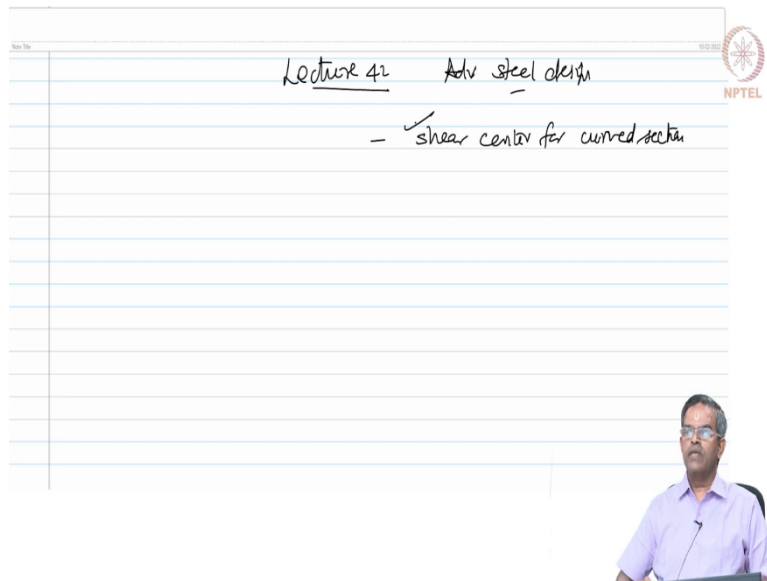


Advanced Design of Steel Structures
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Lecture - 42
Shear center for curved sections

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Friends, welcome to the 42nd lecture on Advanced Steel Design course, we are continuing to do estimate or locating shear center for curved sections. So, we are in the middle of a problem we will do that now in this lecture, hope you have done this. So, this was the last expression we had for V_1 am I right.

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$$V_1 = \int_0^{b_1} \frac{V}{It} (tz) \left(R + b_1 - \frac{z}{2}\right) t dz$$

$$= \frac{Vt}{I} \int_0^{b_1} \left(R + b_1 - \frac{z}{2}\right) z dz$$

$$= \frac{Vt}{I} \left(\frac{Rb_1^2}{2} + \frac{b_1^3}{3} - \frac{b_1^3}{6} \right)$$

$$V_1 = \frac{Vt}{I} \left(\frac{Rb_1^2}{2} + \frac{b_1^3}{3} \right) \quad (1)$$

So, we found out V_1 which is given by this equation which is $V t$ by $I R b_1$ square + b_1 cube by 3, so that is V_1 here. Now let us try to find out V_2 . So, we will copy this figure again rub this.

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To find V_2

$$V_2 = \int_0^b \frac{V}{It} (ay^2) da$$

$$da = t dx$$

$$ay = (b_1 - t) \left(R + \frac{b_1}{2}\right) + (t - x)R$$

$$V_2 = \int_0^b \frac{V}{It} \left[(b_1 - t) \left(R + \frac{b_1}{2}\right) + (t - x)R \right]^2 t dx$$

$$V_2 = \frac{V}{I} \left[t \left(b_1 R b + b_1^2 b + b_1^2 R \right) \right]$$

Now, I want to find V_2 ok? Let me cut a new strip now which is here at a distance x from here thickness of the strip is dx and I want the hatched area is this, hatched area has got 2 components. I got 2 components now. So, I divide this into 2 components and try divide this

into 2 components here, I divide this into 2 components here and then try to find out. Let us see how we do it.

So now, V_2 is integral 0 to b because that is the dimension we have for V_2 . V by $I t$ thickness is uniform a y bar of $d a$ ok, $d a$ is very simple $d a$ is area of this small strip which is being considered which I am hatching here. Which is $t d x$ there is no doubt, a y bar has got 2 components. So, first let us do it for b_1 , so b_1 into t of $R + b_1$ by 2. So, I have done for this sample I am just marking it here I am marking it here I have done for the sample + this is for $1 + t x$ into R this is for 2 what is this one.

