Advanced Design of Steel Structures
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Lecture - 46
Curved beam with large curvature - 2

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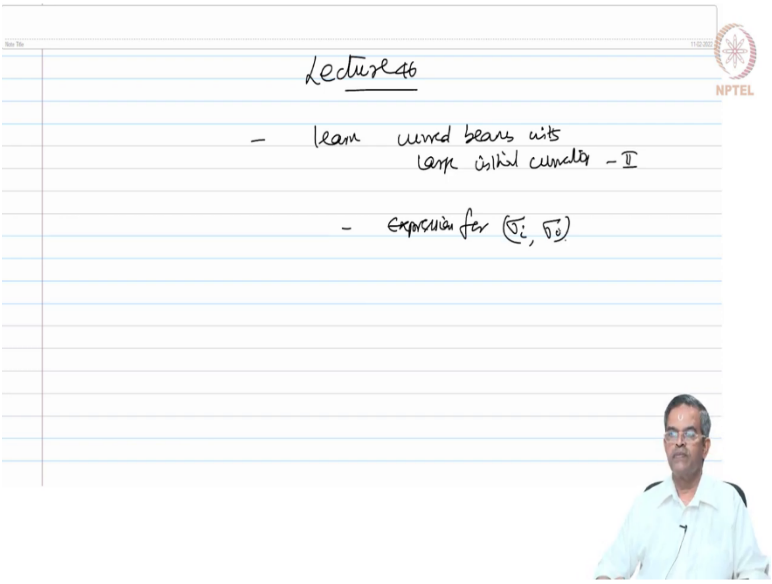

So, friends, welcome to the lecture-46 of the course Advanced Steel Design. We are now going to continue to learn Curved beams with large initial curvature. I will put this II. So, in this we are going to find out the expression for stress at intrados and stress at extrados.

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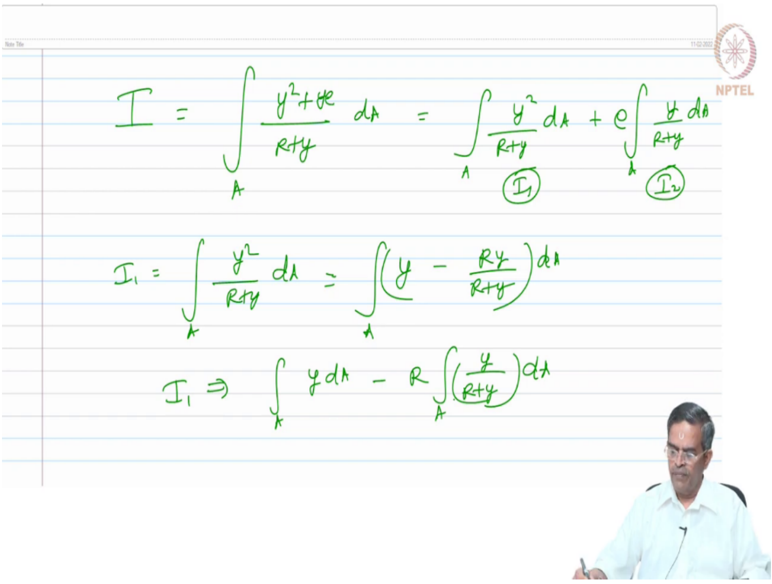

So, in the last lecture we said that the integral I is

$$I = \int \frac{y^2+ye}{R+y} dA = \int \frac{y^2}{R+y} dA + e \int \frac{y}{R+y} dA$$

$$I = \int \frac{y^2}{R+y} dA = \int \left(y - \frac{Ry}{R+y}\right) dA$$

Now this can now become I_1 can become split as 2 integrals

$$I_1 = \int y dA - R \int \left(\frac{y}{R+y}\right) dA$$

This is where we left and I hope you would have done the integration by this time and substituted back in the original equation you must have got the I value. Let us continue from this point.

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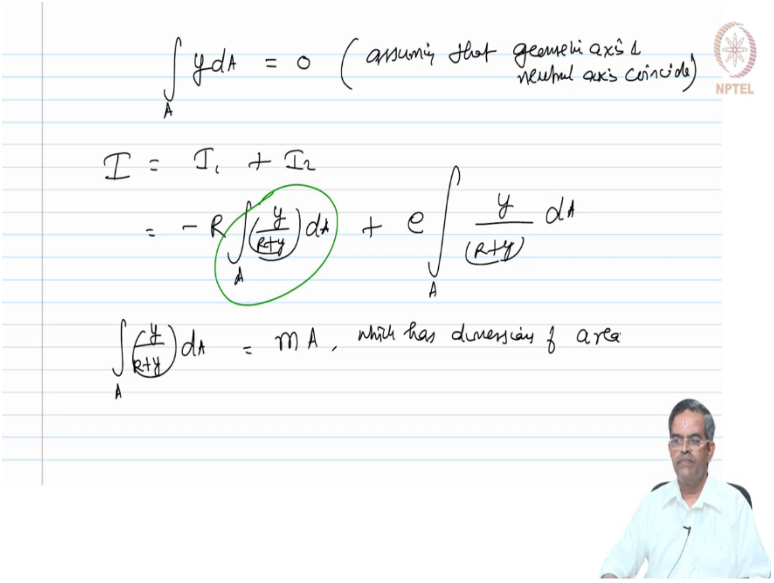

So, now integral y dA for the entire area A should be 0. Why? We are assuming that the geometric axis and neutral axis coincide. So, I will be $I_1 + I_2$ which is. So, this goes away.

$$\int y dA = 0$$

$$I = I_1 + I_2$$

$$I =- R\int\left(\frac{y}{R+y}\right)dA + e\int\frac{y}{R+y}dA$$

Furthermore, friends we already have this relationship with us. You recall it we have this

$$\int\frac{y}{R+y}dA = mA$$

is called as mA which has dimensions of area.

