Advanced Quantum Mechanics with Applications
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Lecture – 02
Stern Gerlach Experiment, Spin Quantization, Young's Double Slit Experiment

So, we shall substantiate some of the things that we have learnt in the previous class and in order to do that let us start with an experiment which is known as the Stern-Gerlach experiment.

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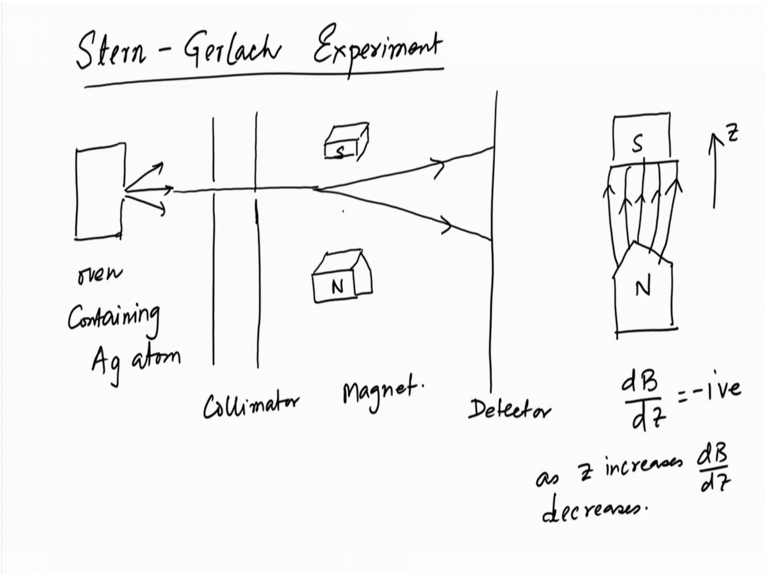

And this experiment is the first one to actually establish the quantization of angular momentum. And let us see how this experiment actually talked about the quantization of angular momentum and hence that all these quantities or these physical observables could take quantized values including energy and other things.

So, they considered a simple setup which is this is an oven and it contains silver atoms. So, there silver atoms are emitted in all directions and they are further sent through collimators. So, that this thing can pass through this and now, there are two pole pieces of magnets. So, this is the South Pole, and it is a specially designed magnet where one of them is flat, say the south is flat and the North Pole has sharp edges. And so basically these silver atoms coming out of the collimator is they undergo a deflection because of the inhomogeneous magnetic field that exists in this region, and then they are collected on the screen and they get deflected or deviated like this, and they are so this is a detector or a screen, and this is a collimator and this part is the magnets as told earlier.

So, this magnet let us see a schematic representation of what this magnet does. So, there is a cross section of the two magnets here they look like this and there is a gradient of magnetic field that is created and this gradient of magnetic field. So, this is say the Z direction. So, this dB dZ are actually negative which means that as Z increases dB dZ decreases.

So, once again I just repeat that this is an oven containing silver atoms we will discuss this silver has 47 as the electronic configuration. They are made to pass through they actually are coming out in all possible directions. However, they collimated through a series of collimators and then they are made to pass through a magnet arrangement which one of the pole pieces of the magnet is say the south pole is flat while the north pole is having pointed edges so that one can have the inhomogeneous distribution of the magnetic field as a function of Z. And so basically this is the in this figure; so, basically this is you know; and they are made to fall on the detector and so they in principle as we discuss that they should go and hit anywhere on the screen here between two extreme values pardon me. So, this is the detector. However, they chose to actually impinge on two distinct locations and how that corresponds to quantization of angular momentum is what we shall see.

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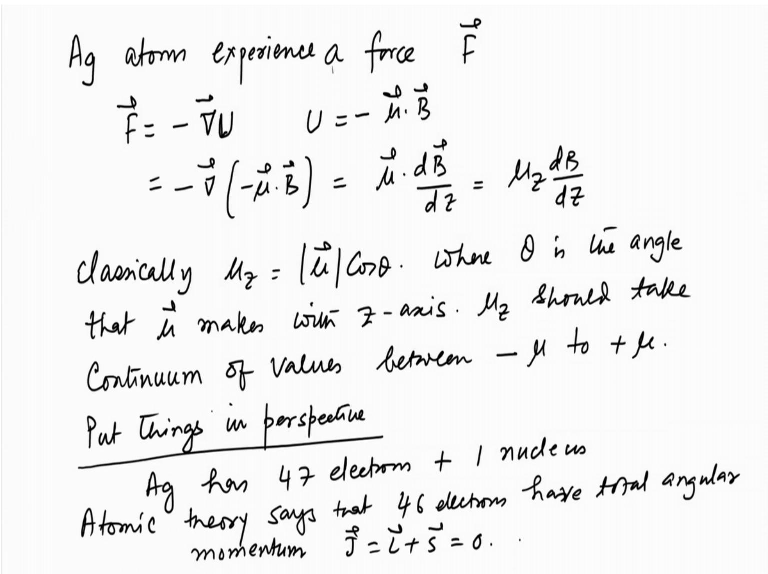

