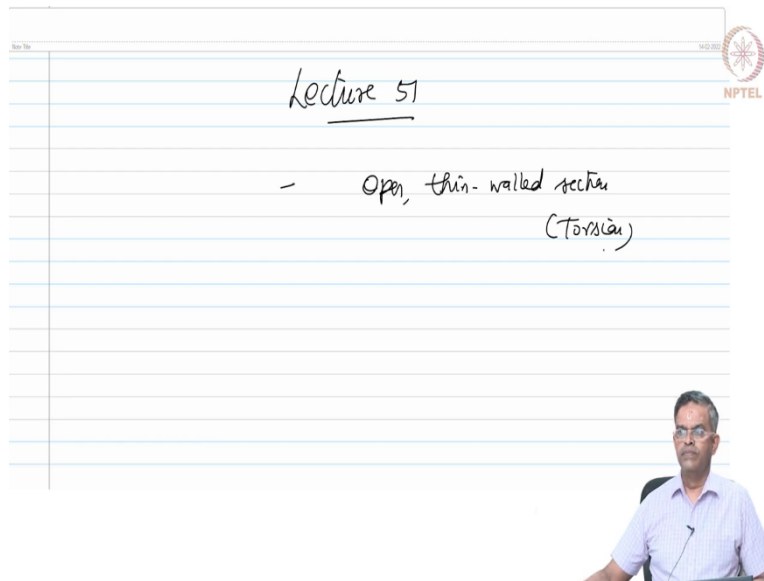


**Advanced Design of Steel Structures**  
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**Indian Institute of Technology, Madras**

**Lecture - 51**  
**Open thin-walled section**

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Friends, welcome to the 51st lecture of Advanced Steel Design. In this lecture, we will continue to learn more about torsion in Open thin-walled sections.

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Additional stresses will be developed due to warping

- applied torsion will be pure

$T = T_v + T_w - (H/d)$

$T_v = GJ \frac{\theta}{L} = GJ \frac{d\beta}{dz}$  (8)

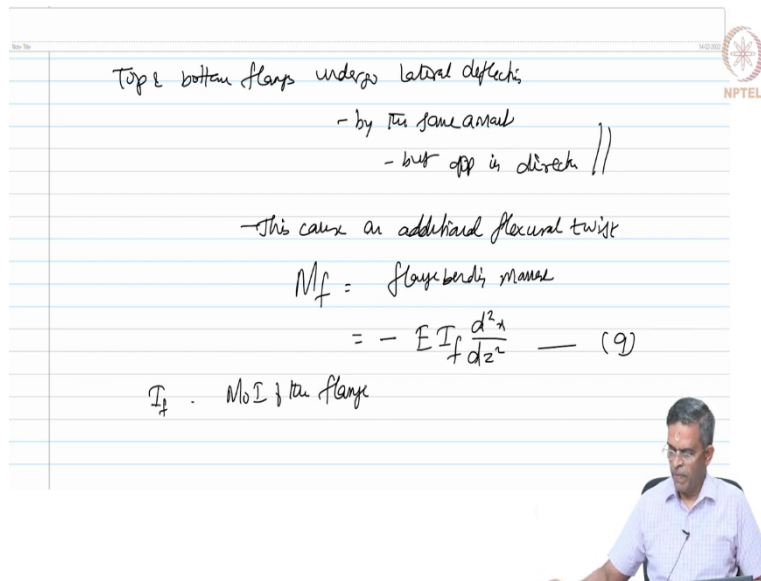
When  $\frac{d\beta}{dz}$  is the change in twist @ every xch, along the length of the member (z axis).

We already said that in case of open thin-walled sections additional stresses will be developed due to warping. Hence therefore, the applied torsion will be now equal to torsion due to pure shear + warping. This equation 7, we already have with us. Now, referring to this figure, we say the torsion is supplied and an axis X, Y, Z and it is about Z axis which can be a sum of pure torsion + torsion caused because of warping.

And, this is the shift what we have seen which is equally displaced both top and bottom flanges, in the opposite direction by delta by 2 or x by 2. They create a couple which is H into d, as we see in the figure. So, T w will be now counteracted by this additional moment. Now, let us see what is T v, T v is (Refer Time: 02:53) torsion which is  $GJ\theta$  by L which can be said as  $GJ d \beta$  by dz, where we call this equation number 8.

Where,  $d \beta$  by dz is the change in twist at every section along the length of the member. In the figure, you can easily see the length of the member is measured along the Z axis, and I right see here. In addition to this, top and bottom flanges undergo later deflection by the same amount which resulted in additional fractional twist.

