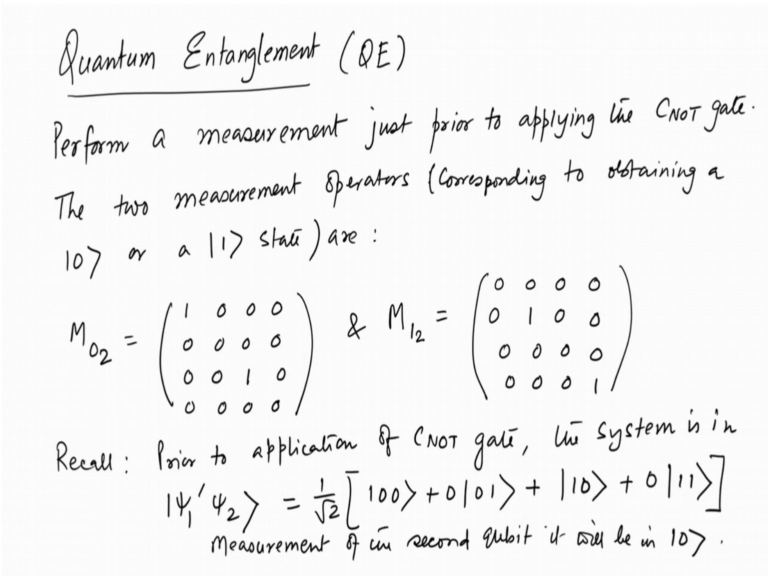
`Advanced Quantum Mechanics with Applications
Prof. Saurabh Basu
Department of Physics
Indian Institute of Technology, Guwahati

Lecture – 17
Quantum Entanglement (QE)

So, we have been discussing about Quantum Entanglement. Now, let us see that why quantum entanglement is so surprising. So, let us perform a measurement just prior to application of the C NOT gate which had been discussed earlier.

(Refer Slide Time: 00:47)



We will in short, we will call it as Q E. So, perform a measurement just prior to applying the C NOT gate.

So, the 2 measurement operators are. So, this is corresponding to obtaining 0 or 1 state are. So, it is a M 0 2. So, which is written as 1 0 0 0 0 0 0 0 0 0 1 0 and 0 0 0 in addition to that. So, there is a M 1 2 which is equal to 0 0 0 0 0 1 0 0 0 0 0 0 and 0 0 0 1. So, these are the 2 measurement operators.

So, if you recall that, prior to application of the C NOT gate, the system is in psi 1 prime and psi 2 which is equal to 1 by root 2 0 0 plus 0 0 1 plus a 1 0 plus a 0 1 1. So, this is very apparent that the result of measuring the second qubit it will always be in the 0 state. So, it will be in this 0.

(Refer Slide Time: 04:44)

Also note that: $M_{02}^{\dagger} M_{02} = M_{02}$

$$p(0) = \langle \psi_1' \psi_2 | M_{02}^{\dagger} M_{02} | \psi_1' \psi_2 \rangle = \langle \psi_1' \psi_2 | M_{02} | \psi_1' \psi_2 \rangle$$

$$\begin{pmatrix} \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \frac{1}{\sqrt{2}} \\ 0 \\ \frac{1}{\sqrt{2}} \\ 0 \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} & 0 \end{pmatrix} \begin{pmatrix} \frac{1}{\sqrt{2}} \\ 0 \\ \frac{1}{\sqrt{2}} \\ 0 \end{pmatrix} = 1$$

