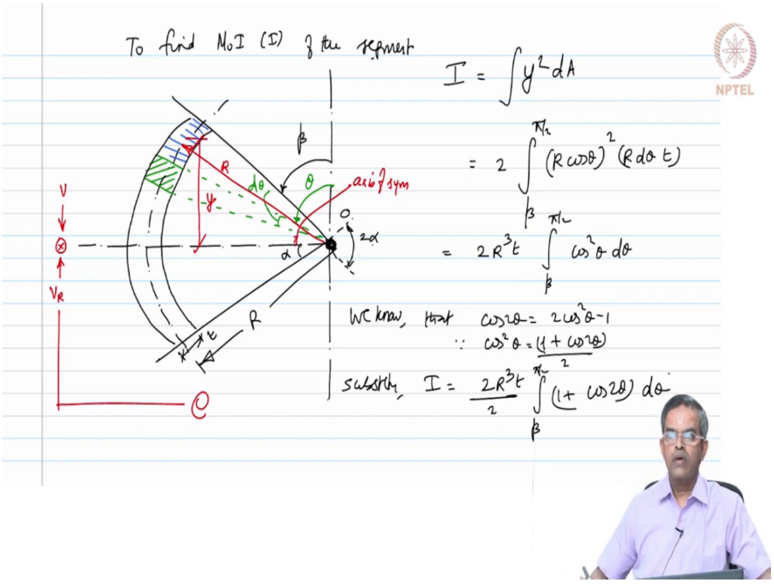

Now, the issue is to find the moment of inertia I of the segment. So, let us draw this figure again. I will take this figure again,. We know I is y square dA, and I want to integrate it for the entire section. So, when I integrate I can do this as integration of β to$\pi/2$ and 2 times of this. y we already have with us, it is R cosθ, am I right. Look at this figure, R cosθ will be the y value. That is this value, this is R.

And this value R cosθ, is it not. So, R cosθ the whole square and dA is R dθ into t, am I right? So, now, let us simplify this, so 2 R cube t integral β 2π/2 cos squareθ dθ. We know from trigonometry, that cos 2θ is 2 cos squareθ - 1. So, hence cos squareθ is 1 + cos 2θ /2. So, substituting, I is 2 R cube t /twice βπ/2, 1 + cos 2θ dθ.

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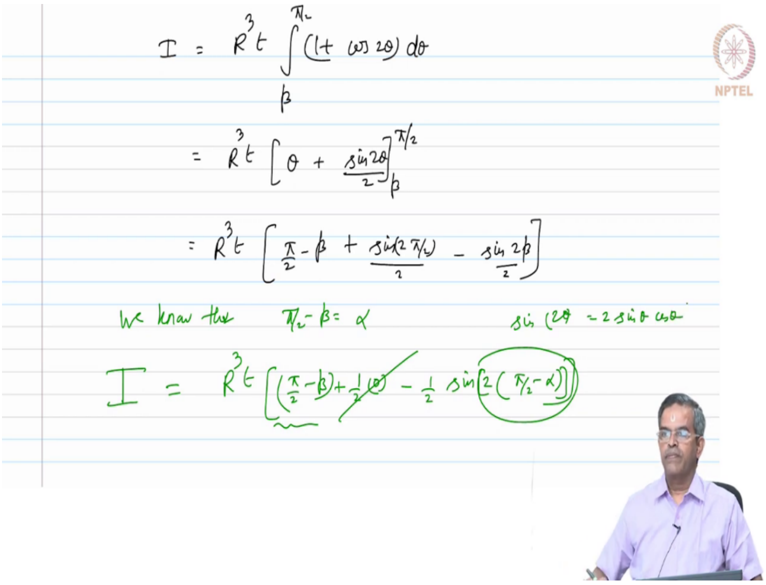

So, I is R cube t integral βπ/2, 1 + cos 2θ dθ, which is R cube tθ + sinθ /2 β 2π/2, which is R cube tπ/2 - β + sin 2π/2 /2, - sin 2 β /2. We know alreadyπ/2 - β is α, is it not. We know already have it here, see here. Hence, I will now become R cube t, sayπ/2 - β. So, this becomes 0, half of 0 - half of sin twice ofπ/2 - α. Can I say that? So, this is closed.