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Top & bottom flange undergo lateral deflection  
- by the same amount  
- but opp in direction //

- This causes an additional flexural twist

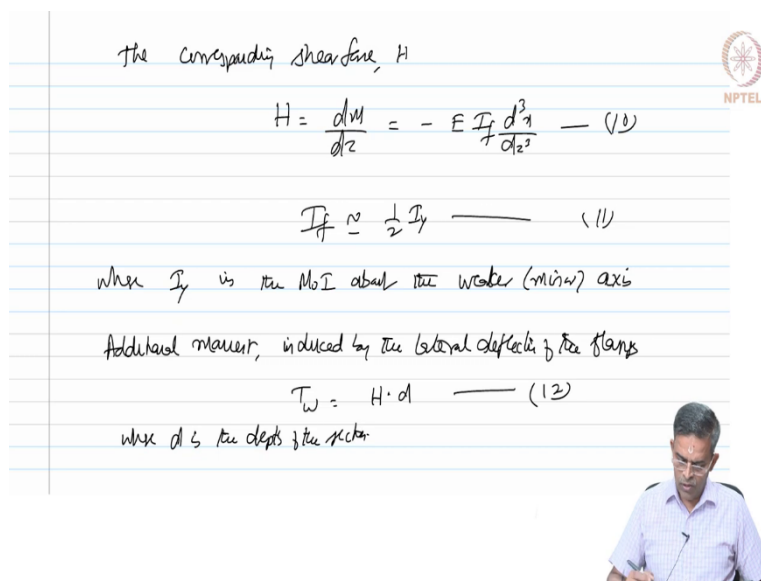
$$M_f = \text{flange bending moment}$$
$$= -E I_f \frac{d^2 x}{dz^2} \quad \text{--- (9)}$$

$I_f$  . MoI of the flange

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Top and bottom flange undergo lateral deflection by the same amount, but opposite in direction correct. So, these results or this cause an additional flexural twist. So, this additional flexural twist is expressed as  $M_f$  which is the flange bending moment which is given by  $-EI_f \frac{d^2 x}{dz^2}$ . We call this equation number 9.  $I_f$  indicates the moment of inertia of the flange and - indicates, it is opposite to the applied twisting moment. Also note the second derivative is along the X axis which is the lateral deflection, as we have seen in the figure.

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the corresponding shear force,  $H$

$$H = \frac{dM}{dz} = -E I_f \frac{d^3 x}{dz^3} \quad \text{--- (10)}$$
$$I_f \approx \frac{1}{2} I_y \quad \text{--- (11)}$$

where  $I_y$  is the MoI about the weaker (minor) axis

Additional moment, induced by the lateral deflection of the flange

$$T_w = H \cdot d \quad \text{--- (12)}$$

where  $d$  is the depth of the section

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Therefore, the corresponding shear force is given by  $H$ , which is actually equal to  $dm / dz$  because that is the length of span which is  $- E I_f d^3x / dz^3$ , equation 10. So,  $I_f$  is approximately half of  $I_y$ , that is usually the design practice, where  $I_y$  is the moment of inertia about the weaker axis, I should say minor axis.

Also, please understand that the web contribution in moment of inertia is neglected. Now, the additional moment induced by the lateral deflection of the flange  $T_w$  is given by this equation. The additional moment induced by the lateral deflection of the flanges is  $T_w$  which is  $H$  into  $d$ . We call this equation number 12, where  $d$  is the depth of the section.

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for a small angle of twist, following Eq 10 and 11.

$$\chi = \frac{\beta d}{2} \quad \text{--- (13)}$$

Hence,  $T_w = - E I_y \frac{d^2}{4} \frac{d^3 \beta}{dz^3}$  --- (14)

The total applied torsion,

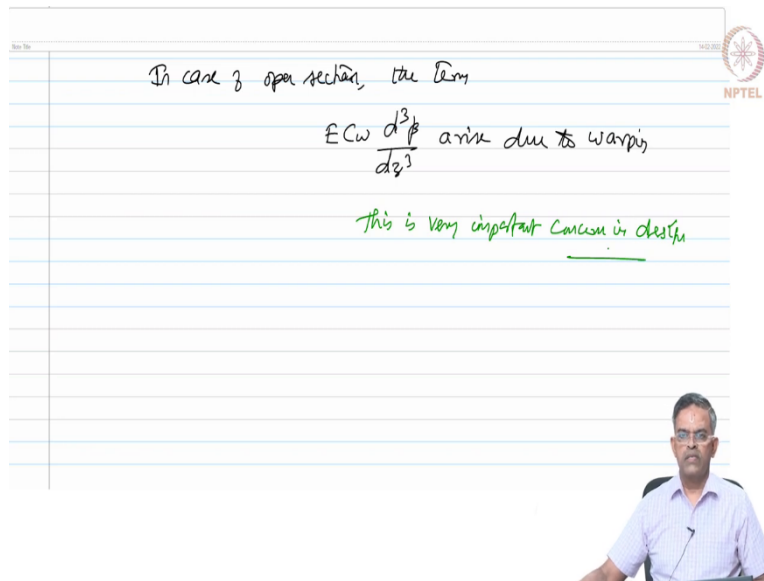
$$T = GJ \frac{d\beta}{dz} - E C_w \frac{d^3 \beta}{dz^3} \quad \text{--- (15)}$$

where  $C_w =$  warping constant ( $\approx I_y \frac{d^4}{4}$ )

