Advanced Design of Steel Structures
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Lecture - 48
M factor for curved beams

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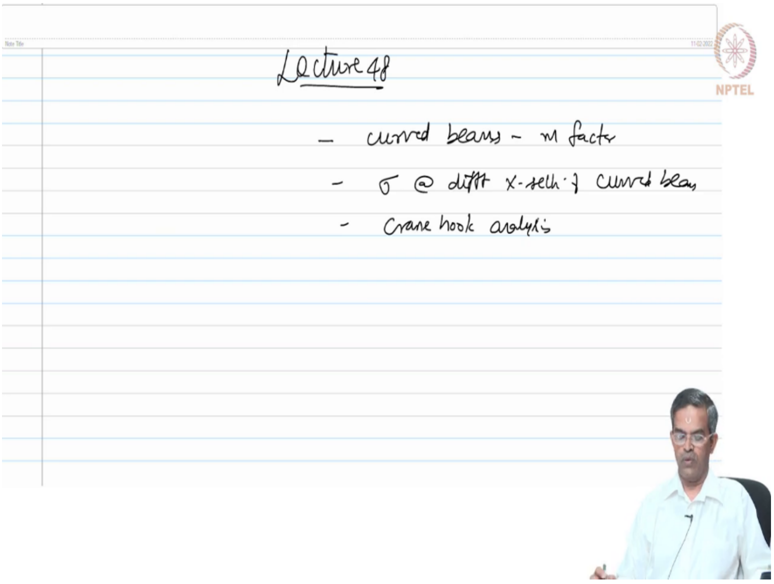

Friends, welcome to the 48th lecture on Advanced Steel Design course. Here, we are going to discuss about more on Curved Beams. We will also talk about finding out the stresses and different cross sections of curved beams. So, we will talk about M factor, we will also learn more about the crane hook analysis.

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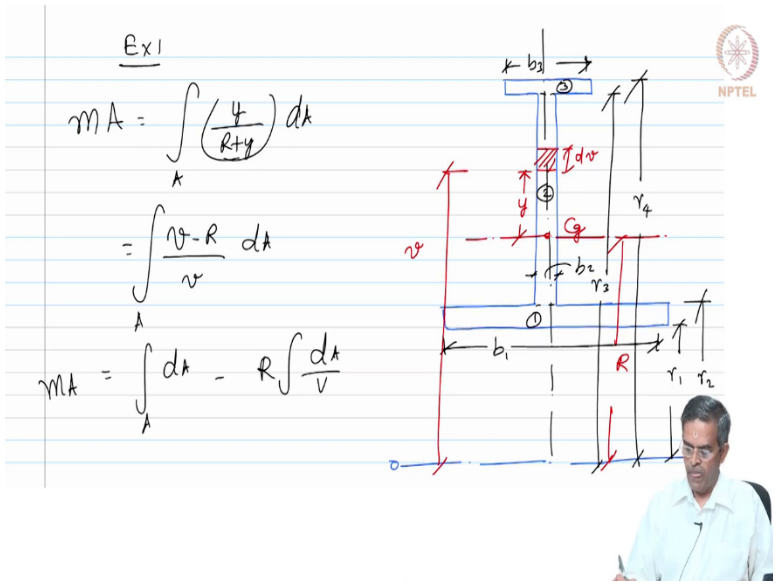

So, we will say example 1 for this lecture. We will take up a section, an unequal high section is a very common case for curved beams are played with large moments. Write down the properties of this. This becomes my plane of reference o-o and will mark the centroidal axis somewhere here. We know this distance actually is always measured as R and we call this dimension as b_1, this as r_1 and this as r_2. And of course, this dimension is r_3.

This is r_4 and this is r_3, this is b_1, this is b_2 and this is b_3. This piece number 1 for us 2 and 3 for our reference. Then, we will cut a small strip which is measured a distance y from here, we call this as dv where this distance is v from here. So, general expression for mA is given as

$$mA = \int \left(\frac{y}{R+y}\right) dA$$

$$mA = \int \frac{v-R}{v} dA$$

$$mA = \int dA - R \int \frac{1}{v} dA$$

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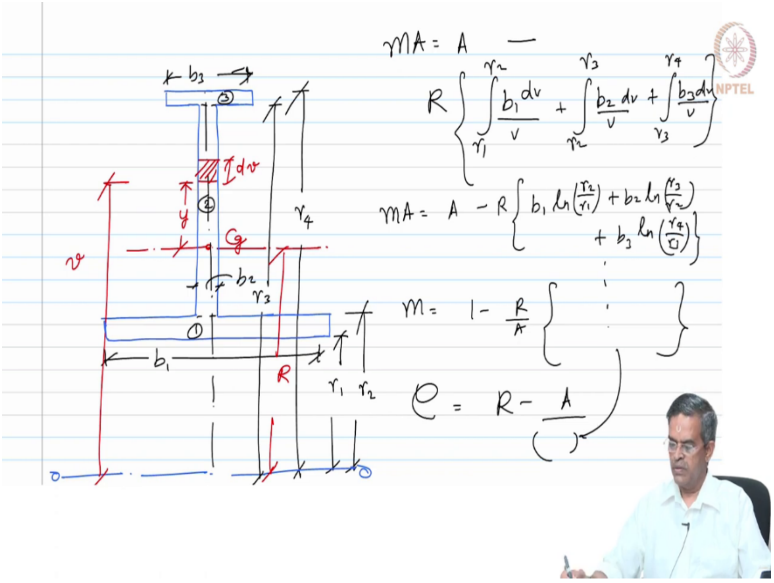

So, now m A is equal to

$$mA = A - R\left\{\int_{r_1}^{r_2} \frac{b_1 dv}{v} + \int_{r_2}^{r_3} \frac{b_2 dv}{v} + \int_{r_3}^{r_4} \frac{b_3 dv}{v}\right\}$$

So, I have got different components. Let me I think copy this figure its better and copy this figure, put it here then I will write m A here. m A is going to be equation is dA which is A minus R times of see we have integral over the entire area A, I am dividing the three parts. So, R times of , I will do it here minus R times of let us do from integral r_2 to r_1. So, that area will be b_1 dv by v. See here, I am doing it for dA.

