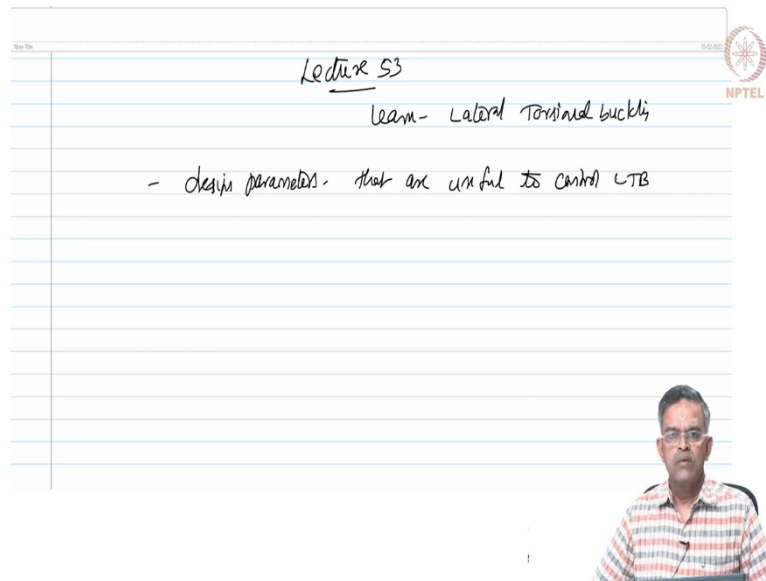


Advanced Design of Steel Structures
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Lecture - 53
Design for LTB - 1

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Lecture 53
Learn - Lateral Torsional buckling
- design parameters that are useful to control LTB

Friends, welcome to the lecture-53 of Advanced Steel Design Course, we are now continuing to learn the Lateral Torsional Buckling. So, in the last lecture we discussed about the design parameters that are useful to control a section against lateral torsional buckling. Now, let us quickly see what are the various buckling modes with lateral restraints that can be offered to avoid LTB?


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Buckling modes - with different lateral constraints that are provided to avoid LTB.

a) no restraints
- free to buckle due to LTB



b) compression flange restrained
No LTB is possible.




The slide contains handwritten text and diagrams. At the top, it says 'Buckling modes - with different lateral constraints that are provided to avoid LTB.' Below this, there are two cases: 'a) no restraints - free to buckle due to LTB' with a diagram of an I-section beam that has buckled laterally; and 'b) compression flange restrained No LTB is possible.' with a diagram of an I-section beam where the top flange is fixed to a wall, preventing lateral movement.

So, now let us see what are the various buckling modes that can happen with different lateral constraints that are provided to avoid lateral torsional buckling, suppose if you have an I section which has no restraints then the I section is free to buckle due to lateral torsional buckling and the buckling will happen like this.

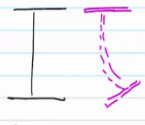
Suppose one restraint the compression flange like for example, I have an I section where restrain the compression flange. So, in this case no LTB is possible.

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c) Flange in tension is restrained
distortional buckling is possible



d) No restraints
Lateral distortional buckling is possible
(when the beam is slender)
flexible webs rigid flanges



The slide contains handwritten text and diagrams. Case 'c' is 'Flange in tension is restrained distortional buckling is possible' with a diagram of an I-section beam where the bottom flange is fixed to a wall, and the top flange has distorted. Case 'd' is 'No restraints Lateral distortional buckling is possible (when the beam is slender) flexible webs rigid flanges' with a diagram of an I-section beam that has buckled laterally and distorted.

Suppose, a tension flange restrained for example, this is an I section, we restrain the tension flange then there is a possibility that it may deflect like this. So, distortion buckling will happen then we do not provide any restraints as usual then lateral distortion buckling is also possible. This will happen only when the beam is slender.

So, it has got flexible webs web thickness is very small and rigid flange if this combination is there then it can result in distortion buckling as you see on the picture.

