Advanced Design of Steel Structures
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Lecture - 44
Curved beam - 1

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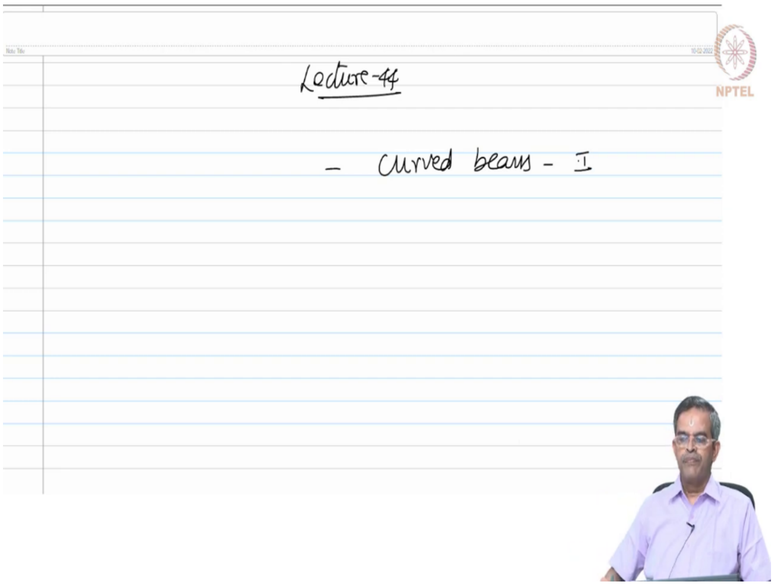

Friends, welcome to the lecture 44 of the course Advanced Steel Design. In this lecture we are going to learn more about Curved Beams, I will call this lecture as curved beams - I.

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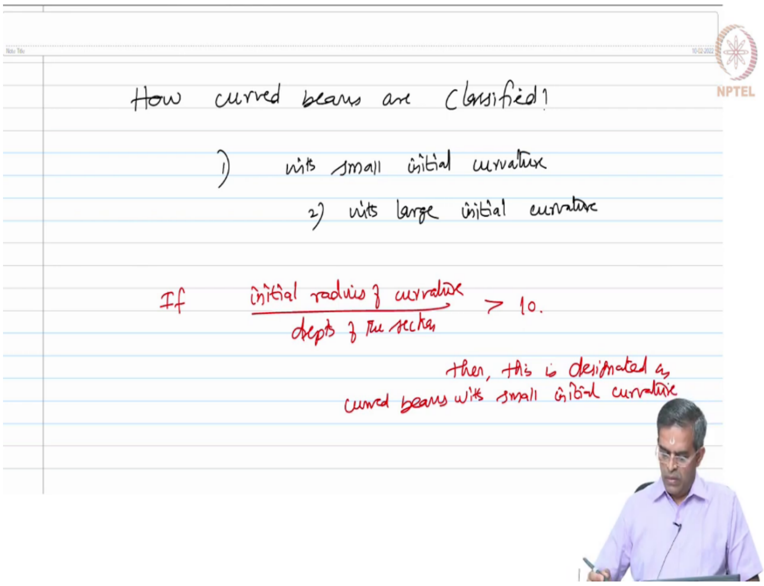

The foremost question comes in mind is, how curved beams are classified? Curved beams are classified in 2 ways, beams with small initial curvature and beams with large initial curvature. Now, what do you understand by small and large? How do you quantify this? If the initial radius of curvature divided by depth of the section is greater than 10, we call this designated as beams or curved beams with small initial curvature.

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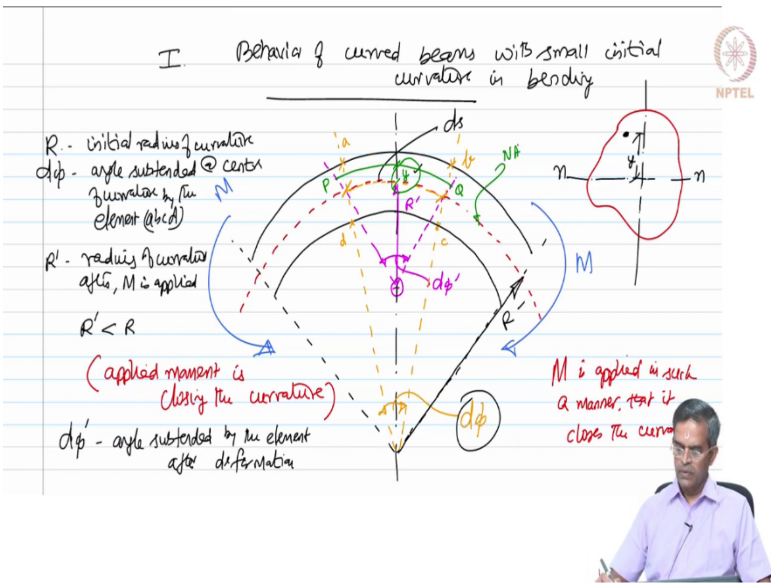