So, small strip cut here and that strip is going to vary from r_1 to r_2, that is what I am doing plus integral r_3 to r_2, for this strip it is b_2. Similarly, integral r_4 to r_3 and this is b_3 which will now become

$$mA = A - R\left\{b_1\, ln\left(\frac{r_2}{r_1}\right) + b_2\, ln\left(\frac{r_3}{r_2}\right) + b_3\, ln\left(\frac{r_4}{r_3}\right)\right\}$$

$$m = 1 - \frac{R}{A}\left\{b_1\, ln\left(\frac{r_2}{r_1}\right) + b_2\, ln\left(\frac{r_3}{r_2}\right) + b_3\, ln\left(\frac{r_4}{r_3}\right)\right\}$$

$$e = R - \frac{A}{\left\{b_1\, ln\left(\frac{r_2}{r_1}\right) + b_2\, ln\left(\frac{r_3}{r_2}\right) + b_3\, ln\left(\frac{r_4}{r_3}\right)\right\}}$$

So, I can easily calculate this.

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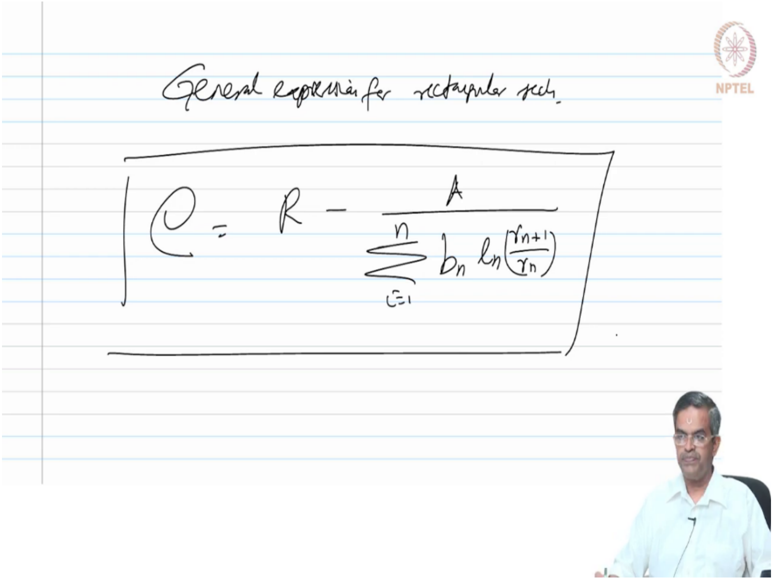

So, therefore, friends if I have a general expression for rectangular sections e will be

$$e = R - \frac{A}{\sum_{i=1}^{n} b_n ln\left(\frac{r_{n+1}}{r_n}\right)}$$

This is what I get the e is general expression. Let us take a triangular section.

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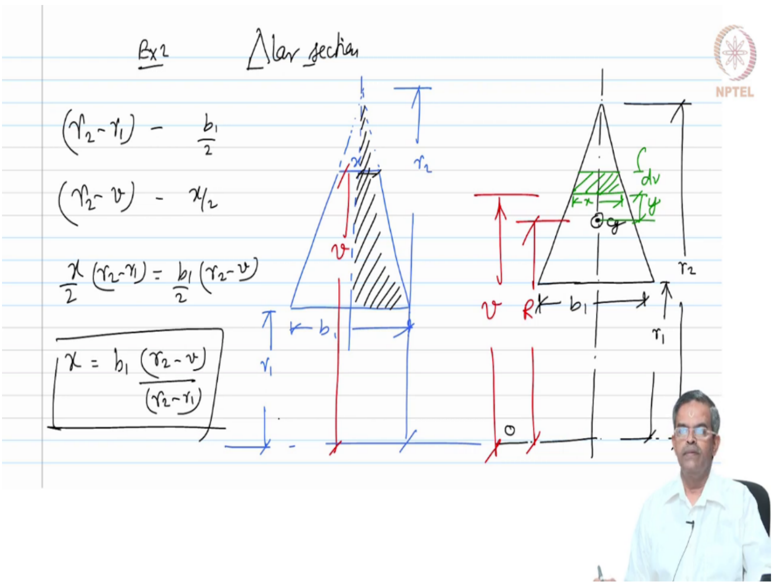

So, we will draw a triangle plane of reference, triangle has value b_1 here and this is r_1 and this value is termed as r_2 and triangle has a cg somewhere here and we mark that distance from here as R. Let us cut the strip at a distance y from here. Let the thickness of the strip be dv and the strip be measured v from here. Let me draw the frustum of that separately, I will mark this distance as x, because I want to find the area that is tricky here.

$$\left(r_2 - r_1\right) \sim \frac{b_1}{2}$$

$$\left(r_2 - v\right) \sim \frac{x}{2}$$

$$\frac{x}{2}\left(r_2 - r_1\right) = \frac{b_1}{2}\left(r_2 - v\right)$$

$$x = b_1 \frac{\left(r_2 - v\right)}{\left(r_2 - r_1\right)}$$

Let me mark that figure separately. So, anyway this getting converge to become a triangle. So, this is x and this is b_1, let us say this is r_2 and this is r_1. So, we now write a relationship r_2 minus r_1. The variation is for b_1 by 2, is it not if you look at this triangle. Let us say I am looking for this part, for a variation of r_2 minus r_1.

Now, we know that this strip is at v is at v. So, therefore, for r_2 minus v, this value will be x by 2. So, now, x by 2 into r_2 minus r_1 is b_1 by 2 r_2 minus v. So, can I say x as b_1 times of r_2 minus v by r_2 minus r_1, that is my x; width of the strip is x.