So, then there is a the silver atoms while passing through the magnetic region where the magnets exist, there is the experience force a force F and that F is given by equal to minus gradient of U, where U is the potential energy. Now, here U is equal to minus mu dot B where mu is the magnetic moment of the silver atoms.

So, if you put this u here. So, there will be a minus a gradient of minus mu dot B which since mu the magnetic moment can be taken to be a constant. So, this becomes equal to mu dot dB dZ say for example, and this can be written in a scalar form such as mu Z and a dB, dZ. So, this is the force that is exerted on the atoms. And we have shown that this del B del Z is negative. So, if mu Z is a negative, then we will have a positive force which is exerted on the atoms and maybe they will be shifted up from this mean position let us call this as O.

And in the other case when mu Z is positive then there will be a negative force that will be exerted on the atoms and they will be pushed down which is below O on the screen. So, this is the idea, and if we follow classical physics then mu Z is equal to a mu a cosine theta where theta is the angle that mu makes with Z axis. And so mu Z that way mu Z should take continuum of values of values between minus mu to plus mu because the cos theta has limits of plus 1 and minus 1. So, there should have been all the atoms should have been distributed between these two values, but however, it does not happen, what happens is that there are two spots on the screen, so all these atoms they go get deflected and they impinge on the screen at two discrete points.

Now, what is happening and how is it related to the quantization of angular momentum let us try to understand that. So, basically put things in perspective and for that silver has 47 electrons and of course, will say that there is a nucleus and how that comes into the discussion we will just see. Now, the atomic theory says that 46 of those electrons have total have total angular momentum J which is equal to L plus S is equal to 0, also the orbital angular momentum of the 47th electron is 0 as well.

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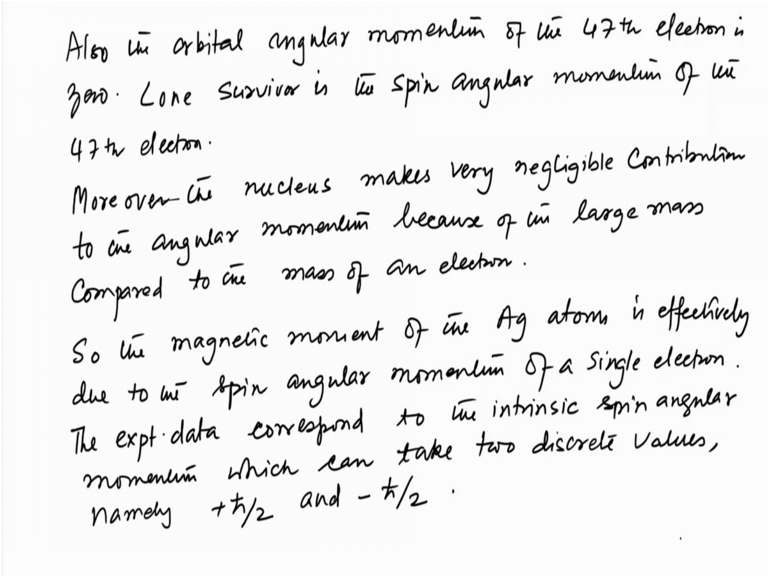

So, the lone survivor is the spin angular momentum of the 47th electron. Thus what should happen is that, so we should get signature of the spin angular momentum of the 47th electron on the screen if we do this experiment. And moreover the nucleus makes very negligible contribution to the angular momentum because of because of the large mass compared to the mass of the mass of an electron.