Now, let us look into the discussions of behavior of curved beams with small initial curvature in bending. We will look into this now. Let us say, I have a curved beam, , shown in the figure here. Let us say this is now subjected to in a moment as shown the figure. Let us say the radius indicated is R. So, R is the initial radius of curvature d ϕ is angle subtended at the center of curvature. I will mark d ϕ, by the element abcd. Let us first mark the element abcd, say this is the angle subtended, we call this as d ϕ and this element this is a, this is b and this is c and this is d.

Then, R prime is a radius after applying the moment. See when you apply a moment as shown in the figure; the radius of curvature now changes. Let us say, we mark a new center here. And let us see the new angle subtended by this is d ϕ prime and the radius is R prime. Now, I will take a fiber. Please, note, that R prime is much lesser than R that is it is very important applied moment is closing the curvature.

See before M is applied, the radius of curvature was R; after M is applied it reduced to R prime in such a condition that R prime is smaller than R. So, the applied moment actually closes the curvature. We have to remember this very carefully, right. This is a very important convention we need to. And, of course, d prime d ϕ prime is now the angle subtended by the element after deformation.

Let us mark the element layer P, Q and mark it here at a distance y, this is P, Q at a distance y away from the center. So, let us mark the cross-section somewhere here. Let us mark the axis. We call this as n-n axis which is the neutral axis. And we take any point which is above this measured as y, that is what we have marked. So, it is important M is applied in such a manner that it closes the curvature. It does not open it does not open the curvature. It closes the curvature.

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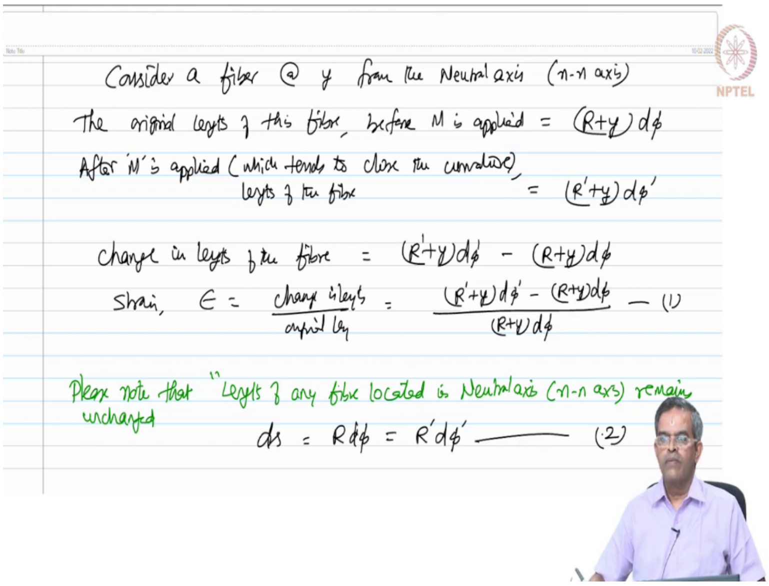

We consider a fiber at y from the neutral axis. Now, let us see what the original length of this fiber is. The original length of this fiber will be R + y, d ϕ is it not, before M is applied will be equal to R + y, d ϕ sees this figure. R is this value, and this is y, d ϕ is a angle that is the original thing.

Now, after applying this moment M, which is closing the curvature the length of this fiber is now reduced to R prime + y d ϕ prime, am I right. After M is applied, which tends to close the curvature length of the fiber is now given by R dash + y d ϕ dash. Therefore, friends change in length of the fiber because we are looking for strain which will be R dash + y d ϕ dash - R + y d ϕ.