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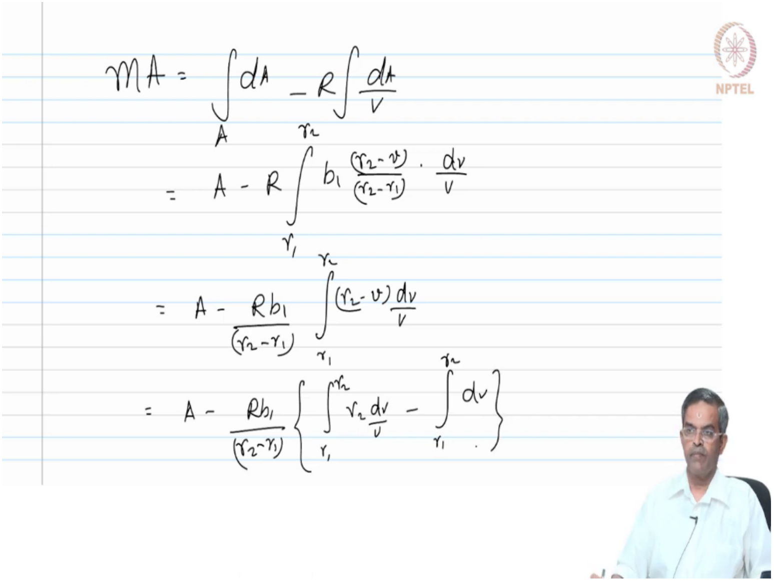

Now, m A is

$$mA = \int dA - R\int \frac{dA}{V}$$

$$mA = A - R\int_{r_1}^{r_2} b_1 \frac{(r_2 - v)}{(r_2 - r_1)} \frac{dV}{V}$$

$$mA = A - \frac{Rb_1}{(r_2 - r_1)} \int_{r_1}^{r_2} (r_2 - v) \frac{dV}{V}$$

$$mA = A - \frac{Rb_1}{(r_2 - r_1)} \left\{ \int_{r_1}^{r_2} r_2 \frac{dV}{V} - \int_{r_1}^{r_2} dV \right\}$$

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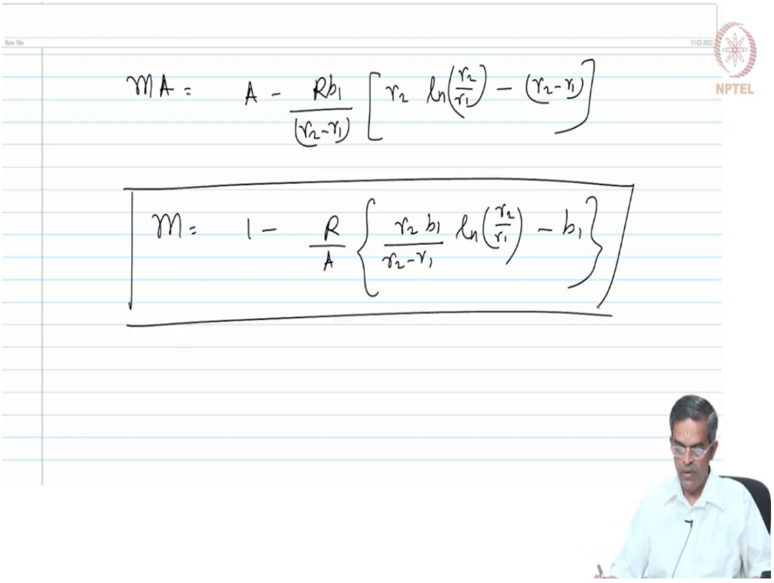

$$mA = A - \frac{Rb_1}{(r_2 - r_1)}\left[r_2\, ln\left(\frac{r_2}{r_1}\right) - \left(r_2 - r_1\right)\right]$$

$$m = 1 - \frac{R}{A}\left\{\frac{r_2 b_1}{(r_2 - r_1)} ln\left(\frac{r_2}{r_1}\right) - b_1\right\}$$

So, that is my m value for the triangular section.

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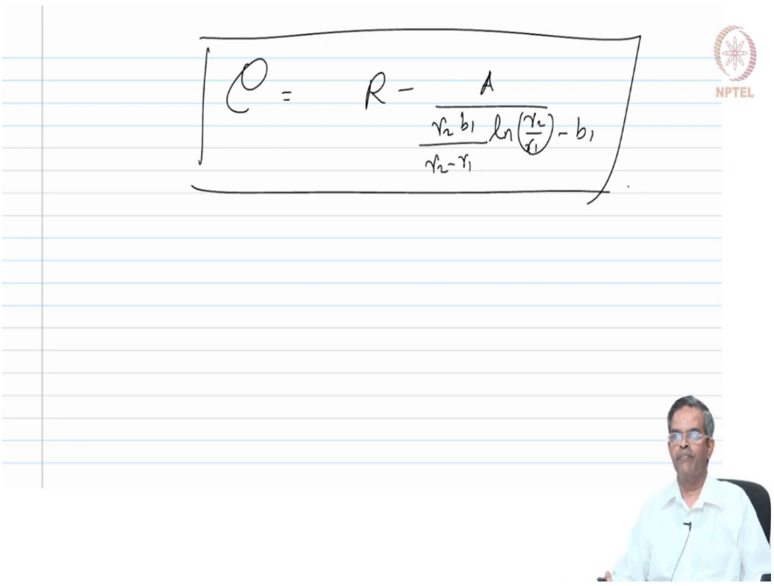

Once I have a m value, I can find the e value using this relationship

$$e = R - \frac{A}{\frac{r_2 b_1}{(r_2 - r_1)} ln\left(\frac{r_2}{r_1}\right) - b_1}$$

that is my e to locate the neutral axis.