So, the magnetic moment of the silver atoms silver is written by Ag is effectively due to the due to the spin angular momentum of a single electron. So, this should correspond to, so what we mean by this is that the experimental data that one gets which is a too bright spots on the screen where the silver atoms they impinge on. So, this data should correspond to the intrinsic spin angular momentum which can take two discrete values namely plus h cross by 2 corresponding to maybe the spot above the mean level and a minus h cross by 2 corresponding to the spot that appears below the mean level.

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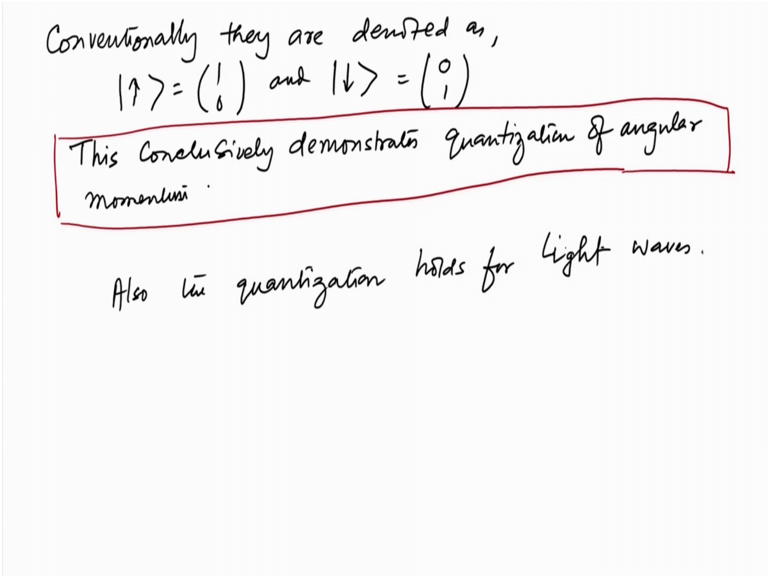

And these are conventionally written as up which is also written as a two component spinner and down which is 0, 1. So, this conclusively demonstrates a quantization of angular momentum, ok. So, but now, it is shown for matter waves or other particles such as electrons and so on, but it also happens for light waves such quantization of angular momentum and that is related to its polarization properties that is the direction in which the amplitude is pointing at. And so there could be right circularly polarized light waves which are photons or there could be left circularly polarized photons, and they are associated with integer number of the Plank's constant that is a left circularly say corresponds to a plus h cross and the right circularly polarized photon corresponds to an angular momentum of minus h cross.

Thus, this quantization is not a priority for the matter waves, but it can also happen for light waves. So, also the quantization holds for light waves, ok. So, this is the birth of quantum mechanics in which some of the observables are related to the measurement of subatomic particles are found to be quantized. And then we have also talked about the wave particle duality at length and it was conclusively told that in some experiments particles show the particle nature of flight. Whereas, in some other experiments they show the wave nature of light and there is an ambiguity and without this the description is not complete.

There are examples of particle nature or the corpuscular nature in which the Compton effect and their photoelectric effect which shows this particular characters whereas, the interference and diffraction etcetera they show the wave character. And we have seen elaborately the Young's double slit experiment which is basically which is because of the interference of light waves or the photons the waves associated with the photons. So, we want to understand that what are the other features associated with the wave function.

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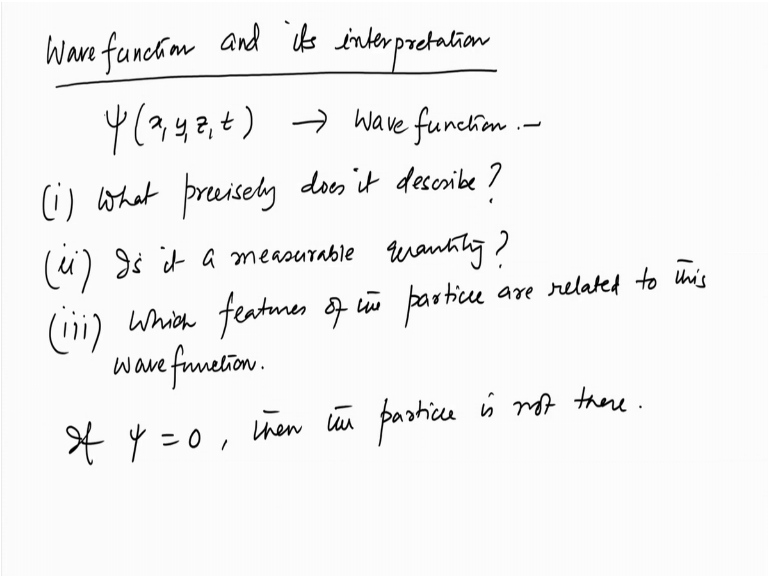

So, let us get into a discussion called as the wave function and its interpretation. So, we are going to talk about mainly what it means, what does a wave function mean, but also there are issues related to superposition or the specific directional properties of the amplitude which are known as polarization etcetera. However, some of these issues can be left out of discussion for now, and we may come back to it later or to them later and right now, let us talk about what is a wave function and what is its interpretation.