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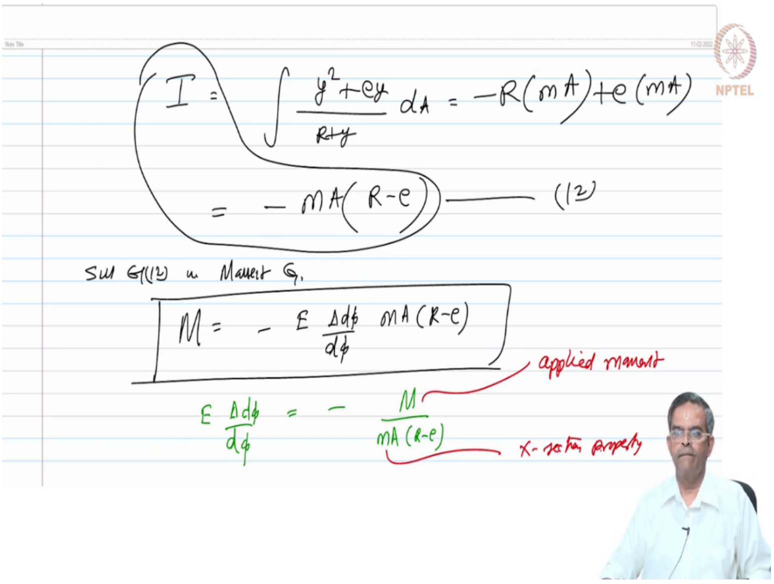

Having said this the integral now I becomes

$$I = \int \frac{y^2+ey}{R+y} dA = - R(mA) + e(mA)$$

This is the original integral here. So, now, I can say this integral is now going to be $- mA(R - e)$equation 12.

$$I = - mA(R - e)$$

Now, substitute equation 12 this equation 11 we have substitute equation 12 in the moment equation. What is the moment equation we had? Please turn your notes. M was equal to $- E\frac{\Delta d\phi}{d\phi} mA(R - e)$. So, now, I have substituted this integral back and got a new equation 1 here.

$$M = - E\frac{\Delta d\phi}{d\phi} mA(R - e)$$

$$E\frac{\Delta d\phi}{d\phi} = - \frac{M}{mA(R-e)}$$

So, please understand this equation; this is the applied moment and this is cross sectional property. So, do not get confused with the notation M.

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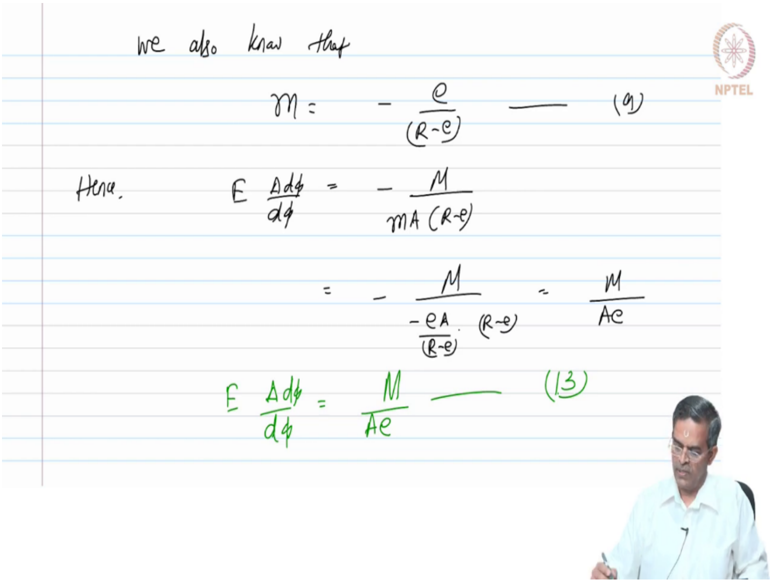

We also know that m is given by $-\frac{e}{(R-e)}$ I think this is equation number 9, please check that. So, I have somewhere mA here let me substitute that.

$$m = -\frac{e}{(R-e)}$$

$$E\frac{\Delta d\phi}{d\phi} = -\frac{M}{mA(R-e)}$$

$$E\frac{\Delta d\phi}{d\phi} = -\frac{M}{-\left(\frac{eA}{(R-e)}\cdot(R-e)\right)} = \frac{M}{Ae}$$

$$E\frac{\Delta d\phi}{d\phi} = \frac{M}{Ae}$$

The small m have gone is substituted back is equation 13.

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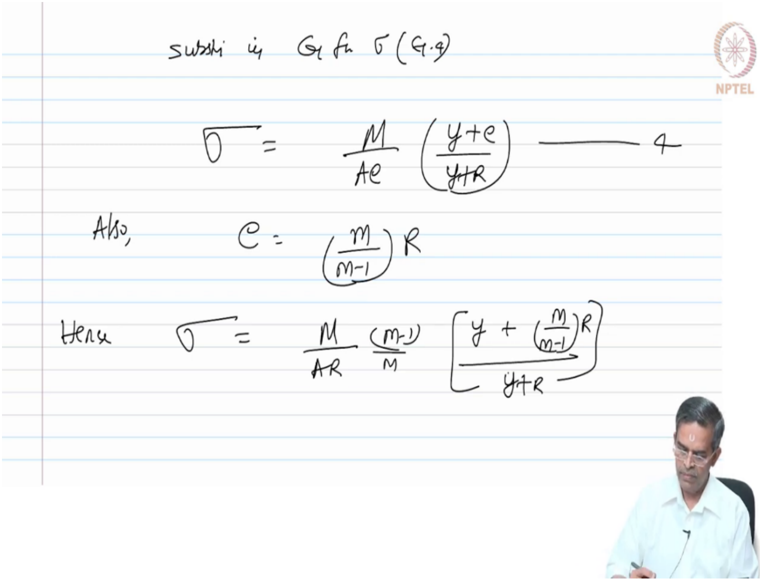

So, substitute this in equation for the stress. Substitute in equation for final stress. It is the possibly equation 4 you can check that. So, the stress is now given by

$$\sigma = \frac{M}{Ae}\left(\frac{y+e}{y+R}\right)$$

$$e = \left(\frac{m}{m-1}\right)R$$

$$\sigma = \frac{M}{AR}\frac{(m-1)}{m}\left[\frac{y+\left(\frac{m}{m-1}\right)R}{y+R}\right]$$

