Quantum Technology and Quantum Phenomena in Macroscopic Systems
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Lecture – 23
Josephson Junctions-I

Hello, welcome to lecture 7 of the module-2 and lecture 17 of this course. In this lecture we are going to discuss about Josephson junctions in somewhat details and the Josephson Junctions as you know it is one of the integral components of any circuit QED based devices. So, let us begin.

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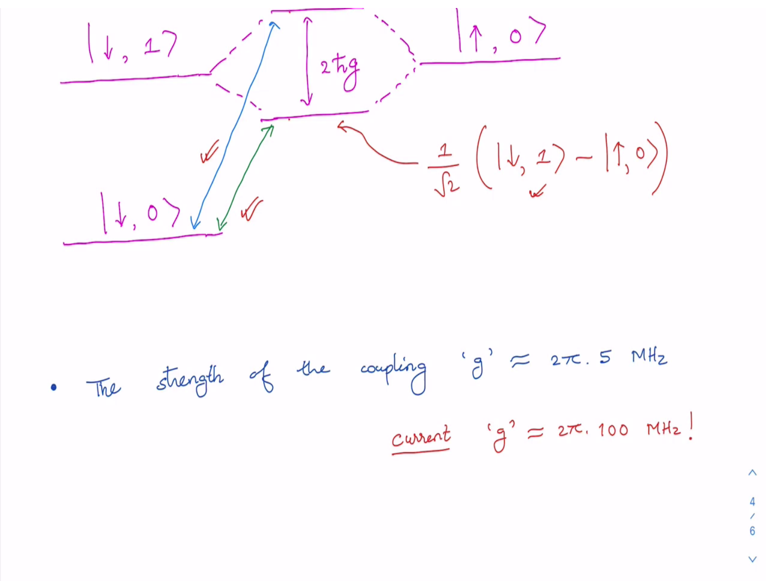

In the last class we discussed dynamics of the Jaynes Cummings model. We started with the question that say at time t is equal to 0 if the atom is in the ground state and there are n number of photons in the field mode then how the system is going to evolve in time. So, we analyze it by using very elementary quantum mechanics and we found out the state vector of the system at an arbitrary time t.

And then also we found out what would be the probability that we are going to get the system such that the atom is in the ground state at a later time t and we worked out this particular expression and we saw that this particular quantity g into square root of n that acts like a Rabi frequency that we discussed earlier. And when we have plotted it we saw that this frequency of oscillation is obviously g into square root of n and there are 2 cases of importance when the number of quanta in the field is extremely large very very large then the whole thing actually mimics the classical case of Rabi oscillation right, the driving field actually behaves like a classical driving field. On the other hand, there is another extreme case where suppose there are only one quanta in the field mode then it turns out that the system oscillates between these 2 state where atom is in the ground state.

And the fields there are one photon in the field mode and the atom is in the excited state and there is no quanta or photon in the field mode. So, because of this oscillation and particularly because there are no quanta in the field mode that is basically vacuum. So, this oscillation is known as the vacuum Rabi oscillation. Then we went on to discuss the dispersive case where the detuning parameter, the detuning parameter is difference between the transition frequency and the field frequency and if that is much larger than the coupling strength between the field and the 2 level atom we discussed that and we found that the energy levels now gets modified according to these 2 expressions and what is interesting is that now as per this expression you see that the field mode gets modified depending on whether the atom is in the excited state just like here.

If the atom is in the excited state the field mode is getting enhanced. And on the other hand, if the atom is in the ground state the field mode is getting you know de-excited or what is actually can be observed spectroscopically and the whole thing system in this dispersive case can be the Hamiltonian can be written by expressed by an effective Hamiltonian.

And in fact, all these things that we have discussed was experimentally observed by a yell group in the year 2004 and that experiment is considered to be a landmark experiment and we discussed that and as per the experiment what is this scheme is like this where a microwave is irradiated on a transmission line resonator and this transmission line resonator is now coupled to a cooper pair box or a 2-level atom or a qubit.

