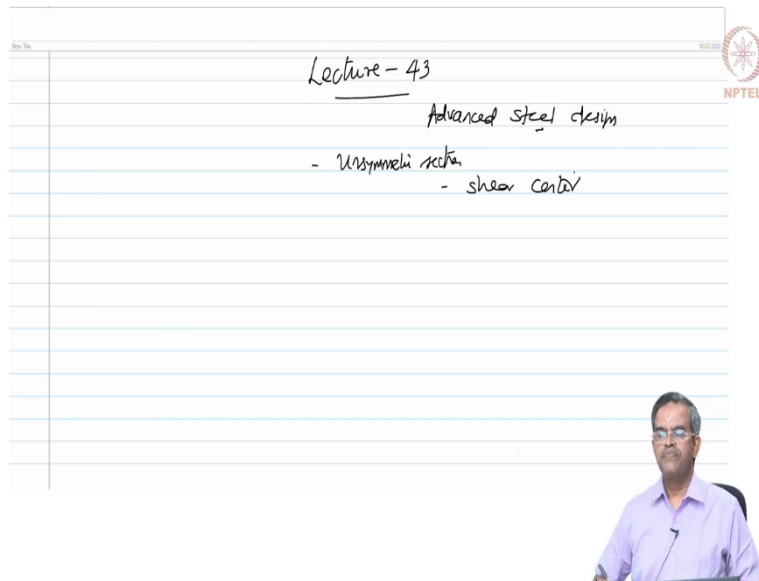


Advanced Design of Steel Structures
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Lecture - 43
Shear center for unsymmetric section

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Friends welcome to lecture 43 of Advanced Steel Design course. We are now continuing the problem of unsymmetric section, and we are trying to locate the shear center.

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1) Cg of the sects (\bar{y}, \bar{z})

2) computed I_y, I_z, I_{yz}

3) computed α - located u, v axes

4) Take moment about (\bar{x})
 (V_1, V_2) are concurrent
 To find v_1
 $\bar{v} = 1$

(all dimen in mm)

So, this was the unsymmetric section we had. We located the u and v axes and we first located the cg of the section that is we found out \bar{y} and \bar{z} , then we located. We computed I_y I_z and yz and then computed alpha angle and located u & v axes.

Then we are in the process of finding out the shear center, then we attempted to take moment of point A where V_2 and V_3 are concurrent, and no moments will be created. So, we wanted to only find V_1 . To do that we wanted to find out the distance v bar because we are using, or we are transforming the problem to the principal planes. So, we wanted to find out v bar.

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$$V_1 = \frac{V}{I_u} \int \frac{A \bar{v}}{k} (\cancel{k} dz)$$

$$= \frac{V'}{I_u'} \int (\bar{z}' \bar{v}) dz$$

$$\bar{v} = -z' \sin \alpha + y' \cos \alpha$$

$$= - \left[-(20 - 10.8) - \frac{3}{4} \right] \sin(-26.31)$$

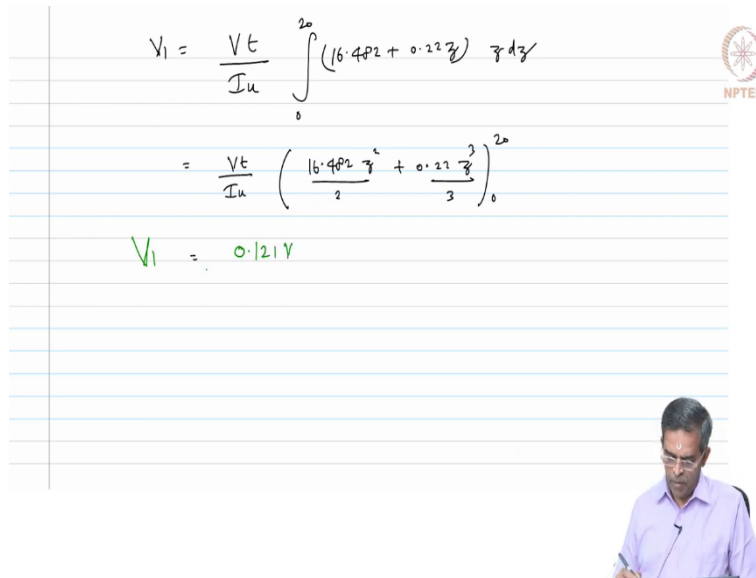
$$+ (22.9) \cos(-26.31)$$

$$= - \left[- (9.2 - \frac{3}{4}) (-0.44) + 20.53 \right]$$

$$V_1 = - 4.048 + 0.22 \bar{z}' + 20.93 = \underline{16.482 + 0.22 \bar{z}'}$$

And this was the expression we had to find out v bar. So, v bar is 16.48222 z . So, then we substitute that v bar here in this equation and get V_1 .