$GJ$  - Torsional rigidity  
 $E C_w$  - warping rigidity

Now, friends for a very small angle of twist, we can assume the following  $x$  is  $Bd/2$ . Hence,  $T_w$  will be now  $-E I_y d^2 / 4, d^3 \beta / dz^3$ , equation 4. Now, we can say the total applied torsion is given by  $T$  which is  $GJ dB/dz - EC_w d^3 \beta$  by  $dz^3$ , where  $C_w$  is called warping constant which is approximately equal to  $I_y d^2 / 4$ . In the above equation,  $GJ$  is called torsional rigidity and  $E C_w$  is called warping rigidity.

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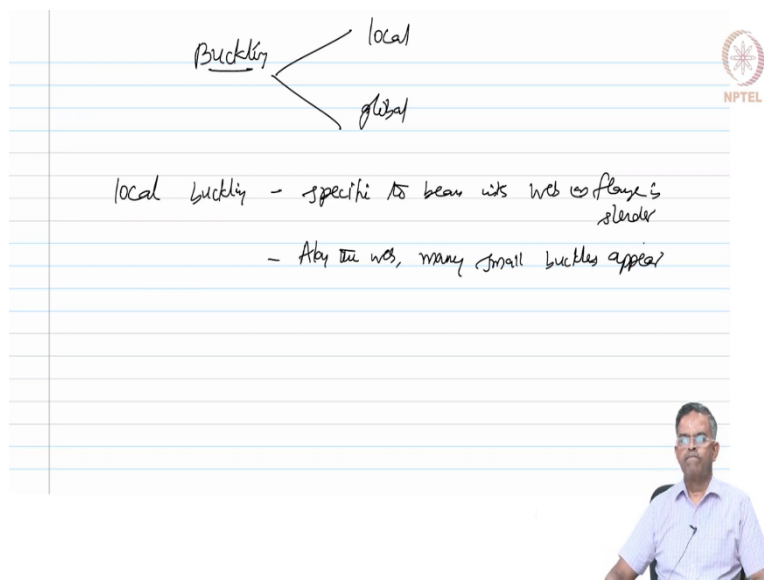


In case of open section, the term  $E C_w \frac{d^3 \beta}{dz^3}$  arise due to warping. This is very important concern in design.

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So, in case of open sections which has an additional flexural twisting, the term  $E C_w \frac{d^3 \beta}{dz^3}$  arise due to warping. This is a very important concern in design. Having understood how torsion is initiated and how warping is built in open sections; let us pay attention to buckling.

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Buckling

- local
- global

local buckling - specific to beam with web is flange is slender  
- At the web, many small buckles appear

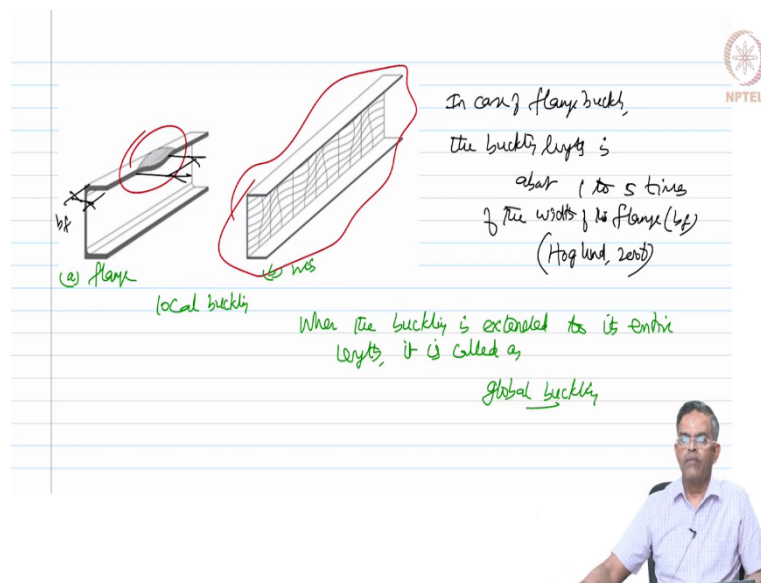
The slide features a white background with blue horizontal lines. The text is handwritten in black ink. The NPTEL logo is visible in the top right corner. A small inset image of a man in a light blue shirt is positioned at the bottom right of the slide.