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Behavior of real beam

The lateral Torsional buckling theory is applicable to

- 1) Elastic condition
- 2) Ideal beam
- 3) No geometric imperfection

- Ideal beam is undeformed until the load reaches elastic, critical moment, (M_{cr})

- In real beam, lateral deflection (δ) will occur due to imperfection

$(M_{capit})_{real\ beam} < M_{cr}$

The graph shows Moment (M) on the vertical axis and Deflection (δ) on the horizontal axis. A horizontal dashed line represents the critical moment M_{cr} . A red curve labeled 'beam without imperfections' starts at the origin and reaches M_{cr} at a certain deflection. A purple curve labeled 'real beam' starts at a non-zero deflection on the horizontal axis and reaches a lower moment value than M_{cr} .

Now, let us try to compare the behavior of a real beam and see how it is different in LT, let us now see behavior of a real beam. Now, we have seen that the lateral torsional buckling theory is generally applicable to ideal elastic conditions for beam possessing perfect geometry, but in reality it is not possible to have an ideal beam like this. So, one cannot get an idealized design conditions because they will be different in reality. So, now, you will always have a beam with imperfections.

So, let us try to mark the displacement moment curve and we will mark the real beam behaves like this whereas and the critical moment beams without imperfections will behave like this. So, this is the real beam and this is an ideal. So, there is a significant difference in the response behavior of this and of course, this is what we call as initial deflection del naught.

So, we comparing both the behavior as a real beam and ideal beam, it can be seen as ideal beam is laterally undeformed until the load reaches the elastic critical moment M_{cr} . So, one can say ideal beam is undeformed until the load reaches elastic, critical moment marked as M_{cr} . So, when M_{cr} is reached the beam experiences a different state of instability and results in significant deflection occurs in the lateral direction since the material is ideally elastic infinitely large deformations can occur.

So, a new state of equilibrium will be developed in the deflected position every slight increase in load will cause significant deflection. On the contrary in case of a real beam which has got lot of imperfections and residual stresses present in the beam during welding when such beam is subjected to apply load a lateral imperfection initial deflection δ_0 will occur.

So, in real beam, lateral deflection δ_0 will occur due to imperfection, the lateral deformation increases with the increase in magnitude applied load closer to M_{cr} the deflection increases spontaneously without reaching theoretical value of M_{cr} . So, the moment capacity of the real beam is far lesser than M_{cr} . So, this is governed by material property in plastic range non-linear geometry and possible local buckling.

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Real beam has not reached Moment Capacity ($M < M_{cr}$)

Factors that cause this?

- 1) Nonlinear material response (plastic behavior of the material)
- 2) Initial imperfections (not straight...)
- 3) Residual stresses during fabrication, manufacturing
- 4) Local buckling of the beam, in class 4 members
- 5) Prolonged, asymmetry & defects (Jerrin buckling curve)

Now, let us quickly see what are the factors that cause reduction of capacity? So, we have understood that the real beam has got reduced moment capacity is it not that is M is lesser

than M_{cr} , let us see what are the factors that cause this. It may be due to non-linear material response that is plastic behavior of the material that could be one reason.

Second reason could be initial imperfections the beam may not be initially straight etcetera, thirdly can also be due to presence of residual stresses during fabrication, during manufacturing etcetera. The fourth reason could be local buckling of the beam in class 4 sections. The last reason could be piercings, unsymmetry and defects in the fabrication.

Now, these effects can be considered in design through design buckling curves which simulate the real beam behavior. There are design buckling curves which are given the design codes which can help you to have a modified factor of reduced moment capacity on account of each one of these factors.

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Design procedure for LTB

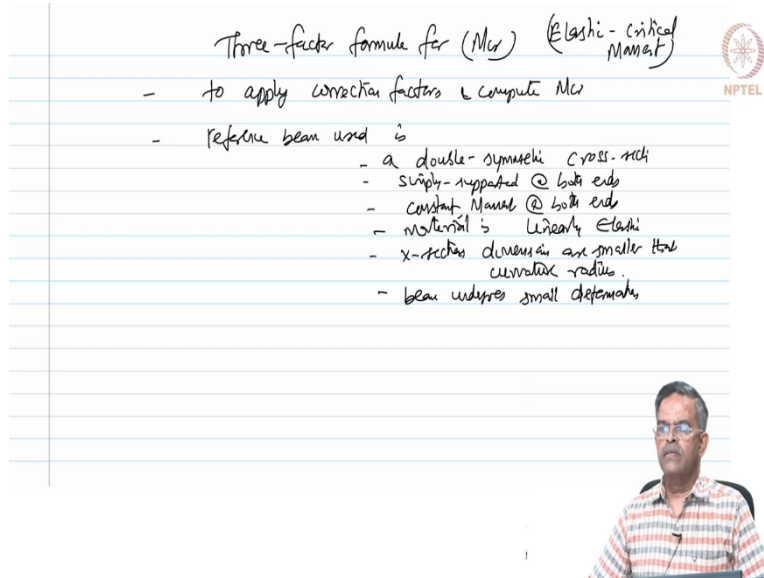
- The Elastic critical Moment (M_{cr}) is the primary design parameter
- M_{cr} is defined as the max^m value of BM supported by the beam, free from any imperfections
- Eurocode - Three-factor formula (Lopez et al, 2006)

