Advanced Design of Steel Structures
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Lecture - 45
Curved beam with large curvature - 1

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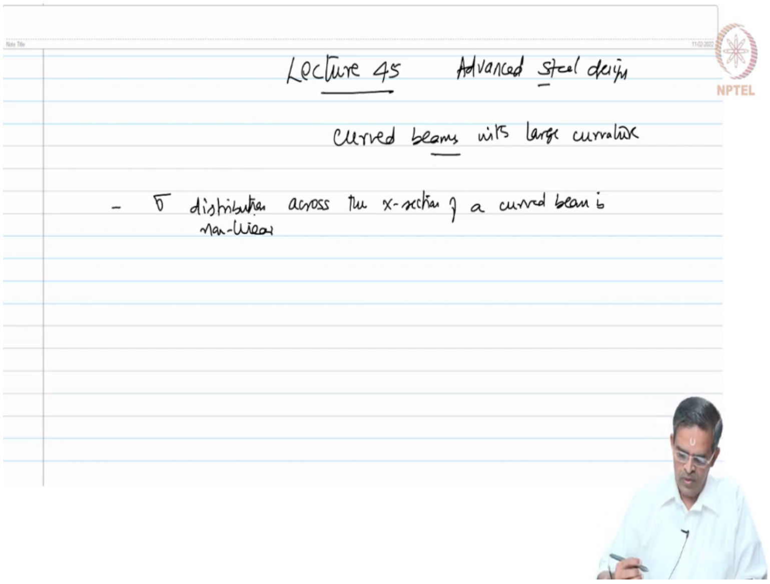

Friends, welcome to this lecture 45 on Advanced Steel Design. We are now continuing to discuss the derivation for finding out stresses on Curved beams under large initial curvature. So, we are looking for curved beams with large curvature. So, we discussed the derivation for finding out stresses at the intrados and extrados of the curved beam for small initial curvature.

And we also said very clearly that the stress distribution along the depth of the section in curved beams is non-linear. The stress distribution across the cross section of a curved beam is non-linear therefore, the classical bending equation cannot be used to find out the stresses. So, we need to really derive the control expression for finding out the stresses in the extrados and intrados that is the maximum tension and maximum compression on the concave and convex side respectively.

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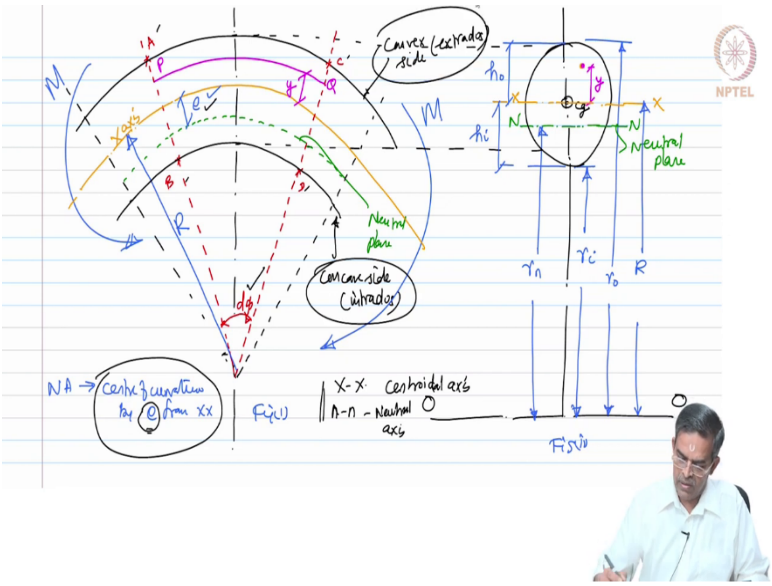

So, this was the discussion what we had in the last lecture I urge that you should please redraw this figure again I will hold the screen for a second. So, we have got the condition for a large initial curvature, please remember that the moment is applied at the ends in such a manner that it tends to decrease the cross section. So, the convex side and concave side are marked on the screen which is also called as extrados and intrados and we have marked the neutral plane with an offset from the x axis by the amount e which is indicating a shift of neutral plane towards the centre of curvature.

There is a very important statement friends. This is moving towards the centre of curvature by an amount e. We pick up a fibre at a distance y measured from the x axis which is away from the centre of curvature and we mark that fibre as pq. So, the figure 2 shows the cross section of the member any arbitrary shape where the extrados and intrados are marked as Ri and RO on the screen R stands for the radius and O stands for extrados or outer end I stands for intrados or the inner end.

Whereas, the xx plane is measured at a distance R from the radius from the centre of rotation O. Please understand x axis is different from the neutral axis it cannot be same because x-x the centroidal axis x-x is the centroidal axis and n-n is a neutral axis they cannot be same for a simple reason the stress distribution along the depth is non-linear its not going to be same in that case there is no coincidence of this and this shift is indicated as e in the figure.