So, now I can find the strain, epsilon which is change in length by original length which will be equal to R prime + y d ϕ prime - R + y d ϕ by R + y d ϕ. We will call this equation number 1. But please note that, length of any fiber located in the neutral axis remains unchanged. There is no change in the neutral axis.

So, friends if I take any fiber in the neutral axis let us say I am marking that fiber here, if I take any fiber in the neutral axis that is this fiber, this length will remain unchanged. So, therefore, ds will be R d ϕ which is same as R dash d ϕ dash see in this figure. If I call this as ds before and after applying the moment, there is no change; because this is a fiber measure the neutral axis. So, I call this equation number 2.

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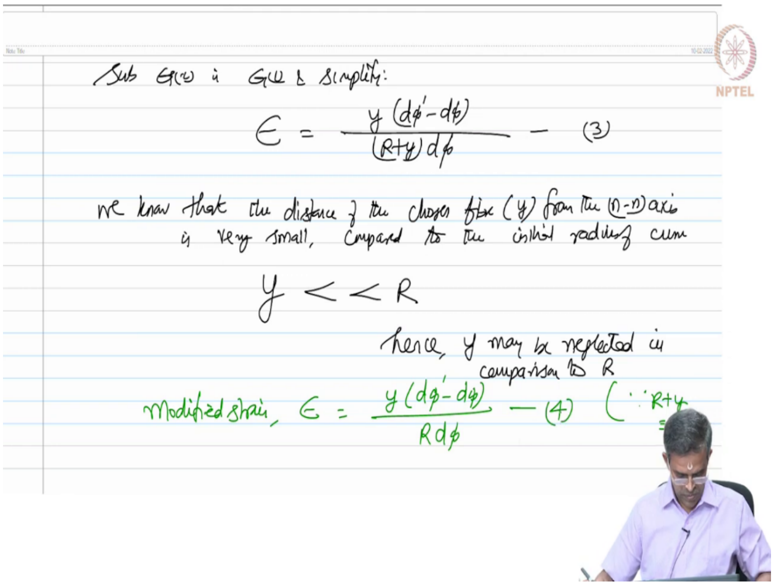

Now, I substitute 2 in equation 1. And simplify. Let us see what do we get? We get strain; because this equation is for the strain equation 1, , which will become y times of d ϕ dash - d ϕ by R + y d ϕ is it ? , call this equation number 3. Now, we know that the distance of the fiber, y from the n-n axis is small compared to the initial radius of curvature.

We have given a small displacement. Therefore, y is very very less compared to R. Hence, y may be neglected in comparison to R wherever we have this formation. Therefore, I write modified strain, epsilon will be y times of d ϕ prime - d ϕ by simply R d ϕ because R + y is identically equal to R for y being small. We call this equation number 4.

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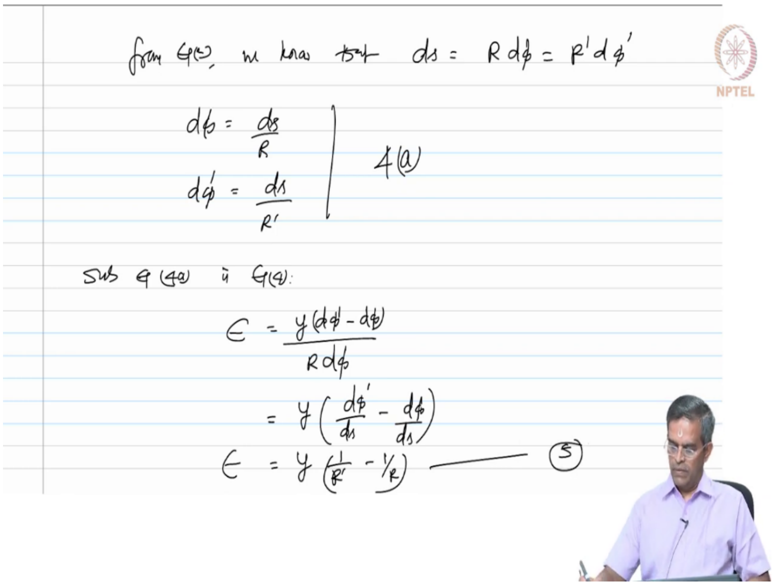

Now, from equation 2, we know that ds is R d ϕ. Let us see what is equation 2, which is also equal to R dash d ϕ dash. So, d ϕ is ds by R and d ϕ dash is ds by R dash, we call this equation number 4 a. So, we have equation 4, where d ϕ dash is there substitute equation 4 a in equation 4. So, equation 4 is for epsilon which is y times of d ϕ dash - d ϕ by R d ϕ, which will now become y times of d ϕ dash by ds - d ϕ by ds which is equal to y times of 1 by R prime - 1 by R equation 5.

