Advanced Design of Steel Structures
Dr. Srinivasan Chandrasekaran
Department of Ocean Engineering
Indian Institute of Technology, Madras

Lecture - 47
Modified area factor for curved section

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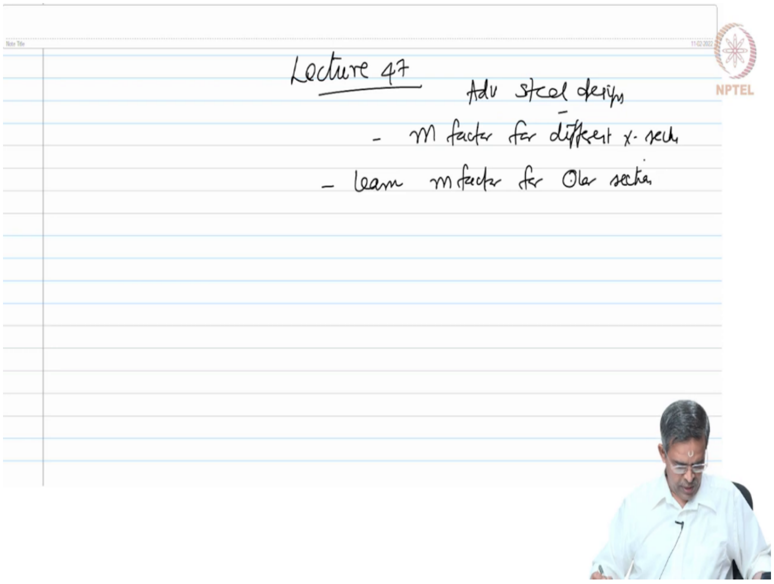

Welcome to the lecture-47 of Advanced Steel Design course. We are continuing to discuss about the m factor for different cross sections. So, in this lecture we are going to learn m factor for circular section and few more geometric shapes.

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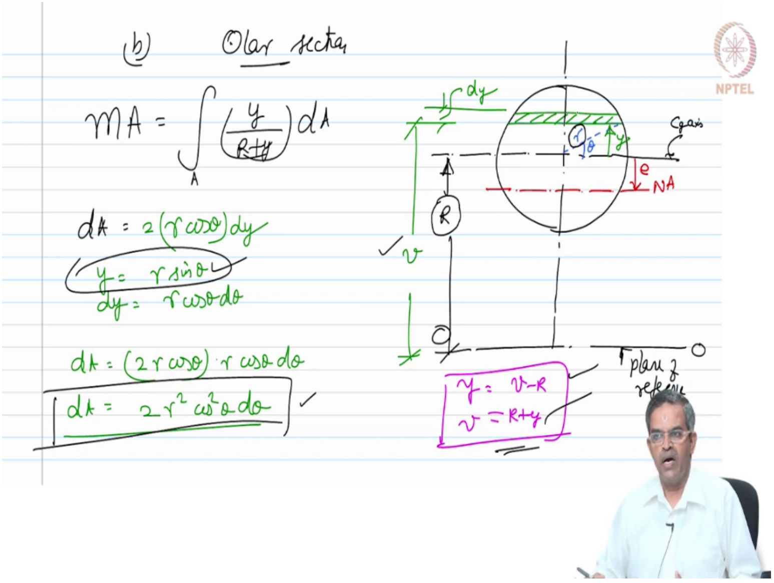

So, this was the circular section what we considered. Look at the screen and we say that v is a point where the strip is being measured from the plane of reference. And we have derived the equation written the equation for dA and so on, continue from this point and work ahead.

$$mA = \int \left(\frac{y}{R+y}\right) dA$$

$$dA = 2(r \cos cos\, \theta)dy$$

$$y = r \sin sin\, \theta$$

$$dy = r \cos cos\, \theta\, d\theta$$

$$dA = (2r \cos cos\, \theta) r \cos cos\, \theta\, d\theta$$

$$dA = 2r^2 cos^2 \theta d\theta$$

$$y = v - R$$

$$v = R + y$$

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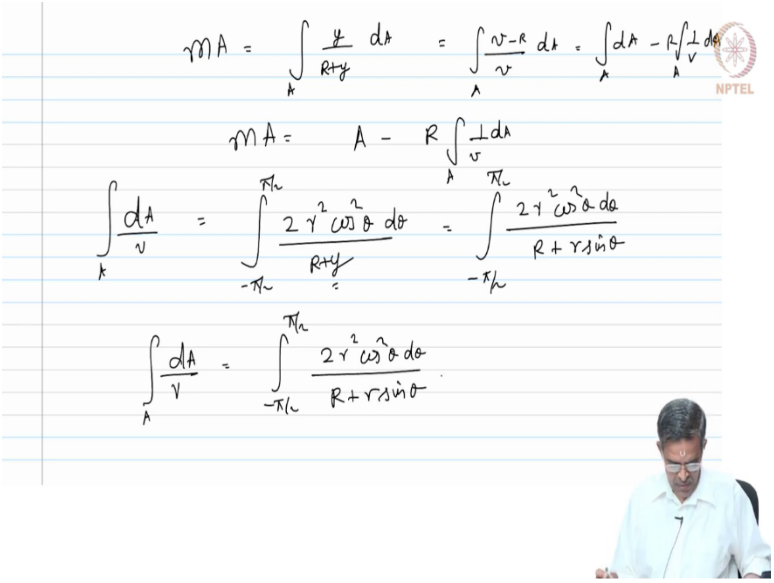