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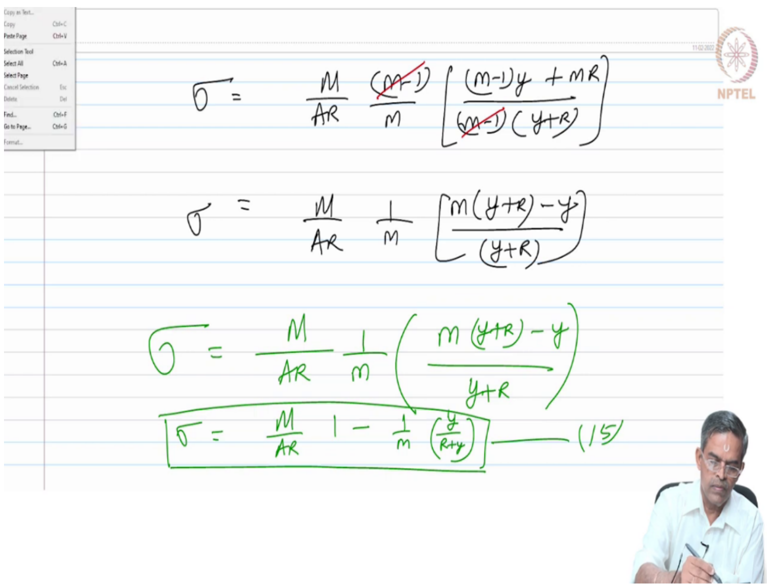

$$\sigma = \frac{M}{AR}\frac{(m-1)}{m}\left[\frac{(m-1)y+mR}{(m-1)(y+R)}\right]$$

$$\sigma = \frac{M}{AR}\frac{1}{m}\left[\frac{m(y+R)-y}{(y+R)}\right]$$

$$\sigma = \frac{M}{AR}\ 1 - \frac{1}{m}\left(\frac{y}{R+y}\right)$$

The classical equation we have for curved beams which is equation 15.

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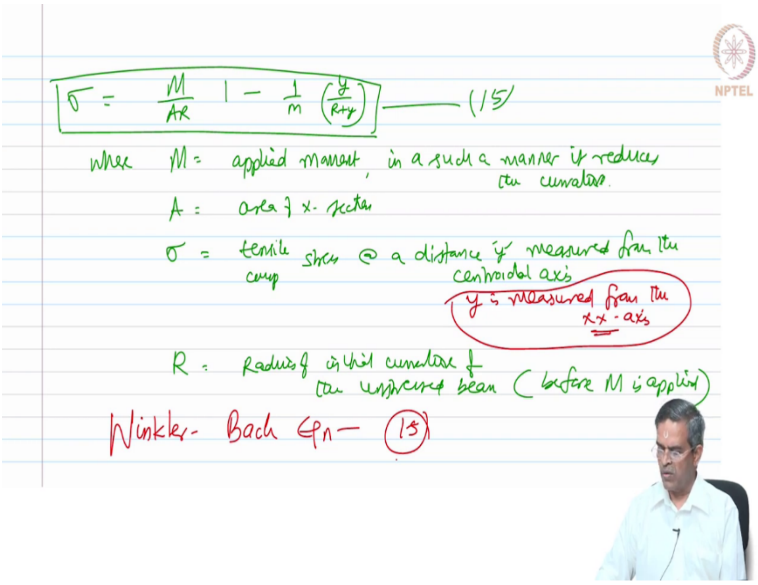

$$\sigma = \frac{M}{AR}\ 1 - \frac{1}{m}\left(\frac{y}{R+y}\right)$$

Let us copy this equation to the next screen. Where we know where m is the applied moment in such a manner it reduces the curvature. A is an area of cross section of the membrane. Sigma is the tensile or compressive stress at a distance y measure from the centroidal axis. Now remember y is measure from the centroidal axis not from the neutral axis. That is a common confusion people will have.

Generally in classical equation of bending for finding out stresses the distance of extreme fiber is measured from the neutral axis, but here it is from central axis. Please understand this. So, very important deviation we have. Then of course, R is a radius of initial curvature we should say of the unstressed beam before the M is applied the moment is applied; original initial curvature.

Friends this equation 15 is classically known as Winkler back equation; equation 15 is classically known as Winkler back equation. Winkler back formula Winkler back equation. So, I think we have derived this stress to be found out at any point distance y from the centroidal axis. If I know the geometric properties of the curved beam and the value of the applied moment acting on the curve.

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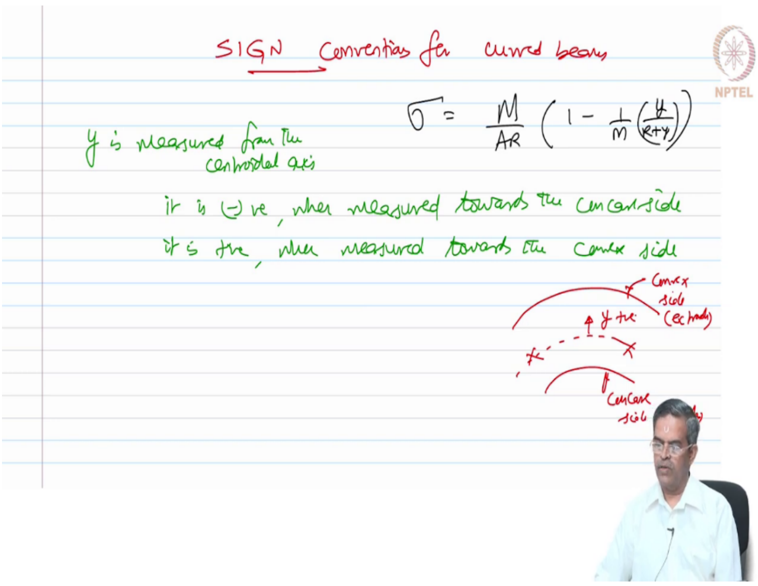

$$\sigma = \frac{M}{AR}\left(1 - \frac{1}{m}\left(\frac{y}{R+y}\right)\right)$$

Let us revise some sign conventions for curved beams. Let us say the equation is very generic stress is given by $\frac{M}{AR}\left(1 - \frac{1}{m}\left(\frac{y}{R+y}\right)\right)$. Now y is measured from the centroidal axis it is negative when measured towards the concave side. It is positive when measured towards the convex side. Now one will have a confusion what is the convex and concave side.

So, in a given curved beam this is convex side called extrados. This is concave side called intrados from the centroidal axis xx when you move towards this y is positive. When you move towards the convex side.

