Advanced Design of Steel Structures
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Lecture - 50
Thin - walled section

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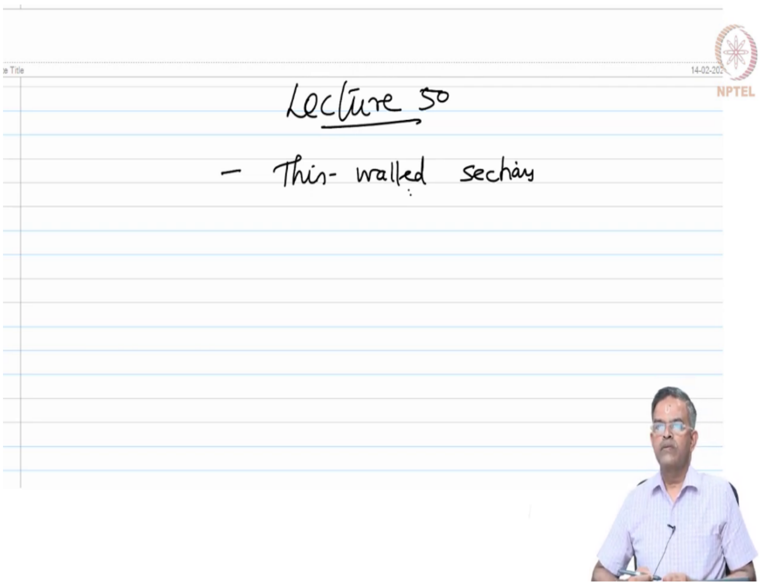

Welcome to the 50th lecture on the course Advanced Steel Design. In this lecture, we are going to learn more about Thin-walled sections which lead to the discussion for lateral torsional buckling (LTB).

We all know that structural steel is the most preferred construction material for many engineering projects. Structural steel offers more advantages compared to other construction material which is popular which is reinforced concrete. In fact, there are certain applications where steel is considered to be a non-replaceable only construction material.

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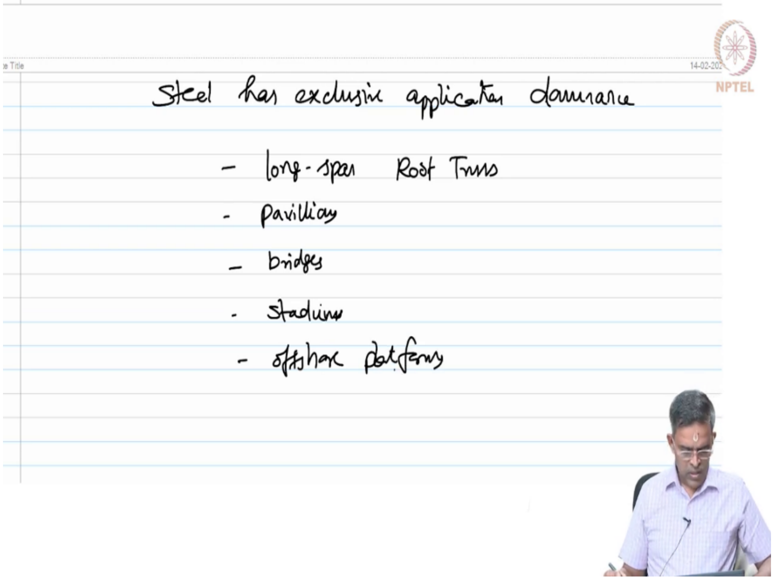

So, steel has exclusive application dominance in certain areas like long span roof stresses, pavilions, cable stayed bridges, stadiums and of course, offshore platforms.

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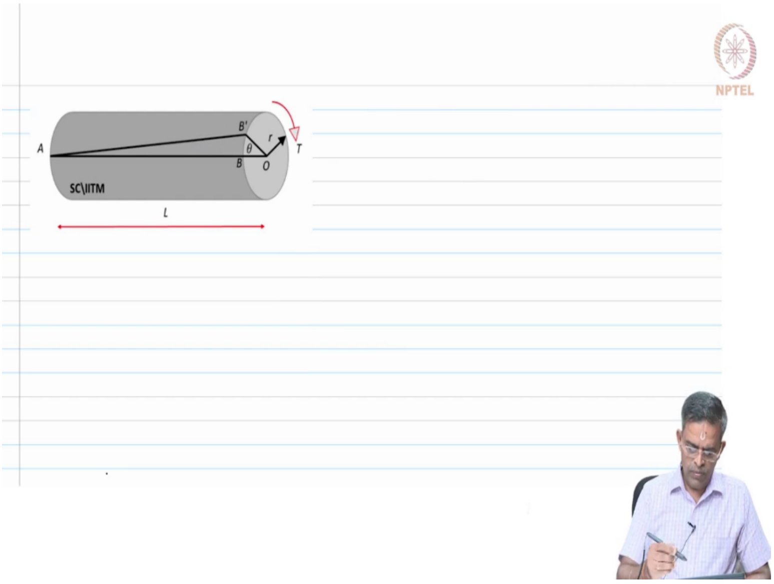

Steel has got very exclusive advantages; we have seen them in repeated lectures.