So, we know that mA is $\int\left(\frac{y}{R+y}\right)dA$. Now, I can rewrite this as you know look at this figure y can be simply said as $v - R$. So, I replace it.

$$mA = \int\left(\frac{y}{R+y}\right)dA = \int\frac{v-R}{v}dA = \int dA - R\int\frac{1}{v}dA$$

$$mA = A - R\int\frac{1}{v}dA$$

$$\int\frac{dA}{v} = \int_{\frac{-\pi}{2}}^{\frac{\pi}{2}}\frac{2r^2cos^2\theta}{R+y}d\theta = \int_{\frac{-\pi}{2}}^{\frac{\pi}{2}}\frac{2r^2cos^2\theta}{R+rsin\theta}d\theta$$

$$\int\frac{dA}{v} = \int_{\frac{-\pi}{2}}^{\frac{\pi}{2}}\frac{2r^2cos^2\theta}{R+rsin\theta}d\theta$$

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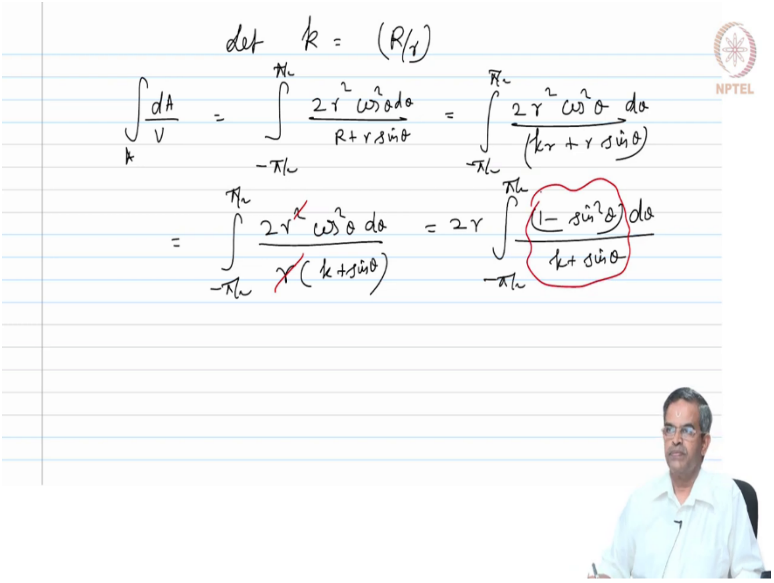

We will take this equation forward. So, let us say introduce a variable k as a ratio of this $\frac{R}{r}$. Let us imagine and understand what is $\frac{R}{r}$. Look at this figure, R is a radius of curvature of the initial level of the unstressed curved beam whereas, small r is a radius of the cross section of the circular section. So, there are two different things here. And you know R by r will be very large number, because R compared to small r will be very large.

$$k = \left(\frac{R}{r}\right)$$

$$\int \frac{dA}{v} = \int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} \frac{2r^2 cos^2\theta}{R+rsin\theta} d\theta = \int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} \frac{2r^2 cos^2\theta}{(kr+rsin\theta)} d\theta$$

$$\int \frac{dA}{v} = \int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} \frac{2r^2 cos^2\theta}{r(k+sin\theta)} d\theta = 2r \int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} \frac{(1-sin^2\theta)}{k+sin\theta} d\theta$$

Now, let us find the simplified value of this expression let us do that.

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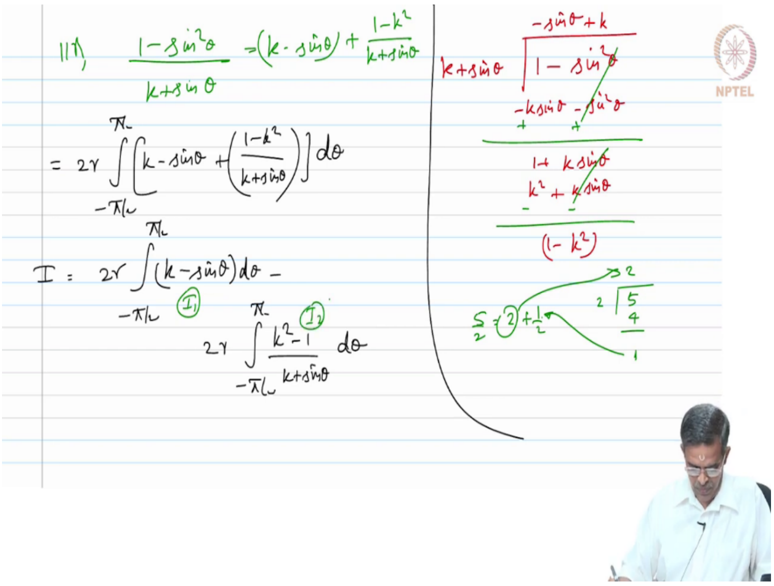