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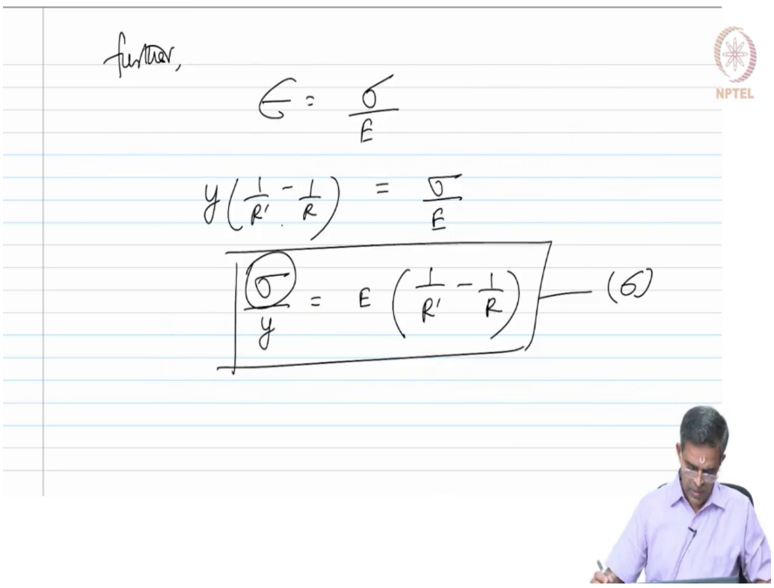

Further, epsilon is also ratio of stress to this. So, therefore, y of 1 by R prime - 1 by R should be equal to stress by E. So, stress by y is E times of 1 by R prime - 1 by R we call this equation as equation number 6. So, we make certain assumptions, while doing this derivation; let us see what are they.

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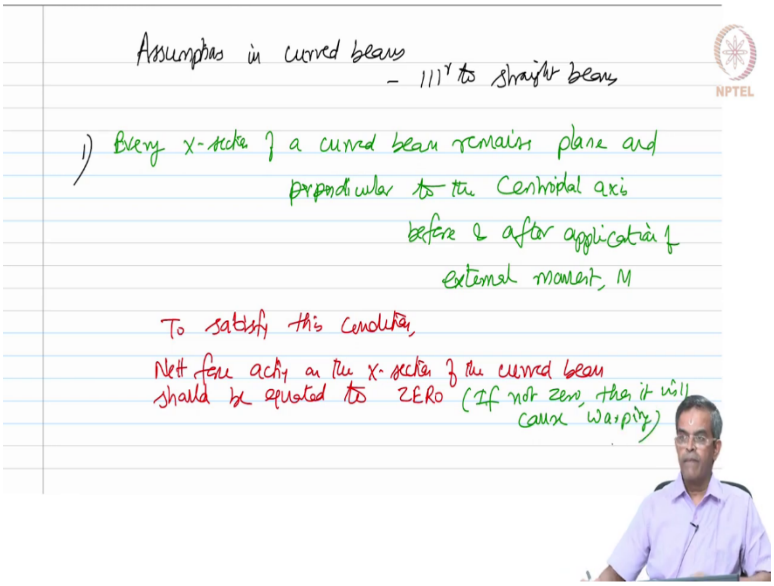

There are certain assumptions made in curved beams, which are similar to straight beams. 1. Every cross section of a curved beam remains plane and perpendicular to the centroidal axis before and after application of external moment. Now, to satisfy this condition net force acting on the cross section of the curved beam should be 0. If it is not equated to 0, then it will cause warping.