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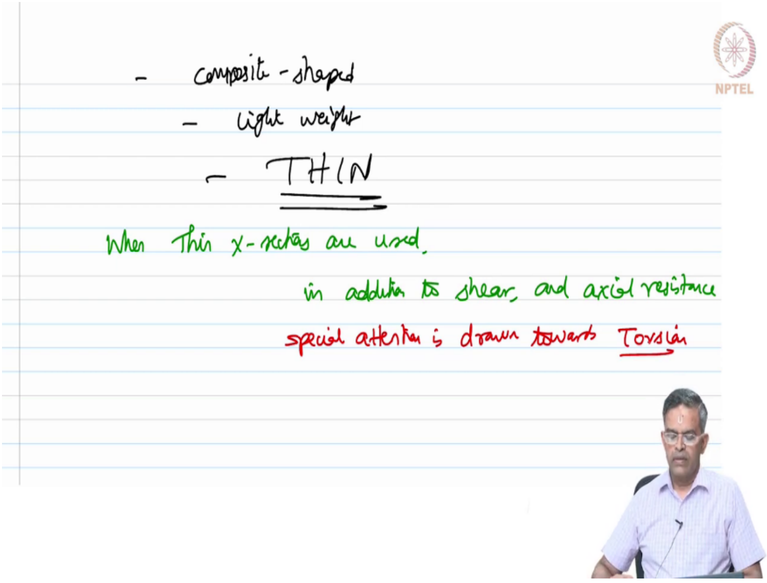

Usually, structural steel is available in different shapes. So, we can say they are composite shaped they have light weight and of course, thin sections are also used. When we talk about thin cross-sections there is a complexity. In addition to shear and axial resistance, other parameters should also be checked for design of steel beams and columns when you use thin sections.

So, there are well laid design procedures in design course design checks are carried out with advanced application under the structural response. Therefore, in the thin sections special attention is paid is drawn towards torsion.

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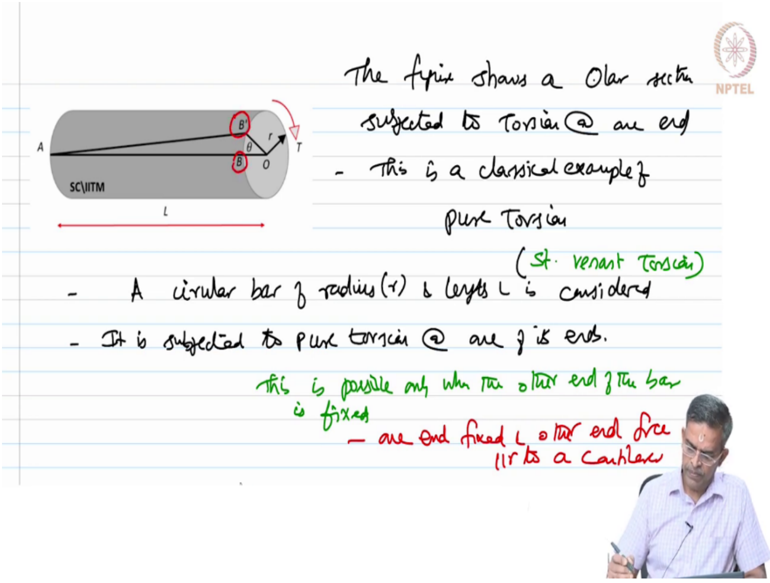

Let us look at the figure. I will copy this figure here. This is a circular section subjected to torsion at one end. In case of pure torsion which is referred as Saint – Venant torsion, so, this is a classical example of pure torsion which is referred also as Saint – Venant torsion.

So, let us try to understand this slightly in a better manner a circular bar of radius r and length L is considered. This bar is subjected to pure torsion at one of its ends. This is possible only when the other end of the bar is fixed. So, this is a case of one end fixed and other end free which is similar to a cantilever.

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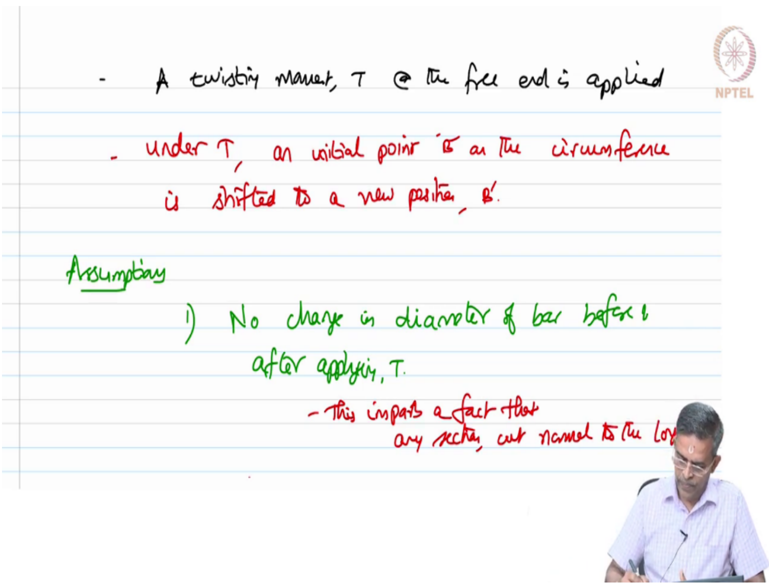

Now, we are applying a twisting moment of magnitude T at the free end. This simulates a cantilever beam as well; we have to keep it in mind. Now, under the influence of this torsion an initial point is shifted to a new position, see this figure. The initial point B is shifted to a new position B prime. So, under torsion on the circumference is shifted to a new position $B^{'}$.