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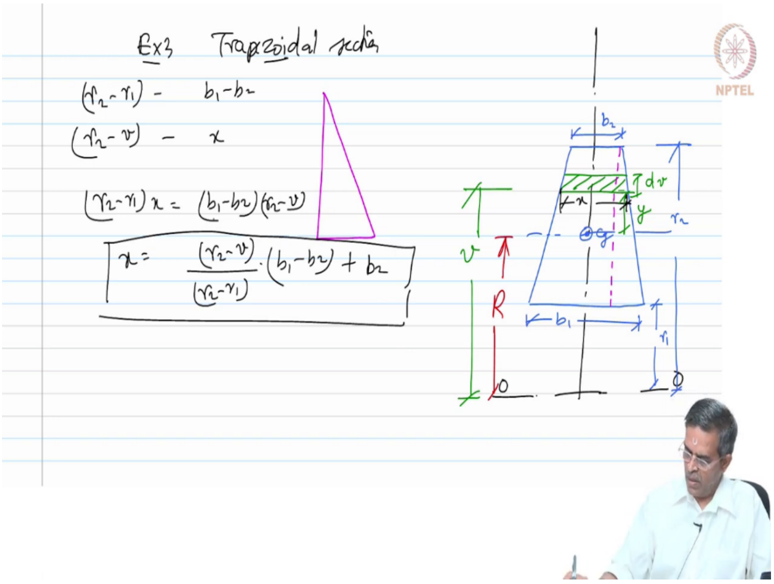

In the same logic, let us do for a trapezoidal section. This will be the plane of reference. Let us say this is b_1, this is b_2, r_1, this is r_2, let us say this has got a cg somewhere here. So, this becomes a cg axis which is measured R from the plane of the radius R.

Then, we will also then mark a strip at a distance y from the central axis. We call this as dv and let the strip be cut at the distance v from the plane of reference. Now, I divide this into a segment like this and I will extrapolate this separately. Then, I can write the similar triangle equation for this which will be r_2 minus r_1 on the height gives me the variation as b_1 minus b_2.

$$(r_2 - r_1) \sim (b_1 - b_2)$$

$$(r_2 - v) \sim x$$

$$(r_2 - r_1)x = (b_1 - b_2)(r_2 - v)$$

$$x = \frac{(r_2 - v)}{(r_2 - r_1)}(b_1 - b_2) + b_2$$

So, if it is r_2 minus v, let us call this variation as x, r_2 minus r_1 into x is b_1 minus b_2 into r_2 minus v. Therefore, x will be r_2 minus v by r_2 minus r_1 of b_1 minus b_2 plus b_2, because I am looking for this distance is x. So, that is my x value.

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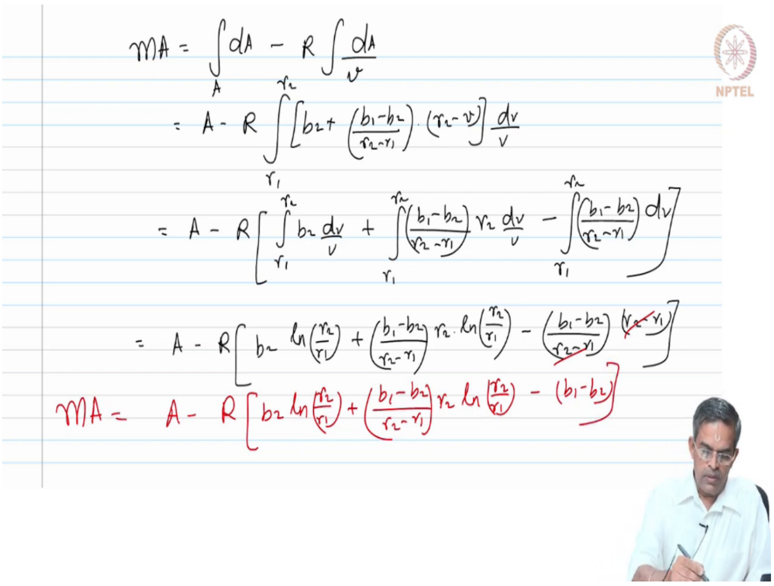

Having obtained this, I can now use the standard relation m A is

$$mA = \int dA - R\int \frac{dA}{V}$$

$$mA = A - R\int_{r_1}^{r_2}\left[b_2 + \left(\frac{b_1 - b_2}{r_2 - r_1}\right)\left(r_2 - v\right)\right]\frac{dV}{V}$$

$$mA = A - R\left[\int_{r_1}^{r_2} b_2\frac{dV}{V} + \int_{r_1}^{r_2}\left(\frac{b_1 - b_2}{r_2 - r_1}\right)r_2\frac{dV}{V} - \int_{r_1}^{r_2}\left(\frac{b_1 - b_2}{r_2 - r_1}\right)dV\right]$$

$$mA = A - R\left[b_2 ln\left(\frac{r_2}{r_1}\right) + \left(\frac{b_1 - b_2}{r_2 - r_1}\right)r_2 ln\left(\frac{r_2}{r_1}\right) - \left(\frac{b_1 - b_2}{r_2 - r_1}\right)\left(r_2 - r_1\right)\right]$$

$$mA = A - R\left[b_2 ln\left(\frac{r_2}{r_1}\right) + \left(\frac{b_1 - b_2}{r_2 - r_1}\right)r_2 ln\left(\frac{r_2}{r_1}\right) - \left(b_1 - b_2\right)\right]$$

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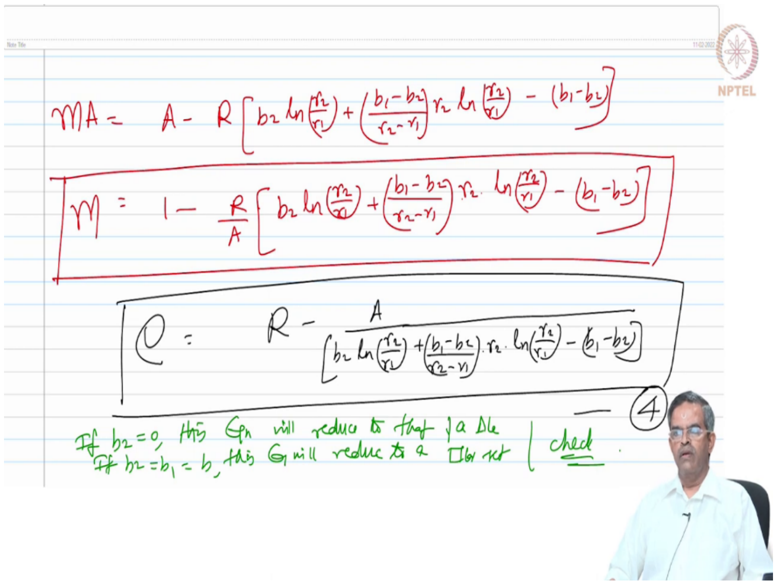

$$mA = A - R\left[b_2 ln\left(\frac{r_2}{r_1}\right) + \left(\frac{b_1 - b_2}{r_2 - r_1}\right) r_2 ln\left(\frac{r_2}{r_1}\right) - \left(b_1 - b_2\right)\right]$$