Now, there is a specific way by which we assume m; m is closing the curvature right please understand these are all standard conventions which has a positive convention in the analysis. We pick up any point y on the fibre P Q which is offset measured from the centroidal axis it is nothing to do with the neutral axis. I am always measuring the point respect to the geometry neutral axis are layer or a plane respect to the stress distribution there are two different things here we have to be very clear about that.

From the geometry the section depth is marked from the central axis as h i and h o that is height or depth of the intrados from the centroidal axis and extrados from the central axis it can be equal may not be equal depending upon the location of cg because the sections can have also an asymmetric cross sections.

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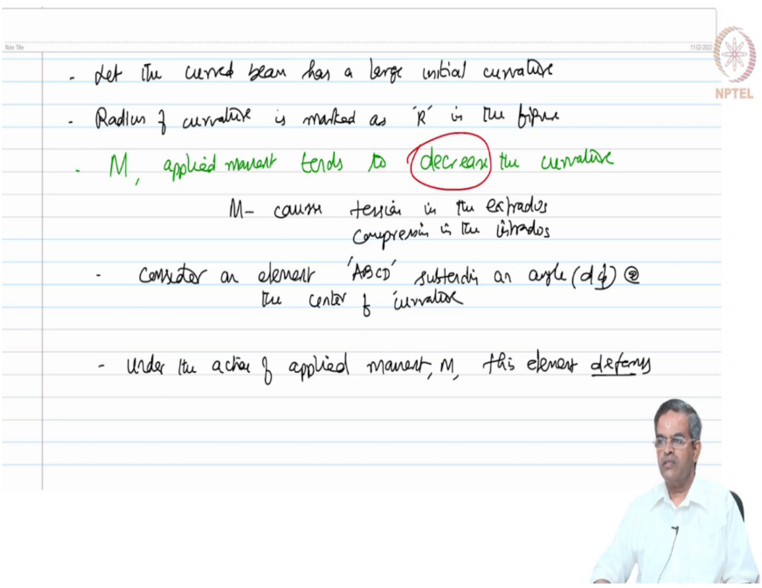

Having said this we will say let the curved beam has a large initial curvature. The radius of curvature is indicated as R in the figure where M the applied moment tends to decrease the curvature.

So, friends, in curved beams there is no hogging and sagging bending moments you have to classify the applied moment in terms of does it increase or decrease the initial curvature of the curved beam.

Therefore, friends, m causes tension in the extrados is it not and of course, compression in the intrados. Now let us consider an element ABCD we will consider an element ABCD we are considering an element A B C D. So, let us consider an element A B C D subtending an angle d ϕ at the centre of curvature. We see this figure the member or the segment ABCD is making an angle d ϕ.

Now, under the action of M this element deforms is it not this element will deform now because the radius is going to change; obviously, this element will deform.

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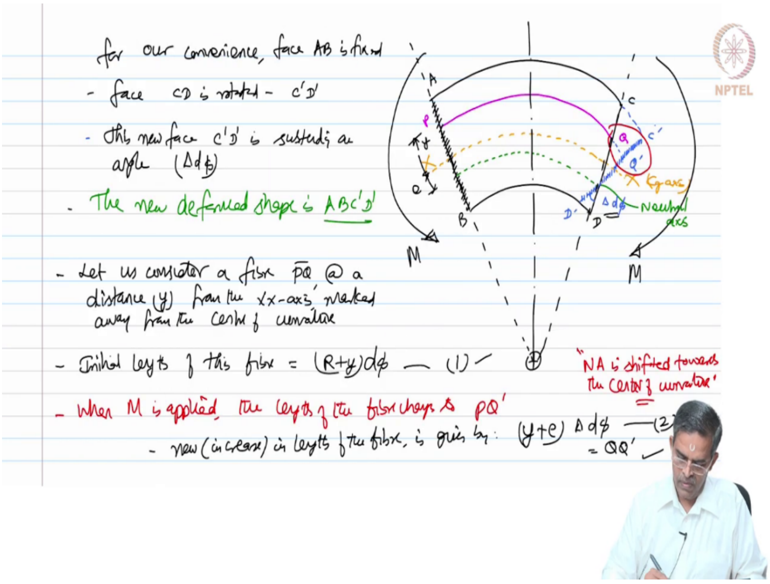

Let us now draw the deformed shape of this now let me draw the element we call this as A this is C that is what we have is not yeah and let us call this as B and D this is my element let me also mark P Q. So, P Q was in crayon color let us mark P Q this is P Q.