So therefore, V_2 now will become integral 0 to b V by $I t$ b_1 into t of $R + b_1$ by 2 + $t x$ into R of $t d x$. So, we can simplify apply the limits please check we will get V_2 as V by $I t$ times of $b_1 R + b_1^2$ square b by 2 + b square R by 2 this.

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$$V_2 = \frac{Vt}{I} \left[b_1 b R + \frac{b b_1^2}{2} + \frac{b^2 R}{2} \right] \quad (2)$$

To find V_3

$$\tau = \frac{V}{I t} \int (\mu_0 d i + \mathcal{Q}_1 + \mathcal{Q}_2)$$

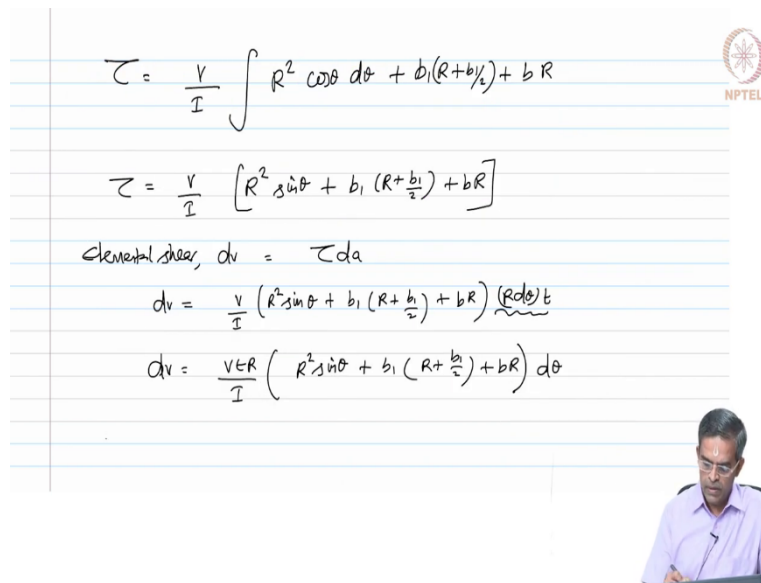
$$= \frac{V}{I t} \left[(R \omega \theta) (R d \theta t) + [b_1 t (R + \frac{b_1}{2})] + (b t R) \theta \right]$$

The diagram shows a semi-circular wire of radius R and thickness t . A small strip of width dx and height t is hatched in green. The distance from the center of the wire to the strip is x . The angle subtended by the strip is $d\theta$. The diagram also shows the wire's cross-section with dimensions b_1 and t .

We can simplify this I get V_2 in a simplified form as $V t$ by $I b_1 b R + b b_1^2$ square b by 2 + b square R by 2 equation 2. So, equation 1 we had V_1 equation 2 we had V_2 . Now I want to find for the semicircular portion V_3 ? We already know this equation let us draw this figure we are looking for the curved portion, which is this, I will hatch it here maybe with this color I am looking for this portion we already have an expression for this ok, if you know the angle, I can do it.

So now, the elemental shear stress V by $I t$ will be integral $y d a + \theta 1 + \theta 2$. Let us say we will put this $Q 1 + Q 2$ where $Q 1$ and $Q 2$ are the shear of first and second pieces. So, let us do that V by $I t$ we already have this equation. So, I am writing it here $R \cos \theta$ into $R d \theta$ into t am I right $+ b 1$ times of t into $R + b 1$ by 2 this is for piece number 1 $+ b t$ into R this is for piece number 2.

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$$\tau = \frac{V}{I} \int R^2 \cos \theta d\theta + b_1 \left(R + \frac{b_1}{2} \right) + b R$$

$$\tau = \frac{V}{I} \left[R^2 \sin \theta + b_1 \left(R + \frac{b_1}{2} \right) + b R \right]$$

Elemental shear, $dv = \tau da$

$$dv = \frac{V}{I} \left(R^2 \sin \theta + b_1 \left(R + \frac{b_1}{2} \right) + b R \right) (R d\theta) t$$

$$dv = \frac{V t R}{I} \left(R^2 \sin \theta + b_1 \left(R + \frac{b_1}{2} \right) + b R \right) d\theta$$

So, therefore, I can now expand this which will become τV by $I R$ square $\cos \theta d \theta + b 1 R + b 1$ by $2 + b$ into R , which will now become V by $I R$ square $\sin \theta + b 1 R + b 1$ by $2 + b R$.