We already know buckling failure is one of the dominant modes of failure in many steel structures. Buckling is one of the modes of failure, when a structural member is displaced laterally out of its plane under compressive stresses. The lateral displacements are associated

with the flexural stresses whose magnitude depends on slenderness ratio of the member. We also know buckling of the beam can be categorized into local and global buckling.

Local buckling is specific to beams whose web or flange is slender. So, buckling will have two components, local and global. Local buckling will be specific to beams with web or flange is slender. So, therefore, many small buckles tend to appear along the web. So, along the web many small buckles appear.

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So, one can see this figure, can see along the web there are lot of small buckling. If the flange is slender can also happen in the flange. So, in case of flange buckling, the buckling length varies between 1 to 5 times of the width of the flange. In case of flange buckling, we have a we have a condition, the buckling length which is we call this as the buckling length, is about 1 to 5 times of the width of the flange whereas, is the width of the flange.


This is an experimental observation made by Hogland in 2006. When the buckling of the beam is extended to its entire length, it is termed as global buckling. This type of buckling is further classified based on type of loading and nature of displace. So, this is essentially the local buckling figure. This is on the flange. This is on the web.

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Global buckling modes

- due to buckling, steel beam can fail in different modes
- Modes of deformation are : Bending, torsion, warping

If we need to estimate buckling resistance, we should estimate the flexural, torsional and warping stiffness of the beam.

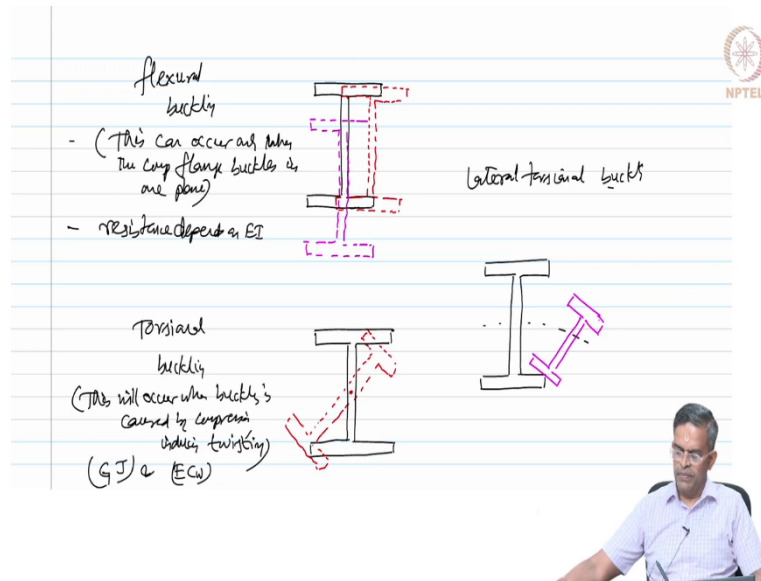


Now, let us pay more attention to the global buckling modes. Due to buckling steel beam can fail in different modes. The failure modes are governed by type of the load and deformation shape of the member. Furthermore, they can be subject different types of deformation simultaneously, bending, torsion and warping. The resistance of the beam depends on the stiffness and different modes of the deformations.

So, the possible modes of deformations are bending, torsion and warping, each one of them has different kind of stiffness. So, the resistance of the depends against this kind of bending or deformation depends on the respective stiffness. Therefore, flexural, torsional and warping stiffness are essential for estimating buckling resistance.

So, if one need to estimate buckling resistance, one should estimate the flexural, torsional, and warping stiffness of the beam. Therefore, beams overall buckling resistance is a combination of flexural, torsional buckling and lateral torsional buckling. Let us see how they look like in different modes.