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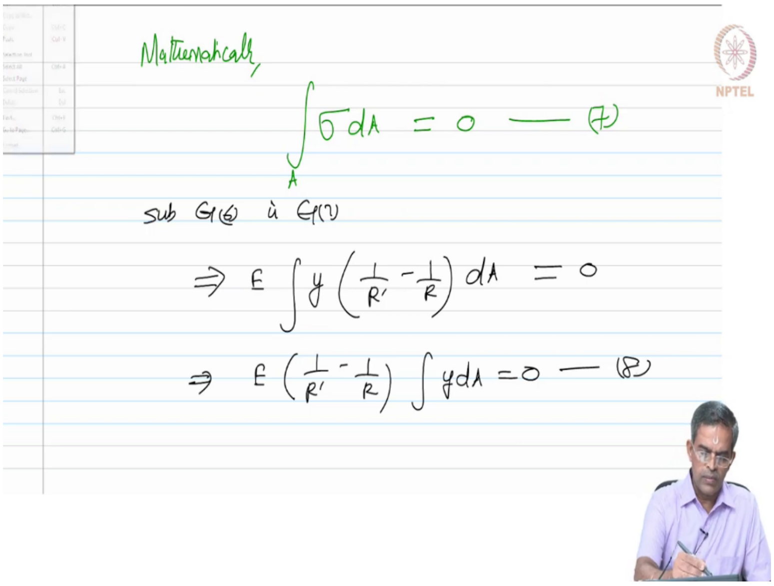

So, mathematically integral stress d A for the area should be 0 equation number 7. Now, let us substitute equation 6 in equation 7. What is equation 6? We have the stress value in equation 6; here let us substitute that in equation 7. So, you know this should be now equated E times of integral of y 1 by R dash - 1 by R of d A and equate this to 0, which means E times of 1 by R dash - 1 by R integral y d A should be equal to 0. So, if you call this equation number 8, let us copy this equation.

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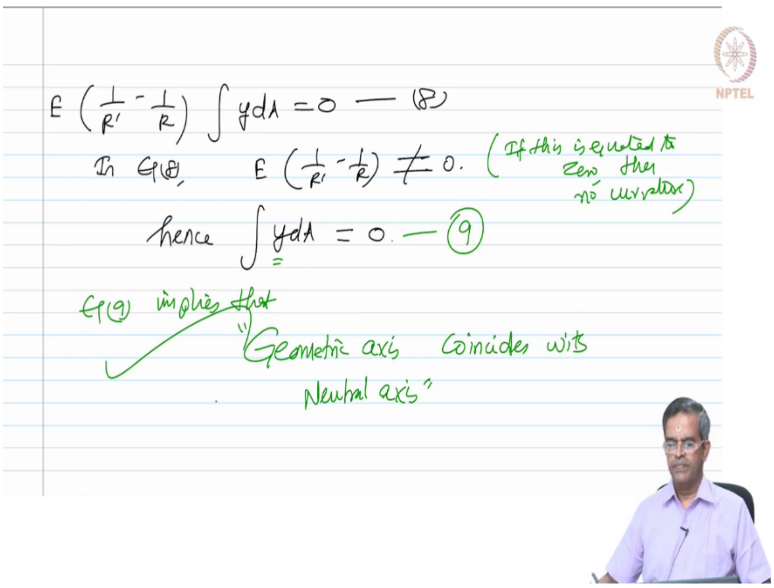

In equation 8, E times of 1 by R dash - 1 by R cannot be equated to 0. Hence, integral ydA should be only equated to 0; because, if this is equated to 0 if this is equated to 0; then no curvature. What does it mean? We call this equation number 9. Equation 9 implies that geometric axis coincides with neutral axis.

How can we say this? You know y is measured from the geometric axis. See this figure. And this is my neutral axis. So, this condition is now applied, and it is very interesting this is forming the basis for the derivation of the curved beams.