So, I can say which will be equal to

$$m = 1 - \frac{R}{A}\left[b_2 ln\left(\frac{r_2}{r_1}\right) + \left(\frac{b_1 - b_2}{r_2 - r_1}\right) r_2 ln\left(\frac{r_2}{r_1}\right) - \left(b_1 - b_2\right)\right]$$

$$e = R - \frac{A}{\left[b_2 ln\left(\frac{r_2}{r_1}\right) + \left(\frac{b_1 - b_2}{r_2 - r_1}\right) r_2 ln\left(\frac{r_2}{r_1}\right) - \left(b_1 - b_2\right)\right]}$$

that is my shift of the neutral axis from the centroidal axis which is equation 4 in our original derivation.

So, friends if you call this as equation 4, in this equation if b_2 is 0; what is b_2? See, the figure if b_2 is 0, we get a triangle. This equation will reduce to that of triangle. If b_2 and b_1 are b, see this figure if they are b reduced to a rectangle; this equation will reduce to a rectangular section. You can check that, I will leave it to you. Please check that and check the derivation carefully.

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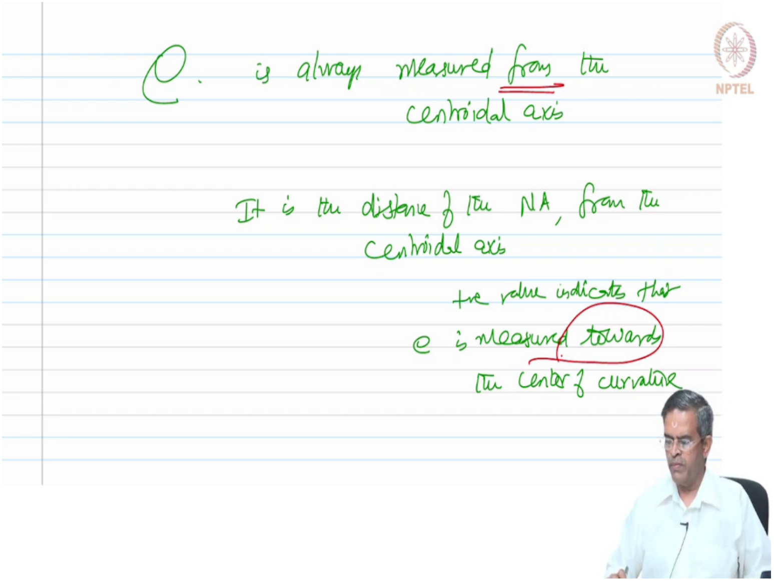

So, there are some comments which we need to highlight very clearly before we do some more numerical. e is always measured from the centroidal axis and e it is the distance of the neutral axis from the centroidal axis, positive value indicates that e is measured towards the center of curvature that is very important friends. So, now, for curved beams there are some more simplified equations given in the literature. Let us also try to appreciate them and see.

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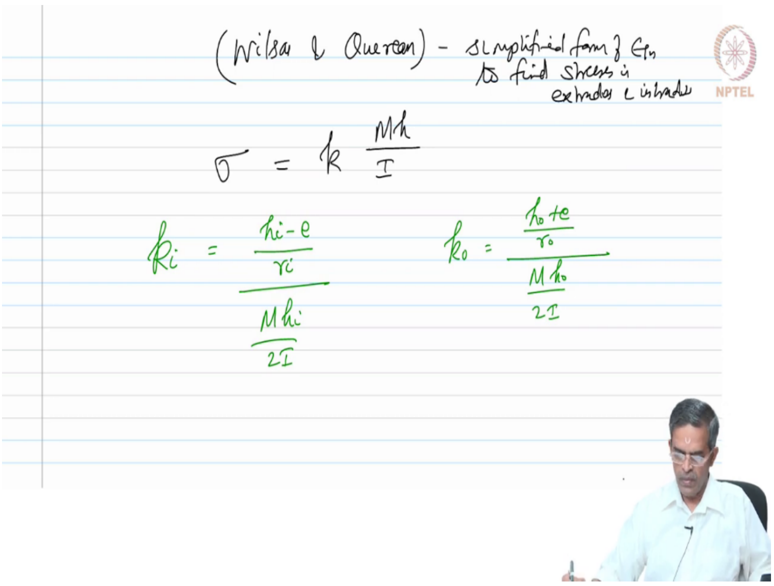

So, Wilson and Querean have given simplified form of equations to find the stresses in extrados and intrados of different cross sections and say the equation is given by σ is equal to k times of applied moment h by I. So, they say the stress you have to use k_i, say k_i is a factor for intrados which is h_i minus e by r_i divided by M h_i by 2 I, k_o is a factor used for extrados which is h_o by e by r_o divided by M h_o by 2 I. So, now I will give you the values of these tables as recommended by these researchers for our usefulness.

$$\sigma = k\frac{Mh}{I}$$

$$k_i = \frac{\frac{h_i - e}{r_i}}{\frac{Mh_i}{2I}}$$

$$k_o = \frac{\frac{h_o - e}{r_o}}{\frac{Mh_o}{2I}}$$

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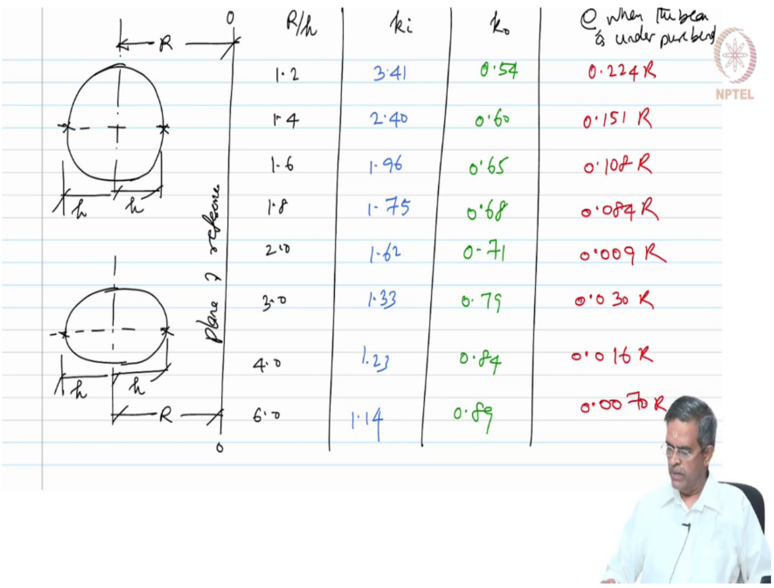