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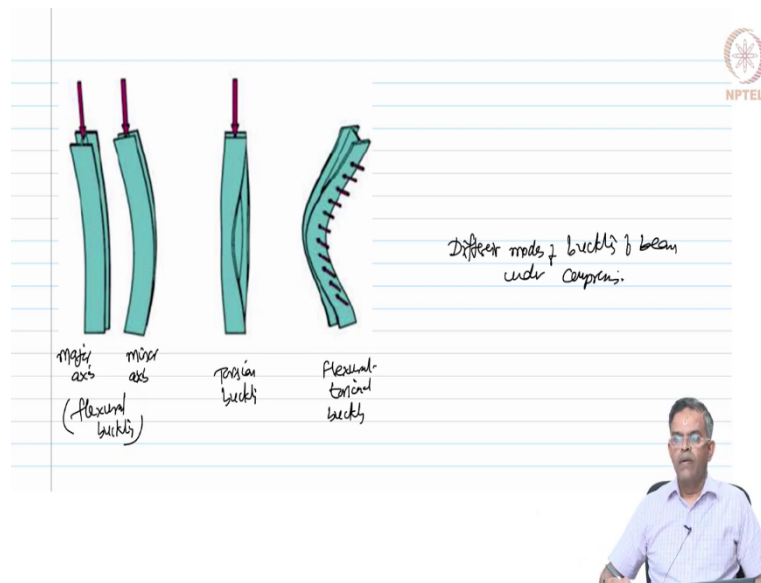
So, if I say flexural buckling, if this is my open I section. There will be a shift either in this format or in this format. If we talk about torsional buckling, the original I section will get rotated about its cg. When we talk about lateral torsional buckling, then I section is completely displaced like this.

Flexural buckling can occur only when the compression flange buckles in one plane. Resistance of the beam to this kind of buckling depends on EI, the flexural rigidity where I is the moment of inertia of the plane about the plane of flexural. When the beam is caused, buckling of the beam is caused by compression and induces only twisting then it is resulting torsional buckling. So, this will occur when the buckling is caused by compression inducing twisting.

In this case, torsion rigidity will play a role or torsion rigidity and warping rigidity are important for design. In some cases, when the beam is subjected to axial compression it may experience both flexural and torsional buckling. This combined mode is called flexural torsional buckling, also known as lateral torsional buckling.

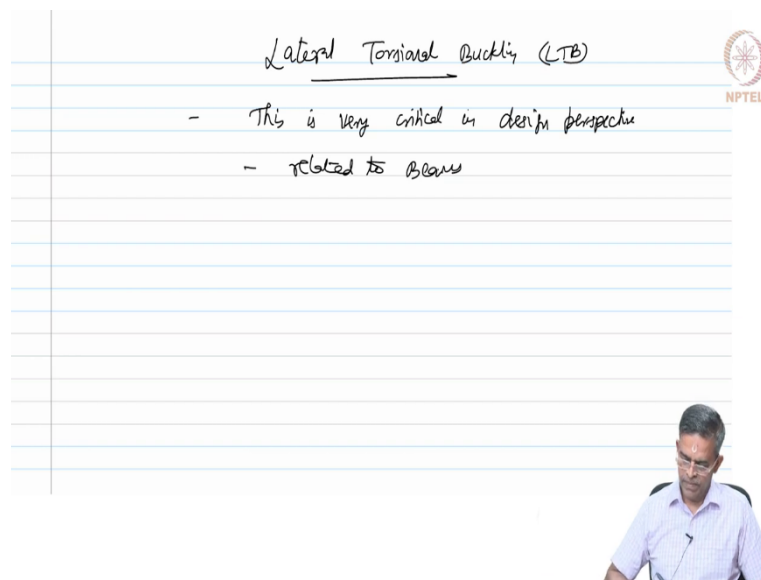


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So, you can also see the picture on the screen. This is about the major axis. This is about the minor axis. This is flexural buckling. This is torsion buckling. This is flexural torsional buckling. These are all different modes of buckling of beam under compression. Now, let us pay attention to lateral torsional buckling.

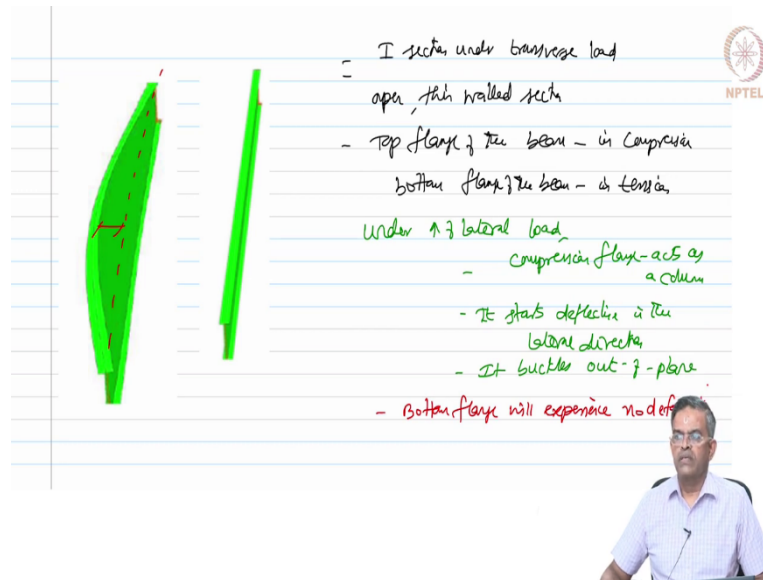
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Amongst all the modes of buckling, lateral torsional buckling calls for a special attention because this is very critical in design. Lateral torsional buckling is essentially related to

beams. Let us draw a figure to understand the lateral torsional buckling of an I section. Let us see this figure.