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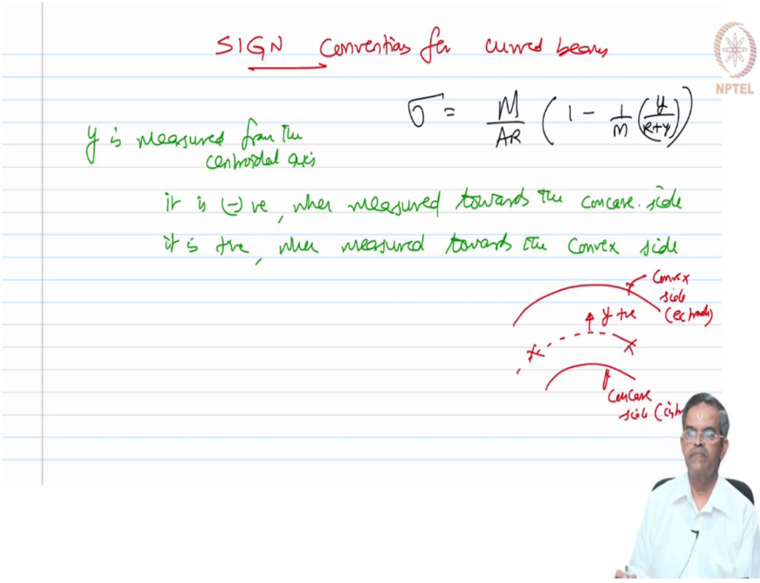

Let me that is clear, towards the convex side towards the concave side. So, let us apply this condition and try to find out the equations for introduction next class.

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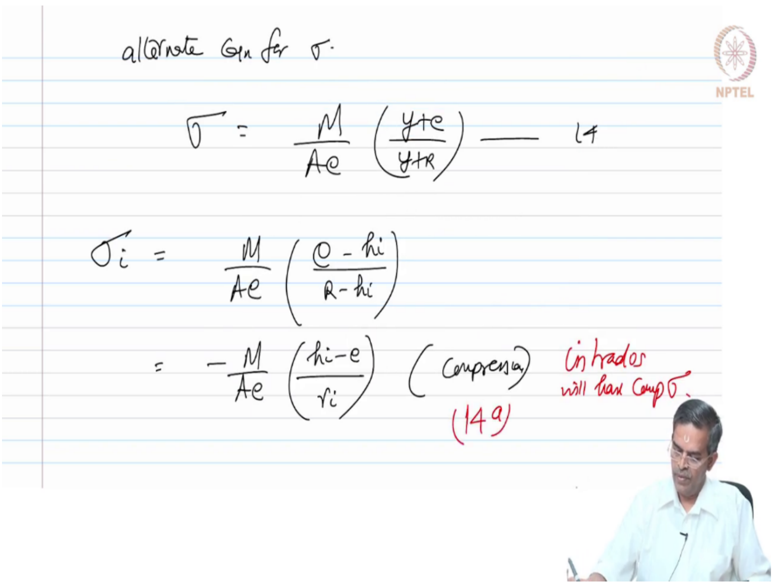

We also have an alternate equation for stress. Stress is also given by

$$\sigma = \frac{M}{Ae}\left(\frac{y+e}{y+R}\right)$$

If you remember we derived this equation this was equation number 14 we derived this. So, using this stress is intrados will be

$$\sigma_i = \frac{M}{Ae}\left(\frac{e-h_i}{R-h_i}\right)$$

Look at the figure hi is the depth of the intrados point measured from the central axis. I can rewrite this equation as which is equal to

$$\sigma_i =- \frac{M}{Ae}\left(\frac{h_i-e}{r_i}\right)$$

I can I write this as Ri negative indicates this is compression. So, intrados will have compression compressive stress.

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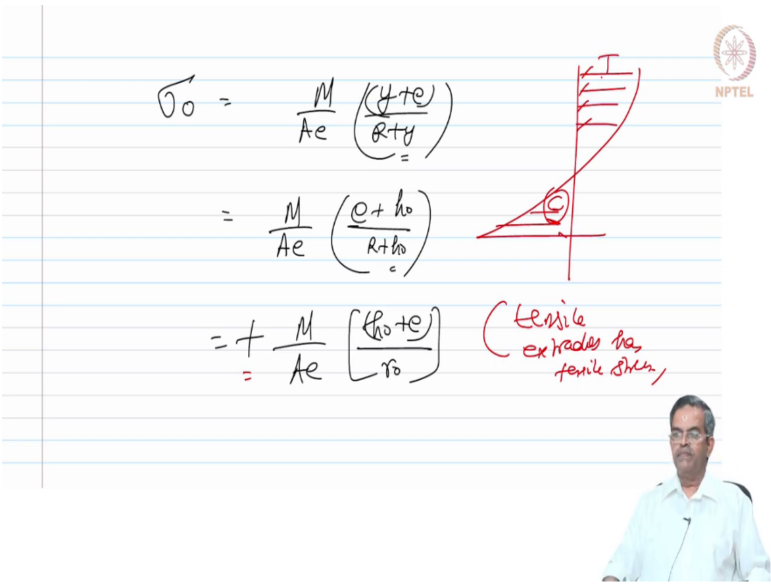

$$\sigma_o = \frac{M}{Ae}\left(\frac{y+e}{R+y}\right)$$

I am looking for the extrados point. So, I am substituting this as h_0 that is what I am doing.

$$\sigma_o = \frac{M}{Ae}\left(\frac{e+h_0}{R+h_0}\right)$$

$$\sigma_o =+ \frac{M}{Ae}\left(\frac{h_{o+e}}{r_o}\right)$$

So, this is tensile because it is positive and extrados has tensile stress that is what we also plotted is not it. The plot was like this. So, this is all compression and these are all tension. it was 14 b. So, this was 14 a and this is 14 b.

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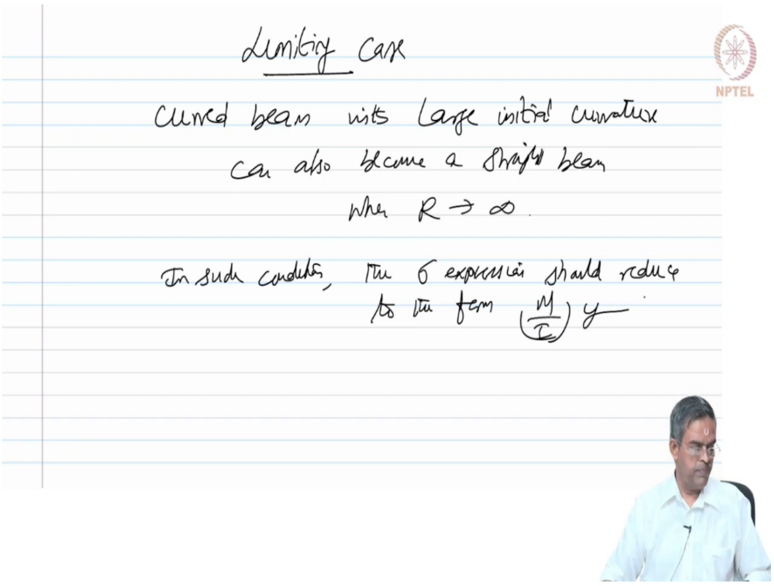

Let us look at a limiting case. Curved beams with large initial curvature can also become a straight beam when R tends to infinity, in such condition the stress expression should reduce to the form $\left(\frac{M}{I}\right)y$, there is a classical thing for a straight beam.