Now, we make certain assumptions we will say that there is no change in diameter of the bar before and after applying T. So, this imparts a very important condition. This imparts a fact that any section cut normal to the longitudinal axis of the member is circular despite torsion is being applied that is a very interesting assumption.

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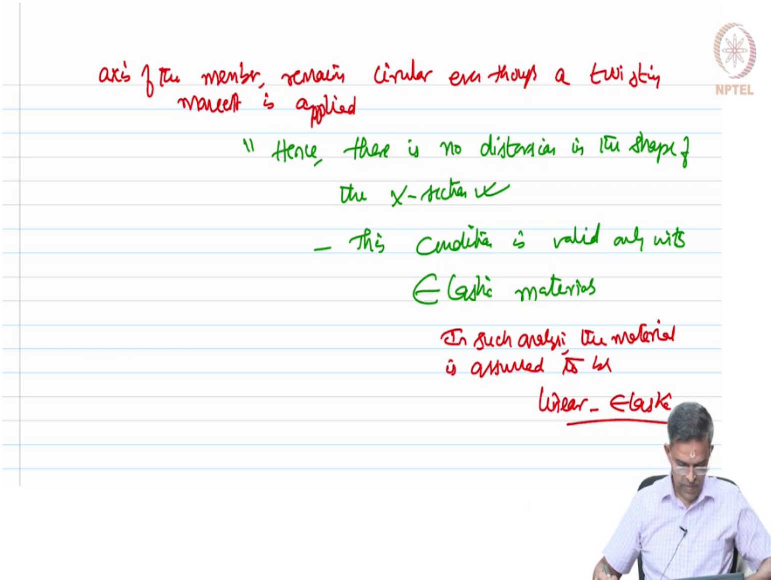

So, any section cut normal to the longitudinal axis of the member remains circular even though your twisting moment is applied. So, there is no distortion in the shape of the cross-section because the cross section initially was circular later on also it remains circular even though it is subject to torsional collinear. Now, interestingly friends, this condition is valid only if the material is elastic.

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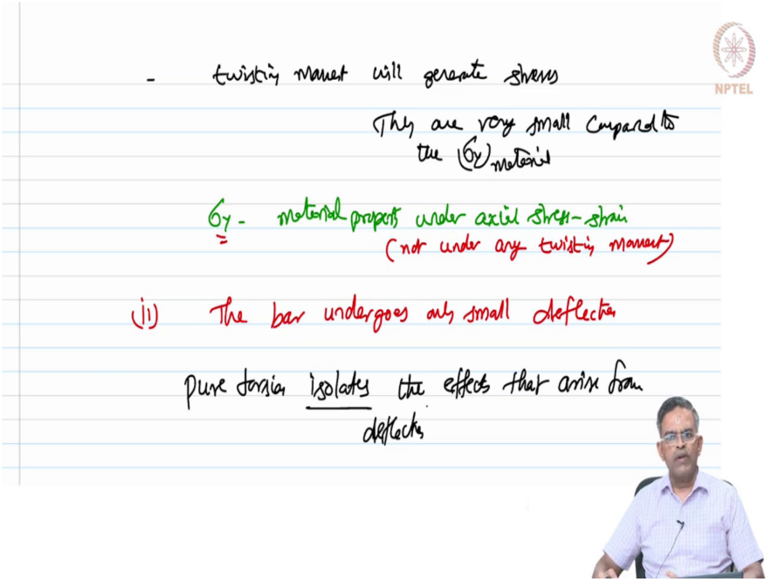

Therefore, in such analysis the material is assumed to be linear elastic. Also, friends please note twisting moment will generate stresses. They are very small compared to the yield stress of the material please understand this the magnitude of stresses developed because of this twisting moment is very small.

Now, please note yield stress is a material property under axial stress strain, please understand that. It is not under any twisting moment. Yield stress is calculated based upon axial stress strain, but this is a very important assumption we make, so that is helpful for our analysis.

The second assumption what we make is the bar undergoes small deformation let us say not deformation let us call this as a deflection. It is a cantilever bar; it undergoes small deflection. This is a very important assumption because it conforms to the fact that if large deflection is expected cantilever, then the behavior is of a different phenomenon.

Therefore, friends, pure torsion isolates the effects that arise from deflection of the member which will be inherently present in the member, but that is being neglected. In simple terms, when we focus on effect of twisting moment applied at the free end this end has already undergone a concept deflection which is oversight, that is a very important statement we want to make here.

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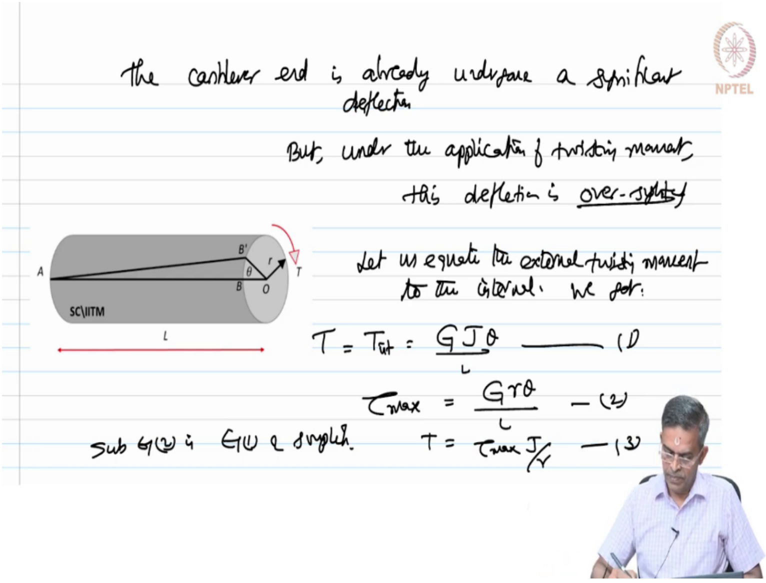