Let us say I draw the shapes here, this is my plane of reference o o. I write here plane of reference. So, let us say circular section and for elliptical sections because, elliptical node sections are also not coming very common now, because they have lot of advantages on eddies.

We will talk about this shape in some other future lectures. So, let us say we know if this is my centroidal axis, we know the measurement of the centroidal axis always R from the plane of reference. This is also R from the plane of reference and from the cg we call this as h is also h, because it is symmetric and the major axis is h minus axis h again. So, he is trying to work out only in the stresses in the intrados and extrados, only here, not anywhere in between.

So, he said the values of R by h and he has given the coefficients k_i and k_o, then he also compare e when the beam is under pure bending. So, let us see what are these values given by researchers. So, at different intervals of 1.2 1.4 6 a 2, then 3, 4 and 6; k_i 3.41 2.40 1.96 1.75 1.62 33 23 14; k o 0.54 60 65 68 71 79 84 89.

e value is also given in terms of R when the beam is under pure bending. Its says it is 0.224 R 0.151 R 0.108 0.084 0.009 0.030 0.016 and 0.0070. For rectangular sections, this table is again given in a different format, becomes the plane of reference.

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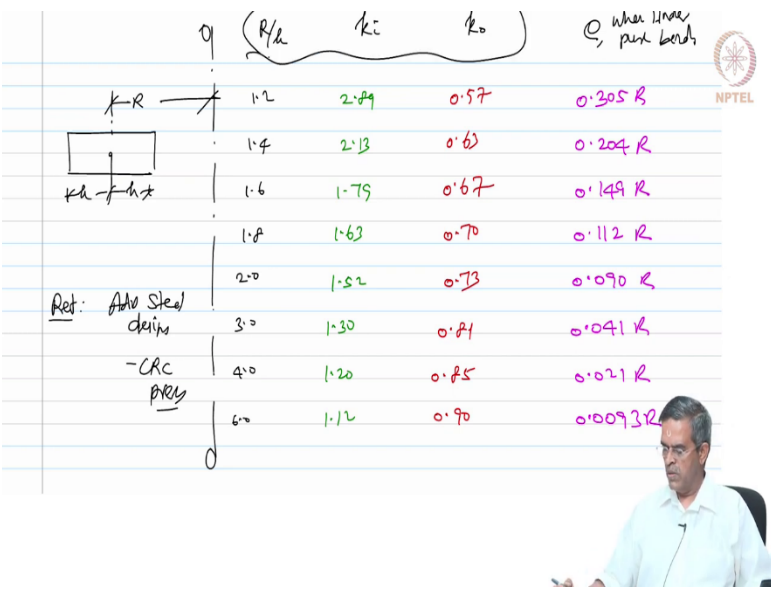

Say o o, we draw a rectangle centroidal axis measurement R from the plane of reference. Let us say this is h is also h. So, R by h k_i k_o and e when the beam is under pure bending. So, 1.2, k_o 0.305 R 0.204 R 0.149 R 0.112 0.090 0.041 0.021 0.00 93.

So, this is not very interesting reference, my own book advanced steel design written for CRC. Please refer this book to get more details about the experimental studies conducted by the authors and how they obtain these coefficients. Let us do a couple of more examples on circular sections.

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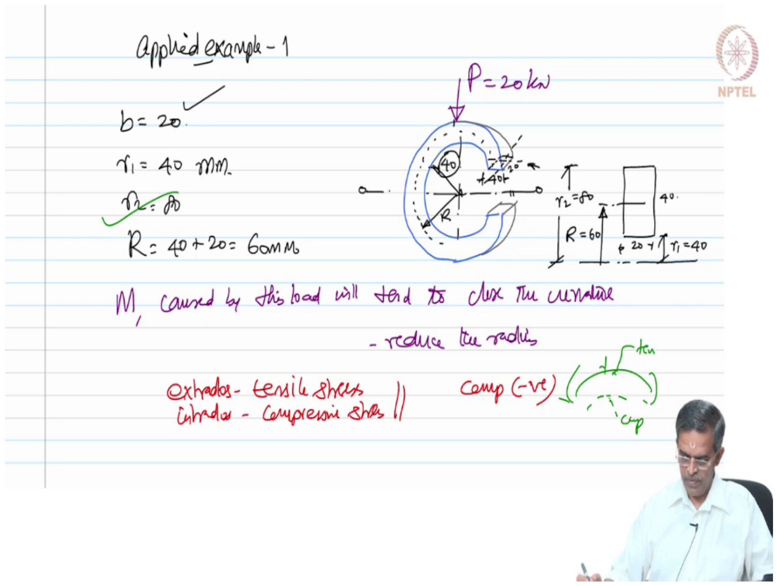

Let us say we have a curved beam, we have an applied example now. So, let us say there is a curved beam as shown in the figure here. Let me mark the gray. Let us have it like, this is better. Let us mark the cross section, this is 40, this is 20. Let us say this radius is also 40 point, this is central axis. Now, I call this as my o plane of reference because the curve is measured from here.

And, for the center, I will mark R as usual. I am doing an applied example. Now, let us draw this cross section separately. So, the cross section has got 40 parallel to plane, see 40 mm is parallel to o o. So, let me draw the line o o, let us say from the edge from the edge see from here, see from here this cross section is 20 and this is 40.