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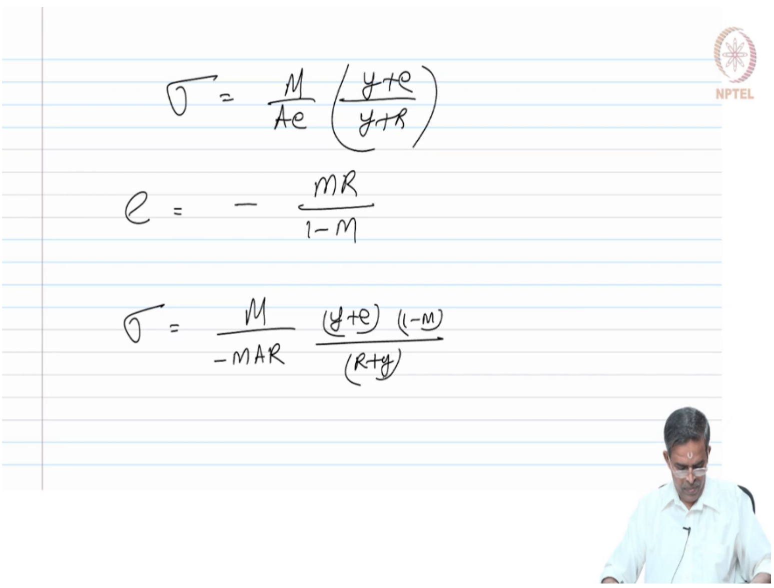

Let us show how this is happening. So, we know stress is

$$\sigma = \frac{M}{Ae}\left(\frac{y+e}{y+R}\right)$$

$$e = -\frac{mR}{1-m}$$

$$\sigma = \frac{M}{-mAR}\frac{(y+e)(1-m)}{(R+y)}$$

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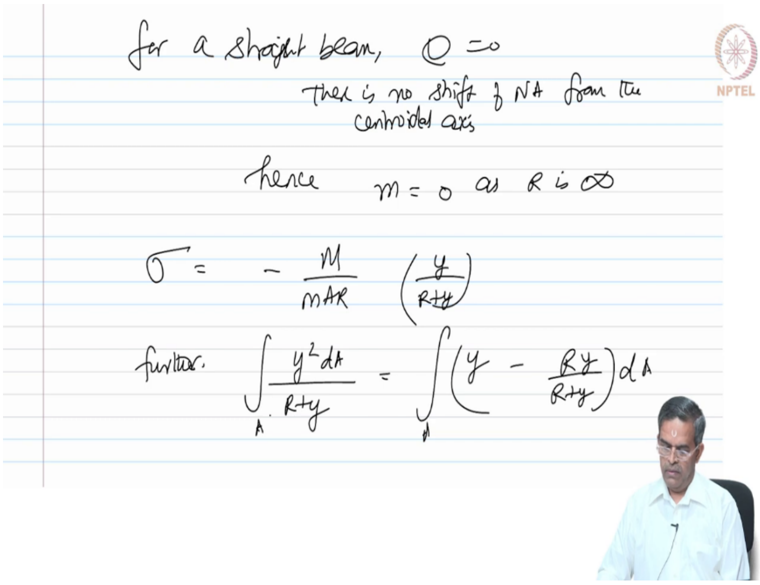

Now, for a straight beam e is 0. That is there is no shift of neutral axis from the centroidal axis shift is 0. Hence small m will become 0 as R is infinity therefore, stress now becomes $-\frac{M}{mAR}\left(\frac{y}{R+y}\right)$.

$$\sigma = -\frac{M}{mAR}\left(\frac{y}{R+y}\right)$$

$$\int \frac{y^2 dA}{R+y} = \int\left(y - \frac{Ry}{R+y}\right)dA$$

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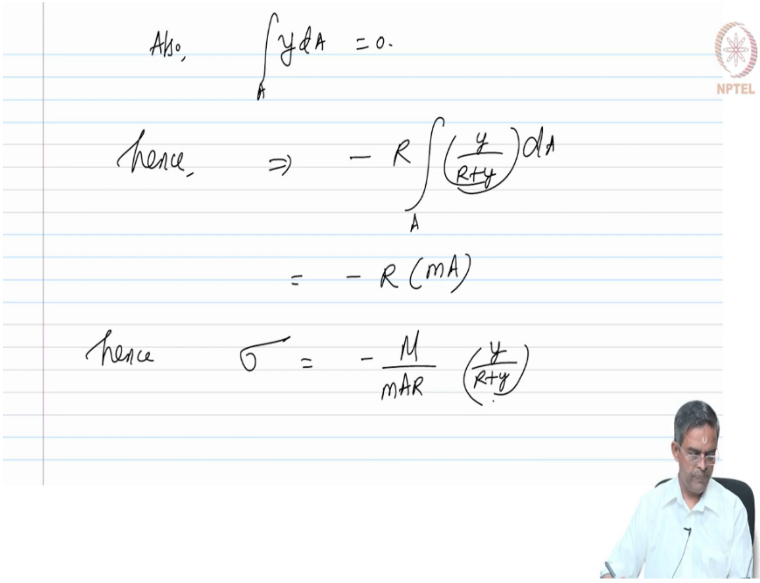

We also know integral y dA for the area a is 0. Hence this will now become

$$\int y dA = 0$$

$$= - R\int\left(\frac{y}{R+y}\right)dA$$

$$= - R(mA)$$

Hence the stress equation IS

$$\sigma = - \frac{M}{mAR}\left(\frac{y}{R+y}\right)$$

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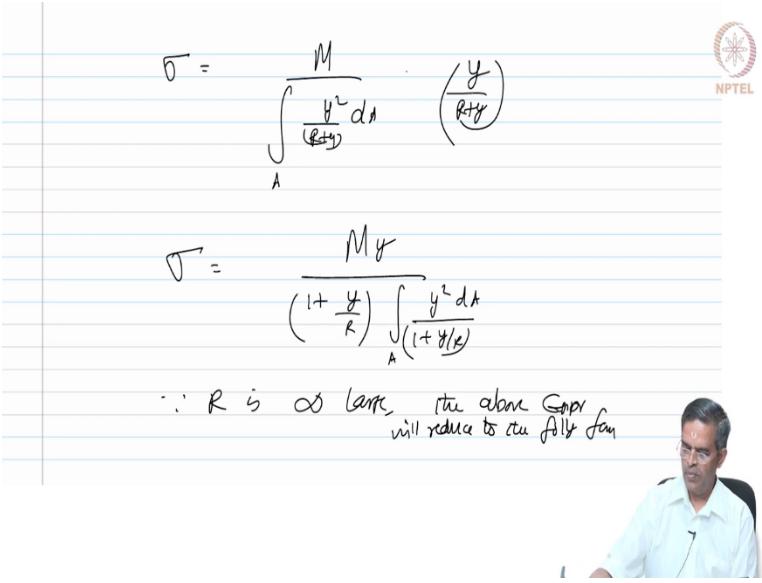