The cantilever end is already undergone a significant deflection. But, under the application of twisting moment this deflection is over sighted. We are not looking into it, we are considering it. Now, let us copy this figure. Now, we will equate the external twisting moment to the internal.

$$T = T_{int} = \frac{GJ\theta}{L}$$

we call equation number 1.

$$\tau_{max} = \frac{Gr\theta}{L}$$

Equation 2. Now, let us substitute equation 2 in 1 and simplify we get T as let us say G theta by L is tau max. So,

$$T = \tau_{max} \frac{J}{r}$$

this calls equation number 3.

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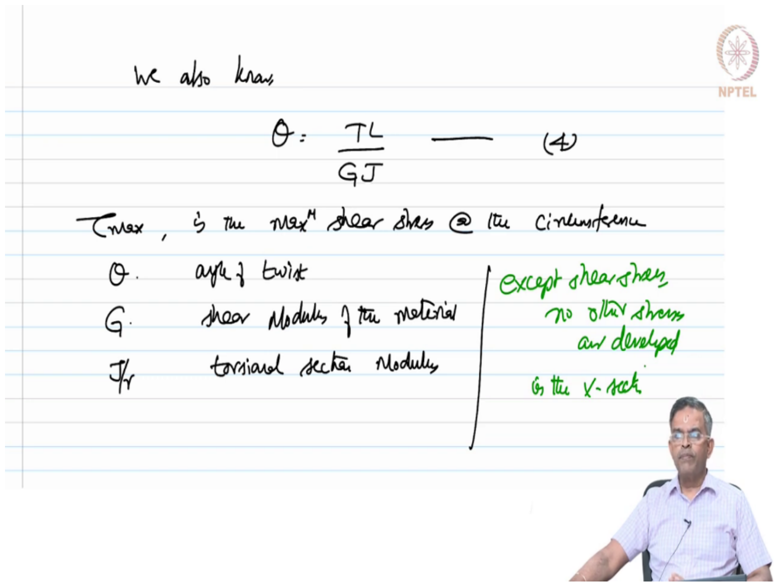

Having said this, we also know that

$$\theta = \frac{TL}{GJ}$$

which can be written from the simplification. Now, τ_{max} in this expression is the maximum shear stress at the circumference. θ is the angle of twist; G is the shear modulus of the material; J by r is termed as torsional section modulus. It is very important to note friends, except shear stress, no other stresses are developed in the cross-section, in the cross-section.

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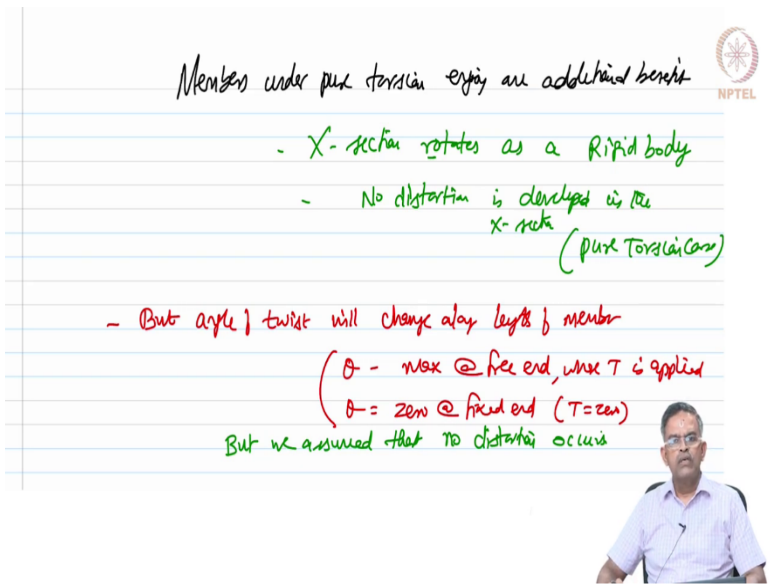

Now, when you look at a member under pure torsion, it enjoys one more benefit. Members under pure torsion enjoy one additional benefit. What is that? Cross-section rotates as a rigid body. So, what does it mean? No distortion is developed in the cross section when T is applied, if it is a pure torsion case.

But, the angle of twist will change along length of the member. Probably, theta will be maximum at the free end where T is applied and theta will be 0 at the fixed end. So, theta will keep on changing along the length of the member. It keeps on decreasing or the section travels towards the fixed end where the twist is 0.

So, we should say theta is maximum at T is maximum; θ=0 where t is 0, there is no twisting moment at the end, but we have assumed that no distortion occurs. So, then what will happen to the cross-section? See, you are having a change in theta. So, it has to twist the section, but we said that section, cross section shape does not change at all under pure torsion, then what should have happened to the section? The section undergoes warping.