And depending on the location of the cooper pair box that decides the coupling strength what is observed is you know when the microwave is getting irradiated a part of it get transmitted and a part of it get reflected. And typically, the transmitted beam is observed and people do 2 things one is they either find out the intensity as well as they measure the frequency of this transmitted wave.

And what they observe that when there is no cooper pair box, the system is detuned in such a way that the effect of the cooper pair box is not there then observe only one peak here as you can see and when the coupling is on it is switched on coupling is switched on they found that there are 2 peaks and these 2 peaks are separated by a frequency 2g where g is the coupling frequency or the coupling parameter as we know.

And they also observed the case and this was the case when they have the detuning was there that means field frequency is exactly matches the atomic transition frequency and they check the case for when they varied the detuning parameter and they of this observe this results and you see the intensity actually varies depending on the detuning parameter how you are getting it varied and here actually the things that we have discussed for the dispersive case was experimentally observed and they found it in the in their experiment.

And this were very simply explained on the basis of this level scheme that we discussed for Jaynes Cummings model and the 2 peak that appeared is primarily due to the reason that there are 2 transitions are possible. And in these 2 transitions are associated with the creation of a photon in the field mode and that is responsible for the observation of these 2 peaks.

And you see because if you look at these 2 transitions, they are separated by frequency 2g and that is the origin of the of the peak you know the difference in the peak by frequency is 2g and that is clearly explainable in terms of this level diagram one thing I forgot to tell in the last class was that the strength of the coupling parameter they took in their 2004 experiment was 2 pi into 5 megahertz.

Nowadays it is typically 100 megahertz or even more than that so because of technological development. Now there is around 20 times or even 30 times larger than what it was at that time in 2004. And this landmark experiment they reported in this they published it in this journal nature. Now as I mentioned in the last class ,we will have a somewhat detailed discussion on Josephson junctions.

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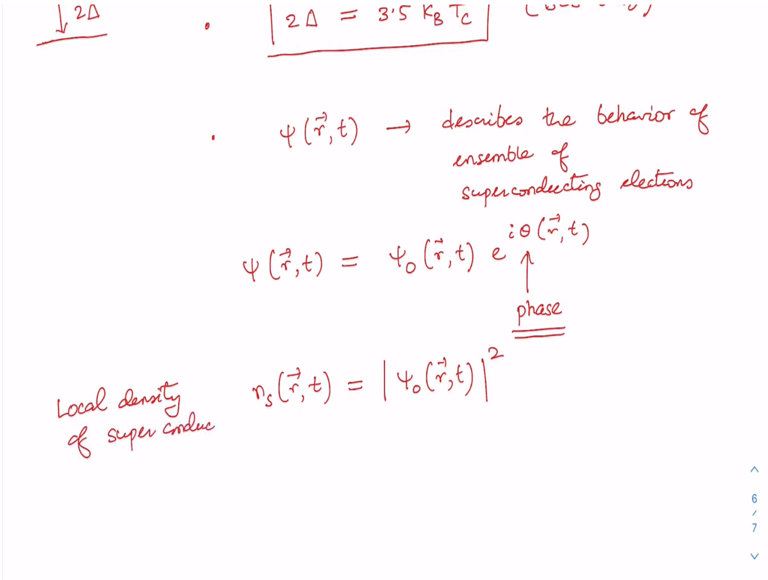

So, Josephson junctions and Josephson junctions is a fundamental element in any superconducting qubit design it basically consists of 2 superconducting electrodes. So, these are say our super conducting electrodes separated by a thin insulating layer. So, this is the insulating layer. So, far we discussed only on one type of Josephson qubit that is the so-called cooper pair box.

And cooper pair box is also known as charge qubit cooper pair box we discussed in somewhat details earlier and it is also called charge qubit. So, in this class we will talk about other variants such as flux cubit we will talk about flux cubit and also, we will talk about Transmon qubit. However, before that let us discuss about one important theory called Ginzberg Landau theory which is responsible Ginzberg Landau theory and this theory is responsible for much of the development regarding superconducting circuits.