Which means stress is

$$\sigma = \frac{M}{\int \frac{y^2}{(R+y)} dA}\left(\frac{y}{R+y}\right)$$

$$\sigma = \frac{My}{\left(1+\frac{y}{R}\right)\int \frac{y^2}{\left(1+\frac{y}{R}\right)} dA}$$

Now, since R is infinitely large the above expression will now reduce to the following form.

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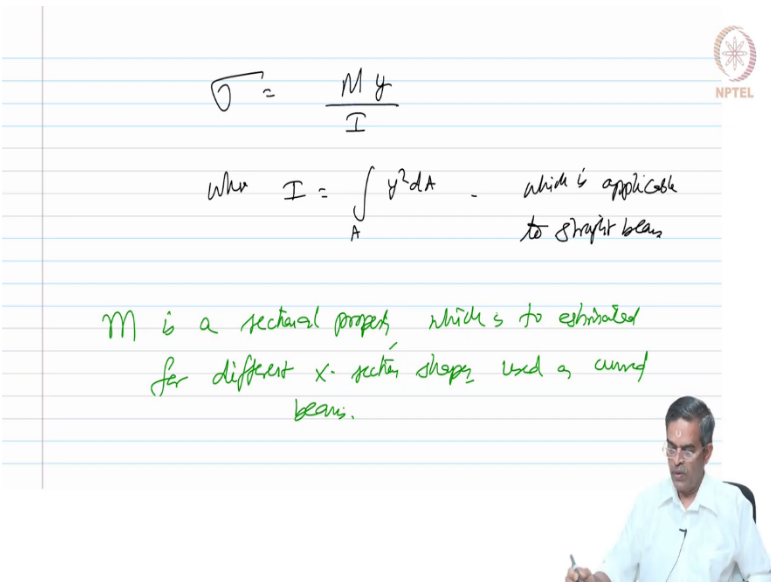

$$\sigma = \frac{My}{I}$$

$$I = \int y^2 dA$$

σ is $\frac{My}{I}$ where I is y^2 dA second moment of area which is applicable to straight beams. Now friends there is an important statement here; m is a sectional property which is to be developed or which is to be estimated for different cross section shapes used as curved beams. That is the job we have additional. Let us do that now for 1 or 2 and see how they can be estimated.

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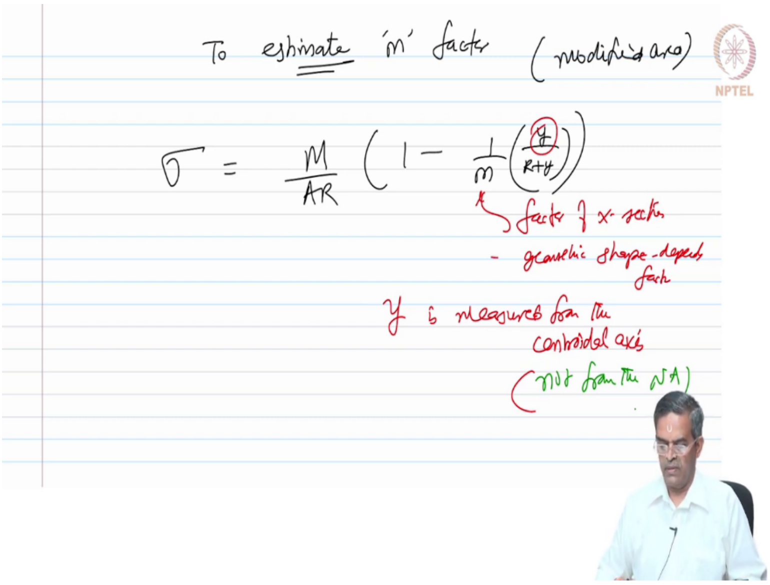

$$\sigma = \frac{M}{AR}\left(1 - \frac{1}{m}\left(\frac{y}{R+y}\right)\right)$$

So, we know we are looking for estimating m factor which is otherwise called as modified area. How it is modified? You are applying a moment it changes the curvature area is getting modified. So, where it is being used in the stress equation see the equation we have for the curved beam 1 minus 1 by m y by R plus y.

So, this is the factor of the cross sectional area. So, it is a geometric shape dependent factor. Furthermore, y is measured from the centroidal axis not from the neutral axis you must remember this. There is a common confusion and a common mistake which people generally make when estimate stresses.

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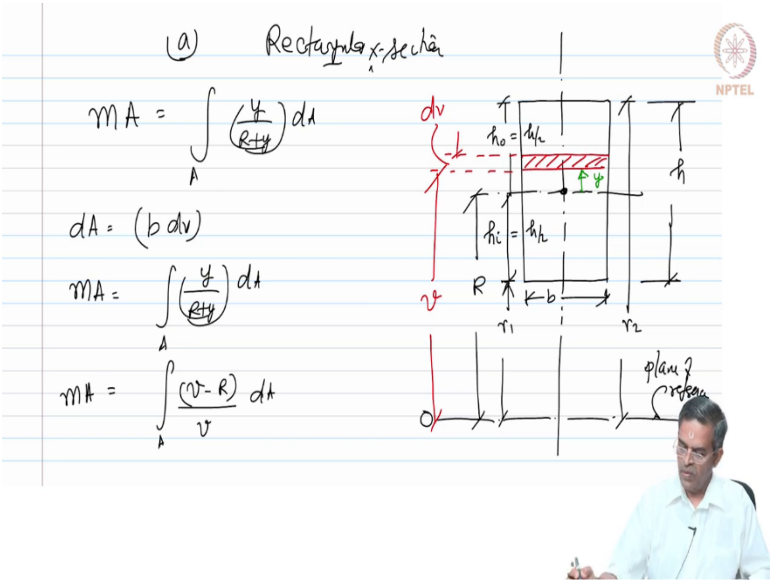