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$$V_1 = \frac{Vt}{Iu} \int_0^{20} (16.482z + 0.22z^2) dz$$

$$= \frac{Vt}{Iu} \left(\frac{16.482z^2}{2} + \frac{0.22z^3}{3} \right)_0^{20}$$

$$V_1 = 0.121 V$$

So, now, V_1 will be seen here Vt by Iu integral. Now we can say this will be for length of 20. So, I can say 0 to 20 right. So, see here we have A which is $z dz$ that is what we are looking for $z dz$ is what we are looking for so $z dz$. So, let us say Vu Vt by Iu 16.482 + 0.22 z , $z dz$ ok that is what we have here, is it not? V by Iu $z dz$ v bar, v bar is already here.

We will evaluate this integral, and we will find out this as Vt by Iu 16.482 z square by 2 + 0.22 z cube by 3 of 0 to 20. If I do that and t is 2 millimeters V_1 will now become 0.121 V . Now we will take moment about this.

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Take moment about A

$$V e_u = V_1 (38)$$

$$e_u = 0.121 (38)$$

$$e_u = 4.59 \text{ mm}$$

(all dimensions in mm)

NPTL

So, we will draw this figure. Now we will take moment about A. So, V we will call this as e_u , $V * e_u$ will be equal to this is V 1. So, V 1 and this distance 38.

So, already we have V 1 with us. So, V 1 is 0.121 V. So, V into e_u is 0.121 V of 38. So, V goes away e_u is 0.121 into 38 which is about 4.59 millimeters. So, this is about 4.6 millimeters, that is how we can find out.

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In case of unsymmetric axis.

locate (u, v) before proceeding to locate the shear center

NPTL

So, friends when you have a problem of unsymmetric sections one need to locate the principal axes before proceeding to locate the shear center, that is an important point which we must remember.

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Ex 4 locate the shear center

By symmetry, $V_1 = V_4$
 $V_2 = V_3$

$$V_1 = \int_0^{b_1} \frac{V}{I t} (a \bar{y}) da$$

$$Q_0 = t \bar{z}$$

$$dQ_0 = t dz$$

$$\bar{y} = (b \sin 45^\circ - b_1 \sin 45^\circ) + \frac{z}{2} \sin 45^\circ$$

$$= (b - b_1 + \frac{z}{2}) \sin 45^\circ$$

NPTEL

We will do one more example. Say example 4 draw this figure. So, the problem is locating the shear center of the given field ok section like this. Of course, this has an axisymmetric which I am marking here. Let us mark the dimensions I think here in mark the dimensions of this. First let us mark the center lines and mark the shear flow. Let us say this is V 4 this is V 3 this is V 2 this is V 1. Let us mark this dimension that is b 1 and this dimension as b is also b, this is 45 degrees.

Of course, this is also b 1. The section has uniform thickness t this is a point which we call as point A and the shear center will be located somewhere on this line. Let us say this is my shear center C. The vertical force will pass through this, and we measure the distance from this center from the point A as offset of shear center which is e. So, from this figure by symmetry we know that V 1 is equal to V 4 and V 2 is equal to V 3 because of the dimension similarity.

So, we will cut a section. Let the section be cut from here at a distance z and let the thickness of the section be d z and of course, this thickness is already given as t. We are looking for the c g of this from here. This is my y bar for piece number 1 ok, for piece number 1. So, now, V 1 is given by integral 0 to b 1 V by I t a y bar of piece 1 into d.

So, a of piece 1 is t into z da of piece 1 is t into dz y bar is b sin 45 - b 1 sin 45 + z by 2 sin 45 say b sin 45 let us say you see here. So, looking for this distance ok b sin 45 will be this, b 1 sin 45 you will be reaching here. Then if you do this is you know z by 2 will give to the center get this right. So, which will be b - b 1 + z by 2 of sin 45 which is y bar.

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$$V_1 = \int_0^{b_1} \frac{V}{I} (t \, dz) \left(b - b_1 + \frac{z}{2} \right) \frac{1}{\sqrt{2}}$$

$$= \frac{Vt}{\sqrt{2} I} \left(\frac{b b_1^2}{2} - \frac{b_1^3}{2} + \frac{b_1^3}{6} \right)$$

$$= \frac{Vt}{\sqrt{2} I} \left(\frac{b b_1^2}{2} - \frac{b_1^3}{3} \right)$$

$$V_1 = \frac{Vt b_1^2}{\sqrt{2} I} \left(\frac{b}{2} - \frac{b_1}{3} \right) \equiv V_4 \quad I = \text{MoI of the whole section}$$

By taking moment of all forces about O, (V₂, V₃) are concurrent & hence moment

Substituting let us get V 1 as integral 0 to b1 V by I t t z into t dz b - b 1 + z by 2 of 1 by root 2 that is sin 45. Integrating and substituting the limits we get V t by root 2 I b 1 b 1 square. It is not b 1 is b, b b 1 square by 2 - b 1 cube by 2 + b 1 cube by 6.