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Now, friends let us see the design procedure for lateral torsional buckling. Let us see this. Now, the elastic critical moment the elastic critical moment M_{cr} is the primary design parameter. So, how do you define M_{cr} ? M_{cr} is defined as the maximum value of the bending moment supported by the beam which is free from any imperfections. In practical design M_{cr} can be estimated using a software or by performing hand calculations.


Euro code helps the estimate M_{cr} using a three factor formula. It is one of the most used analytical formula to estimate the elastic critical moment as suggested by Lopez et al in 2006.

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Three-factor formula for (M_{cr}) (Elastic-critical Moment)

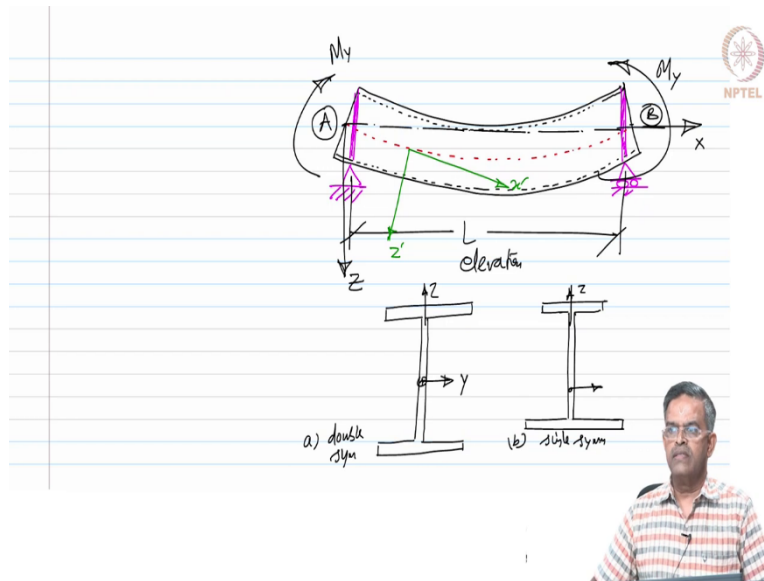
- to apply correction factors & compute M_{cr}
- reference beam used is
 - a double-symmetric cross-section
 - simply-supported @ both ends
 - constant Moment @ both ends
 - material is Linearly Elastic
 - x-section dimensions are smaller than curvature radius.
 - beam undergoes small deformations



Now, let us see, what is the three factor formula for M_{cr} here? M_{cr} is referred as elastic critical moment, which is the design parameter to control LTB in beams. So, the basis of this formula is to apply correction factors and compute M_{cr} . Now, the reference beam used as the basis is a double symmetric cross section which is simply supported at both the ends subjected to constant moment at both ends that is the ideal condition for which this material is linearly elastic.

The cross sectional dimensions are assumed to be smaller than the curvature radius and the deformations are considered to be very small. Let us try to see a simply supported beam which is considered for the analysis.

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So, we will have a simply supported beam which is undergoing an initial deflection will mark the beam with this. So, there are n constraints let us call this end as A this end as B let us say this beam is subjected to a momentum M_y at the ends as shown. Let us say the span of the beam is L meters and if I draw a neutral axis of this beam at any section let us draw the axis normal and tangent we call this as x dash and z dash whereas, the original axis is X and Z this is an elevation.