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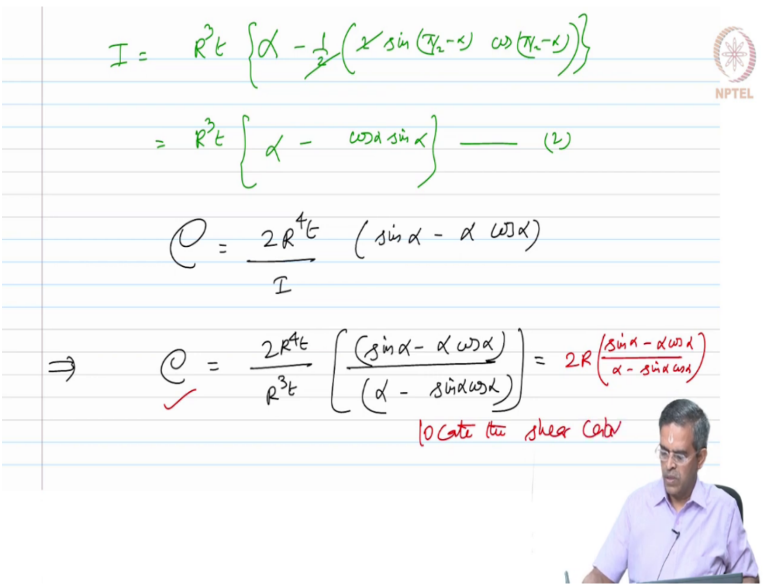

So, now I can say I is R cube t. See, look at this figure. So, this becomes α - this is half of sin 2θ, so 2 sinθs are Rθ. So, half of 2 sinπ/2 - α, cosπ/2 - α. Can I write like this? This is argument is 2θ, let us say sin 2θ is 2 sinθ cosθ. That is what I am writing here. So, these two goes away now, which will become R cube t α - cos α sin α. So, this is equation number 2.

So, now, e is 2 R 4 t /I, sin α - α cos α. That is what we have written here, 2 R 4 t /I sin α - α cos α. So, now, therefore, substituting e will become 2 R 4 t /R cube t, sin α - α cos α /α - sin α cos α,. So, which will now become 2 R times of sin α - α cos α /α - sin α cos α, that is my e. So, we have all the values. So, geometric parameters, I can find e.

Now, I can locate the shear center for the given geometry. So, as a small example, let us copy this figure, somewhere see this figure.

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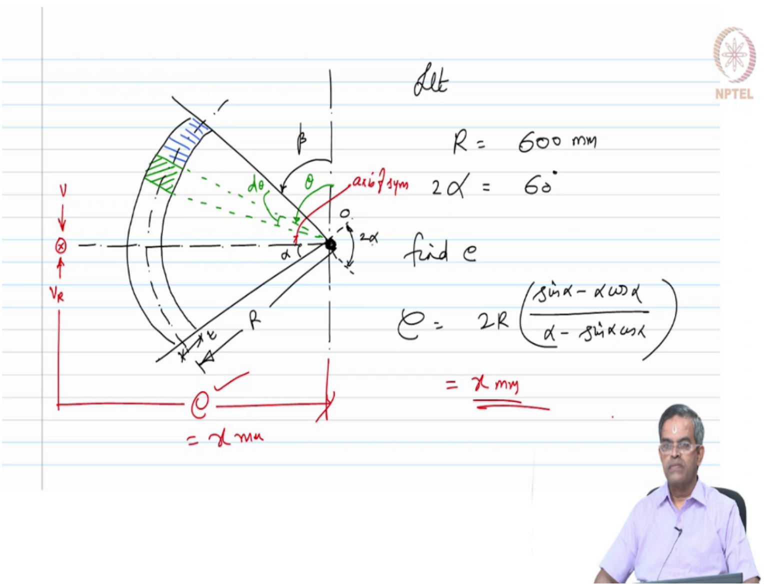

So, let us say, let R be 600 mm. A or 2 α be 60 degrees. Find e. So, e is straight away 2 R times of sin α - α cos α /α - sin α cos α. And compute this, mark the value and e will be maybe x millimeters. So, friends, it is very simple that this particular numerical example explains how to compute the shear center for a curved section as you see here.