Now, let us mark the centroidal axis which is in orange color. I call this as X axis this is the centroidal axis and we know that the element PQ is at a distance y from the centroidal axis is it not let us mark that this is y away from the centre of curvature now let us also mark the neutral axis which is in green color. So, this is my neutral axis and we know this is at a distance e shifted towards the centre of curvature.

Now, under the applied moment M which is tending to close the curvature or decrease the curvature the element is going to deform. Let us draw the deformed condition of this element. So, what we do is, A B C D is deforming we will not disturb one face we will just disturb only the other face for our simplicity let us mark that. So, at the neutral axis there is no change now let us say it is going to deform. We call this point as C dash at this point D dash. So, Q will also get extended to Q dash sorry.

So, we call this angle as $\Delta d\ \phi$. So, now D is shifted to D dash and let us note neutral axis is shifted towards the centre of curvature. There is an important observation which we need to understand the shifting towards the centre of curvature.

Now, with reference to this figure for our convenience face AB is fixed that is this face is fixed face CD is rotated and to C dash D dash. As you can see in this figure you can see here this is what you have rotated here we have rotated. Now this new face C dash D dash is subtending an angle del d ϕ sorry $\Delta d\ \phi$ as marked I mark this angle.

Now, I can say the new deformed shape is A B C dash D dash. I think this is there is no confusion in it. Now, let us consider the fibre PQ or the distance y from the xx axis at a distance y from the x x axis marked away from the centre of curvature as shown in the figure. The initial length of this fibre will be R + y of d ϕ call the equation number 1.

Now, when M is applied the length of the fibre changes to PQ dash you can see here this has become PQ dash this has become PQ dash Q has moved to Q dash. So, the new length of the fibre or increase in length of the fibre which is the consequence of the applied moment M is given by y + e d ϕ equation 2, which I call as QQ dash from the geometry can I say QQ dash is y + e of $\Delta d\ \phi$.

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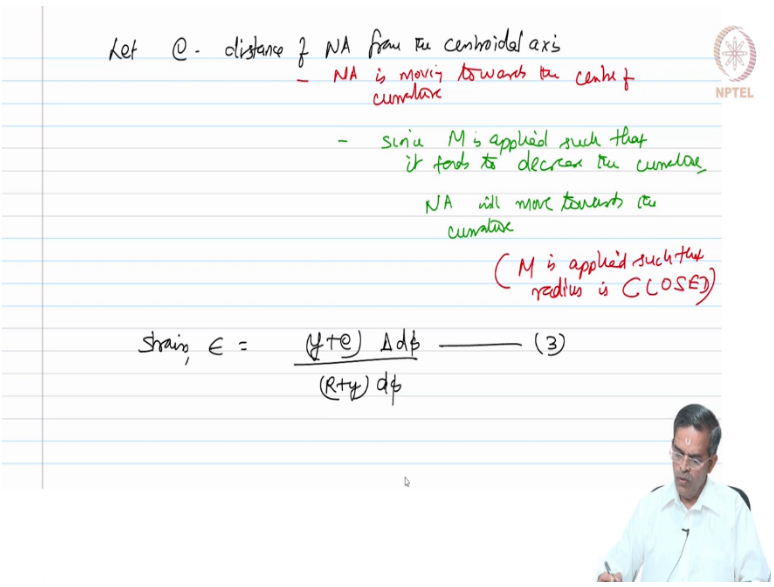

Having said this let e be the distance of neutral axis from the centroidal axis. Please note centroidal axis sorry neutral axis is moving towards the centre of curvature. You may ask me a question how this has happened why it cannot move away from the curvature very interesting. Since M is applied such that it tends to decrease the curvature neutral axis will move towards the curvature.

So, we can also put a note M is applied such that radius is closed we are decrease the curvature. Now, let us write down the equation first here strain epsilon is y + e Δd ϕ change in length by original length. See here original length is R + by d ϕ change in length is this. So, I have written a strain equation call this equation as 3.

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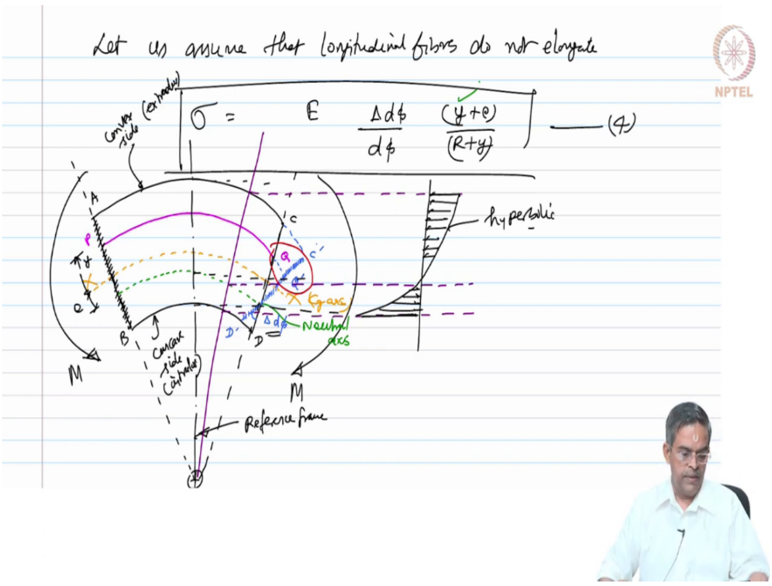

Let us assume that longitudinal fibers do not elongate we are only talking about the cross section. So, I can write stress as E into del d ϕ by d ϕ of y + e by R + y can I write this equation 4.