The cg from here is R which is 60. Because this is 40 and furthermore another 20 is from 60 point. So, we now say r_1 is 40 and r_2 will be 80 and b so, we say b is 20, r_1 is 40 and r_2 is 80 and R is 40 plus 20 which is 60 mm. Let us say this is subjected to a low P without 20 kN.

Now, this 20 kN P will try to compress this ring. So, M caused by this load will tend to close the curvature. It will reduce radius. Under that condition extrados will be under tension, on intrados will be under compressive stress. So, compression is negative for us.

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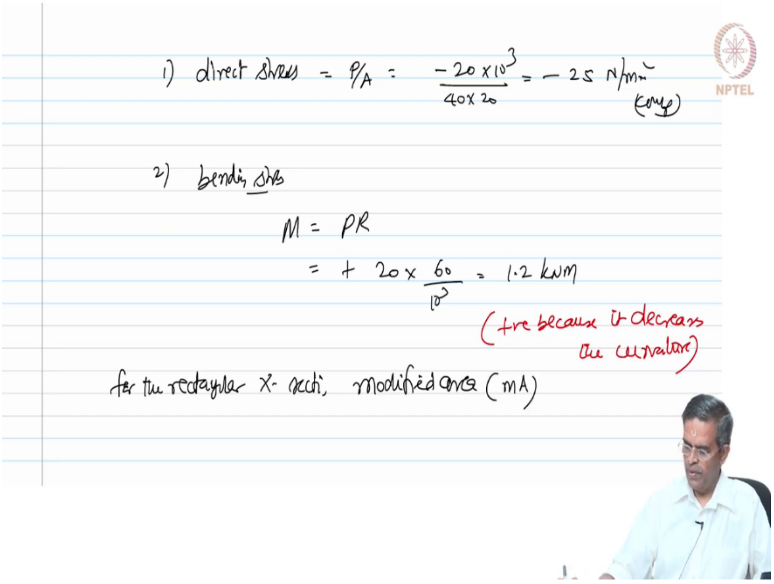

Let us try to find out the direct stress. What will be the direct stress? If you do not apply a curved beam concept which will be simply P by A which will be minus 20 into 3 by 40 into 20. Why minus? Compression. So, minus 25 Newton per mm square, we say negative stands for compression.

$$\frac{P}{A} = \frac{-20\times10^{3}}{40\times20} =- 25\frac{N}{mm^{2}}$$

$$M = PR =+ 20\times\frac{60}{10^{3}} = 1.2kNm$$

So, we can find the bending stress. Now, M will be actually P into R which will be plus because it is tending to close the curvature. So, 20 into 60 by 10 power 3 which is 1.2 kilonewton meter. Why it is positive? The moment is positive because it decreases the curvature, that is the reason. Now, we want to find for the rectangular cross section, what is the modified area m A.

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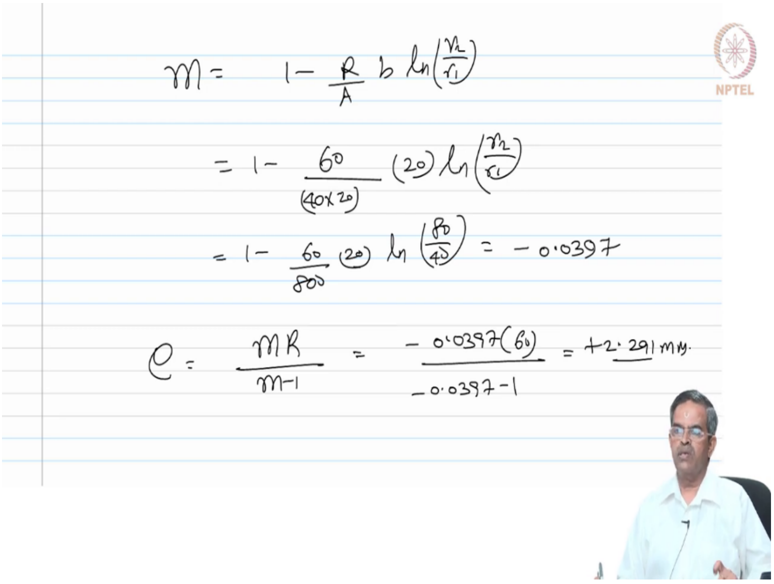

You have the equation m is given by 1 minus R by A b ln of r_2 by r_1, we have standard equation for this. So, 1 minus R by 40 into 20 that is A, b is 20 see we already said that b is 20 of ln of r_2 by r_1 which will be 1 minus 60 by 800 20 ln r 80 by 40 which will become minus 0.0397.

$$m = 1 - \frac{R}{A} bln\left(\frac{r_2}{r_1}\right)$$

$$m = 1 - \frac{60}{(40\times20)}(20)ln\left(\frac{r_2}{r_1}\right)$$

$$m = 1 - \frac{60}{(80)}(20)ln\left(\frac{80}{40}\right) =- 0.0397$$

$$e = \frac{mR}{m-1} = \frac{-0.0397(60)}{-0.0397-1} =+ 2.291mm$$

So, e will be mR by m minus 1 which will be minus 0397 into 60 by minus 0397 minus 1 which will become plus 2.291 m. So, there is a shift of neutral axis from the centroidal axis by about 2.3 mm though it is small, but look at the cross sections only 40 by 20 point. So, for the transaction is significant.