So, double symmetric section let us say the general cross section is looking like this having a C_g at the center. So, this becomes my y and z axis, if I have a wide flanged bottom when the C_g is shifted this becomes y and z and C_g is shifted down, both are double symmetric sections anyway double symmetric and single symmetric you can say. So, this is double symmetric, and this is single symmetric, is it not?

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The Elastic critical moment of the standard section (Double-sym sect)

$$M_{cr} = \frac{\pi}{L} \sqrt{G I_T E I_z \left(1 + \frac{\pi^2 E I_w}{L^2 G I_T} \right)} \quad - (23)$$

The Eq(23) will be modified to apply for single-sym sects using the correction factors (C_1, C_2, C_3) - account for imperfection



Having said this the elastic critical moment of the perfect section that is of the standard section is given by M_{cr} is $\pi/L \sqrt{(G I_T E I_z (1 + \pi^2 E I_w / L^2 G I_T))}$ equation 23 we are continuing from the previous derivation. So, the standard section what we say is a double symmetric section that is the standard symmetric.

If you want to expand this M_{cr} equation for a single symmetric section then equation 23 will be modified to apply for single symmetric sections using the correction factors C_1 , C_2 and C_3 . These correction factors account for imperfections, these factors can be determined either from the tables or from the figures they can also be estimated using closed form expressions available in the literature.

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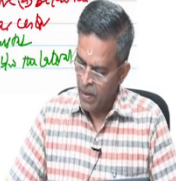
Considering the warping degrees-of-freedom and lateral rotation @ the supports, the Eqn is modified with 2 more factors (k_z, k_w)

The modified, 3-factor Eqn for singly-sym section is given by

$$M_{cr} = C_1 \frac{\pi^2 EI_z}{(k_z L)^2} \left\{ \left[\left(\frac{k_z}{k_w} \right)^2 \frac{I_w}{I_z} + \frac{(k_z L)^2 GI_T}{\pi^2 EI_z} + (C_2 Z_g - C_3 Z_j)^2 \right]^{0.5} - (C_2 Z_g - C_3 Z_j) \right\} \quad (24)$$

k_z = Effective length factor (lateral bracing)
 k_w = " " " (related to warping)
Limitations of Eq(24) 1) applicable only to sym & singly-sym section
 2) includes the effects of load applied above/below the shear center

E = Modulus of Elasticity
 I_z = second MoI about z-axis
 G = shear modulus
 I_T = torsion constant
 I_w = warping constant
 L = span (4x the member length)



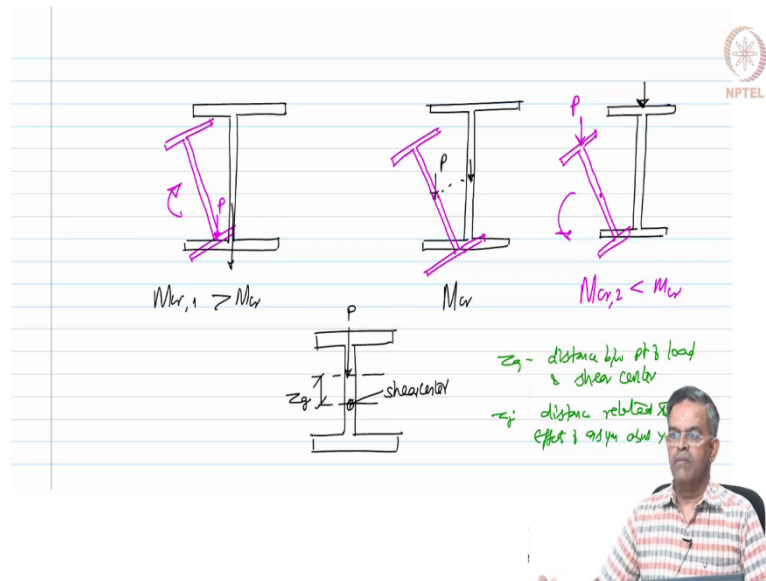
Now, considering the warping degrees-of-freedom also and lateral rotation at the supports, the equation is further modified with two more factors namely k_z and k_w . So, now the modified 3 factor formula for singly symmetric sections is given by M_{cr} equals

$$C_1 \frac{\pi^2 EI_z}{(k_z L)^2} \left\{ \left[\left(\frac{k_z}{k_w} \right)^2 \frac{I_w}{I_z} + \frac{(k_z L)^2 GI_T}{\pi^2 EI_z} + (C_2 Z_g - C_3 Z_j)^2 \right]^{0.5} - (C_2 Z_g - C_3 Z_j) \right\} \quad \text{was equation}$$

number 24.