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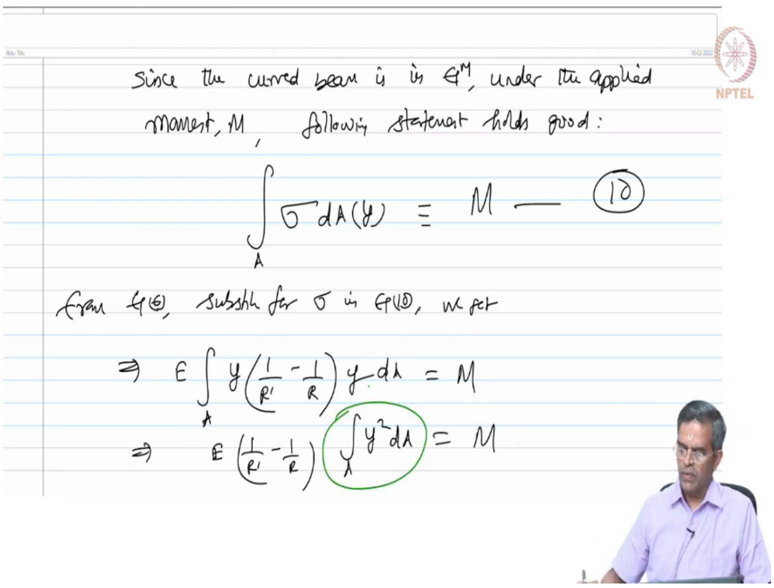

Now, since, the curved beam is in equilibrium under the applied moment M, one can, following statement holds good d A into y should be equated to the moment. Now, from equation 6, substitute for stress in equation 10 we get, E times of A y 1 by R dash - 1 by R of y dA is here, which implies that E 1 by R dash - 1 by R integral y square dA should be M. We have interesting term arrived here, what is this y square d A called second moment of area. That is a very standard expression we have in mechanics. We know integral A y square dA is the moment of inertia.

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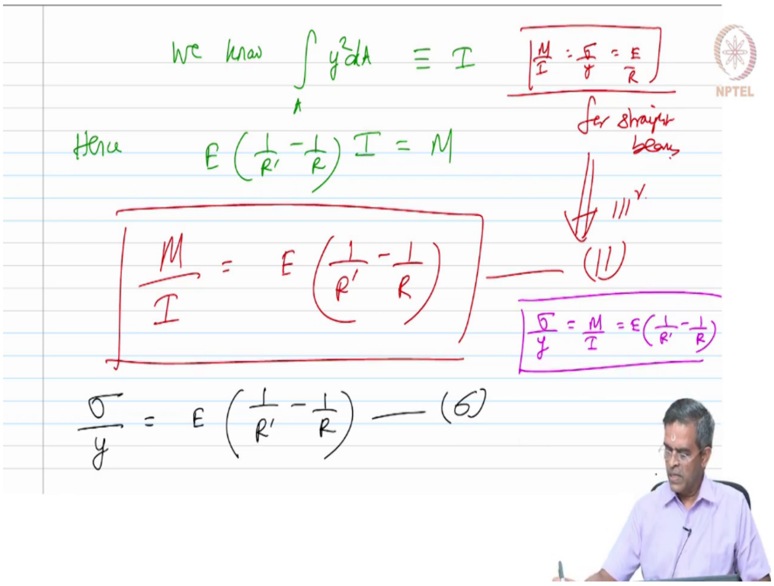

Hence E times of 1 by R dash - 1 by R into I is M, am I right, see here. So, now, I have a very interesting and standard relationship. M by I is E times of 1 by R dash - 1. I mean this is a familiar equation similar to that of a straight beam; we call this equation number 11. Now, let us copy equation 6 here, now I want to copy this equation and then compare. I want to compare.

Now, let us compare equation 6 and equation 11. So, right-hand side is same. Can you now say, stress by y M by I is E times of 1 by R dash - 1 by R standard relationship friends for a curved beam which is more or less looking similar to that of a straight beam is it not, whereas, we have a new term here 1 by R dash, , that is the theory of flexure which we recollect that equation M by I is stress by y is E by R the standard relationship we have for straight beams, is it not. It is more or less similar to that. This is what this is interesting. So, the curve beams are conserved.