$$\frac{1-sin^2\theta}{k+sin\theta} = (k - sin\theta) + \frac{1-k^2}{k+sin\theta}$$

$$I=2r\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}}\left[(k - sin\theta) + \left(\frac{1-k^2}{k+sin\theta}\right)\right]d\theta$$

$$I = 2r\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}}(k - sin\theta)d\theta - 2r\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}}\left[\frac{k^2-1}{k+sin\theta}\right]d\theta$$

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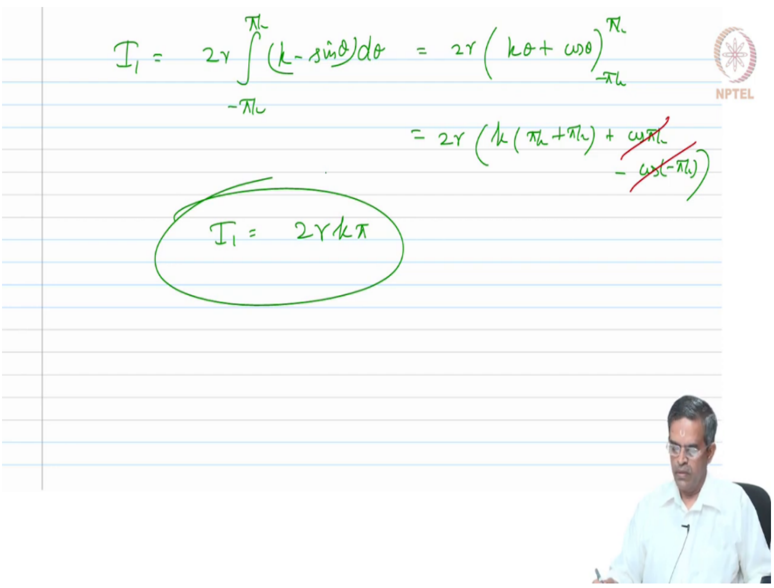

$$I_1 = 2r \int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} (k - sin\theta)d\theta = 2r(k\theta + cos\theta)\Big|_{\frac{-\pi}{2}}^{\frac{\pi}{2}}$$

$$I_1 = 2r\left(k\left(\frac{\pi}{2} + \frac{\pi}{2}\right) + \cos cos\left(\frac{\pi}{2}\right) - \cos cos\left(\frac{-\pi}{2}\right)\right)$$

$$I_1 = 2rk\pi$$

This gets cancelled 2r of $k\pi$ is my I_1.

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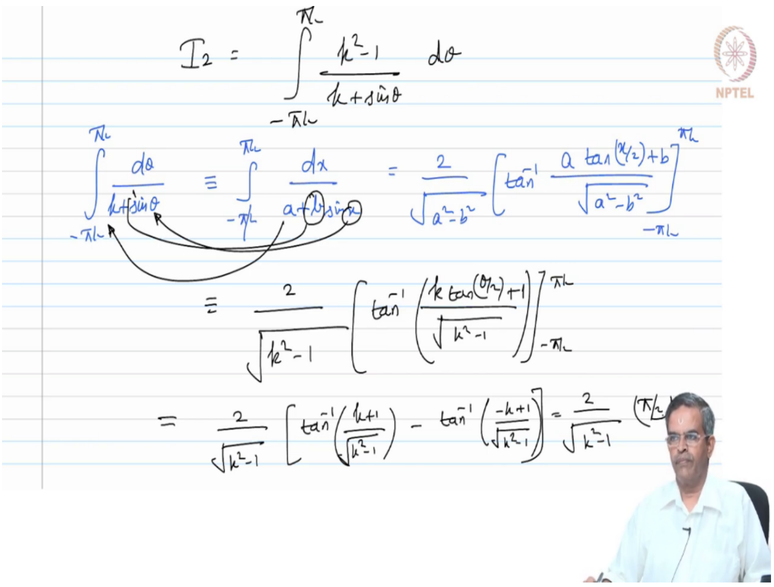

Let us try to find I_2 which is

$$I_2 = \int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} \left[\frac{k^2-1}{k+sin\theta}\right] d\theta$$

$$\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} \left[\frac{d\theta}{k+sin\theta}\right] \equiv \int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} \left[\frac{dx}{a+bsinx}\right] = \frac{2}{\sqrt{a^2-b^2}}\left[\frac{atantan\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right]_{\frac{-\pi}{2}}^{\frac{\pi}{2}}$$

$$= \frac{2}{\sqrt{k^2-1}}\left[\left(\frac{ktantan\left(\frac{\theta}{2}\right)+1}{\sqrt{k^2-1}}\right)\right]_{\frac{-\pi}{2}}^{\frac{\pi}{2}}$$

$$= \frac{2}{\sqrt{k^2-1}}\left[\left(\frac{k+1}{\sqrt{k^2-1}}\right) - \left(\frac{-k+1}{\sqrt{k^2-1}}\right)\right] = \frac{2}{\sqrt{k^2-1}}\left(\frac{\pi}{2}\right)$$