So, one can see in this equation there are different factors C_1, C_2, C_3 which are accounting for the imperfection conditions in addition to that by allowing warping degrees of freedom and lateral rotation at the ends because the ends are simply supported k_z, k_w are also involved. Now, this above equation has got some limitations, now this equation is applicable only to symmetric and singly symmetric sections. This equation includes the effects of loading apply above or below the shear center.

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Let us see how this included. So, let us talk about the application of law, let us say I have the original section of this format nicer. Now, this can also have a marginal twist when the load is applied at P which can be causing a restoring moment initially the load is applied here. So, one can say here $M_{cr,1}$ is greater than M_{cr} because we have restoring moment available.

Let us take another example where the standard I section twists to this form where initially the loading is applied at the shear center and now the loading is here. So, in this case M is actually equal to M_{cr} . Suppose we have a section where the load is applied here and the section is twisted this will cause an additional moment in this case $M_{cr,2}$ will be less than M_{cr} . Ideally speaking, if you have an I section is this the point where load is applied and this is the point where the shear center is located the shift of the shear center from here is actually Z_g .

So, friends in this equation we can also write down that E is modulus of elasticity, G is the shear modulus, I_z the second moment of area about the weaker axis, I_t torsion constant, I_w warping constant, L span of the beam or center distension distance between the lateral constraints.

Let me write here k_z effective length factor, k_w is the effective length factor related to lateral bending, k_w is again effective length factor, but related to warping. Z_g the distance between the point of application j center as I show here and shear center, Z_j is the distance related to effects of asymmetry about the y axis.

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$$Z_j^* = Z_s - \frac{0.5 \int_A (y^2 + z^2) z dA}{I_y} \quad (25)$$

Z_s - distance b/w shear center & C_g

C_1 - factor accounts for shape & moment diagram

C_2 - factor accounts for point of application of load with respect to shear center

C_3 - factor accounts for asymmetry about y axis



And this can be given by a separate equation Z_j is Z_s minus 0.5 times of integral over A $y^2 + z^2$ $z dA$ by I_y equation 25, where Z_s is the distance between the shear center and center of gravity. C_1 is the factor that accounts for shape of the moment diagram, C_2 is a factor which accounts for point of application of load with respect to the shear center, C_3 is a factor which accounts for asymmetry about y axis. Now, let us quickly see; what are these moment correction factors C_1 , C_2 and C_3 ?

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Moment correction factors (C_1, C_2, C_3)

C_1 - moment gradient factor

- This is valid when load acts at the shear center

C_2 - is applied to account for the effect of not loading @ the shear center

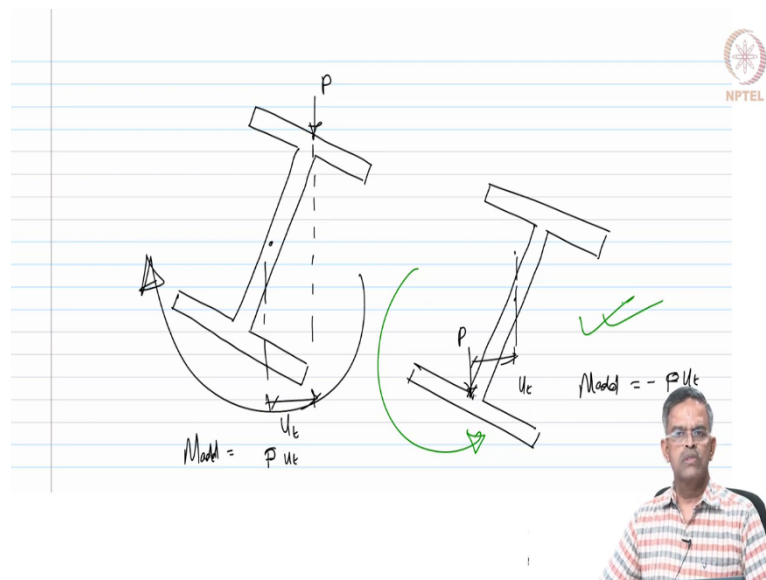
- (when point of application of load is not @ the shear center, it causes an additional twist; moment
- load applied below shear center stabilizes member
- load applied above shear center destabilizes member