Let us see the nature of the stress, I want to copy this figure. Let me project this extrados point an intrados point and the neutral axis point at any specific reference frame. The reference frame may not be at the centre there is a reference frame, it need not be at the centre it can be anywhere I am just putting the point. So, for your convenience if you really want we can also do it the other way let us stop this, let us put the reference frame somewhere here and let us project this for our convenience.

So, this is my neutral axis. So, let me draw the figure. Now, we know this is going to be tensile is going to be compressive and the stress is going to be non-linear. So, this is tensile, this is compressive and this variation will be hyperbolic its non-linear and let us for our understanding we already know that this is the convex side which is extrados and this is the concave side which is intrados M is applied a specific style. This is the stress variation and the stress intensity at any distance y at any distance y is given by this equation and is non-linear.

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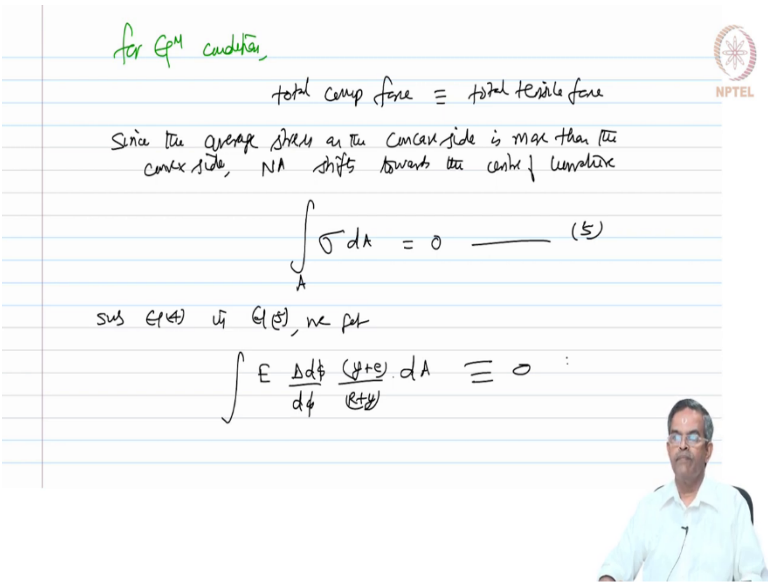

Now, let us say for equilibrium condition total compressive force should be identically equal to the total tensile force. Now, since the average stress on the concave side on the concave side is more than the convex side neutral axis shifts towards the centre of curvature. So, stress dA for the area A should be 0 equation 5 let us substitute equation 4 this is the equation 4 in 5. So, substituting equation 4 in equation 5 we get what is equation 4 ?

See integral E d ϕ by d ϕ y + e + R + y dA should be set to 0.

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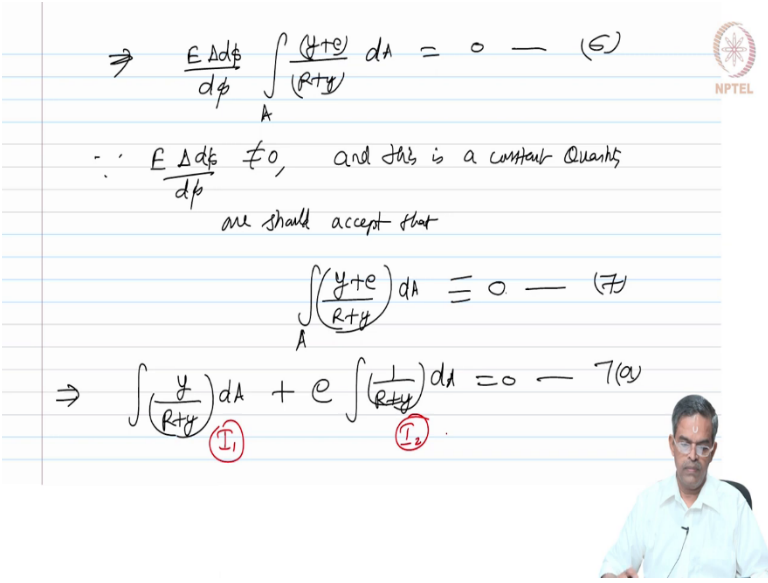

Which means E d ϕ by d ϕ integral y + e by R + y of dA should be set to 0 we call this equation number 6. Now friends since E d ϕ by d ϕ cannot be set to 0 and this is a constant quantity I must. So, one should accept the integral y + e by R + y dA should be only set to 0 equation 7.