Which will now become V t root 2 I b b 1 square by 2 - b 1 cube by 3. Therefore, we get v 1 as V t b 1 square by root 2 I of b by 2 - b 1 by 3 which is identically same as V 4. Now we taking moment about the point A I do not have to work out V 2 and V 3 because they are concurrent about the point t.

So, by taking moments of all forces about A V2 and V3 or concurrent and hence no moment. So, there is no need to calculate these 2. So, directly I can use V 1 and V 4, but there is a issue here V 1 need to calculate I of the entire section. So, this I is a moment of inertia of the whole section. So, that is a tricky thing here. So, let us do one by one. So, we need to find I of the whole section.

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To find I of the whole section

$$I_{uu} = \frac{tb^3}{12} \quad I_{vv} = \frac{bt^3}{12}$$

$$I_{x'x'} = I_{uu} \cos^2 \theta + I_{vv} \sin^2 \theta \quad (\theta = 45^\circ)$$

$$= \frac{tb^3}{12} \cos^2 45^\circ + \frac{bt^3}{12} \sin^2 45^\circ$$

$$= \frac{tb^3}{12} \left(\frac{1}{\sqrt{2}}\right)^2 + \frac{bt^3}{12} \left(\frac{1}{\sqrt{2}}\right)^2$$

$$I_{x'x'} = \frac{tb^3}{24} + \frac{bt^3}{24}$$

$$I_{xx} = I_{x'x'} + ak^2 = \left(\frac{tb^3}{24} + \frac{bt^3}{24}\right) + bt \left(\frac{b}{2} \sin 45^\circ\right)^2$$

So, now, to find I of the whole section whole X dash X dash and we call this small axis as uu and vv. We know and this dimension is of course, b and this thickness is t, this angle is 45 degrees.

We know I uu is tb cube by 12 and I vv is bt³ by 12 and I x dash x dash is I u cos square θ + I v sin square θ where θ is 45 degrees here. So, I can say it is tb³ by 12 cos square 45 + bt³ by 12 sin square 45 which will be tb cube by 12 1 by root 2 the whole square + bt cube by 12 1 by root 2 the whole square which becomes tb cube by 24 + bt cube by 24, this is I X dash X dash.

Now, I have an axis here which is XX. So, now I can say I XX will be I X dash X dash + ak square which will be tb cube by 24 + bt cube by 24 + ak square which is b into t of the distance. You know XX axis is not here. We should mark it here look at this figure XX axis is the tip ok let us mark it here XX. So, which will be b by 2 sin 45 the whole square.

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$$I_{xx} = \frac{tb^3}{24} + \frac{bt^3}{24} + \frac{b^3t}{8}$$

$$= \frac{4tb^3}{24} + \frac{bt^3}{24}$$

$$I_{xx} = \frac{tb}{24} (4b^2 + t^2)$$

So, which will give me I_{xx} as tb cube by 24 + b t cube by 24 + b cube t by 8 ok, b cube t by 8. So, which will now become 4 tb cube by 24 + bt cube by 24. So, we should say tb by 24 4 b square + t square is my I_{xx} of that arm. Let us do it for the arm b 1 because we have no we have done for this arm. Let us do it for this arm.

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$$I_u = \frac{b_1 t^3}{12}$$

$$I_v = \frac{t b_1^3}{12}$$

$$I_{xx'} = I_u \sin^2 45 + I_v \cos^2 45$$

$$= \frac{b_1 t^3}{12} \left(\frac{1}{\sqrt{2}}\right)^2 + \frac{t b_1^3}{12} \left(\frac{1}{\sqrt{2}}\right)^2$$

$$I_{xx'} = \frac{b_1 t^3}{24} + \frac{t b_1^3}{24}$$

$$I_{xx} = I_{xx'} + A k^2 = \left(\frac{b_1 t^3}{24} + \frac{t b_1^3}{24}\right) + b_1 t \left(\frac{b_1^2 + t^2}{8}\right)$$

Let us draw. So, this is my V 1 I am looking for an axis which is XX . Let us also mark centroidal axis as u u and v v for this and we call this horizontal axis as X dash X dash and the dimensions are of course, this is b 1 and this is t . So, now we can say I_u will be b 1 t cube

by 12 and I v will be $t b^3$ by 12. So, I_{XX} will be $I_u \sin^2 45 + I_v \cos^2 45$ which will be $\frac{b^3 t}{12} + \frac{b^3 t}{12}$ by $\sqrt{2}$ the whole square $\frac{b^3 t}{12}$ by $\sqrt{2}$ the whole square.