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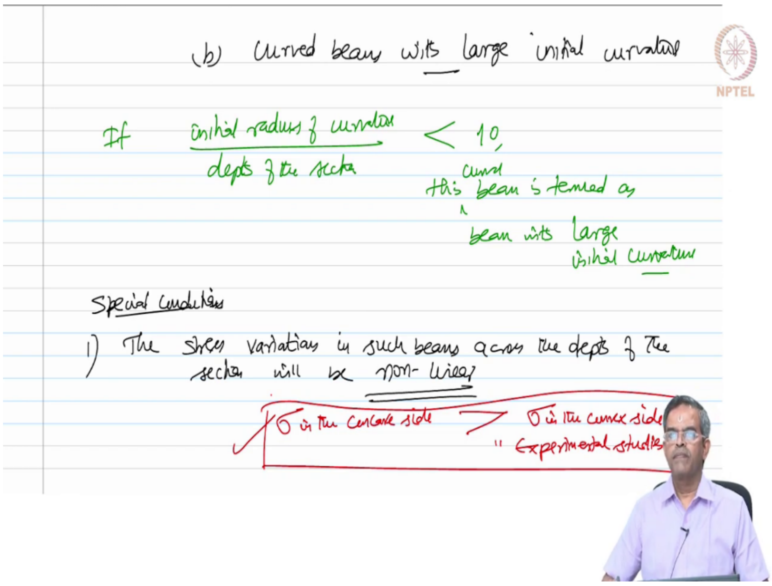

Having said this, let us take our discussion forward for curved beams of large initial curvature. Now, let us recollect if initial radius of curvature to depth of the section is lesser than 10. We can call this beam; this curved beam is termed as beam with large initial curvature.

Can I say that? Because, more than 10 is small, I can say this. So, there are some special applications and properties of this condition. What are they? There are some special conditions. 1 The stress variation in such beams across the depth of the section will be non-linear. What does it mean? Stress in the concave side is generally greater than stress in the convex side, these are experimental observations.

Experimental studies in the literature show that this statement is valid, that is a general information we need.

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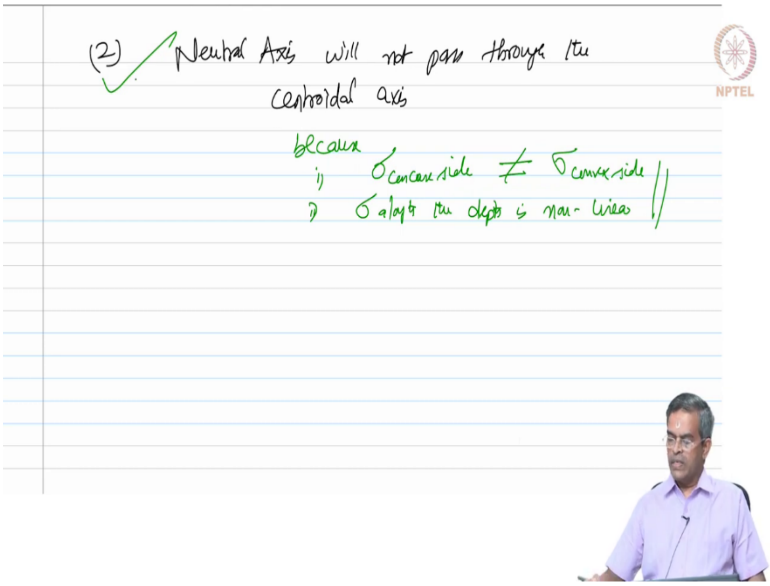

The 2nd point is neutral axis will not pass through the centroidal axis. Why? Because, the stress on the concave side is not equal to the stress on the convex side; further, stress along the depth is non-linear. So, these two conditions make this condition valid.

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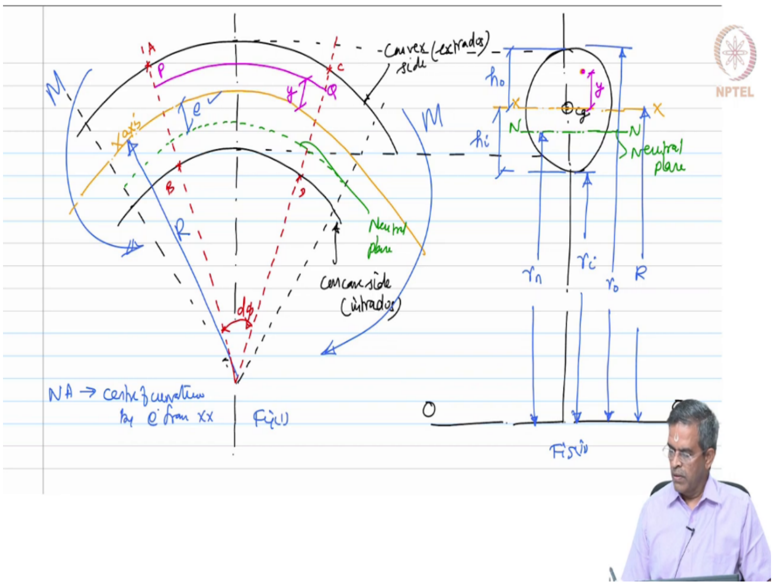

Now, we will take up this for our discussion. So, I am talking about beam with large curvature. Let us draw, let us call this angle as d ϕ. Let us cut a new section as d ϕ prime from the same center. Say, this angle is d ϕ dash. We mark the four corners A, B, C, D. Let us mark a layer P, Q. Let us mark the neutral plane first on the X-X axis let us mark the X-X axis we call this as X axis.