So, please understand this integral $y d a$ is applicable only for the curve section ok, for the remaining section we only have the forces with us into $b R$. So, V by I let us mark it this way this $y d a$ is only for the curved section. So, we have now the value for τ which is V a by $I R$ square $\sin \theta + b 1$ into $R + b 1$ by 2 , let me write it here clearly $+ b R$.

Now the elemental shear dv will be $\tau d a$ which will be V by $I R$ square $\sin \theta + b 1$ times of $R + b 1$ by $2 + b R$ of $R d \theta$ into t am I right look at this figure. We are looking for $d \theta$ as a small segment I will mark it here the segment we are looking for. We know this is θ and this angle is $d \theta$ this is the sigma which is $y d$. So, this $R \theta d \theta$ is very clear from there.

So, now I can find $d V$ as V into $t R$ by I this t has now come into play here of R square $\sin \theta + b 1$ times of $R + b 1$ by $2 + b R$ of $d \theta$. Now we would like to take moment about of this

force about the point o this is the point o take moment of this point, the force V is acting here we will take the moment of this particular d V.



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Take moment of this elemental shear force about 'o'

$$dM = (\tau da) R$$

$$dM = \frac{VtR^2}{I} \left[R^2 \sin\theta + b_1 \left(R + \frac{b_1}{2} \right) + bR \right] d\theta$$

$$\int dM = M = \frac{VtR^2}{I} \int_0^\pi \left[R^2 \sin\theta + b_1 \left(R + \frac{b_1}{2} \right) + bR \right] d\theta \quad \text{--- (3)}$$

$$M = \frac{VtR^2}{I} \left[R^2 (-\cos\theta) + \left\{ b_1 \left(R + \frac{b_1}{2} \right) + bR \right\} \theta \right]_0^\pi$$



So, we will see taking moment of this elemental shear about o. So, we call that as d M which will be $\tau d a$ into R see here the distance will be R is it not into R. So therefore, d M now will become V t R square by I let me write the equation for $\tau d a$, d M will now become V t R square by I R square sin θ + V 1 times of R + b 1 by 2 + b R of d θ . Integral d M will be the total moment M which will be equal to V t R square by I integrated from 0 to pi R square sin θ + b 1 times of R + b 1 by 2 + b R of d θ equation 3.

So, M will now become V R cube sorry R square t by I R square - cos θ + b 1 times of R + b 1 by 2 + b R θ , applying the limits 0 to pi. I will copy this equation.

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$$M = \frac{VR^2t}{I} \left[R^2(-\cos\theta) + \left\{ b_1 \left(R + \frac{b_1}{2} \right) + bR \right\} \theta \right]^\pi$$

$$= \frac{VR^2t}{I} \left(R^2(1+1) + \pi \left(b_1 \left(R + \frac{b_1}{2} \right) + bR \right) \right)$$

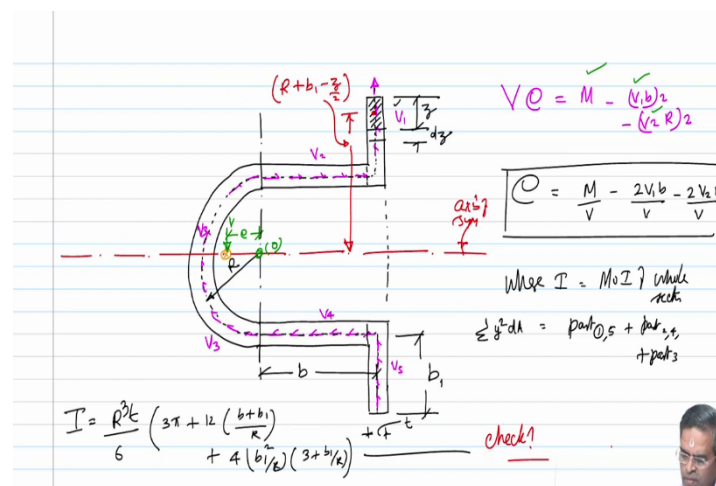
$$\checkmark M = \frac{VR^2t}{I} \left[2R^2 + \pi \left\{ b_1 \left(R + \frac{b_1}{2} \right) + bR \right\} \right] \quad \text{--- (4)}$$