So, as you recall that superconductivity appears in a material when it is cooled below a temperature less than the critical temperature and in this case the physical device which is kept at a temperature less than this critical temperature it exhibits zero resistance and expels the magnetic field. And also, we learned based on the so-called BCS theory that the electron density of the states acquired a small gap of twice delta separating the occupied and the unoccupied you know states.

So, we discussed this earlier in class that this energy gap is two delta and as per BCS theory this energy gap is equal to as per BCS prediction for normal or conventional superconductors that is 3.5 k B T c k B is the Boltzmann constant. While explanation of superconducting phenomena came from BCS theory as I said most practical developments actually came from the so-called Ginzburg Landau theory. So, the silent feature of I am not going to discuss in great details about this theory.

But we will just discuss the main features which would be useful for our purposes first of all what Ginzberg Landau postulated was that the behaviour of the whole ensemble of superconducting electrons is described by many particular wave functions say psi of r t. So, psi of r t describes the behavior of ensemble of superconducting electrons.

And what they postulated is this that this wave function this many particle wave function psi of r t is equal to say psi 0 r t e to the power exponential e i theta here and theta is the phase of this super conducting electrons and what was assumed that the theta this is the phase all the cooper pairs has the same phase theta and therefore macroscopic variables depends on this important quantity that is a phase theta.

And the local density of the superconducting electrons is given by say let me denote it by density of the local electron or superconducting electrons say at a point r and at time t that is say ns is equal to modulus of psi 0 of rt mod square. So, this gives the local density of superconducting electrons. So, this is local density of superconducting electrons. So, this is what we have from BCS theory.

Now let us understand the physical meaning of this very important quantity that is the phase theta and that we can understand if we consider the continuity equation for the probability of charged particle in an electromagnetic field.

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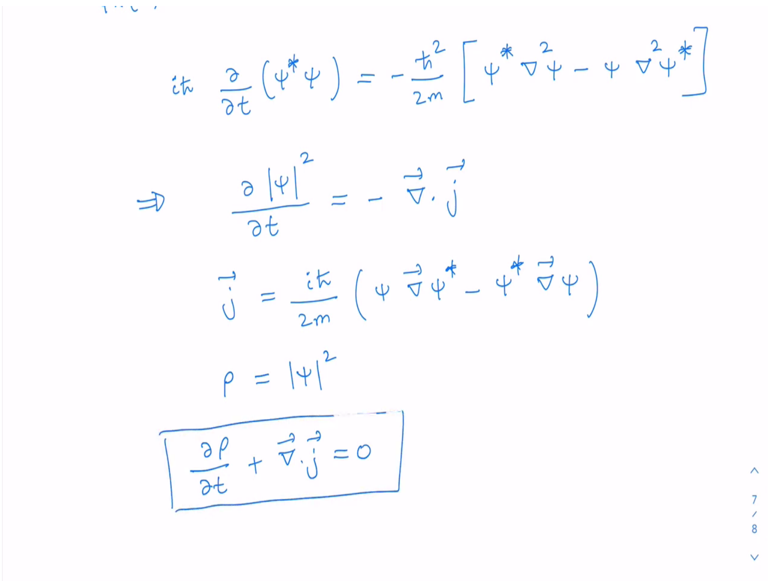

So, for that we need actually the continuity equation I will explain all these things the continuity equation for the probability for the probability of a charged particle in our case these superconducting electrons charge particle in an electromagnetic field. And this continuity equation it can be derived and it would be del rho del t plus divergence of J rho is equal to 0 and where rho is psi of rt mod square I will I think it is maybe it would be useful to digress a little bit here and let me remind you about continuity equation.

Because if you remember that can be worked out using the so-called Schrodinger equation just let us remind ourselves we know how we start we start with this Schrodinger equation i h cross del psi of rt del t is equal to the time dependent Schrodinger equation we write it as h cross by twice m then we have delta two psi of rt and then we have this potential term.

Suppose the potential is just dependent on space only then we have this Schrodinger equation from our elementary quantum mechanics. Then what we do let us say this is my equation number one and then if I take the complex conjugate of this equation then I get psi star of rt del t and then here I have h cross by twice m delta two psi star of rt then I have V of r psi star of rt say let us say this is my equation number 2.