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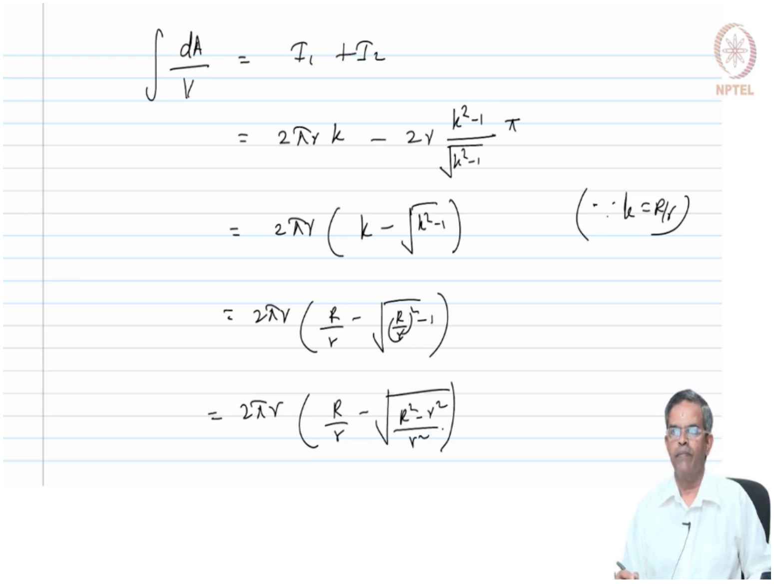

So, now I can say integral dA by V is $I_1 + I_2$ that is what we have here.

$$\int \frac{dA}{V} = I_1 + I_2$$

$$= 2\pi r k - 2r\frac{k^2-1}{\sqrt{k^2-1}}\pi$$

$$k = \frac{R}{r}$$

$$= 2\pi r\left(k - \sqrt{k^2 - 1}\right)$$

$$= 2\pi r\left(\frac{R}{r} - \sqrt{\left(\frac{R}{r}\right)^2 - 1}\right)$$

$$= 2\pi r\left(\frac{R}{r} - \sqrt{\frac{R^2-r^2}{r^2}}\right)$$

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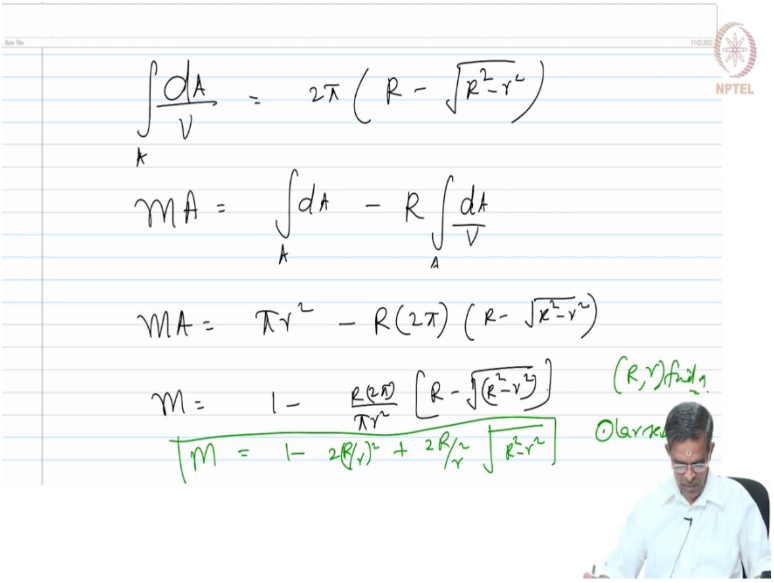

$$\int \frac{dA}{V} = 2\pi\left(R - \sqrt{R^2 - r^2}\right)$$

$$mA = \int dA - R - \int \frac{dA}{V}$$

$$mA = \pi r^2 - R(2\pi)\left(R - \sqrt{R^2 - r^2}\right)$$

$$m = 1 - \frac{R(2\pi)}{\pi r^2}\left(R - \sqrt{R^2 - r^2}\right)$$

$$m = 1 - 2\left(\frac{R}{r}\right)^2 + \frac{2R}{r^2}\sqrt{R^2 - r^2}$$

that is m for the circular section. So, if we know the values of R and r one can find m. So, it is a geometric property.