Take moment about point o



So, we will do $V R$ cube by t by $I R$ square $1 + 1 + \pi$ times of $b_1 R + b_1$ by $2 + b R$ which now become V this is R square sorry $V R$ square t by $I 2 R$ square $+ \pi$ times of b_1 into $R + b_1$ by $2 + b R$ equation number 4. You have the M value now will take moment about the point o take moment about point o we get V into e should be equal to M is actually equal to M , let us look at this figure and write this equation we will copy this figure and write this equation. Selection tool this is V 3 let us mark this this is V 3 curved section.

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So, now V into e because this was V in this is the point o V into e which is anticlockwise should be equal to $M - V_1$ into b by $2 - V_2$ into R these are for sorry let us do it this way $M - V_1$ b twice, because I have V phi also - V_2 into R twice. So, friends I have equation M see here I have equation M which is having the term V right. So, M is there we also have equation for V_1 we had the equation for V_1 which also has V .

So, we are equation for V_1 we also have equation for V_2 which also has V so V_2 also here. Of course, b and R are geometric properties ok, can I find e ? So, e will be so V goes away M by $V - 2 V_1$ b by $V - 2 V_2$ R by V I can find e . Now, friends in all these equations of M V_1 and V_2 see here in V_1 I have I in V_2 I have I in M also I have I , where I is the moment of inertia of the whole section.

So, for first principles you will be able to find out the moment of inertia of all the 5 parts part number 1 part number 5 part number 2 1 4 and part number 3 I think moment of inertia is sum of second moment of area of all the pieces you can do it for component 1 and 5 component 2 and 4 + component 3.

I will give you the equation you please check. The moment of inertia of this whole section is given by $R^3 t^3$ by $63 \pi + 12$ times of $b + b$ 1 by $R + 4$ times of b 1 square by R into $3 + b$ 1 by R is the moment of inertia of the whole section you can check this. So, friends we have learnt how to do the shear center problem, for curved section which has caught a mixture of rectilinear and curvilinear sections from the first principles we found out how to locate e ?

So, locating shear center for geometrically complicated sections not that easy ok, we have got to gradually divide them into parts into finite elements and keep on assembling them and do it slowly and patiently it is not that easy. So, when you have got form dominant designs where the geometric shapes are dependent on the functional requirements obtaining shear center, to check whether unsymmetric bending will be caused whether twisting will happen whether stability will be checked all will become a prerequisite.

So, standard software's cannot handle special kind of structural shapes, it is always better friends to use the cad models develop these geometric shapes, identify the shear center and calculate the necessary values yourself with MATLAB program. So, the whole intention of this course is to make you independent researcher and independent thinker and not making you dependent on using software as a black box.

So, advancement is not only on design process is not only on differing defining equations, not only on checking stabilities, not only an understanding plastic design it also enables you inherently to be an independent thinker. So, the MATLAB program is not that difficult is a very user-friendly software you have got good support from matlab incorporation.

So, they will try to help you answer your queries on time and you can also contact me through discussion forum, but the whole objective is try to learn have a grip of the problem and enjoy your creation as a programmer ok; that is the beauty in the whole exercise right. Having said this let us do a couple of more problems. So now I am going to take a different section I will call this as example 3.

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Example 3 Unsymmetric section

1) To locate C_g (\bar{x}, \bar{y})

$$\bar{x} = \frac{\sum a\bar{x}}{\sum a}; \quad \bar{y} = \frac{\sum a\bar{y}}{\sum a}$$

$$\bar{x} = \frac{(20 \times 2 \times 10) + (40 \times 2 \times 20) + (36 \times 2 \times 1)}{(20 \times 2) + (40 \times 2) + (36 \times 2)}$$

$$= 10.8 \text{ mm}$$

(all dimensions in mm)

So, I am going to use or check an unsymmetric section friends, it is a very tricky problem. So, far we have been discussing the shear center location for a symmetric section, the section at least had one axis of symmetry the shear center was located along the axis. But that is not the true truth all the time you can have unsymmetric sections, unsymmetric sections have their own complications I think we all know that now by this time we have understood how to operate an unsymmetric section.