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So, this figure is a classical example of an I section undergoing lateral torsional buckling. So, now the figure shows you an I section under transverse load. This is a classical case of open thin-walled section. The top flange of the beam is in compression. The bottom flange of the beam is in tension.

Under the gradual increase of transverse magnitude of the load, the compression flange acts as a column and it starts deflecting in the lateral direction. You can see the figure. Just mark, it starts deflecting in the lateral direction. It buckles out of plane, because there are no restraints available. On the other plane web offers a resistance, therefore does not deflect.

But, in the it offers an outer plane bending. The bottom flange will anyway have no deformation, please note that. Practically, the upper half of the section undergoes lateral deflection, while the lower half of the section does not undergo any deflection. We can write this.

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upper half of the section - undergoes lateral deflection  
lower half of the section - No deflection

Inference: bottom flange is less loaded than the top flange ✓

- lower half does not buckle because it is under tension

- This causes overall twist of the x-section

Therefore, one can also state that the bottom flange is less severely loaded than the top flange. So, we can infer, but in the case of a column one can agree that both top and bottom flanges would have undergone the lateral deflection right.

But in this case of an example beam only half of the section is intention does not buckle. So, the lower half does not buckle because, it is under tension, and this causes overall twist of the cross section. There is an overall twist of the cross section and under lateral torsional buckling; we can have the following observations.

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Under lateral-torsional buckling, following are observed

- The top flange tends to undergo flexural buckling  
- i.e. its column  
- This causes out-of-plane bending of the top flange

- The bottom flange is in tension & doesn't deflect

This causes twist of the x-section in addition to out-of-plane bending

This is called lateral-torsional buckling

The top flange tends to undergo flexural buckling which is similar to columns. This causes out of plane bending of the top flange. The bottom flange is in tension and does not deflect and this causes twisting on the cross section in addition to out of plane bending. Please understand, there is an additional effect. This is identified as lateral torsional buckling. Therefore, friends in lateral torsional buckling, the lateral deflection of the compression flange exist along with the twist of cross section.

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In lateral torsional buckling  
is the lateral deflection of the compression flange  
along with twist of x-axis

lateral ✓  
torsional ✓

buckling? - compression flange undergoes lateral deflection  
- x-axis undergoes twist  
- since it's compression flange we term as buckling

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Lateral torsional buckling is the lateral deflection of the compression flange along with the twist of cross section. Now, the question comes we have understood how it is lateral, we have also understood how it is torsional, because there is a twist there is a lateral deflection.

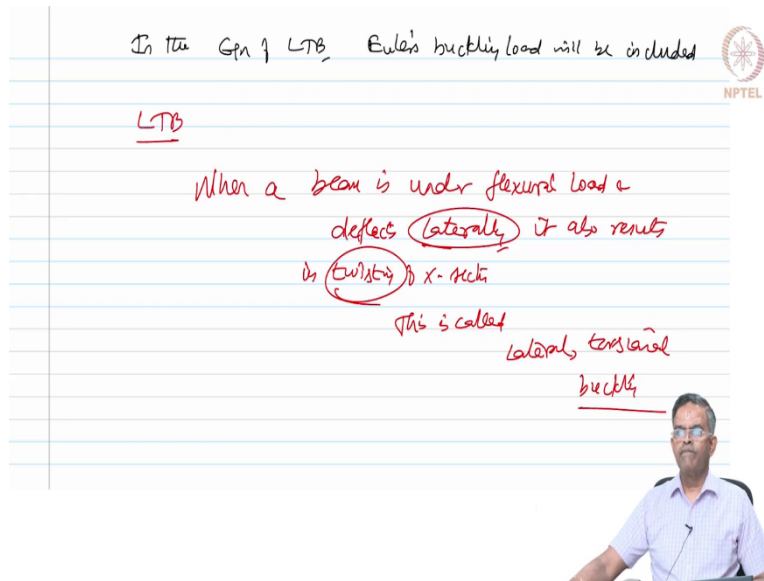
Why is it called buckling? Friends, it is very simple, compression flange undergoes lateral deflection, cross section undergoes twisting. Since, it is compression flange; we term this as buckling. Since, compression flange is involved, we call this term buckling. In case of open thin wall section, the compression flange essentially behaves like a column. Due to this reason, lateral torsional buckling resistance equations have Euler's buckling load involved in this.

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In the Eqn of LTB, Euler's buckling load will be included

LTB

When a beam is under flexural load & deflects laterally it also results in twisting of cross-section. This is called lateral torsional buckling.