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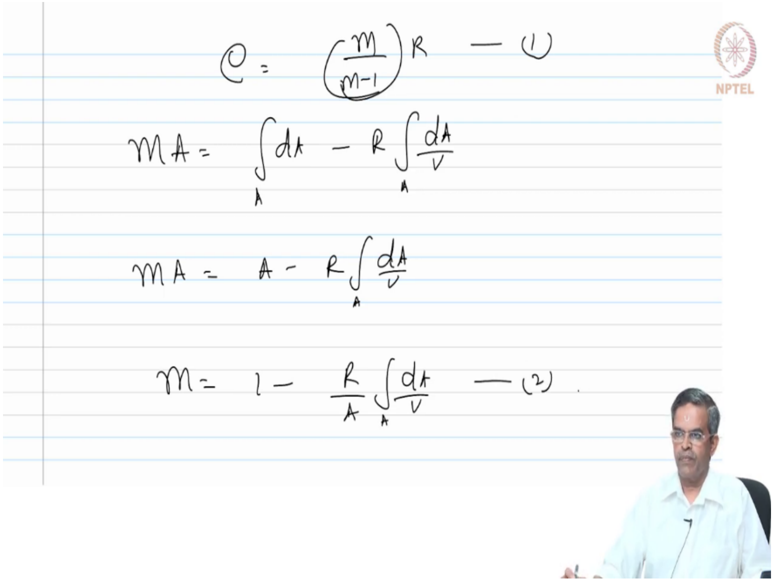

So, now I can find e as

$$e = \left(\frac{m}{m-1}\right)R$$

$$mA = \int dA - R\int \frac{dA}{V}$$

$$mA = A - R\int \frac{dA}{V}$$

$$m = 1 - \frac{R}{A}\int \frac{dA}{V}$$

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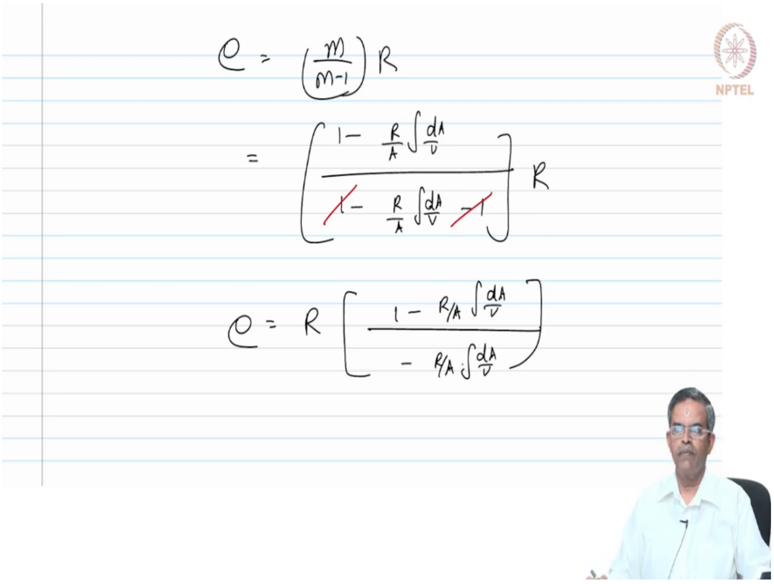

And therefore, e is $e = \left(\frac{m}{m-1}\right)R$

$$e = \left[\frac{1-\frac{R}{A}\int\frac{dA}{V}}{1-\frac{R}{A}\int\frac{dA}{V}-1}\right]R$$

$$e = R\left[\frac{1-\frac{R}{A}\int\frac{dA}{V}}{-\frac{R}{A}\int\frac{dA}{V}}\right]$$

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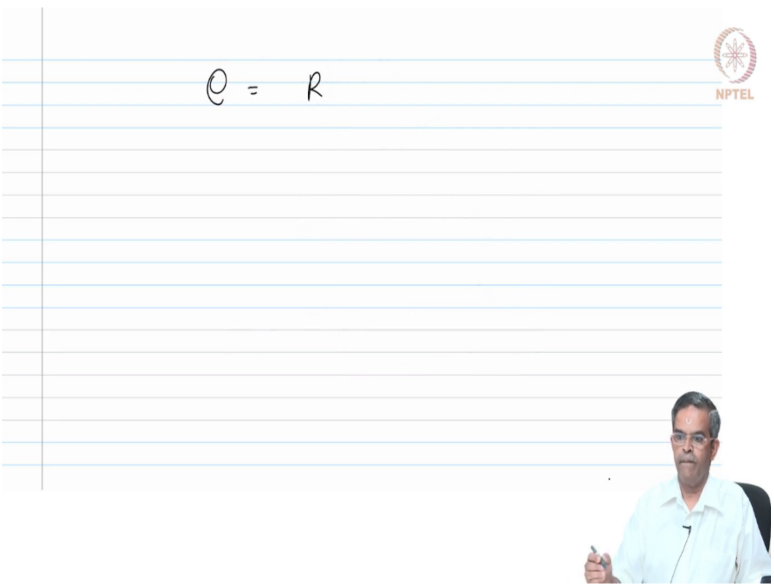