Let us take an example have a section like this unequal channel section with unequal flanges let us mark the dimensions of the section, let us say this is 40 this is also 40 and this is 20 the section has uniform thickness of 2 all dimensions or in millimeters. So, the moment I have an

unsymmetric section I will mark the axis the z and y axis x axis normal to the screen this is my y axis this is my z axis. So, immediately these values are required.

I call this as z bar and I will call this as y bar this means C g not the shear center. So, C g we will divide this as 3 parts this part number 1 component 2 and component 3, can you find the z bar and y bar not to locate C g is the first issue here. That is I want to find z bar and y bar of this problem, we know z bar is given by the simple expression algebra sum of a z bar by a and y bar is a y bar by a we know this.

So, let us work out z bar. So, 20 into 2 into 10 + this is for piece number 1 40 into 2 40 into 2 into 20 this is for piece number 3 + 36 into 2 into 1 piece number 2 divided by 20 into 2 + 40 into 2 + 36 into 2 which will give me z bar as 10.8 mm I will mark this value.

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$$\bar{y} = \frac{\sum Ay}{\sum A}$$

$$= \frac{(20 \times 2 \times 20) + (40 \times 2 \times 39) + (36 \times 2) (18+2)}{(20 \times 2) + (40 \times 2) + (36 \times 2)} = 23.96 \text{ mm}$$

$$I_z = \left[\frac{20 \times 2^3}{12} + 20 \times 2 \times 21.96^2 \right] + \left[\frac{40 \times 2^3}{12} + 40 \times 2 \times (39 - 23.96)^2 \right] + \left[\frac{2 \times 36^3}{12} + 36 \times 2 \times (23.96 - 20)^2 \right]$$

$$I_z = 4.8 \times 10^8 \text{ mm}^4$$

Let me copy this figure. Let us find y bar which can be 20 into 2 into 1 piece number 1 + 40 into 2 into 39 piece number 3 + 36 into 2 into 18 + 2 am I right, this is for piece number 2 divided by 20 into 2 + 40 into 2 + 36 into 2 which will give me y bar as 23.96 millimeters. So, let me mark this value as 23.96.

I also want to find the moment of inertia of this section, because I want to locate the principal axis in the unsymmetric section is it not. So, next step will be step number 2 to find I y and I z and of course I y z for this section, then only I can locate the principal axis. So, I can write I

z directly here that is horizontal axis I can use parallax theorem. So, 20 into 2 cube by 12 + 20 into 2 into 22.96 squares, this is for piece number 1 +.

Let me do it + 40 into 2 cube by 12 40 into 2 39 - 23.96 the whole square all right this is for piece number 3, + 2 36 cube by 12 + 36 into 2 into 23.96 - 20 the whole square this is for piece number 2. If I do this, I will get this value as 4.8 10 power 4 which is I z.

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$$I_y = \left[\frac{2 \times 20^3}{12} + 20 \times 2 \times (10.8 - 10)^2 \right]_0$$

$$+ \left[\frac{2 \times 40^3}{12} + 40 \times 2 \times (20 - 10.8)^2 \right]_0$$

$$+ \left[\frac{36 \times 2^3}{12} + 36 \times 2 \times (10.8 - 1)^2 \right]_0$$

$$I_y = 2.57 \times 10^4 \text{ mm}^4$$

$$I_{zy} = \left[(20 \times 2) (0.8) (22.96) \right]_0 +$$

$$\left[(40 \times 2) (-9.2) (-15.04) \right]_0 +$$

$$\left[(36 \times 2) (9.8) (3.96) \right]_0$$

$$I_{zy} = 1.46 \times 10^4 \text{ mm}^4$$

(all dimensions in mm)

Let us copy this figure let us do I y, I y is a vertical axis which will be 2 into 20 cube by 12 + 20 into 2 into 10.8 - 10 the whole square this is for piece number 1, + 2 into 40 cube by 12 + 40 into 2 into 20 - 10.8 the whole square this is for piece number 3, + 36 into 2 cube by 12 + 36 into 2 into 10.8 - 1 the whole square this for piece number 2.