So, let us say there is a wave function. So, this wave function is associated with the motion of a particle. So, let it is a function and it is a function of space variables x, y and z or you can write it in terms of this spherical polar variables r theta and phi or the cylindrical variables rho phi and z and so on, and also it is a function of time. So, if this is a wave function. So, the question that we ask is that what precisely does it describe, second question is it a measurable quantity?

And so in part in particular which features of the particle are related to this wave function. But certainly this wave function is related to the existence of the particle because in some region if psi is equal to 0 then of course, we say that the particle is not there. But if it has to be directly the measure of finding a particle at a given space time point x, y, z and t then this quantity should always be positive and real. Now, there is no problem with that this quantity can always be positive and real, but we know that this quantity is complex because for it if it is not complex then we cannot explain the interference properties that we have seen in the Young's double slit experiment.

So, this is certainly not directly the measure of a particle or the existence of a particle, but it is somehow related to it. So, we will write down that these observations.

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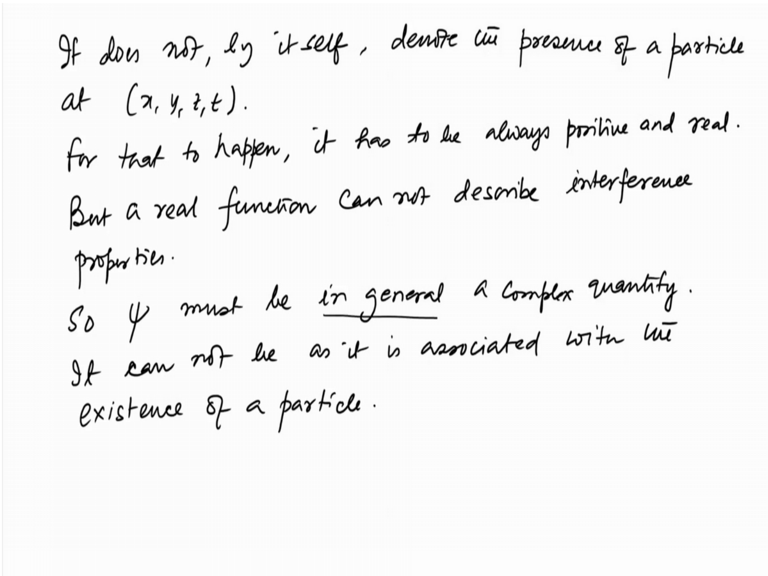

It is not directly related or it is maybe it is a better way of saying it is that it does not by itself denote the presence of a particle at x, y, z and t. For that to happen it has to be always I mean always and everywhere when I say always it means that at all points of x, y, z and t always are positive and real, but the problem is that, but a real function cannot describe interference properties we will come back to this.

So, psi must be in general a complex quantity. Please do not get this message that psi always a complex quantity we will solve Schrodinger equation and more often than not we find psi to be a real quantity the at least a special part of it and so on. And this does not violate any of the things that have being told here about the interpretation of the wave function, it can still be a real quantity, but in general it is a complex quantity. So, that is why this in general it is written. So, if it is in general a complex quantity it cannot be associated it cannot be, as it is associated with the existence of a particle, ok.

So, then what should we do then how do we get this psi into perspective or make it useful for understanding the behavior and the motion of the particle.