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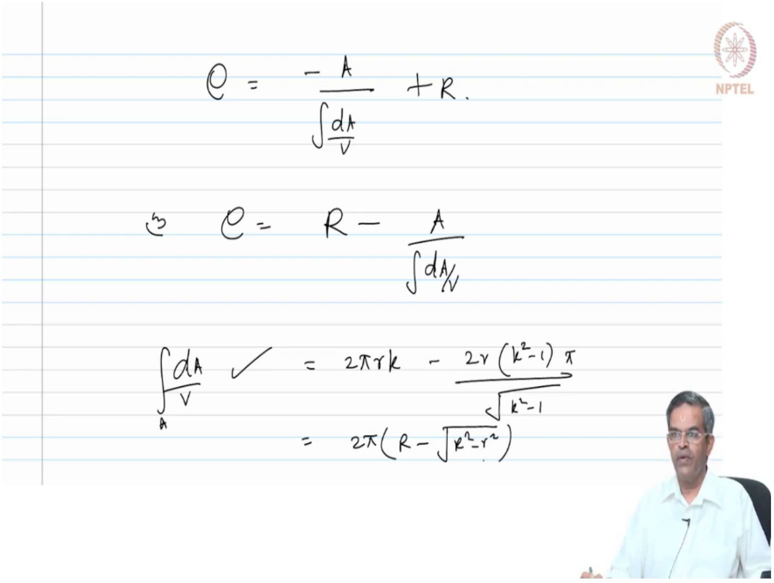

$$e = \frac{-A}{\int \frac{dA}{V}} + R$$

$$e = R - \frac{A}{\int \frac{dA}{V}}$$

$$\int \frac{dA}{V} = 2\pi rk - \frac{2r(k^2-1)\pi}{\sqrt{k^2-1}}$$

I am substituting for k.

$$\int \frac{dA}{V} = 2\pi\left(R - \sqrt{R^2 - r^2}\right)$$

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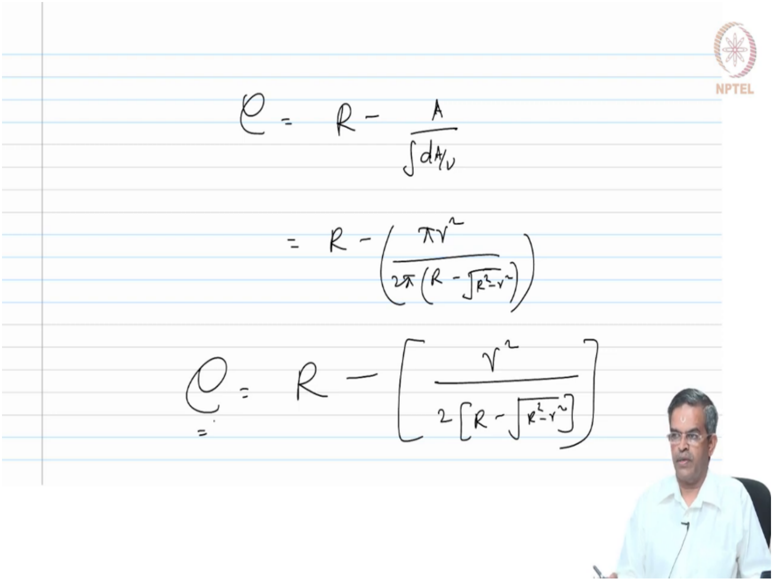

$$e = R - \frac{A}{\int \frac{dA}{V}}$$

$$e = R - \left(\frac{\pi r^2}{2\pi \left(R - \sqrt{R^2 - r^2} \right)} \right)$$

$$e = R - \left[\frac{r^2}{2\left(R - \sqrt{R^2 - r^2} \right)} \right]$$

So, if you know R and r, you can find e. So, now, I know the modified cross section m, I know how to locate the neutral axis from the centroidal axis because that is what e is, e is offset of that. So, all are geometric properties now.

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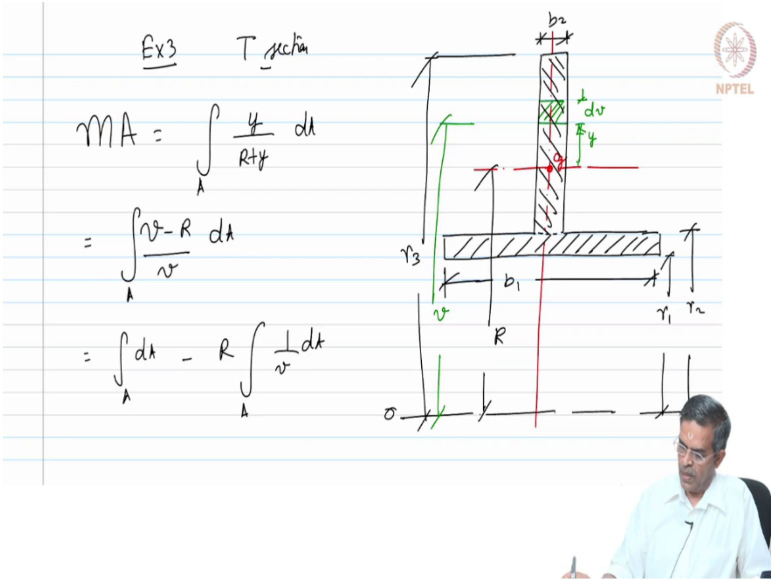