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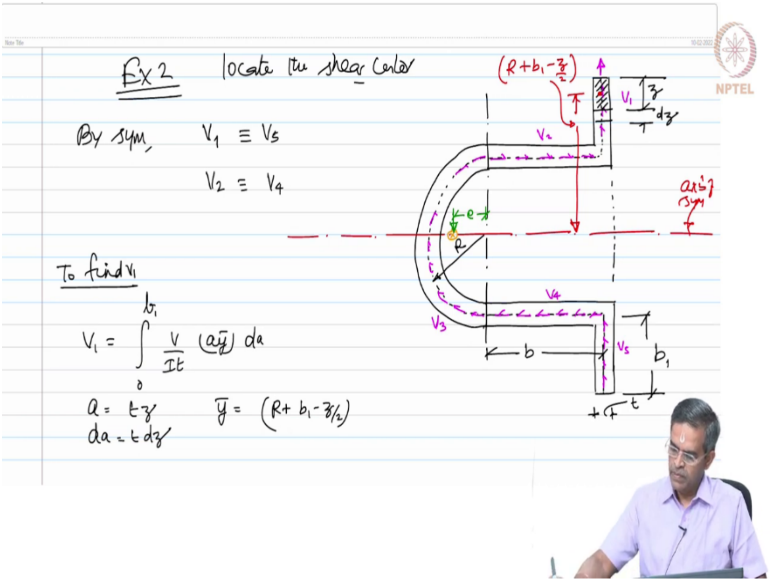

We will do one more example. By a slightly a tricky section, let us see how we handle this section. Locate the shear center of the section shown. So, section we have, ok; let us marks on the dimensions of this section. Of course, we can locate there is an axis of symmetry for the section. The section has uniform thickness as t. Let us mark the center line of the section for our calculation purposes.

And we mark this dimension as b 1 and this dimension as b and this radius as R. We will say the shear center is located somewhere here, and this is at an eccentricity e from here. Let us mark the shear flow. We call this arm as V 5, this as V 4, this as V 3, this as V 2 and this as V 1, this as V 1. So, from this figure, we can easily say by symmetry, V 1 will be same as V 5. Further, V 2 will be same as V 4. Let us cut the section at z from here, that is the thickness of the strip b dz. This what we hatch. We want to find the cg of this, ok from here axis of symmetry.

From the figure, you can always write this value as R + b 1 - z /2. Can I say that? Can I say that? So, now, I can find V 1. V 1 will be the force in the vertical arm which will be integrated from 0 to b 1, V /It, a y bar of da. So, a will be t into z and da will be t into d, and y bar will be R + b 1 - z /2. I can substitute this.

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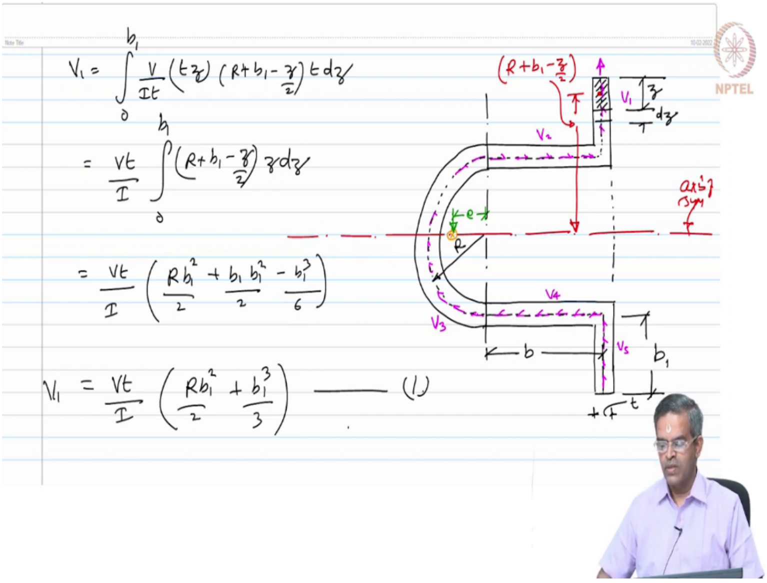

So, V 1 is going to be integral 0 to b 1, V /It, t z, R + b 1 - z /2 of t d z. Let me rub this. So, we can straight away say it is V t /I now integral 0 to b 1, R + b 1 - z /2 of z, d z, which can be V t /I, R b 1 square /2 + b 1 b 1 square /2 - b 1 cube /6. After substituting the limits, which becomes V t /I of R b 1 square /2 + b 1 cube /3. This is my V 1, is equation 1. Ok, friends.

So, we will continue this problem in the next lecture. But I want you to do a small homework on this. I want you to find V 2. And now we have a curved section, we can find V 3 also. So, try to do this exercise, then we will recollect it how we are able to do it. I leave a small gap here intentionally, so that I want you to practice this. And try to do for V 2 then V 3 and so on.

Thank you very much. And have a good day. Bye.