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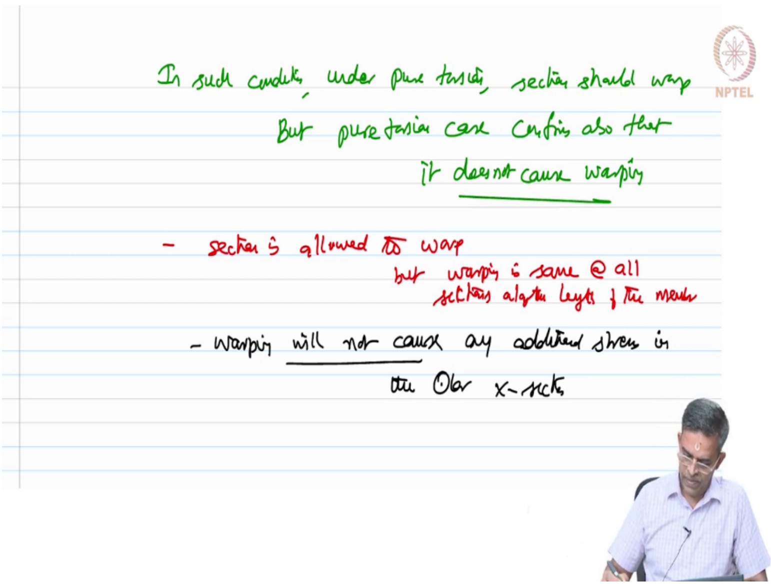

So, in such condition under pure torsion the section should warp, but pure torsion case confirms also that it does not cause warping. Therefore, the angle of twist is allowed to change emphasizing the fact along the length of the member is allowed to warp, but warping is same at all cross-sections along the member that is a beautiful assumption technically valid.

Section is allowed to warp, but warping is same at all sections along the length of the member. Therefore, friends warping will not cause any additional stresses in the circular cross-section.

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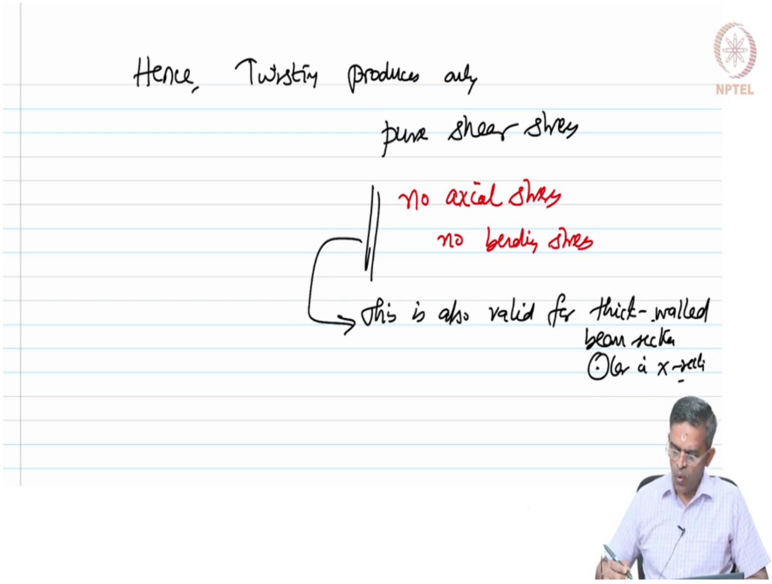

So, testing produces only pure shear stress no axial, no bending stress no axial stress and no bending stress. Such a behavior under pure tension is perfectly valid for circular cross section in addition to thick-walled beam sections. So, this is also valid for thick-walled beam section which is circular in cross-section. Having said this, let us see what happens in case of a closed thin wall section.

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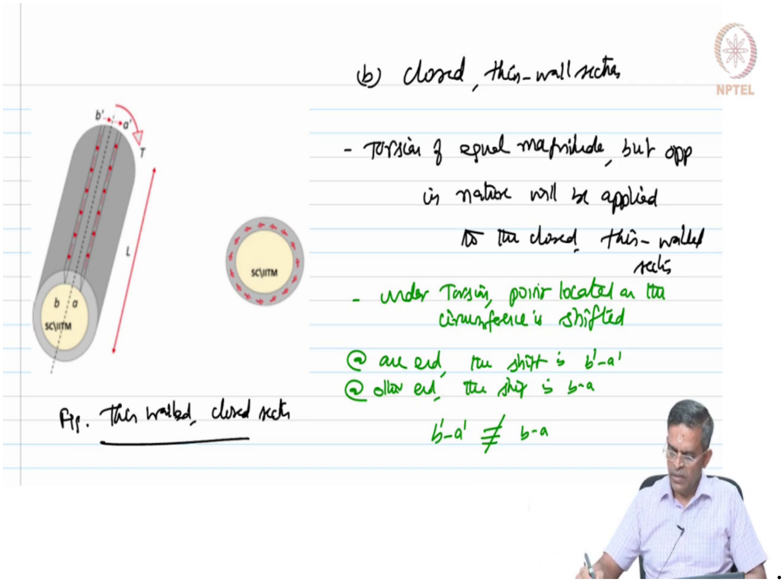

Let us say a closed thin-walled section as shown in the figure. In case of a closed thin-walled section the behavior is different. Torsion of equal magnitude, but opposite in nature will be applied to the closed thin-walled sections. Let us consider a thin-walled section as shown the figure.

Let the length of the section be L under the action of applied torsion one can notice that the point located on the circumference is shift. The amount of shift is not same at both ends of the member. For example, at one end the shift is $b^{'}a^{'}$, at the other end the shift is b-a. Now, the shift $b^{'}a^{'}$ is not same as b, they are different.