And what we do, we actually multiply this equation one with say psi star and then multiply this equation two with psi. So, we multiply equation two with psi and then if we subtract then from this equation, I am just giving you the procedure only here I do not want to go into the detail details what you will get if you do it very straight forward you will get i h cross you will be able to write it as delta t psi star psi and that would be equal to minus h cross by twice m.

You will have it is easy to see I encourage you to please do it delta psi psi delta two psi star and from here you can actually build up the equation of continuity and this is ultimately going to give you equation of this type delta of mod psi square delta t which I can finally write it as minus divergence of j, j is the so-called probability current density and you will find that j is equal to i h cross by twice m.

Maybe I will give it as an assignment problem and you will be able to have a practice over it. So, this is what you should get. Now in this case as you see this is the current density probability current density for a particle a single particle placed in a potential V of r and the probability density rho is given by mod psi square. So, in terms of rho we can now write down the continuity equation in this most familiar form del rho del t plus divergence of j is equal to zero.

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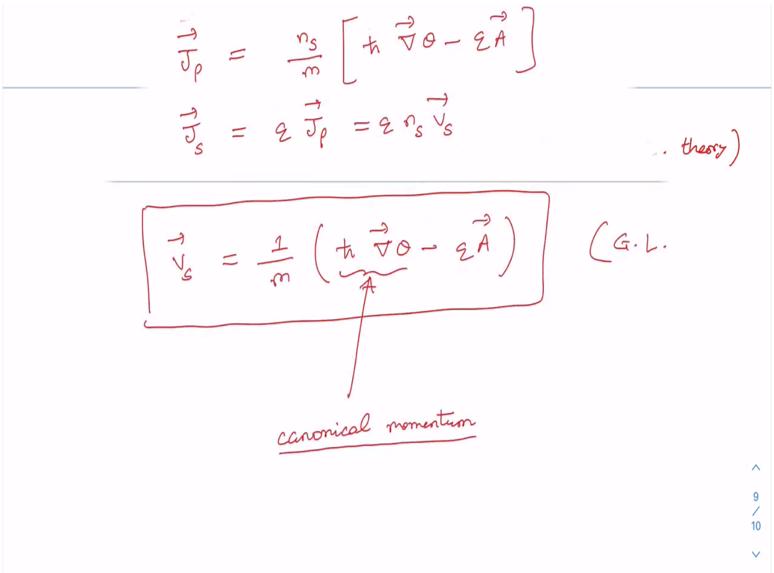

Now let us consider one important case which is of great relevance to us that is the charged particle if a charged particle is placed in a constant magnetic field. So, in this case we can know from our quantum mechanics or we can actually show it that the Hamiltonian for the charge particle placed in a magnetic field would be given by p minus q A whole square divided by twice m plus q into phi, phi is the scalar potential and A is the vector potential.

And then we can write down the Schrodinger equation for the particle which is now placed in a magnetic field and that would be simply equal to you know p I can write it as minus i h cross delta minus q A whole square divided by twice m plus q phi and then it is operating on this wave function. Now, exactly applied in applying the same procedure that we have done for the normal Schrodinger equation.

This continuity equation can be worked out and only thing here would be this quantity j rho is going to be different and we will found it can be worked out and you will find that this would be simply i h cross by twice m and here you will have it as psi grad psi star minus psi star grad psi and you have an additional term that would be q by m psi star A psi and if you compare this expression now with the current density expression that we have we have worked out for the normal case you see this there this extra term here is not there because we did not consider the particle to be placed in the magnetic field in this case when we discussed. Now if the particle is placed in a magnetic field. Now an extra term is now coming and this is the equation that now we are going to utilize.

Now because of this Ginzberg Landau theory what we have that the local density of the super conducting electrons is given by mod of psi of rt square right and because of this the interpretation of j rho if you look at it carefully and what you will have that j rho refers to the fact that it is actually number of condensate or superconducting electrons per unit area per unit area per unit time or we can also say that is the flux of electrons per unit time.