So, we will take up an example of a rectangular section. Draw a rectangular section will say this is my reference plane from where I am going to measure the curvature. Remember beam is a curved beam the cross section is rectangular please do not get confused I should say rectangular cross section. So, it has got the centroidal axis as usual the central axis will be measured from o o at distance R and if I have this as my width and this as my depth h of the cross section. We know this is indicated as r_1 in our nomenclature and this is indicated as r_2 in our nomenclature.

$$mA = \int \left(\frac{y}{R+y}\right) dA$$

$$dA = (b\, dv)$$

$$mA = \int \left(\frac{y}{R+y}\right) dA$$

$$mA = \int \frac{(v-R)}{v} dA$$

And from the cg this is hi which is h by 2 and from the cg this is h o which is h by 2 again. Now let us consider a strip at a distance v from here. Let the thickness of the strip be dv the strip is measured at a distance y from the centroidal axis as usual y is positive and is measured towards a specific extrados side we know that. So, the equation for mA is integral y by R plus y dA for the area this is general equation we have.

Let us find out what is dA in this case the hatched area can I say the dA is b into dv? Can I say that? Now mA becomes integral for the area a y by R plus y dA which I write this as can I write y as v minus R from the figure can I write y I mean R plus y as v from the figure dA for the area this has got plane of reference with which the curvature is being measured. I think this is mA is given by this.

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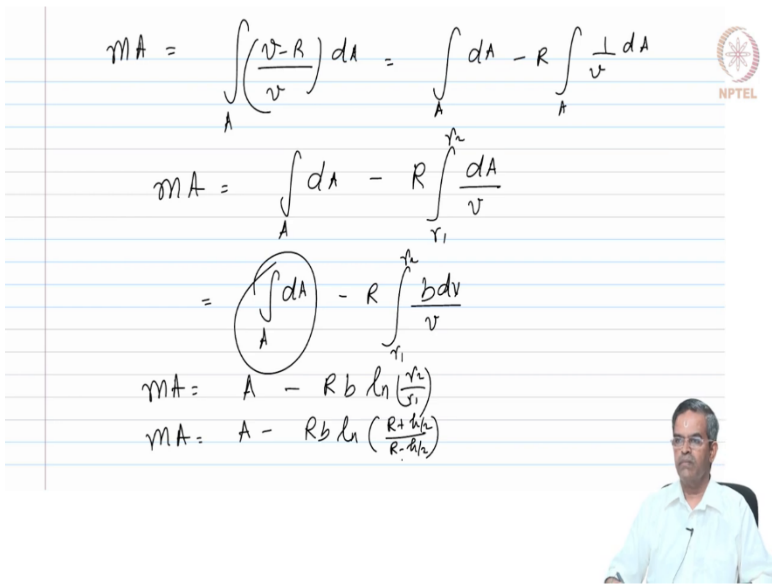

$$mA = \int \frac{(v-R)}{v} dA = \int dA - R \int \frac{1}{v} dA$$

$$mA = \int dA - R \int_{r_1}^{r_2} \frac{dA}{v}$$

$$mA = \int dA - R \int_{r_1}^{r_2} \frac{bdV}{v}$$

$$mA = A - Rb\, ln\left(\frac{r_2}{r_1}\right)$$

$$mA = A - Rb\, ln\left(\frac{R+\frac{h}{2}}{R-\frac{h}{2}}\right)$$

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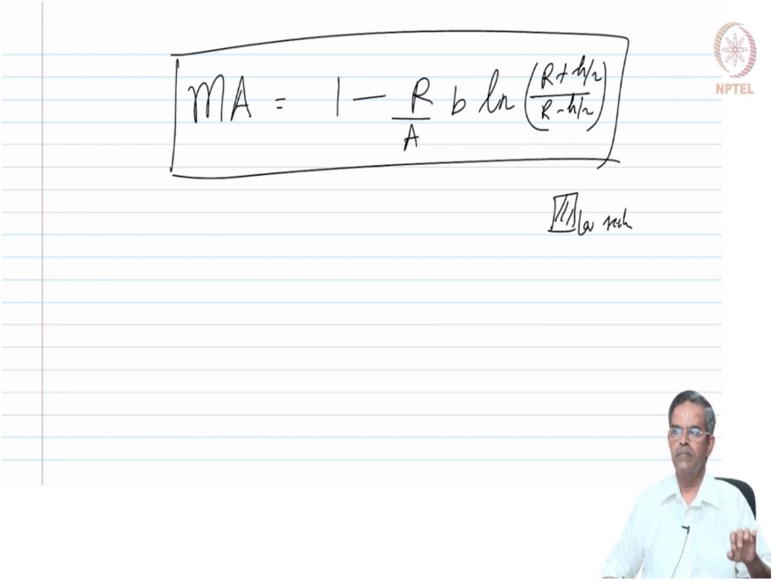

$$mA = 1 - \frac{R}{A} b \, ln\left(\frac{R+\frac{h}{2}}{R-\frac{h}{2}}\right)$$

This for a rectangular section.

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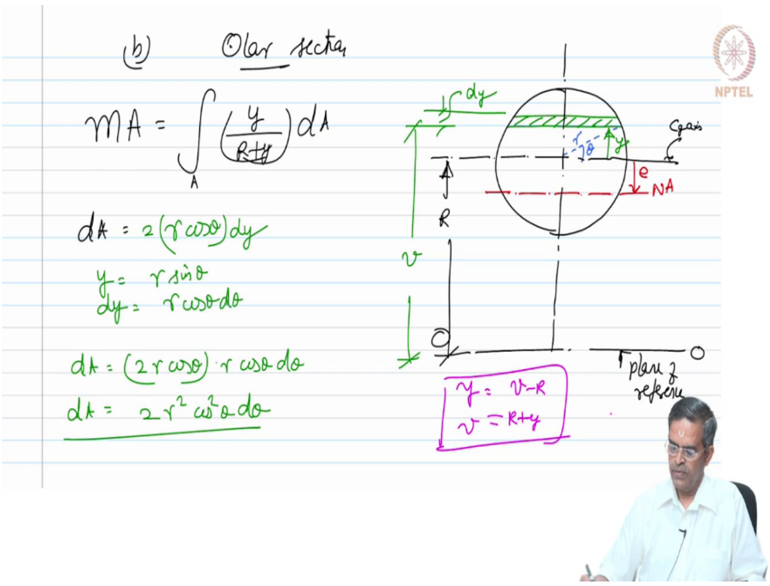

Let us derive it for a circular section. Let me draw a circular section, let me draw the plane of reference. Let us mark the critical points we say this is my centroidal axis as usual and that is always measured distance R from the plane of reference let me mark the neutral axis at a distance e from the centroidal axis towards the center of curvature let us mark a strip at a distance y measured from the centroidal axis.