The neutral plane is lying below this. This is a neutral plane. Let us take a fiber P, Q marked at a distance y from X axis. Say, this is P, Q ϕ. This is the convex side also called as extrados. This is the concave side also called as intrados. We are applying a moment which is closing the curvature that is the nature of the moment. These are the standards in convention we have.

Now, the offset of the neutral axis from the X axis is marked as E. and, the distance of neutral axis, sorry, X-X axis from the center is marked as R. We will call this as d ϕ we will not have marked this. We calls this d ϕ that is a strip being cut, that is a strip. Let us say, I project this project this and draw a cross section in a cross section whatever may be the shape.

I draw a reference line call this line as O O. So, on this I mark the cg. So, the X-X axis will pass through this cg,. This is my X-X axis. I pick up any point on the line P, Q let us be here and we know this distance is measured from the X-X axis at y. I can also mark the green line as N-N axis which is the neutral plane. Now, from the cg, I mark this X-X here; from the X-X axis I mark the tip as h extrados, I mark this step as h intrados.

And the X axis is marked at a distance R from this plane. So, I mark this dimension as radius intrados and this dimension as radius extrados and, this as radius neutral axis,. This is a reference figure here, is that clear. So, this is figure 1, this figure 2. So, we have neutral plane, we have X-X axis and the neutral plane is offset from the X-X axis towards the center of curvature.

Neutral plane is shifted towards the center of curvature by y from the X-X axis, correct, see here by e, not y by e. Having said this, we will now derive the governing equation for this section. friends, we will discuss that in the next lecture, we will put a summary here for this lecture.

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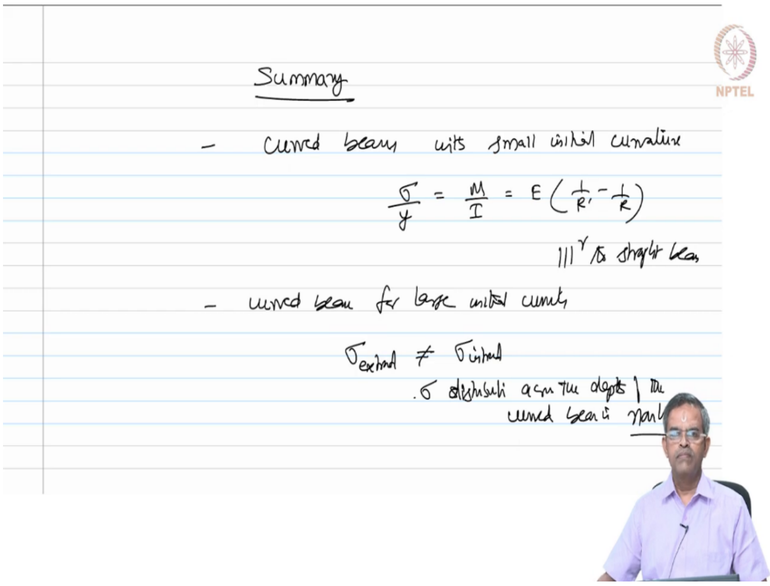

Friends in this lecture we learnt curved beams with small initial curvature. We have derived the equation stress by y M by I, E by 1 by R prime - 1 by R and it is more or less similar to the straight beam, is it not equation. We have also learnt the conditions of curved beam for large initial curvature. We have also learned a fact stresses in extrados are not equal to stress in intrados.

And therefore, the stress distribution across the depth of the section of the curved beam is non-linear. We have learnt this fact. We will continue to discuss this in the next lecture and derive the control equation for finding out these stresses in extrados, intrados using what we call as Winkler Bach equation.

Thank you very much and have a good day, bye.