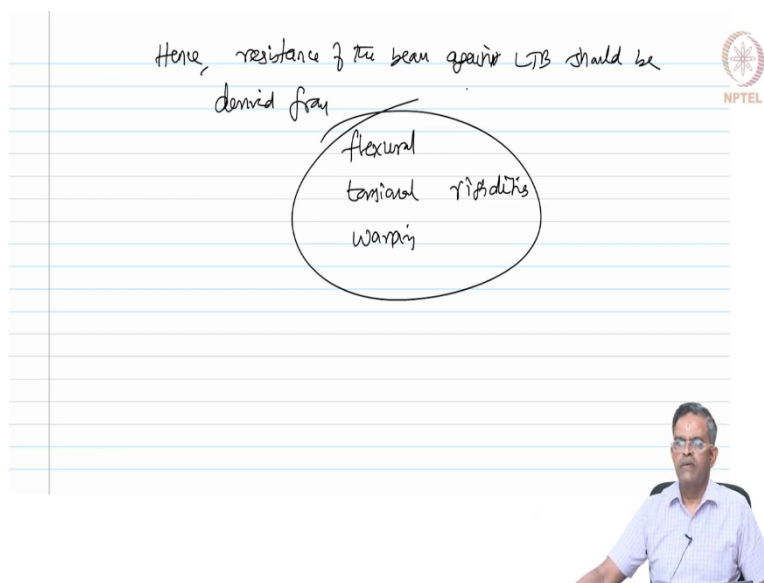


So, friends in the control equation of lateral torsional buckling, Euler's buckling load term will be included. So, in simple terms LTB can be explained like this. When a beam is under flexural load and deflects laterally, it also results in twisting of cross section. This is called Lateral Torsional Buckling, shortly known as LTB. When a beam is subjected to lateral torsional buckling, the cross-sectional resistance of the beam should be derived from the following quantities.

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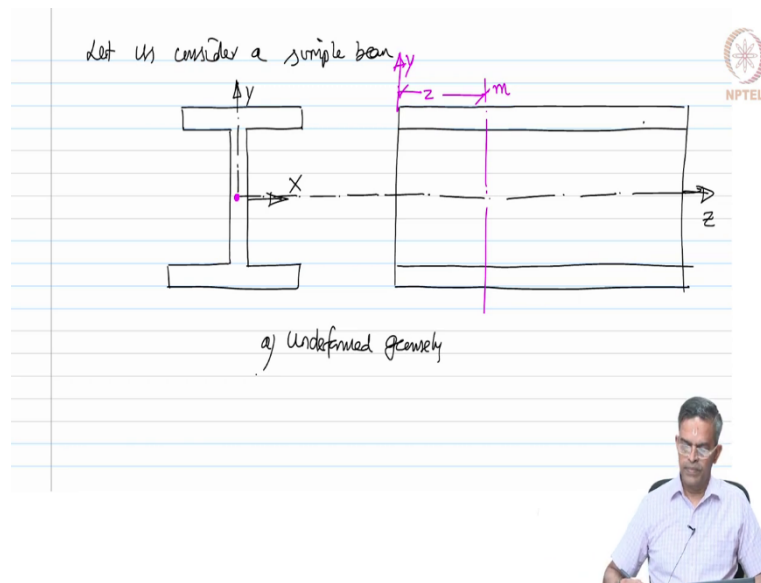
Hence, resistance of the beam against LTB should be derived from

flexural  
torsional rigidity  
warping



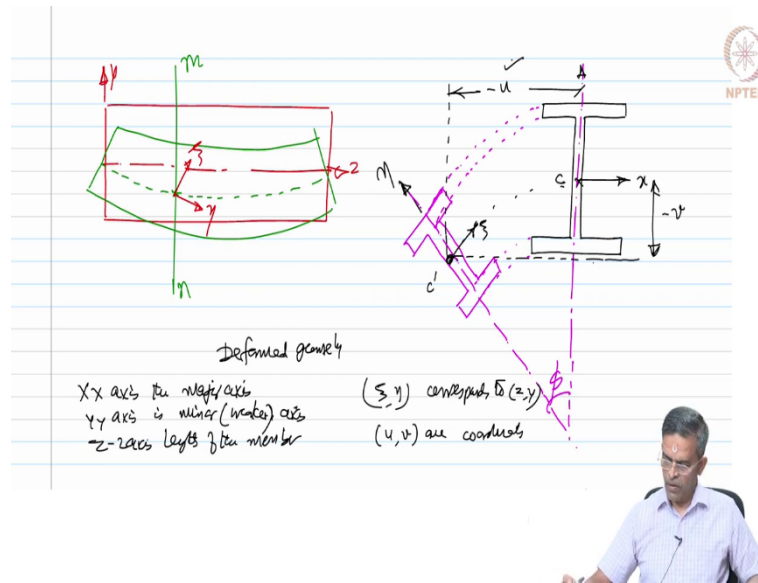
Hence, resistance of the beam against lateral torsional buckling should be derived from fractional, torsional, and warping rigidities that is very important. The three components that contribute to the resistance of the beam subjected to lateral torsional buckling. The phenomenon of lateral torsional buckling is important in design and it should be controlled, that is a very serious concern in design. Let us see how we explain this using a simple beam.