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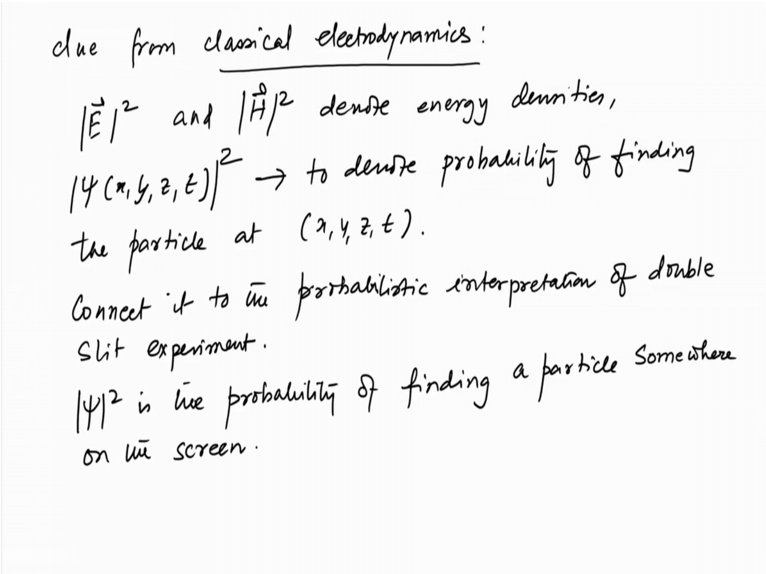

So, this clue is obtained from classical electrodynamics. Just like E square that is a electric field square and H square which are the electric and the magnetic energy densities denote while the vector E and the vector H are just mathematical forms of the electric and the magnetic fields. The measurable quantities or the quantities which have physical interpretation are these densities or when they are integrated over volume there called as a electric electrostatic energy or the energy associated with the electric field and the energy associated with the magnetic field and so on. So, that way we can write down psi mod square to denote probability of finding the particle at x, y, z and t.

In fact, this was one bottlenecks with the birth of quantum mechanics many people including some of the renowned scientists at that time felt very uncomfortable with the non deterministic doctrine or the idea associated with quantum mechanics that it has to have a probabilistic interpretation. But however, after a lot of deliberation and looking at many experiments and looking at and the development of the field during that time they had to adopt these probabilistic interpretations. And so this is the meaning or the physical interpretation associated with the wave function that a mod square of that is related to the probability of or it is the probability of finding a particle at a given a point a or at a given space time point

And, so if we want to connect these to the probabilistic interpretation of the Young's double slit experiment then. So, we can understand that the interference pattern occurs due to the statistical effects are coming from a large number of photons which are interfering from the two slits on and impinging somewhere on the screen, ok. So, psi square is the probability of finding a particle at a given place on the screen.

So, psi square is the particle somewhere on the screen and we are superposing a large number of photons coming and interfering from the two slides and making that pattern occurring on the screen which is kept at a distance from the slits. So, the appearance of the interference fringes that depends on the passage of the wave through both the slits, but now, if these waves are associated with the particle it implies that the particle has gone through either of the slits, ok.

And if there is a careful detection mechanism or a careful monitoring mechanism which finds out that which slit has it gone through, then the interference pattern will be wiped off because then it becomes deterministic and determinism does not give rise to this interference pattern as we know.

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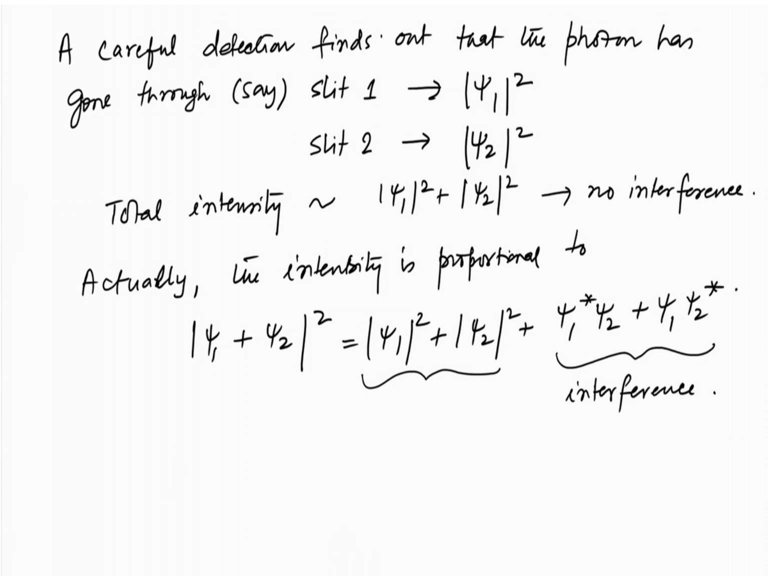

And the way we can understand this is this that suppose a careful detection finds out that the photon has gone through, say slit 1. Then the intensity would be proportional to a psi 1 square and on the other hand, it can also go through slit 2 in which case the intensity of the pattern would be proportional to a psi 2 square. So, the total intensity is proportional to psi 1 square plus psi 2 square and there is no interference.