Let us say this strip is at a distance v from the plane of reference. Let us say the thickness of the strip is dv. And let us say this is small R and this angle is θ.

$$mA = \int \left(\frac{y}{R+y}\right) dA$$

$$dA = 2(r \cos cos\, \theta) dy$$

$$y = r \sin sin\, \theta$$

$$dy = r \cos cos\, \theta\, d\theta$$

$$dA = (2r \cos cos\, \theta) r \cos cos\, \theta\, d\theta$$

$$dA = 2r^2 cos^2 \theta d\theta$$

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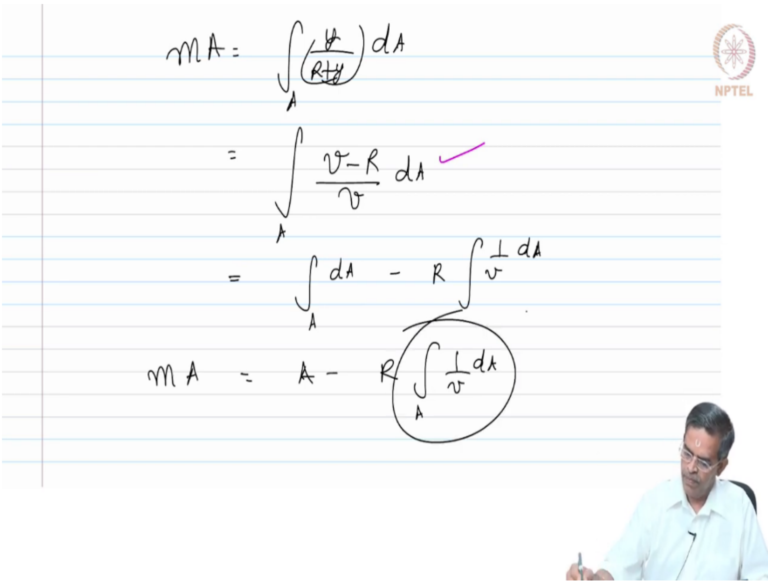

$$mA = \int \left(\frac{y}{R+y}\right) dA$$

$$mA = \int \frac{v-R}{v} dA$$

$$mA = \int dA - R \int \frac{1}{v} dA$$

$$mA = A - R\int \frac{1}{v} dA$$

This is my mA.

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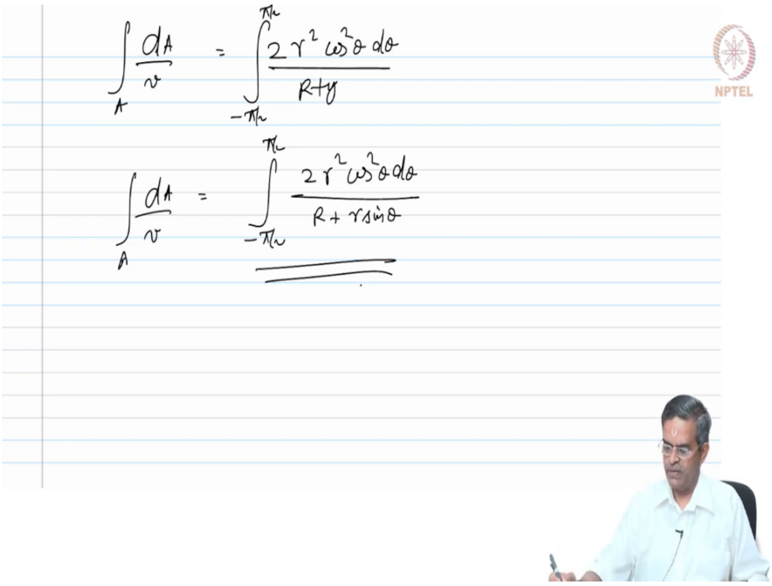

So, now let us pick up this integral which will be

$$\int \frac{dA}{v} = \int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} \frac{2r^2 cos^2\theta}{R+y} d\theta$$

but now the variable is changing θ is the variable. So, θ has to now vary from

$$\int \frac{dA}{v} = \int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} \frac{2r^2 cos^2\theta}{R+rsin\theta} d\theta$$

I am covering the whole section.

friends it is a very interesting integral which is dA by v integral a I will stop here. I want you to evaluate this integral I will continue this in the next lecture.

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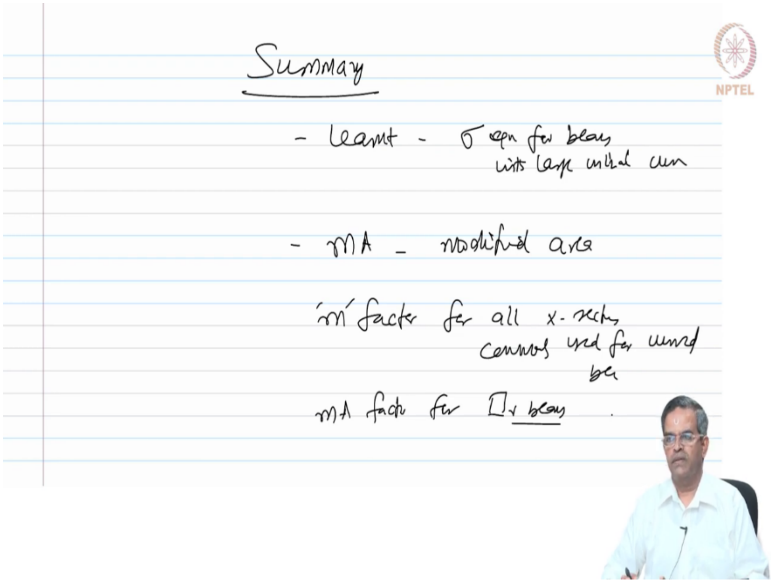

So, friends in this lecture we understood the stress equation for beams with large initial curvature. We understood that mA is a modified area is a cross section property. So, we need to find this m factor for all cross sections which are commonly used for curved beams. So, we found out this mA factor for rectangular beam. We also showed how the stress the equation for curved beams get converges to a straight beam when R or the radius of curvature becomes infinity.

Thank you friends, have a good day bye.