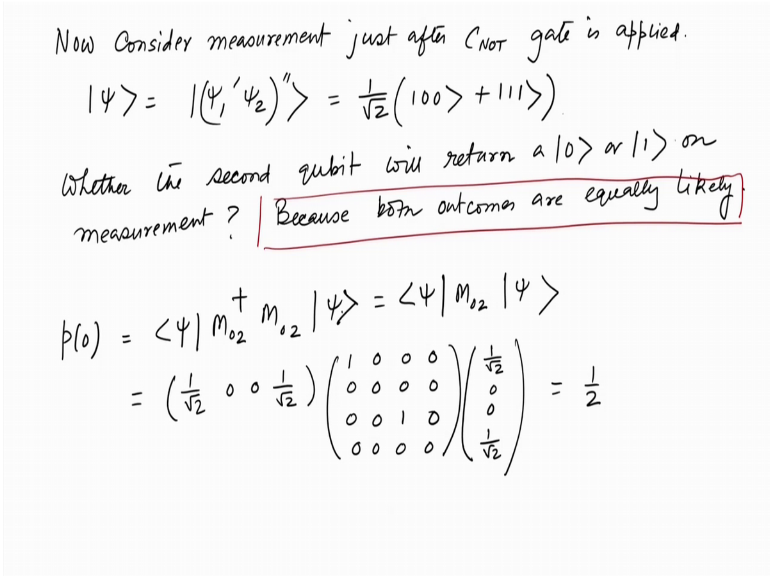
Thus after measurement the system of qubits is again found in $|\psi_1' \psi_2\rangle$.

So, let us also note that M 0 2 dagger M 0 2. It is equal to M 0 2 and so, the probability of getting 0 for the measurement of the second qubit which we just saw that it could be 0. But, we can also prove that. So, this is p 0 which is equal to psi 1 prime psi 2 and M 0 2 dagger M 0 2 and psi 1 prime psi 2 which is nothing but because of this relationship above. So, it is psi 1 prime psi 2 and M 0 2 and psi 1 prime psi 2.

So, this if you do it explicitly, then it is raw of these. This is that psi 1. So, this is that psi 1 prime psi 2 which is here and then, will write the M 0 2 which is what we have written and so, this is M 0 2 it is 1 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0. And now, I will have a 1 by root 2 0 1 by root 2 0 and if you simplify this, it becomes equal to 1 by root 2 0 1 by root 2 0 which should check and 1 by root 2 0 1 by root 2 0 which is equal to 1.

So, after the measurement, the system of qubits is again found in the state psi 1 prime psi 2. So, essentially what we are saying is that, the measurement had no effect on the first qubit. It remain in the superposition of 0 and 1. So now, consider the same measurement, but just after the C NOT gate is applied.

(Refer Slide Time: 08:01)



So, that would mean that, we have a new state psi which is psi 1 prime psi 2 and then, this double prime which is equal to 1 by root 2 and a 0 0 plus 1 1 as we have seen earlier.

Now, it is not clear whether the second qubit will return to a 0 state or a 1 state on measurement because, both seem to be equally likely. So, the question is whether the second qubit. We will return a 0 or 1 on measurement because both outcomes are equally likely alright. So, will now apply the C NOT gate and do the same measurement p of 0.

So, p of 0 on this new state psi, which is written here this new state psi. So, this is equal to M 0 2 dagger M 0 2 and psi and this is equal to psi M 0 2 psi.

So, this is 1 by root 2 0 0 1 by root 2 1 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 and the new state again in the cat form 1 by root 2 and this is if you simplified becomes half. So, what was 1 here on the state that was before application of C NOT. Now, after application of C NOT getting a 0 for the second qubit becomes half.

(Refer Slide Time: 11:51)

Thus after C_{NOT} gate is applied we have only $\frac{1}{2}$ possibility of obtaining a zero for the second qubit.

Let us now ask:

What happens to the state vector of the system?

So, after C NOT gate is applied, we have only half chance or half possibilities or probabilities possibility of obtaining a 0 for the second qubit. So, just to rerun things once more suppose we have the 2 measurement operators which are given as M 0 to and M 1 2 which are corresponding to for the second qubit. If we want to measure it in the 0 state and it to be measure it in the 1 state.

So now, prior to application of the C NOT gate, this is the state that if you remember that we have obtained by applying U or H on the qubit on the bell pair. So, this is equal to half 0 0 and 0 1 and so on. So, the measurement of the second qubit will be 0. So now, in order to see that comprehensively we have applied. So, this is the probability of getting it 0 we have taken the state and have squeezed this M 0 to a dagger M 0 2 which is simply equal to M 0 2 and we get equal to 1.