The C_1 factor is equivalent uniform moment factor also referred as moment gradient factor, this is valid when the load is acting at the shear center. In reality beams are often loaded on the top and bottom flanges and not on the shear center. Therefore, a second correction factor C_2 is applied to account for the effects of not loaded and the shear center. So, what will happen when the loading is not occurring at the shear center?

Very good question, when the point of application of load is not at the shear center it causes an additional moment, we should say additional twisting moment. Therefore, the load applied under shear center stabilizes the beam the load above shear center destabilizes the beam. We can also illustrate this with the figure which we already draw but let us once again do it.

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Let us say if we have a beam which is twisted, and this is my point of C_g and the load is applied here the line of application of load is the shift. So, this is what we call as u_t . So, in this case the additional moment will be equal to F into u_t P into u_t if it is supplied in the bottom flange that is below a shear center if it is applied below the shear center still also the shift is u_t , but M_{add} now is minus $F u_t$ oh sorry $P u_t$.

So, this causes a destabilizing moment, but this causes a stabilizing counteracting moment. So, this is better. Now with the factors C_1 and C_2 the three factor formula can calculate the elastic critical moment M_{cr} for double symmetric beams under various loads and point of application.

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The three-factor eqn can be used to compute M_{cr} for a double-sym section of beam under various loads & points of application (C_1, C_2)

then, an additional (C_3) is used to apply this eqn for singly-sym sections

So, the three-factor formula or equation can be used to compute M_{cr} for a double symmetric section of beams under various loads and points of application of load because C_1 and C_2 are accounted for this, but still the equation is valid only for double symmetric section. Now, if you want to use this equation for single symmetric section then an additional factor C_3 is used to apply this equation for single symmetric sections.

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Correction factors for uniform moment conditions

loads condition	BMD	k_2	C_1	C_2	C_3
		1.0	1.12	0.45	0.525
		0.5	0.97	0.36	0.478
		1.0	1.25	0.59	0.411
		0.5	1.05	0.48	0.338
		1.0	1.04	0.42	0.562
		0.5	0.95	0.31	0.535

So, there are various factors given as correction factors which can be summarized here. So, I should say correction factors for uniform moment conditions. Let us say the loading

condition bending moment diagram k_z C 1 C 2 and C 3. Let us say I have a simply supported beam with UDF of some value, we know the bending diagram is like this.

k_z is 1.0 for 1.0 k_z 1.12, 0.45 and 0.525 or for C 1, C 2, C 3, if k_z is 0.5 for the same condition of loading C 1 is 0.97, C 2 is 0.36 and C 3 is 0.478. If the beam has got central concentrated load P the bending moment diagram is triangular for k_z of 1.0 and 0.5 these values are 1.35, 0.59 and 0.411, 1.05, 0.48 and 0.338.

If the beam is subjected to a load which is 1 by 4, 1 by 4, 1 by 4 and these are the loads which is 1 the bending moment diagram goes like this and for a condition of 1.0 and 0.5 these values are given as you see on the screen. So, now, friends we are going to now perform a design check for lateral torsional buckling which we will discuss in the next lecture.

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The image shows a handwritten summary on lined paper. At the top right is the NPTEL logo. The text is written in black ink and includes the following:

- summary
- 3-factor Eqn to compute (M_{cr})
- to design for beams under LTB
- Adv steel design - CRC
- design aids of Topside - CRC

In the bottom right corner, there is a small video inset showing a man with glasses and a striped shirt, who is the speaker for this lecture.

So, in this lecture we have learnt about the three-factor formula or equation to compute M_{cr} to design for beams under lateral torsional buckling. The lecture and the tables have got good reference Advanced Steel Design book written by me for CRC, then there is another book Design Aids of Topside written by me for CRC. So, these two books are standard references which can help you to learn more about the design against LTB which is explained more in an elaborate manner with an example given from euro codes.

Thank you very much and have a good day, bye.