And J p this quantity can be related to the current density provided we just multiply J p with the charge and then this would be simply the current density J s and you know the current density is related to what is this called number density of the electrons and its velocity. So, that would be q charge into ns into velocity of these super electrons. So, what we can now work out because we have our j rho is equal to i h cross by twice m I have psi grad psi star minus psi star grad psi minus q by m psi star A psi.

So, if I put the expression as per Ginzberg Landau theory psi is equal to square root of n s e to the power i theta. So, let me work it out for you. So, I just have to put this psi this psi here. So, that means I have to work out say first of all what is my grad psi. So, you will see that that would be simply one by twice square root of ns and then I have divergence or a grad of sorry grad of ns e to the power i theta plus i e to the power i theta square root of n s you will have grad of theta.

And this grad of psi star is easy you just have to take the complex conjugate and you will get it simply this here it would be e to the power minus i theta and here you have minus i e to the power minus i theta square root of n s grad theta. Now you multiply this one with psi star and this is with psi. So, you will have psi star grad psi and if you do that you are going to get simply a half grad of n s minus i n s grad of theta.

And then the other one you just have to take again the complex conjugate and you are going to get it as half delta ns plus i n s grad theta. So, you can now you have this expression and with you. So, you just have to put it here and if you do the manipulation you will very simple calculation will give you j rho is equal to ns by m it would be h cross grade theta minus q into A all right.

So, this is what you obtain. Now if you compare this with say J s is equal to q into j rho which already we have written that that is your q into n s into v s. So, if I compare then I will obtain that v s the velocity of the superconducting electrons is equal to one by m h cross del theta minus q into A. So, this is an important expression we have got for the velocity of the conducting electrons from this Ginsberg Landau theory.

And what it says that this particular expression here you see this part is actually we can interpret it as the canonical momentum. So, this is our canonical momentum. So, phase of the superconducting electrons or the cooper pairs basically gives us a momentum and this is an important quantity and we will see its role in flux quantization.

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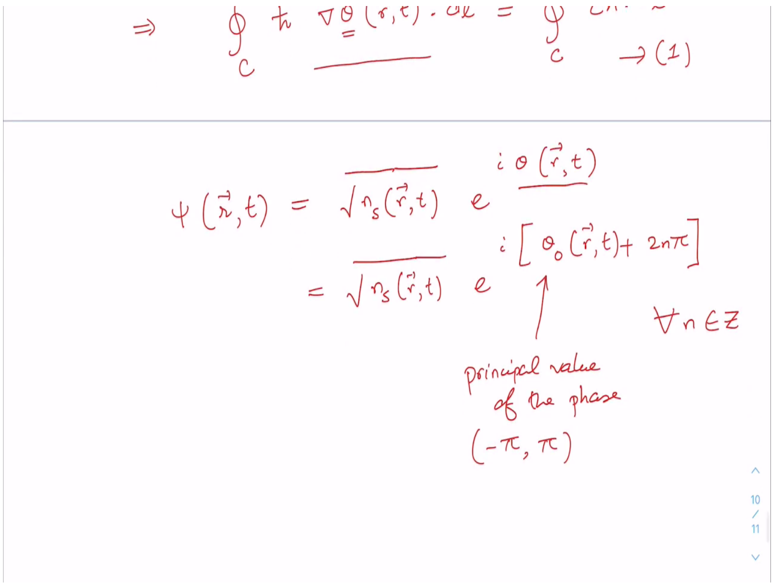

Now let us discuss the important topic of flux quantization. You may recall from an earlier class that magnetic flux can be considered as the coordinate and charges the momentum to quantize an LC oscillator or even the transmission line resonator or transmission line. So, here I am going to show that flux enclosed by superconductor must be quantized must be quantized in units in units of flux quantum say denote it by say phi zero is equal to h by two e

And let us see how we can do that. For this let me consider a closed surface a closed curve in the bulk of a superconductor ring. So, let us say I have a superconductor ring like this and inside this ring let me consider a curved closed curve say c this is the this is our this is the bulk of the superconductor. So, my diagram is not proper but I hope you are getting just to consider this as our superconducting ring.

And here c is the curve closed curve in the bulk of the superconducting ring and in the curve it is a far from the surface far from the surface here or here the current density is a is current density say J s this is our current surface current density J s and the magnetic field B both this quantity vanish far from the surface in the curve in the curve c.