So, the measurement of qubit is the second qubit is always in the 0 state whereas, if we operate the state, that is this psi 1 prime psi 2 on if we operate this psi 1 prime psi 2 by the C NOT gate which it becomes psi 1 prime psi 2 and then, we write it with the double prime it becomes equal to 1 by 2 0 0 and 1 1. Now, this has been discussed earlier.

So, when now again, if we consider p of 0 that is a probability for the second qubit to yield 0, we get only a probability of half and this is what is written there that prior to C NOT, it was 1 and after applying C NOT, it is half. So, let us now ask this question, what happens to the state vector of the system?

(Refer Slide Time: 15:27)

After measurement: $$\frac{M_m|\psi\rangle}{\sqrt{\langle\psi|M_m^{\dagger}M_m|\psi\rangle}}$$

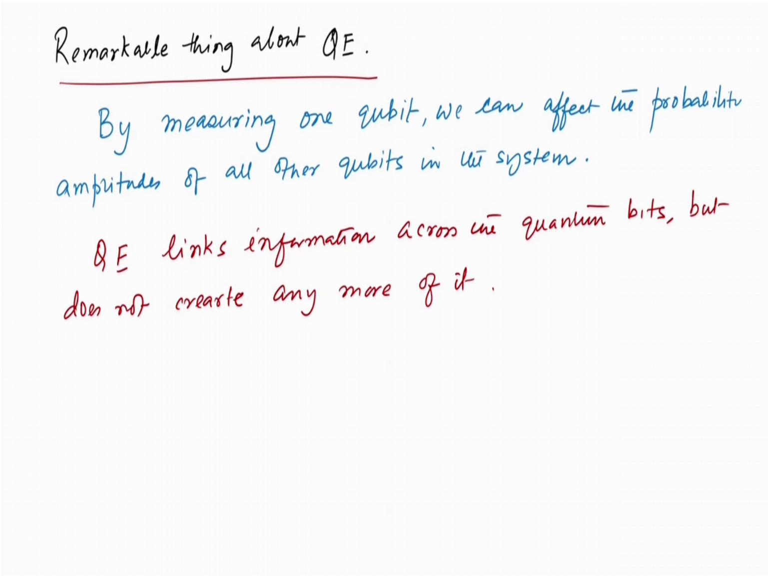
$$= \frac{1}{\sqrt{2}}\begin{pmatrix}1&0&0&0\\0&0&0&0\\0&0&1&0\\0&0&0&0\end{pmatrix}\begin{pmatrix}\frac{1}{\sqrt{2}}\\0\\0\\\frac{1}{\sqrt{2}}\end{pmatrix}$$

$$= \frac{1}{\sqrt{2}}\begin{pmatrix}\frac{1}{\sqrt{2}}\\0\\0\\0\end{pmatrix} = \begin{pmatrix}1\\0\\0\\0\end{pmatrix} = |00\rangle$$

So, after measurement, it is M m psi and divided by root over of psi M m M m psi. In this particular case, if we take M to be equal to that operator the measurement operators that we have just introduced. So, it will be 1 by root 2 and we have a 1 0 0 0 0 0 0 0 0 0 1 0 and 0 0 0 0 and we have 1 by root 2 0 0 1 by root 2 and that is equal to nothing but 1 by root 2 1 by root 2 0 0 0 and this is equal to 1 0 0 0 and with we can simply call it as 0 0 because all other entries are 0.

So, in fact, this is the remarkable thing about the quantum entanglement by measuring 1 qubit we can affect or rather determine the probability or amplitude of the other qubits in the system in principle all other qubits in the system. So, entanglement links the information across the quantum bits, but it does not create any more of them.