Let us do one more example. I will say this as example 3, example 1 is rectangular section, example 2 is circular section. We will go for a T-section. So, let me draw the figure of a T section. Let us say this is my plane of reference o-o this dimension is b_1 and this dimension is b_2 of the T section. Let us say, it has got a centroid axis passing through this point.

This is my cg. I locate a strips at a distance y measured from the cg away from the center. So, I will call this value as r_1, this value as r_2. And of course, we know that this is R radius of curvature to the centroid. Let me call this as r_3. Now, let us cut a section let us say this is my dv and this is at a distance v from the plane of reference.

So, you know the equation mA is given by integral y by R plus y dA for the whole area.

$$mA = \int \frac{y}{R+y} dA$$

$$mA = \int \frac{v-R}{v} dA$$

$$mA = \int dA - R\int \frac{1}{v} dA$$

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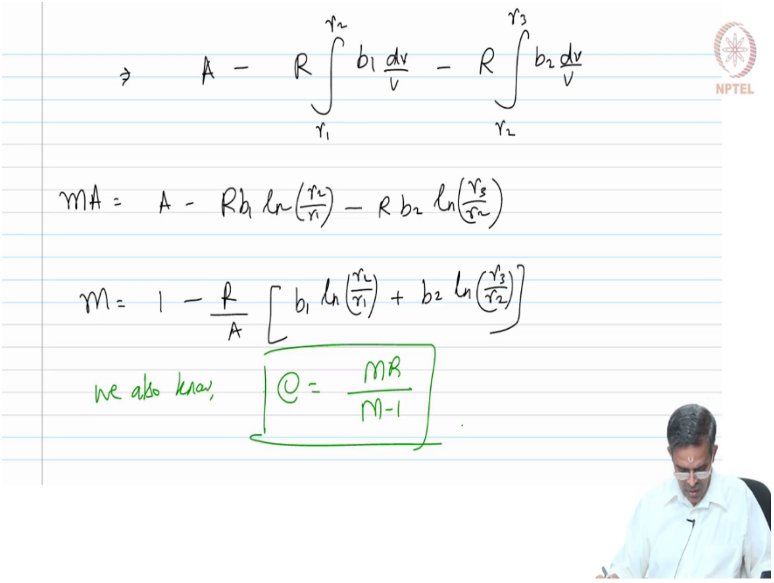

$$mA = A - R\int_{r_1}^{r_2} b_1 \frac{dV}{V} - R\int_{r_2}^{r_3} b_2 \frac{dV}{V}$$

$$mA = A - Rb_1 ln\left(\frac{r_2}{r_1}\right) - Rb_2 ln\left(\frac{r_3}{r_2}\right)$$

$$m = 1 - \frac{R}{A}\left[b_1 ln\left(\frac{r_2}{r_1}\right) + b_2 ln\left(\frac{r_3}{r_2}\right)\right]$$

$$e = \frac{mR}{m-1}$$

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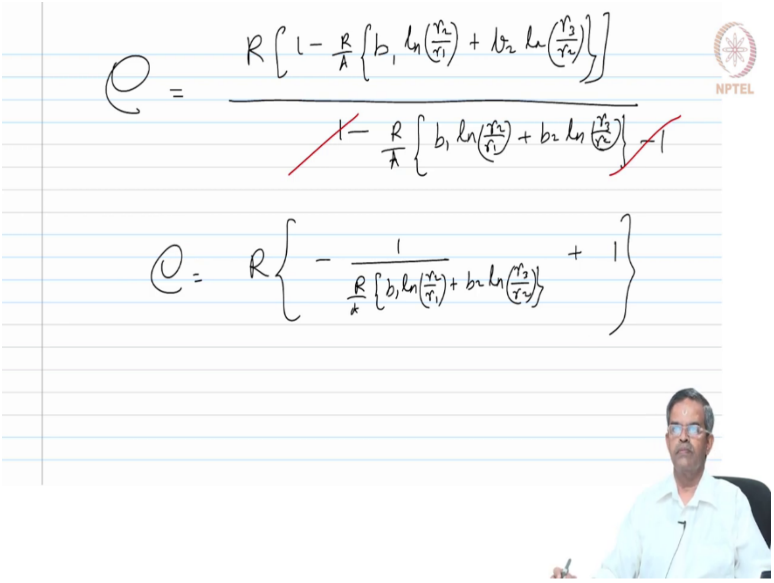

$$e = \frac{R\left(1-\frac{R}{A}\left[b_1 ln\left(\frac{r_2}{r_1}\right)+b_2 ln\left(\frac{r_3}{r_2}\right)\right]\right)}{1-\frac{R}{A}\left[b_1 ln\left(\frac{r_2}{r_1}\right)+b_2 ln\left(\frac{r_3}{r_2}\right)\right]-1}$$

$$e = R\left\{-\frac{1}{\frac{R}{A}\left[b_1 ln\left(\frac{r_2}{r_1}\right)+b_2 ln\left(\frac{r_3}{r_2}\right)\right]} + 1\right\}$$