So, this condition actually it implies that the integral of this current density around the close curve. So, around this closed curve that is going to be equal to zero. This we can write it as because already we have the expression for this current density earlier we have worked out. So, this is what we have as our current density expression using this along with this expression current density J s is equal to q into J p this already we have.

So, utilizing these two we can now write it as q charge into this integral n s divided by m h cross gradient of theta minus q into A that is the vector potential and dot dl is equal to zero. So, we can immediately see that this means that I can write this expression as h cross gradient of theta which is a function of r and t dot dl around this closed curve that must be equal to this integral q A dot dl I think it is straightforward to understand. So, let me say this is my equation number one.

Now in order to evaluate the integral on the left hand side of this equation we have to realize that the real function theta is not well defined since there exists multiple phase variables phase values giving the same result for

the macroscopic wave function let me explain it we know that psi of rt the wave function as per this Ginsberg Landau theory we have taken it as our square root of this local density of this super electron superconducting electrons and we have e to the power i theta, theta is the common phase of this cooper pair electrons. And this you see we are we are going to get the same thing, If we have we write e to the power i if I just write theta as theta zero r of t plus some integral multiple of two pi where for all values of n which is an integer say integer z n is an integer. So, now here this theta zero is the principal value it is the principal value of the phase. So, what I mean to say is that this theta or theta this phase factor is not well defined and this theta zero is the principal value and it values actually its principal value lies between minus pi to plus pi.

And so I think you get the idea here now the integral around the path c is computed by this integral can be computed by taking into account that the integrand is the gradient of the scalar function and also the part is in a is a multiply connected region as you can see that this part c, c is multiply connected. So, using this fact I can now work out this integration.

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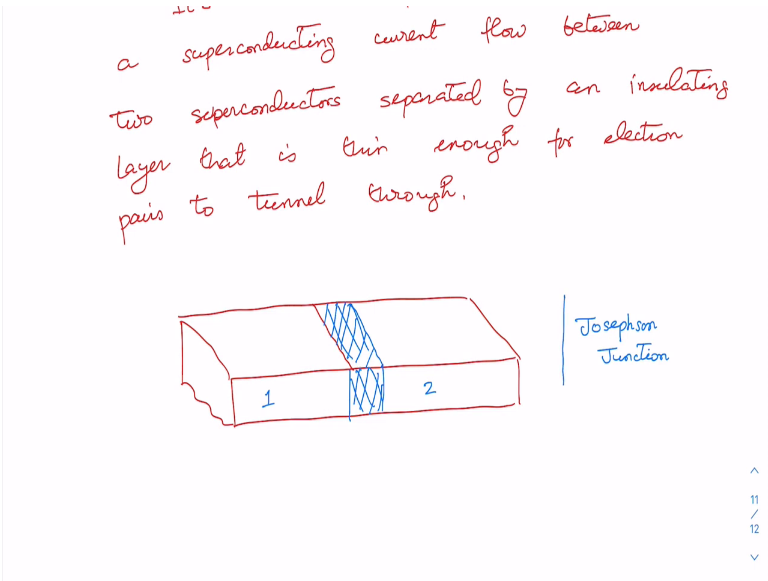

This part is going to give me this integration around this close part h cross gradient of theta r t dot dl I think it is easy to see that you can write it say h cross if I take say limit I will explain r two tends to r one here I can write this theta at somewhere r two I am evaluating it minus theta and evaluating it at r one t and because the radius is basically same r two tends to r one in that limit.

So, you see because of this constraint here because of this condition because theta zero they are defined by this amount varied by this amount. So, therefore immediately the difference as you can see would be h cross and this one is two pi into the integral multiple of two pi. So, therefore I have simply two pi h cross actually same thing I am writing here. Now this part of the left hand side of the integration we have worked out of this equation one.

Now what about the other one this right hand side of this equation let us work that out also that is q A dot d l if you can see I can apply the so-called stokes theorem then I have q is equal to this is the line integral right it is around the closed curve c and then I can convert it to surface integral and you have curl of A dot ds that is the surface integral that is coming from the so-called stokes theorem. So, I have applying stokes theorem stokes theorem I get it.