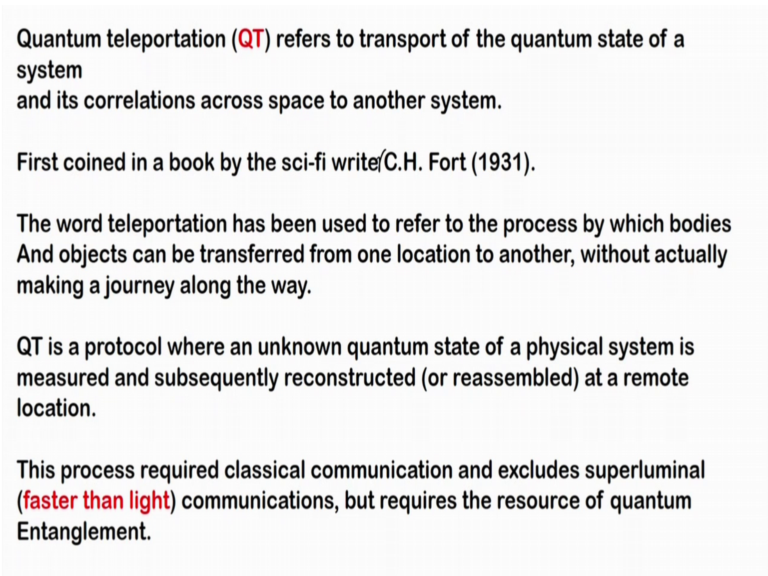
(Refer Slide Time: 17:18)



So, let me write down the most important part by measuring 1 qubit, we can affect the probability amplitude of all other qubits in the system. Thus, quantum entanglement links information across the quantum bits, but does not create any more of it.

So, this abstract concept can be important for quantum teleportation where quantum state can actually be carried from one location to another remote location without having to make journey along the way. So, this will be the next thing that we look at quantum teleportation. Let me give you a very brief introduction to it.

(Refer Slide Time: 19:39)



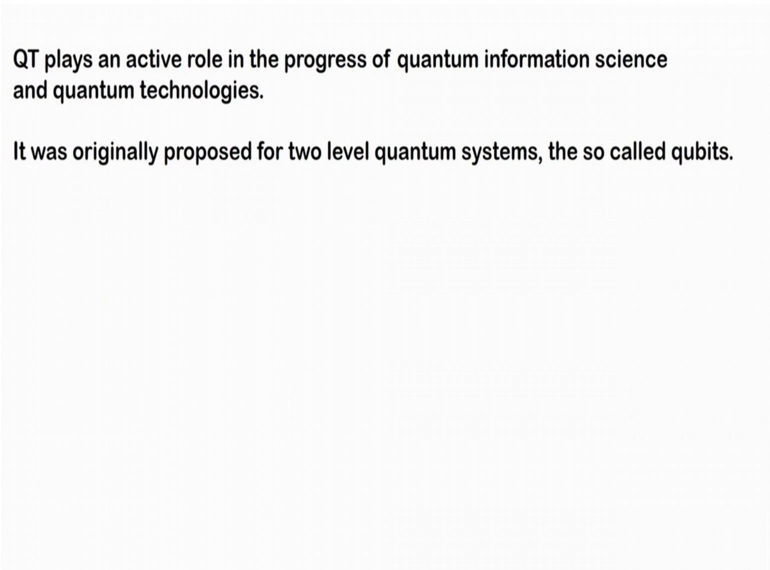
So, a quantum teleportation refers to the trans transport of the quantum state of a system and it is correlation across the space to another system. It was first a coined in a book by the science fiction writer, I just correct the spelling is writer C.H. FORT 1931 the world teleportation has been used to refer to the processes by which bodies and the objects can be transferred from one location to another without actually making a journey along the way.

Quantum teleportation is a protocol where an unknown quantum state of a physical system is measured and subsequently reconstructed or which is also called as reassembled at a remote location. So, this process required classical communication and exclude super luminal, which is what it means is that, the faster than-light which is Einstein has raised a question that is quantum mechanics complete because, nothing moves faster than light is the postulate of the special theory of relativity.

If there is a superluminal communication possible which means that, faster than light communication possible, then quantum mechanics to contradict the postulates of special theory of relativity which is what Einstein and his and the other people as well had their apprehensions.

However, this quantum teleportation does not require or rather it excludes superluminal communication and but it requires the source of or the resource of quantum entanglement which is what we had just see.

(Refer Slide Time: 21:49)



It also plays an active role in the progress of quantum information science and quantum technologies and it will go a long way in terms of research in order to have quantum teleportation possible and. So, it was originally of course, proposed for 2 level system, the so called qubits. But then, it can also be extended to you know the qutraits of the qubits and so on.