For a thin-walled section namely tubular or shaft section one can say the Saint – Venant torsion constant is given by the following equation.

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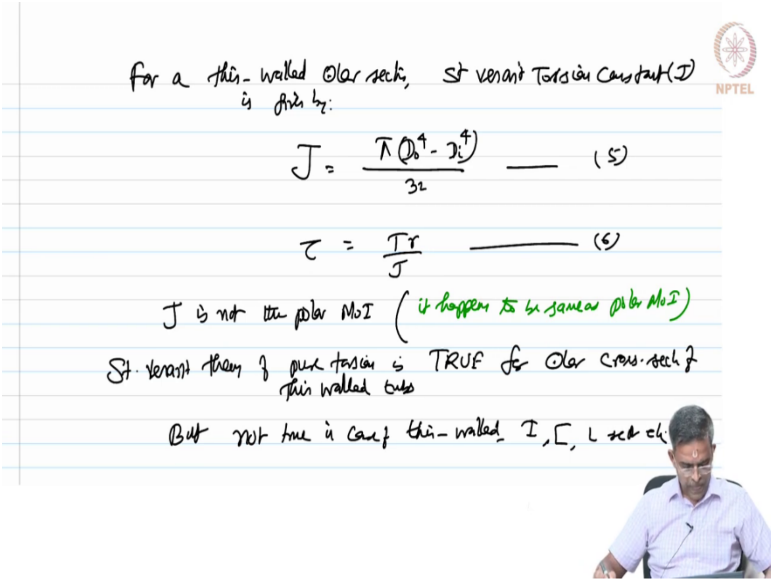

$$\tau = \frac{Tr}{J}$$

Saint – Venant torsion constant J is given by

$$J = \frac{\pi(D_0^4 - D_i^4)}{32}$$

equation number 5 and

$$J = \frac{\pi(D_0^4 - D_i^4)}{32}$$

which is 6. Friends, even in closed thin-walled sections only pure torsion is developed. Most importantly J is not the polar moment of inertia, but it happens to be same as polar moment of inertia.

Hence friends, Saint – Venant theory is perfectly true for circular sections. So, Saint – Venant theory of pure torsion is true for circular cross-sections of thin-walled tubes, but not true in case of thin-walled I section, channel section, L section etcetera. So, only for circular this is true.

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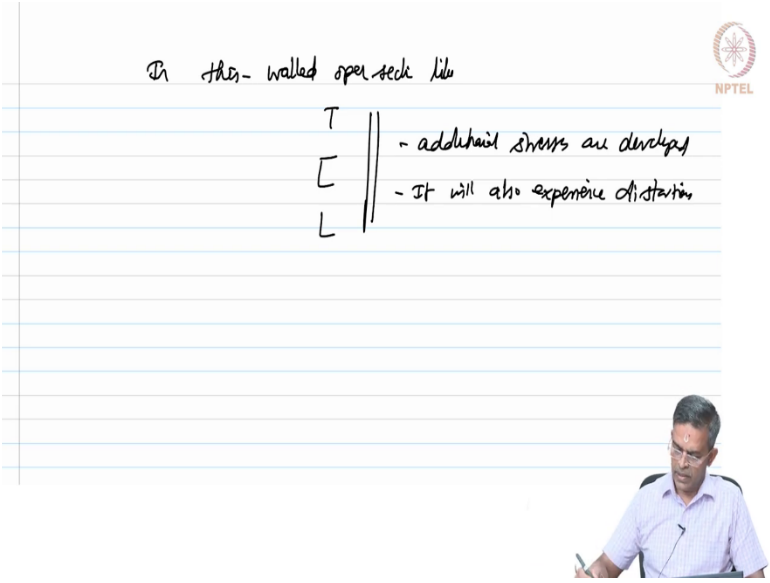

Then you may ask me a question in these following sections. What happens additional in thin-walled open sections like T section, channel section, angle sections. Additional stresses will be developed, it will also experience distortion.

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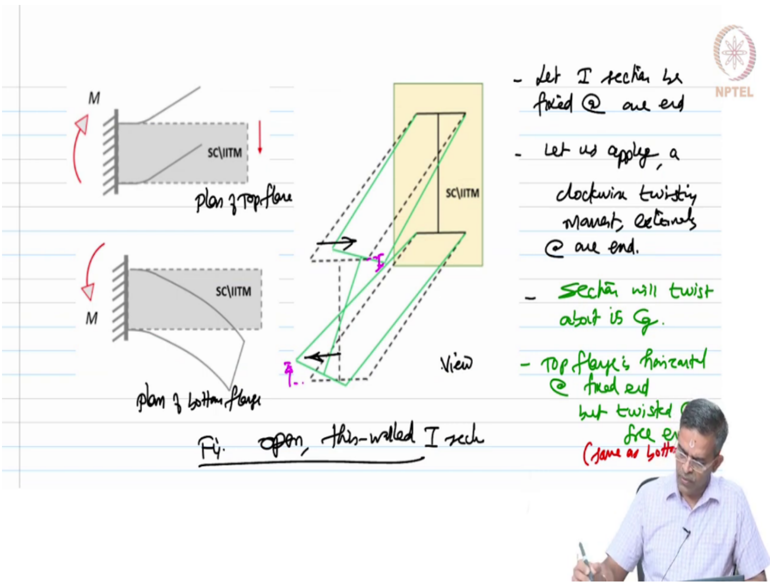

Let us say for example, in I section as shown in the figure. So, look at I section as shown in the figure. So, this is my plan view, of the top flange, this is plan of the bottom flange, then this simply view shown. Here it is an open thin-walled I section.