And you can see that curl of A is nothing but the magnetic field. So, this is I have B dot ds and again this guy is nothing but the magnetic flux. So, it is q into phi B. Now already we have seen that this is our right hand side of this equation here and we have worked out the left hand side of this equation. So, let me equate that. So, what I have is q into phi B that is equal to twice pi h cross into n.

So, this is what I have, also because in superconductors now we know that this charge this elementary unit of charge it is basically twice that of the electron charge. So, immediately we have this. So, therefore so we can write magnetic flux
phi B is equal to 2 pi h cross n divided by twice e and therefore you know 2 pi h cross is nothing but h the Planck's constant h cross is the reduced Planck constant.

So, therefore immediately you see I can write the whole thing as h divided by 2 e into n and I can then write n into phi 0, phi 0 is the flux quantum. So, here we then define that it is h divided by 2 e. So, this is what we mean by flux quantization. Let me now discuss about the Josephson effect. This is a coherent phenomenon, It's a coherent phenomenon, which predicts that a superconducting current a superconducting current flow between two superconductors separated by an insulating layer that is thin enough for electron pairs electron pairs to tunnel through.

So, it is basically a tunneling phenomena. So, say we have a superconductor like this and this for simplicity purposes let us consider that this superconductor, there are two superconductors formed by you know same material, same insulating materials this is the insulator here and both the superconductors are made up of the same materials and it is homogeneous on both sides say this is superconductor number one this is superconductor number two and this actually junction is called the Josephson junction this is called the Josephson junction.

Already we discussed that in the context of copper pair box also it is copper pair box is also a Josephson junction and it is a basic component in circuit quantum electrodynamic based technology.

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$$\rightarrow \; i\hbar \frac{\partial \psi_1}{\partial t} = U_1 \psi_1 + a \psi_2$$

$$\rightarrow \; i\hbar \frac{\partial \psi_2}{\partial t} = U_2 \psi_2 + a \psi_1 \qquad \rightarrow (1)$$

U_1, U_2 are the ground state energies of each S.C.

$$\psi_1 = \sqrt{n_1(\vec{r},t)}\; e^{i\theta_1(\vec{r},t)}$$

$$\psi_2 = \sqrt{n_2(\vec{r},t)}\; e^{i\theta_2(\vec{r},t)} \qquad (2)$$

$$i\hbar \left(\frac{\dot{n}_1}{2\sqrt{n_1}} + i\sqrt{n_1}\,\dot{\theta}_1 \right) e^{i\theta_1} = U_1 \sqrt{n_1}\, e^{i\theta_1} + a\sqrt{n_2}\, e^{i\theta_2}$$

$$i\hbar \left(\frac{\dot{n}_2}{2\sqrt{n_2}} + i\sqrt{n_2}\,\dot{\theta}_2 \right) e^{i\theta_2} = U_2 \sqrt{n_2}\, e^{i\theta_2} + a\sqrt{n_1}\, e^{i\theta_1}$$

Now let us analyze the main features of a Josephson junction say the wave function of the superconducting states on this side say superconductor one is represented by this wave function psi 1. I am actually adopting a very simplified approach and just to explain the physics and psi 2 is the wave function describing the superconductor number two and the actually this is going to lead us to describe the dynamics and this dynamics is given by this couple Schrodinger equation.

Let me write that say i h cross del psi 1 del t for the first superconductor and then you have say U 1 psi 1 + a psi 2 and for the other superconductor number two I have del psi 2 del t this is the Schrodinger equation and U 2 psi 2 + a psi 1 here U 1 and U 2 this U 1 and U 2 are the ground state energies of each superconductor are the ground state or the ground state energies of each superconductor.

And this a is the coupling between the two superconductors and it is a parameter which basically characterizes the overlap between the wave function psi 1 and psi 2. So, you can consider a to be the coupling parameter. Now if a is equal to zero then the dynamics is described by the uncoupled equation as you can see easily and we know that psi 1 we can write it as square root of n 1, n 1 is the density of superconducting electrons in superconductor one which we already know from our Ginsberg Landau theory i theta 1 r t here and psi 2 is equal to square root of n 2 r t e to the power i theta 2 r t.