So, the interference can only come from by superposing. So, actually the intensities to psi 1 plus a psi 1 plus psi 2 square which has in addition to these two terms that are written above it has a term like psi 1 star psi 2 and a psi 1 psi 2 star these are the interfering terms which gives rise to interference pattern. So, this interference is very crucial and for that interference to happen the photon has to be in deterministically passed through both these slits that is, you cannot determine which state it has gone through and the wave function by themselves will interfere and will give rise to an interference pattern, ok.

So, this is, so by this you will not have interference, but with this only included one will have interference pattern. So, this is the way the probabilistic interpretation has shaped up the or the development of quantum mechanics and its very important to say that psi or psi 1 or psi 2 they refer to a single photon here or a single particle for that matter psi mod square is a probability of finding a particle. However, for probabilistic interpretation to be valid or give rise to a valid description one has to statistically superpose many such events and this is what we have seen even earlier that a large number of photons actually come and they interfere and they give rise to the pattern.

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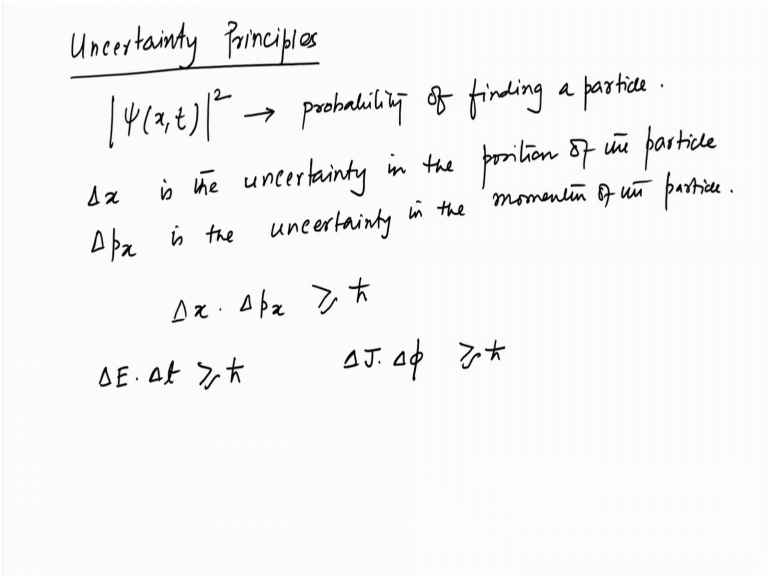

The other thing that is important is this uncertainty principle which we have mentioned and we have seen that how superposing many waves gives rise to a completely deterministic position of the particle. So, the uncertainty in the position of the particle goes away and giving very large uncertainty to the momentum or the velocity of the particle. So, if now, we have said that psi is related to the existence of a particle at a given space time point, and so that given by the probability of finding a particle at a space time point is given by psi mod square x say we are talking about if we talk about in one dimension this is the. So, this is the probability of finding a particle

However, so what does the experiment say on this? So, the experiment say that even if you want to find out the position of the particle there has to be an uncertainty in the position which is not related to the position other or the uncertainty or the measuring device or other the list count of the measuring device it is completely a sort of fundamental uncertainty that cannot be avoided. So, say delta x is the uncertainty in the in the position of the particle, and there has to be an uncertainty in the canonically conjugate variable which in this case in the momentum uncertainty; so is the uncertainty in the momentum of the particle.

And uncertainty principle says that this delta x multiplied by delta p x has to be greater than the quantity called as h cross. So, if you want to localize your particle to a great degree of accuracy you will have to give up a large or, so the momentum uncertainty of the particle will be very large which means that the particle actually can be moving from minus infinity to plus infinity. And if you want to actually talk very precisely about the velocity of the particle at given space time, then you have to give up the notion of specifying with accuracy with any degree of accuracy the position of the particle. And the uncertainty in the momentum and the uncertainty in the position is related by this where h crosses a planks constant.