Let us consider this figure with I section fixed at one end. A clockwise torsion is applied externally. Let us apply a clockwise twisting moment external at one end. The section will twist as you see on the figure, very important observation. Section will twist about it is C g, first observation.

Second observation, you will see that the top flange is horizontal at the fixed end, but twist at free end correct same with the bottom flange also. So, now, one can say the twist is seen in every section along the length of the beam.

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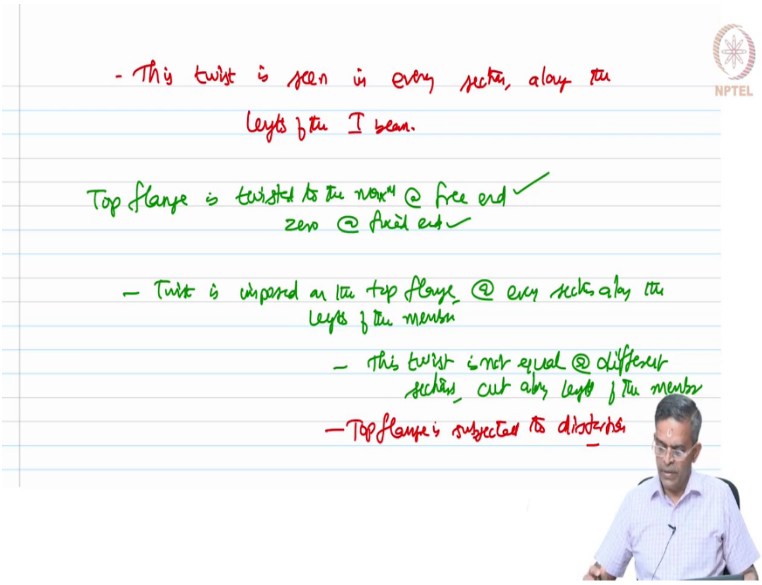

This twist is seen in every section along the length of the beam. In the top flange is twisted to the maximum we can see top flange is twisted to the maximum at the free end and 0 and the fixed end, which means that twist is imposed on top flange at every cross section long length of the member. So, twist is imposed on the top flange at every section along the length of the member.

Further this twist is not equal at different sections cut along the length of the member, because how we can say this? It is maximum with the free end and 0 with the fixed end so, it is not same. So, when I have a twist in the top flange which is happening at every section along the length of the member which is also not uniform. We can say that top flange is subjected to distortion.

It implies the fact that bottom and top flanges have twisted and bent as well, see the figure. There is a downward deflection, I am marking the point here from here to here, there is a downward deflection, but from here to here there is an upward deflection.

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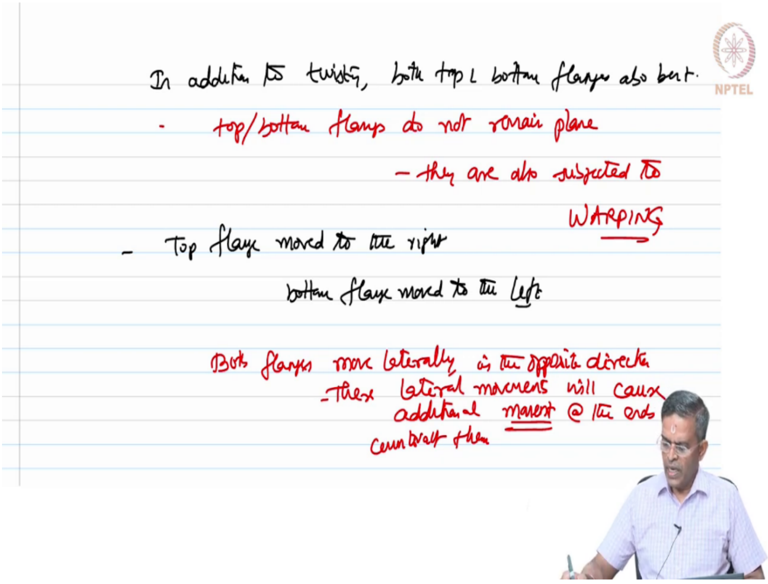

So, what we can say here is in addition to twisting both top and bottom flange planes also bent. So, therefore, the top and bottom flanges do not remain plain anymore. They are subjected to warping.

Furthermore, one can also notice the top flange is literally moved to the right side while the bottom flange move to the left. See the figure here. Top flanges move to the right while bottom flange move to the left, which means that, both top and bottom flanges move in the opposite direction laterally so, therefore, both flanges move laterally in the opposite direction. These lateral movements will now cause additional moments and the ends to contract.