So, if you do this I will get I y as 2.57 10 power 4 mm 4 this I y friends this first principles I mean there is no confusion in this very clear you use parallel axis theorem we have done it. Let us do I z y, I z y is 20 into 2 into 0.8 into 22.96 very simple you can see this value this will be here.

And for the first piece see this value this will be here this is for the first piece + 40 into 2 to - 9.2 into - 14.0 you can very well see this C g is located here. So, it is negative in terms of y it is negative in terms of z and these values can be seen from here, this value is this and this value is this+ this for piece number 3 + 36 into 2 into 9.8 into 3.96 is for piece number 2.

This is 10.8 friends make a correction here I am sorry. So, this value is same as this, and this is 3.96 it is coming here. So, I_{yz} after you compute will become 1.46×10^4 , since I_{yz} is not 0 the section will be now subjected to unsymmetric bending and z and y will not be the principal planes we know that. So, once we understand this, I can straightaway locate the angle of inclination of this figure.

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$$\tan 2\alpha = - \frac{I_{yz}}{I_z - I_y}$$

$$= - \frac{2 \times 1.46 \times 10^4}{(4.8 - 2.57) \times 10^4}$$

$$\alpha = - 26.31^\circ$$
 (measured clockwise from +z to +u)

(all dimensions in mm)

Let me draw this figure again and let us mark the u axis and v axis say this is my v this is my u axis and we know this angle measured from positive z is α . And we know $\tan 2\alpha$ is given by $-I_{yz} / (I_z - I_y)$ now we have all the 3 values with us we will substitute them - twice of 1.46×10^4 by z value is 4.8 and y value is 2.57 .



So, we get α as $- 26.31$ degrees measured clockwise from positive z to positive u . So, this is 26.31 degrees. So, I located the principal axis.

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To find I_u, I_v

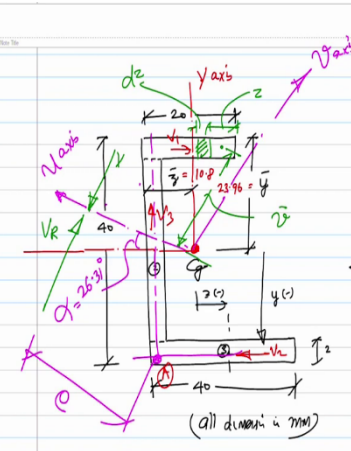
$$I_u = \frac{I_y + I_z}{2} + \frac{I_z - I_y}{2} \cos(2\alpha) - I_{yz} \sin(2\alpha)$$

$$= 64144.83 \text{ mm}^4$$

$$I_v = (I_z + I_y) - I_u = 9645.17 \text{ mm}^4$$



My job is going to find out I_u and I_v because I need to find I_u the moment of inertia about the principal axis now. So, we know I_u is $I_y + I_z$ by $2 + z - y$ by $2 \cos 2\alpha - I_{yz} \sin 2\alpha$ this equation we already derived we know. So, substitute them we will be getting 64144.83 mm^4 I_v can be also said as $I_z + I_y - I_u$. So, I can say 9645.17 mm^4 you can check these values friends. Once we have try this our job is to locate the shear center let us draw this figure again.



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By taking moment about A,
 (V_z, V_y) being concurrent about A
 will cause no moment
 (V_z, V_y) need not be calculated

To find V_1

$$V_1 = \int z da = \int z (t dz)$$

$$z = \frac{VA \bar{y}}{I_z} = \frac{VA \bar{y}}{I_u(t)}$$



So what I am now going to do is ok, I will redraw this figure slightly gentle α is 26.31. So, now if I remove this label and say I am going to apply the load v and the reaction $V R$ here along the $u u$ axis please understand I am writing it normal. So, it is along v axis. So, let us say this is v these are principal axis.