So, let us impose this condition. Now, I split this integral into two parts this integral can be split into two parts as y by R + y dA + e times of 1 by R + y dA is set to 0 equation 7 a. So, I call this as integral I 1 an integral I 2.

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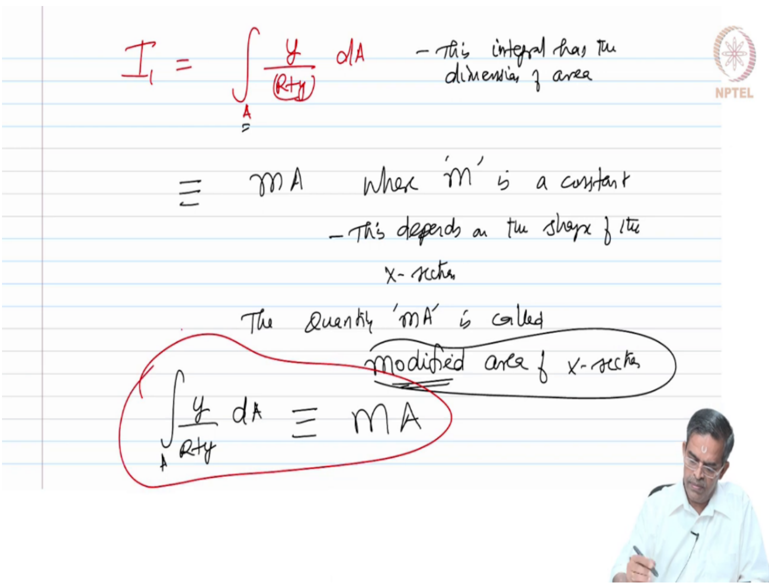

Let us pick up I 1, I 1 is integral y by R + y of dA this integral has the dimension of area because this is over the area A. So, I could say now this is identically equal to mA where m is a constant this depends on the shape of the cross section.

The quantity mA is called modified area of cross section. Why modify? The original area got modified because of application of m in a specific style that is a m is closing the curvature friends. So, we have got a very interesting integral that y by R + y dA over the area A is identified as some factor of A which is a cross section property depends purely on the cross section. So, m should be arrived now for various possible cross sections that can be used in curved beams its a geometric property.

We will do that we will do some illustrative examples to work out m for different cross sections.

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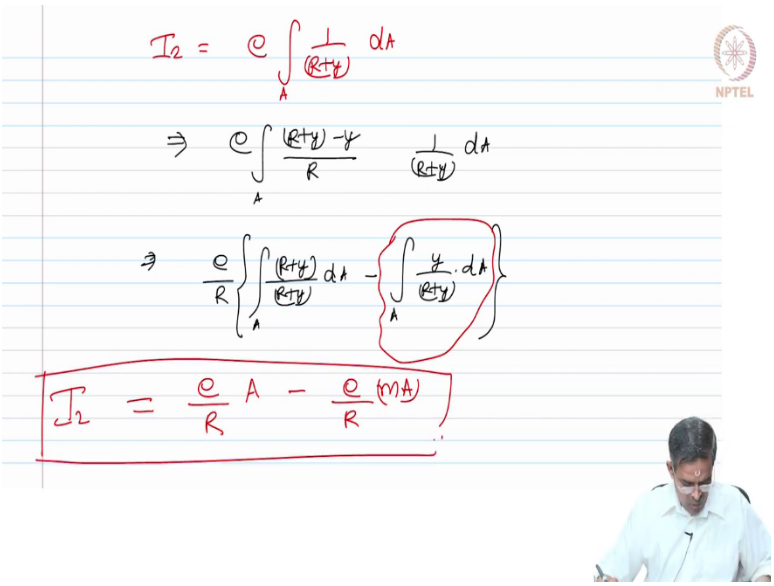

Having said this now let us look into the second integral. Second integral I 2 was given by e times of 1 by R + y dA for the area A which is now equal to e times of R + y - y by R times of 1 by R + y dA can I write like this of the area A which can be said as e by R, R + y by R + y dA integral A - y by R + y dA over A. Friends, if you recollect turn back your notes this particular term is known what is this term? You know this is actually equal to mA.

So, can I now write this as e by R A - e by R mA can I write like this. This is my I 2 integral 2 am I right? So, I have both the integrals now integral 1 I have integral 1 I have this integral

1 which is simply mA and integral 2 also I have. Now can you substitute and write equation 7 again what is 7 equations? Can I write equation 7 again? Let us rewrite I will copy these equations better selection tool.