And similarly there are other uncertainty principles as well like the uncertainty in energy and the uncertainty in time is also related by this. And the uncertainty in the angular momentum multiplied by the uncertainty in the angular variable that is canonical to the angular momentum is also obeys this relation. And these are absolutely fundamental and therefore, nothing to do with the limitations of or the accuracy of the apparatus that are used to calculate these things. Just a small demonstration or rather explanation of how one can find these uncertainties all one can prove for a given case.

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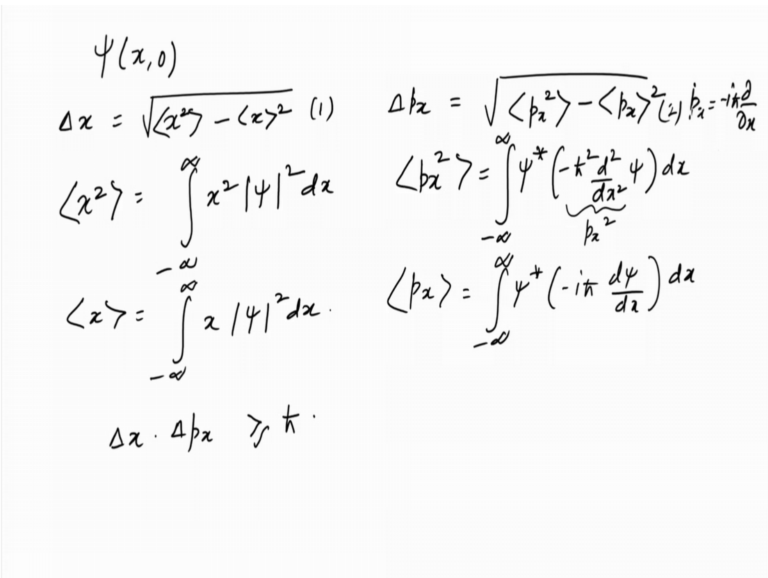

Suppose one has a wave function which is given by psi of x it is it could be a function of t also, but let us just put that time equal to 0. So, a momentum and the position uncertainty delta x is written as x square average or expectation minus x expectation average and similarly a delta p x is equal to p x square average or expectation minus p x expectation square and each one of them can be found. So, a x square can be found from the wave function which is x square. Let us just write it clearly.

X square and the psi mod square from minus infinity to plus infinity dx that should be equal to something which is which should give me the expectation value of x square whereas, the expectation value of x is equal to the same thing with x psi square dx. Now, if psi has a form which is like a symmetric form about 0 then of course, the this term will be equal to 0 and similarly the p x square can be written as. So, the p x operator is equal to minus i h cross del x. Now, this is a representation of the momentum operator in position space this we will talk about soon, and this can be obtained as psi of this you cannot put all these psi's together because this is a psi star and there is a minus h cross square d 2, d 2 dx two and psi dx from again from minus infinity to plus infinity whereas, p x is equal to minus infinity to plus infinity as psi star minus i h cross d psi dx.

So, this is your p x square operator which is related to the kinetic energy of and this is your p x. So, one can use this for a given psi of x one can use this formula in order to calculate all these quantities such as x square expectation and x expectation and so on. And then hence calculate delta x from this formula 1 and delta p x from this formula 2. And then one should be able to show that this is the at least of the order of h cross in order to have the uncertainty principle to be value.

So, these are by and large the foundations of quantum mechanics. We have elaborately explained the probabilistic interpretation and the wave notion or wave particle duality of not only light waves or photons, but also matter waves through examples. And one of the main examples that we have taken is the Young's double slit experiment in which photons do interfere and show an interference pattern on the screen. And these are some of the important way the quantum mechanics actually developed in these days.

And why we are doing this is that some of these key ideas are will be actually applied to modern topics such as quantum computation and so on. And finally, this uncertainty principle which we say that it is related to the two of very fundamental fact; that these are coming because with all these the wave interpretation and so on and the probabilistic interpretation. So to say, because we can still talk about the wave function or a particle with a well defined momentum k, and we can also talk about position of a particle given by psi of x.

However, we know that for a probabilistic interpretation to take place there we have to superpose many of this particle a very large number of them, and in which case these spread in the position of the particle and spread in the momentum of the particle are inevitable. And that is why this uncertainty principle due to Heisenberg is an inevitable thing which is coming out of these fundamental postulates and interpretation of quantum mechanics.