So, I_{XX} will be $\frac{b^3 t}{24} + \frac{t b^3}{24}$. So, now, I_{XX} will be $I_{XX} + a k^2$ we can apply this. So, which will be equal to $\frac{b^3 t}{24} + \frac{t b^3}{24} + b^3 t$ of you know the angle is connected here. This is how it is connected, and this dimension is b , is it not? So, I should say $b \sin 45 - b \cos 45$ the whole square that is my k square here. Let me copy this equation.

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$$I_{XX} = I_{XX'} + a k^2$$

$$= \left(\frac{b^3 t^3}{24} + \frac{t b^3}{24} \right) + b^3 t \left(\frac{b \sin 45}{2} - \frac{b \cos 45}{2} \right)^2$$

$$I_{XX} = \frac{b^3 t^3}{24} + \frac{t b^3}{24} + b^3 t \left(\frac{b}{\sqrt{2}} - \frac{b}{\sqrt{2}} \right)^2$$

$$= \frac{b^3 t^3}{24} + \frac{t b^3}{24} + b^3 t \left(\frac{b^2}{2} + \frac{b^2}{2} - \frac{b b^2}{2} \right)$$

$$(I_{XX})_0 = \frac{b^3 t^3}{24} + \frac{t b^3}{24} + \frac{b^3 t}{2} + \frac{b^3 t}{8} - \frac{b b^2 t}{2}$$

So, I_{XX} you know equal to $\frac{b^3 t}{24} + \frac{t b^3}{24} + b^3 t$ by $\sqrt{2}$ - this is $b^3 t$ by $\sqrt{2}$ - $b^3 t$ by $2 \sqrt{2}$ the whole square is it. Simplify I get this as $\frac{b^3 t}{24} + \frac{t b^3}{24} + b^3 t$ of b^2 by 2 + $b^3 t$ by 8 - $b b^2 t$ by 2 which will be $\frac{b^3 t}{24} + \frac{t b^3}{24} + \frac{b^3 t}{2} + \frac{b^3 t}{8} - \frac{b b^2 t}{2}$ this is my I_{XX} of the first piece.

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$$I_{xx} = \left[\frac{tb}{24} (4b^3 + t^3) + \left[\frac{b_1 t^3}{24} + \frac{t b_1^3}{6} + \frac{b_1 b^2 t}{2} - \frac{b b_1^2 t}{2} \right] \right]^2$$

Tabii Nemat abar (A)

$V_e = (V_1 b)^2$

find e

for example if $b_1 = 40$
 $b = 70$
 $t = 2$ mm

find e



So, now we can say I_{XX} of the whole section will be, let us say the piece number this value I am copying it here ok which I am writing it here $\frac{tb}{24} (4b^3 + t^3)$. Let me rub this then I will copy this down. Let me just write it here $+ \frac{b_1 t^3}{24} + \frac{t b_1^3}{6} + \frac{b_1 b^2 t}{2} - \frac{b b_1^2 t}{2}$ the whole into 2.

Now, I draw this figure take a moment of A this is the point A. So, we can say V into e will be equal to V_1 into b right, because that is V_1 from here and there is same as V_4 so into 2. So, now I have V I have V_1 I have b . So, find e . So, for example, if b_1 is 40 b is 70 t is 2 millimeters ok you can find A . So, friends in this lecture we learnt 2 examples.

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Summary

1) unsymmetric section
steps involved to locate the shear center

2) Example: How to choose the point to take moment of all internal forces to find shear center

crane rail pipe rack

One we learned how to work out the shear center for unsymmetric section and we have seen the steps involved to locate the shear center. We have also learnt one more example and in this example, we learnt how to choose the point to take moment of all internal forces to find the shear center. So, we can choose this point very advantageously, so that our computational effort can be reduced.

So, friends with this we will be completing the discussion on shear center. There are many more problems available in the textbook referred by me for this course. Look at those examples and try to solve them we have also seen curved sections we have seen asymmetric sections. We have seen symmetric complicated sections which very common sections are used in mechanical systems as well as structural systems.

For example, this kind of sections are used for crane rails. This kind of sections are used for pipe racks because you get more space to put the pipe. So, these are all rectilinear shapes, but arranged in a such a fashion they create very complicated geometry for computational purposes. So, you should be able to look into these and practice more problems. So, that you are clear about solving them from the first principles.

Thank you very much and have a good day.