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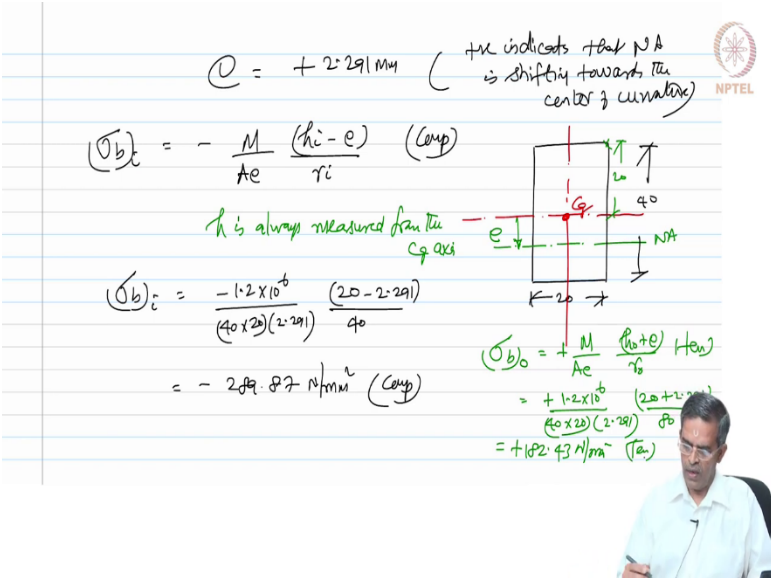

$$e = + 2.291mm$$

$$\left(\sigma_b\right)_i = - \frac{M}{Ae}\frac{\left(h_i - e\right)}{r_i}$$

$$\left(\sigma_b\right)_i = - \frac{1.2\times10^6}{(40\times20)(2.291)}\frac{(20-2.291)}{40}$$

$$\left(\sigma_b\right)_i = - 289.87\frac{N}{mm^2}$$

So, e is plus 2.291 mm. It means positive indicates that neutral axis is shifting towards the center of curvature. So, I have a cross section which is 20, 40, there is a central axis, this is my cg. I mean neutral axis is shifted down by a distance 2.291 m. And, for finding out the stresses for intrados, we know the equation which is minus M by A e h_i minus e by r_i which is compressive. Remember, h is always measured from the centroidal axis, not from the neutral axis please understand that.

So, let me substitute that. So, σ_b bending stress into dos will be minus 1.2 10 power 6 by 40 into 20 of 2.291 of 20 minus 2.291 by r i which is 40. So, I can get this value which is minus 289.87 complex this intrados. Let us do this for extrados. So, bending stress at extrados that is this point which will be plus M by A e h_o plus e by r_o tensile. So, I should say plus 1.2 10 power 6 by 40 into 20 into 2.291 of 20 plus 2.291.

Because, you know this is 20, divided by 80 that is r_o, we already have it here, on this stress is plus 182.43 tensile, extrados tensile. So, I have the hook coming here the ring, the ring is subjected to compression. So, this is the moment. So, extrados will be in tension and intrados will be in compression. So, we have these values with this.

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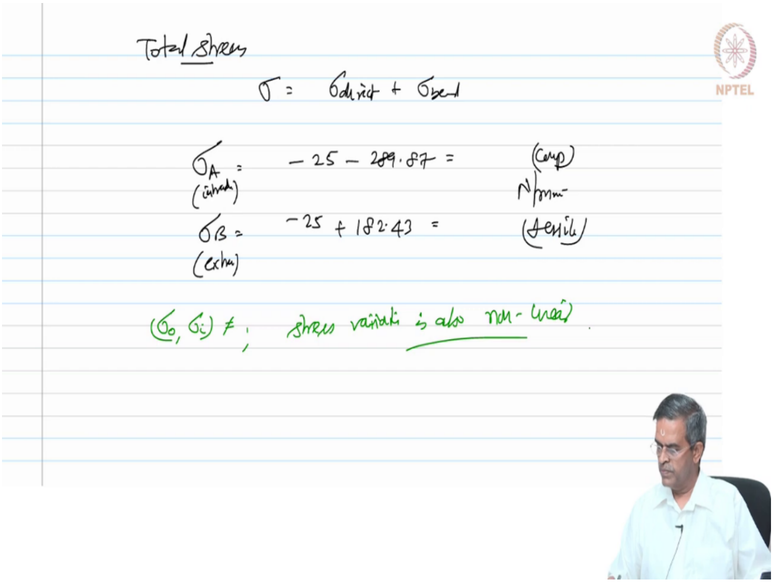

$$\sigma = \sigma_{direct} + \sigma_{bending}$$

So, therefore, the total stress will be stress direct plus stress bending. So, if you want to find at point A which is minus 25 minus 289.87, stress at B which will be minus 25 plus 182.43. So, B is any point on extrados and A is any point on intrados. I can find these values in newton per mm square.

This will be compressive, this will be tensile. So, one can also see here, the stress that extrados and strength intrados are not equal which means the stress variation is also non-linear. So, you cannot predict that, you have to find the exact fiber distance and then keep on working out the stresses.

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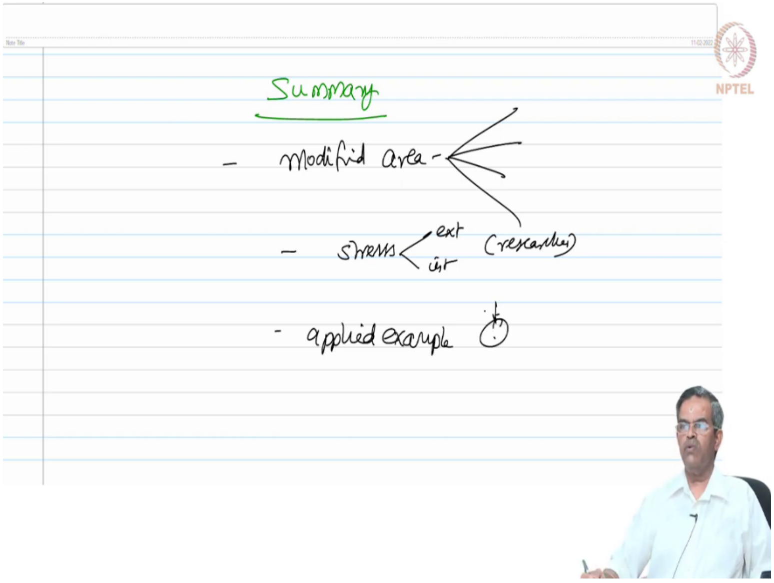

So, friends in this lecture, we learnt the modified area equation for different sections. We also learnt the stresses to find in extrados and intrados by researchers. We also did a applied example of a ring under the compression loop. So, in the next lecture, we will do some more examples on rings and crane hooks etcetera.

We will use MATLAB program and try to find out the use of winkler-bach equation in large initial curvature problems and try to solve and find the stresses in the sections across the curved beam.

Thank you very much and have a good day. Bye.