So, I call this as $V R$. Now if I take moment about this point and I call this distance as e , I have an advantage if I write this as $V 1 V 2$ and $V 3$ and I call this point as a by taking moment about $A V 2$ and $V 3$ being concurrent about A we will cause no moment. So, I do not have to work out $V 2$ and $V 3$. So, $V 2$ and $V 3$ need not be calculated. So, I want to calculate only $V 1$.

So, to do that I will cut the section this section will be at a distance z thickness dz and the $C g$ of this need to be measured from this axis I want this $C g$ to remember that. We want this we call this V bar am I write where I am transforming everything into the principal axis now. So now, to find $V 1 V 1$ is $\tau d a$ integral which is integral $\tau d z$ of t the τ is $V A y$ bar by $I t$ which is $V A$, now V bar by $I u$ into t am I right.

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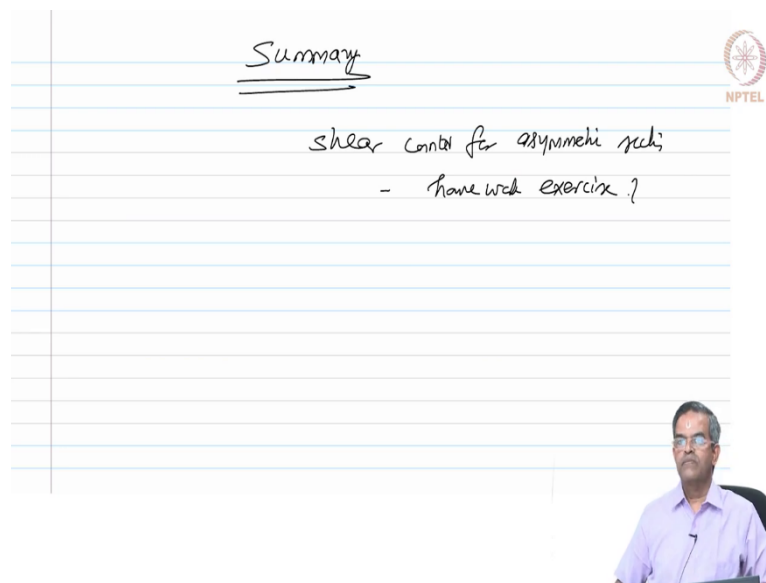
$$\begin{aligned}
 V_1 &= \frac{V}{I_u} \int A \bar{v} dz \\
 &= \frac{V}{I_u} \int (z \sin \alpha + y \cos \alpha) dz \\
 \bar{v} &= -z \sin \alpha + y \cos \alpha \\
 &= - \left[- (20 - 10.8) - \frac{3}{4} \right] \sin (-26.31) \\
 &\quad + (22.9) \cos (-26.31) \\
 &= - \left[- (9.2 - \frac{3}{4}) (-0.44) + 20.93 \right] \\
 V_1 &= -4048 + 0.22z + 20.93 = 16.48z + 0.22z
 \end{aligned}$$

Please look into this equation I am transforming them into principal planes right, therefore $V 1$ will be V by $I u$ integral $A v$ bar by t into $t d z$ which will be V by $I u z t$ into v bar by t in into v bar $d z$ because this t goes away right. So now, I want to find this v bar? This v bar is now equal to we can make use of this figure, if I have 2 axis y and z if you have another 2 axis v and u and this angle is α in simple terms v can be given by $-z \sin \alpha + y \cos \alpha$ is it not.

So, I want to substitute the z value - of - 20 - 10.8 - z by $2 \sin \alpha$ which is - 26.31 degree+ 22.9
 $\cos \alpha$ which is - 26.31 degrees. So, friends we have the v value with us which can be
simplified and said as - of 9.2 - z by 2 this is z let me make it very clear - z by 2 of - 0.44 +
20.5.

So, v now becomes, or v bar becomes - 4.048 + 0.22 z + 20.93 which becomes 16.482 + 0.22
z right.

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So, once we have this value, I can substitute in V 1 I leave a small homework here I want you
to do this before you check this result for my next lecture. So, in this lecture we learnt how to
work core shear center for asymmetric sections. I also left you a homework exercise and you
will check that before you attempt to the next lecture.

Thank you very much and have a good day, bye.