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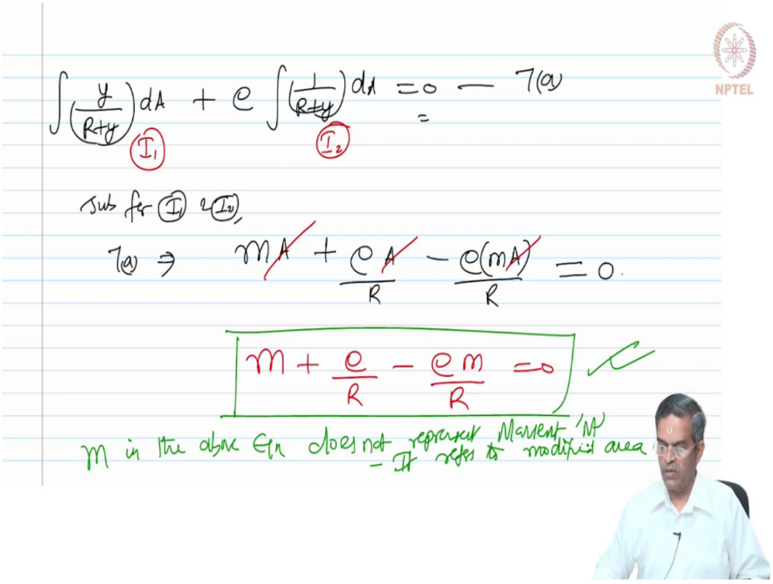

Now, substituting for integral I 1 and integral I 2 7 a now becomes mA + e times of I 2 is actually this value A by R - e mA by R am I right? e mA by R which is now set to 0 that is what we want is it not set to 0. So, if you do that I can cancel this I know I can write m + e by R - e m by R is 0. Friends, please note m in the above equation does not represent the moment. Please understand the common confusion does not represent the moment it represents or refers to modified area important.

So, we have derived this equation closed form I hope there is no confusion in this let us go ahead.

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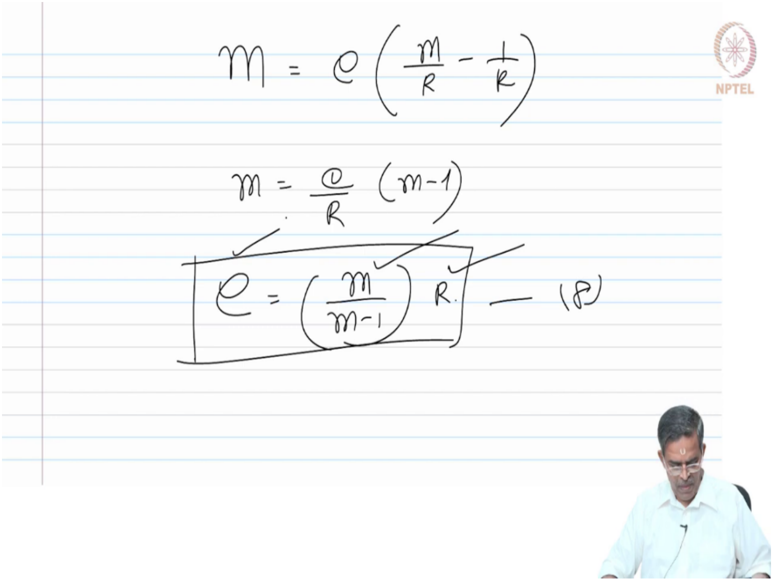

Having said this let us say now m is equal to see here m is equal to let us rewrite this equation or I can say it was m is equal to e times of m by R - 1 by R can I say that? Simplifying so, m is equal e by R of m - 1 or e which is the shift of neutral axis from the centroidal axis given by m by m - 1 of R equation 8 that is a very interesting relationship we have.

If I know the modified sectional area if I know the radius of curvature of the beam I can always find what is the shift of neutral axis. Friends is very interesting now the shift of neutral x also becomes part of the geometric property because m is a modified area m is a modified area or modification in the area because of m capital M applied on the curved beam and e is also now become a geometric property.