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Let us take a simple beam which is undeformed geometry. Let us say it is an I section. Let us say the beam is a centroid axis as now being marked on the screen. We call this as an X axis and this as an Y axis and this will be the Z axis. Let us consider a section at a distance z. Let us consider the section m the distance. So, this is my Y axis, looking for Z axis here, looking for Z Y and this is an undeformed geometry. Now, let us try to draw the deform geometry.

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So, the beam initially was like this. Let us say the beam is now deflected, rotate actually by an angle  $\phi$ . Let us say is the new section. Let us say this was my X axis and let us say the displacement of the new cg from here is  $-u$ , move towards the left. The point  $c$  has shifted and rotated to the point  $c'$  and the axis becomes  $\eta$ . This is the deformed geometry. So, now, I can also draw the longitudinal view of this beam.

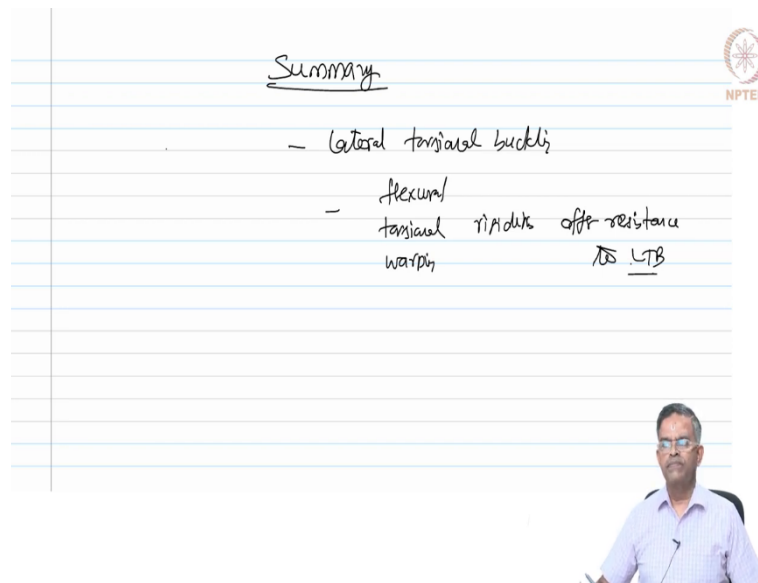
Let me do it in a single line easy. This is my Z and Y axis and my new geometry deforms this way. Let us cut a section  $m$  and let me mark the neutral axis of the new one and let the intersection be  $\eta$  and  $\epsilon$  or  $\zeta$ . This is  $m-n$ . This is the deformed geometry. So, simply supported beam under two conditions deformed and undeformed geometry.

The X axis is the major axis and Y axis is the minor axis or I should say the weaker axis, Z axis is along the length of the member, that is what the figure shows. Let us take a section along the span of the member and view this section before and after deformation, that is what we are drawn. So, as we focus on the lateral deformation, it can be seen that the lateral deformation of the beam is along the X axis correct.

The compression flange has bent out of plane about its minor axis. The torsion section gets twisted which can be seen in the side view of the section. Now, the cross section axis are now  $\zeta$   $\eta$  or  $\zeta$   $\eta$ . Let me mark this  $\zeta$   $\eta$  which corresponds to Z and Y. So,  $\zeta$   $\eta$  corresponds to Z and Y axis and  $u$  and  $v$  are the corresponding coordinates.

So, now in this figure this point is - v and this point is - u. Now, this will redesignate the new position after twisting. So, u and v are the corresponding coordinates of the new cg of the section, after the section is rotated by angle phi. Now, the twisted cross section signifies the fact that the flange is under flexural bending, while the compression flange is under lateral torsional buckling.

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So, friends in this lecture, we started learning about lateral torsional buckling. We said the resistance offered to the beam under LTB has got three components, is it not. It comes from flexural; it comes from torsional, comes from warping rigidity. We will continue derive this section and discuss LTB in the next lecture again. Before that give a thorough reading and understand the figure of deformation and please understand why it is called lateral torsional buckling.

Thank you very much and have a good day. Bye.