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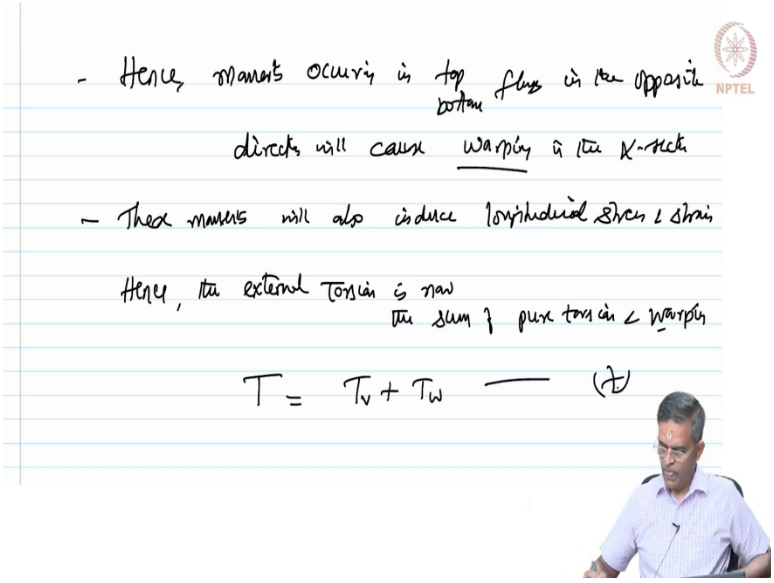

Hence moments occurring at the top and bottom flanges in the opposite direction will cause warping in the cross-section. In addition, these moments will also induce longitudinal stress and strain. So, therefore, friends in an open thin-walled section, other than circular application of torsion resultant shear stress which also caused additional bending stress.

Hence the total external torsion can be expressed as a sum of pure torsion and warping. Hence the external torsion is now the sum of pure torsion and warping. So, T will be equal to $T = T_V + T_W$ equation number 7.

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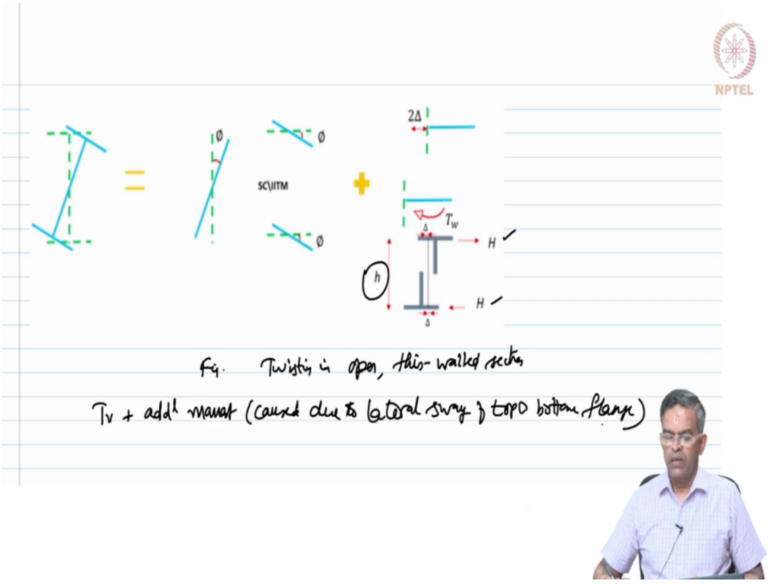

Now, having said this please pay attention to this figure where the rotation is indicated as theta or phi and each displacement laterally of the top and bottom by one delta is summed up as 2 delta and there is a moment cost or a couple which is effective with a horizontal force H separated by height of the member.

So, this is a figure which shows twisting in open thin-walled sections other than circular, it is a nice section. So, the twisting moment applied at one end can be seen as a sum of pure torsion and additional moment T w. So, I can now say pure torsion plus additional moment. This additional moment is caused due to the lateral sway of top and bottom flange. So, they move with the same amount delta. So, the next way is 2 delta, but they are in opposite direction.

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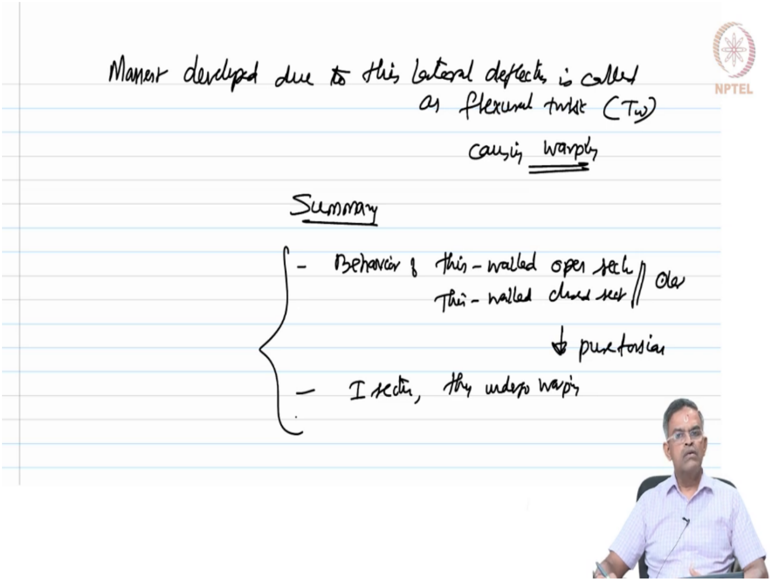

Now, moment developed due to this is called as flexural twist which I call as T w causing warping. It also idealizes that each of the flange bends is a rectangular beam about its minor axis. Each of the flange bends about minor axis rectangular b that is an assumption we make.

So, friends, in this lecture we started learning the behavior of thin-walled open sections, thin-walled closed sections which are circular which represents the fact of pure torsion or otherwise called as Saint – Venant’s torsion. On the other hand, if you have open thin walled, I section for example, they undergo warping. So, now, our job is to find out the behavior of such sections under warping and try to develop a governing equation for the effect of torsion on such sections. So, we will discuss this in the next lecture.

Thank you very much and have a good day, bye.