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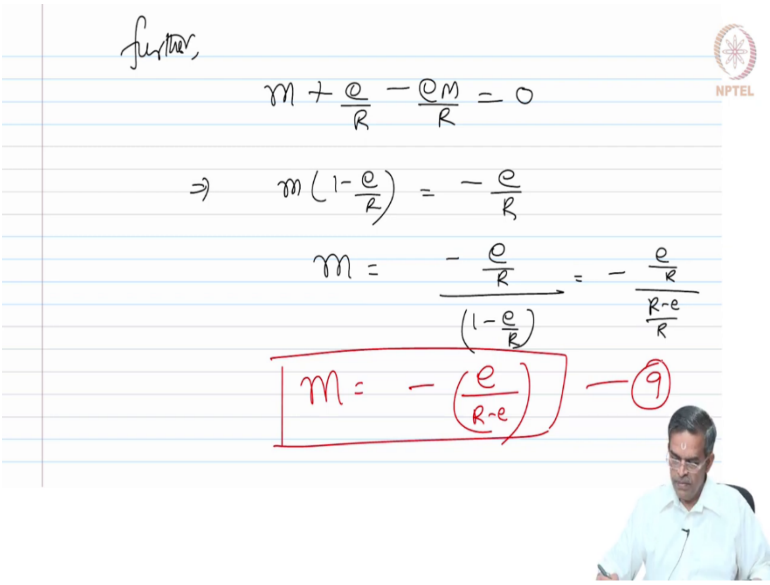

Having said this further, m + e by R - e m by R is 0 we have this equation here just see same equation I am writing m + e by R same equation rewritten. So, now, we can also say m 1 - e by R is equal to - e by R. So, m can also be said as - e by R by 1 - e by R can I say this ? Which can be said as - of e by R by R - e by R. So, therefore, I will get m as - of e by R - e equation 9 is another form to find m or to find e.

This is another form of equation. Having said this let us move forward to work out the stresses. So, I will draw the figure again let us say we have this cross section with us I will draw I will copy this figure.

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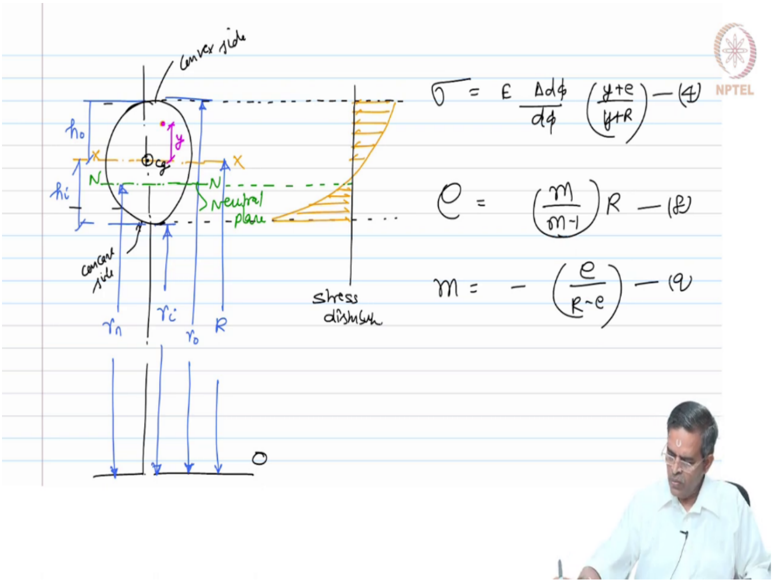

For this figure let me plot the stress distribution. It is my reference plane, and the neutral axis. So, I should say this is my stress distribution and let us also mark this is convex side, this is concave side.

That is what we also marked earlier. The bottom is concave side upper is convex side or extrados and intrados we have marked the same thing here again. Let us now write the equation which we just now derived stress is given by E times of Δd ϕ by d ϕ of y + e by y + R is what we already wrote let me call this number of equation as 4 let me mark it here as 4 I am marking here as 4 because the same equation here written.

We also have e as m by m - 1 of R. I think this equation was 8 let me mark this equation as 8 m is also now given by - of e by R - e which is equation 9. The stress distribution looks like this and this subjected to a moment such that the curvature is decreased in that we have to remember very clearly.

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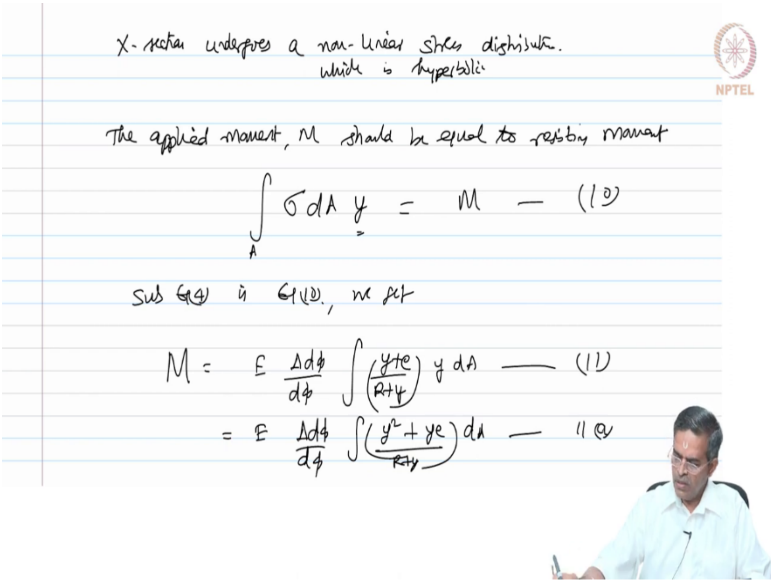

So, now the cross section undergoes a non-linear stress distribution which is hyperbolic. Now, the applied moment M should be equal to the resisting moment. So, we say over area A into y should be equal to M we call equation number 10. So, now, let us substitute equation 4 in equation 10 we get equation 4 is here just wrote it for you.