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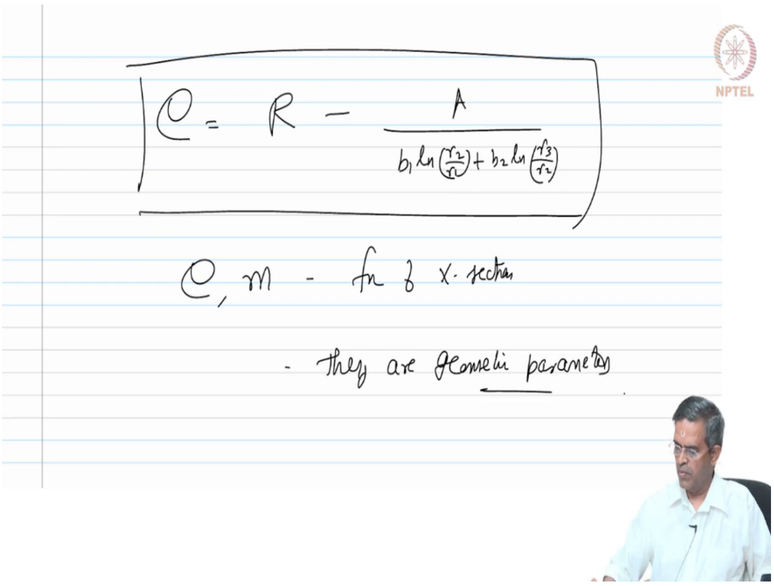

That is my e. So, I can locate the neutral axis. So, e is going to be

$$e = R - \frac{A}{b_1 ln\left(\frac{r_2}{r_1}\right) + b_2 ln\left(\frac{r_3}{r_2}\right)}$$

So, friends you can see from this equation e m are all functions of cross section. They are geometric parameters. So, for a given value let us say for example, I will just draw this figure again.

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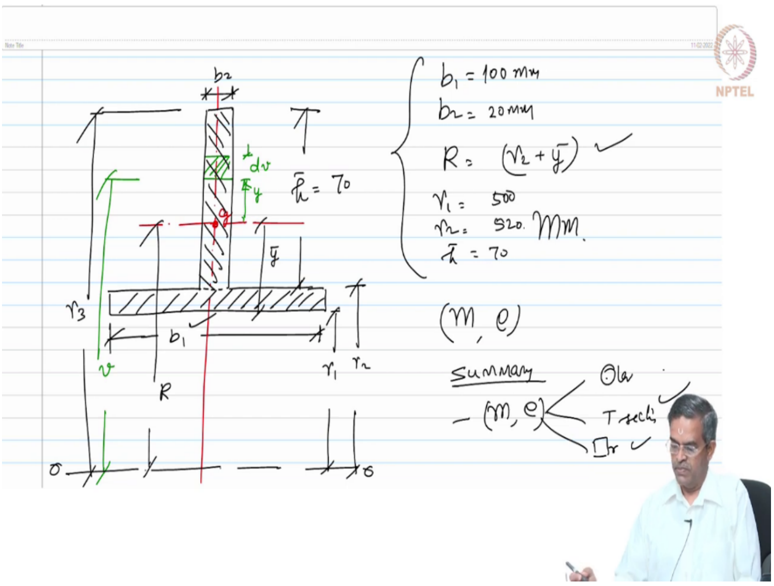

For b 1 as 100 mm, b 2 as 20 mm, R as 1000 mm, r 1 as 500, r 2 as 520. So, you need to find actually R capital R it is a cg. So, you must know this dimension, which will be actually I should say r 2 plus y bar. You can find out this. For b 1 and b 2 and for this dimension which is h bar which is taken as 70 mm, one can find R. Now, all dimensions are known one can find m, one can find e.

So, once you locate e you will be able to locate the neutral axis and then you can find the stresses and plot the stresses. So, friends in this lecture, we learnt how to find the modified area property and the offset of the neutral axis from the centroidal axis for different cross sections. We did for circular; we did for T-section. We already did for rectangular section earlier for first principles.

I think there is no difficulty in handling these kinds of problems and my reference book also gives you the MATLAB program to find out the stresses for various cross sections for curved beams. I strongly promote that you should use this textbook and download the programs and use MATLAB intensively for computing the stresses and the cross sections of these shapes for curved beams of large initial curvature, which is Winkler Bach equation. Please do that.

Thank you very much. Have a good day, bye.