So, now, M becomes E del d ϕ by d ϕ integral y + e R + y dA sorry y dA am I right? Just check this equation 4 and I have one y here y d we call equation number 11 which can be now written as E times of del d ϕ by d ϕ integral y square + y e by R + y of dA we will calls equation 11 a. So, we call this as integral I.

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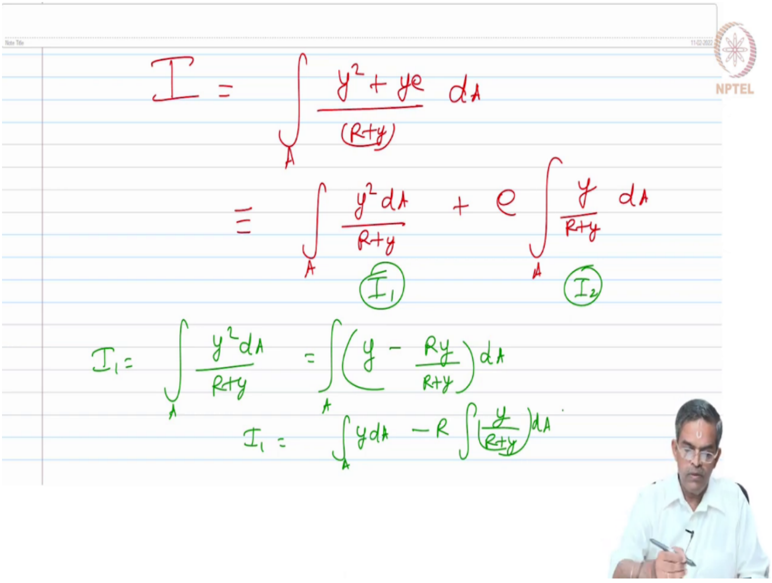

Now the integral I is integral y square + y e by R + y dA over the area which is identically split into two parts integral y square dA by R + y over a + e times of y by R + y dA over A. We call this integral as I 1 thus integral as I 2 where I 1 is integral y square dA by R + y for the area A which is y - R y by R + y of dA over A is check.

This is equal is it not? So, I 1 now becomes integral y dA for A - R times of y by R + y dA I 1 is further split.

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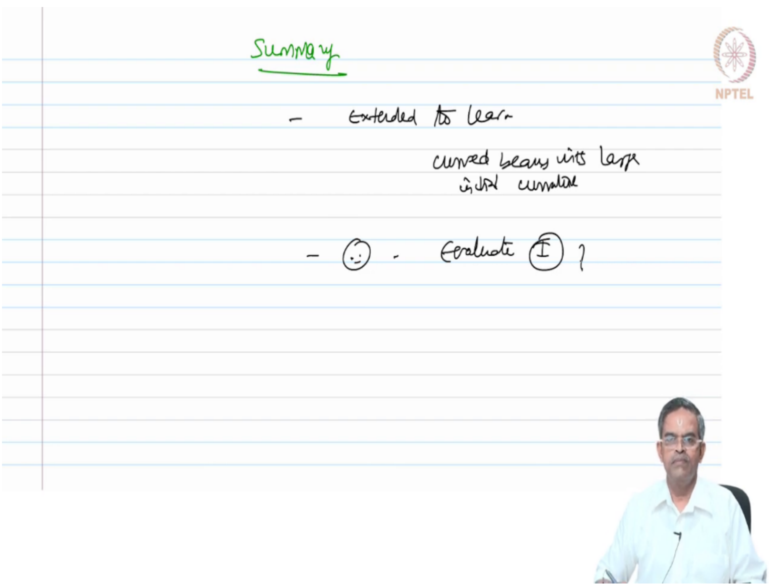

So, friends, we will continue this discussion in the next lecture, I want you to evaluate this integral. So, I leave a small homework. So, in this lecture we extended to learn curved beams with large initial curvature. We are in the process of finding out the equation to obtain the stress and its distribution across the section of a curved beam under large initial curvature. There is a small homework given to you evaluate the integral I. We will discuss in the next lecture.

Thank you very much and have a good day bye.