And theta 1 and theta 2 are the phases of each superconductor. Now let us say this is these are my equation number 1 and these are my equation number 2 let me put equation 2 in equation

1 then if I actually put it then this is going to lead us to this equations you can easily actually see the first equation here this equation is going to give us i h cross n 1 dot, dot refers to the time derivative 2 into square root of n 1 + i square root of n 1 theta 1 dot and then you will have e to the power i theta 1.

And the other side I will have U 1 square root of n 1 e to the power i theta 1 + a into square root of n 2 e to the power i theta 2. Similarly for the second equation for the superconductor number 2 I have i h cross into n 2 dot divided by 2 into square root of n 2 + i into square root of n 2 theta 2 dot and you will have i theta 2 then you will have U 2 square root of n 2 e to the power i theta 2 + a square root of n 1 e to the power i theta 1.

I think you can very easily verify these equations. Now if I equate the real and imaginary parts on both sides we can obtain these equations that I am now going to write.

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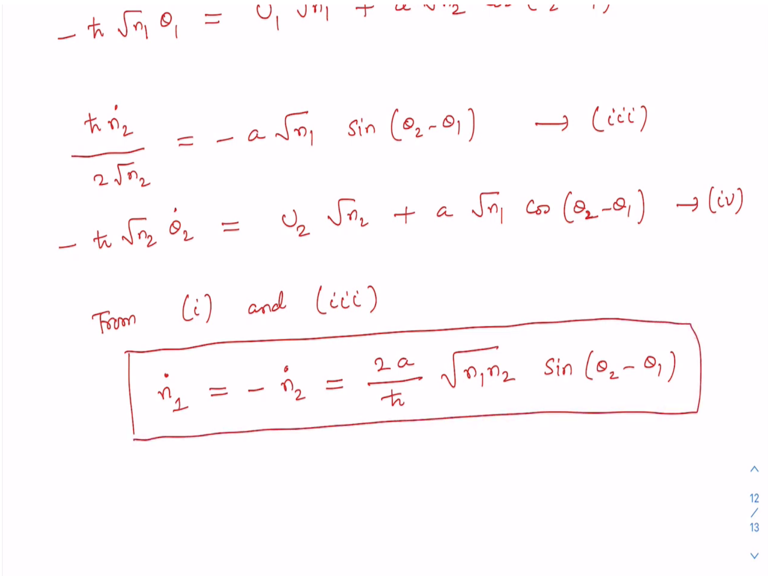

Firstly, you will get h cross say n 1 dot divided by 2 into square root of n 1 is equal to a into square root of n 2 sine theta 2 minus theta 1. You can immediately see I just compared the imaginary part on both sides of equation this equation and then the real part if I compare then I will get h cross square root of n 1 theta 1 dot that is equal to U 1 square root of n 1 + a square root of n 2 cos theta 2 minus theta 1 let me say this is my equation number 1 in this form this is equation number 2.

Similarly from this second equation if I do the analysis I will get similar equation h cross n 2 dot divided by 2 into square root of n 2 is equal to minus a square root of n 1 I will get sine theta 2 minus theta 1 see this is my equation number 4. And then I will get minus h cross square root of n 2 theta 2 dot is equal to U 2 square root of n 2 + a square root of n 1 cos theta 2 minus theta 1.

Let us say this is my sorry this is my equation number 3 this is equation number 3 and this is my equation number 4. Now from equation number 1 and 2 actually 1 and 3 I can get if you look at it I will get that rate of change of the number density of the superconducting electrons in superconductor 1 is equal to minus n 2 dot that is the rate of change of superconducting electrons density of superconductors in the superconductor number 2.

That I can write it as 2 a by h cross square root of n 1 n 2 sine theta 2 minus theta 1 all right. So, this equation basically establishes a relation between the superconducting current density and phase difference across the junction. So, phase difference phi is equal to theta 2 minus theta 1.

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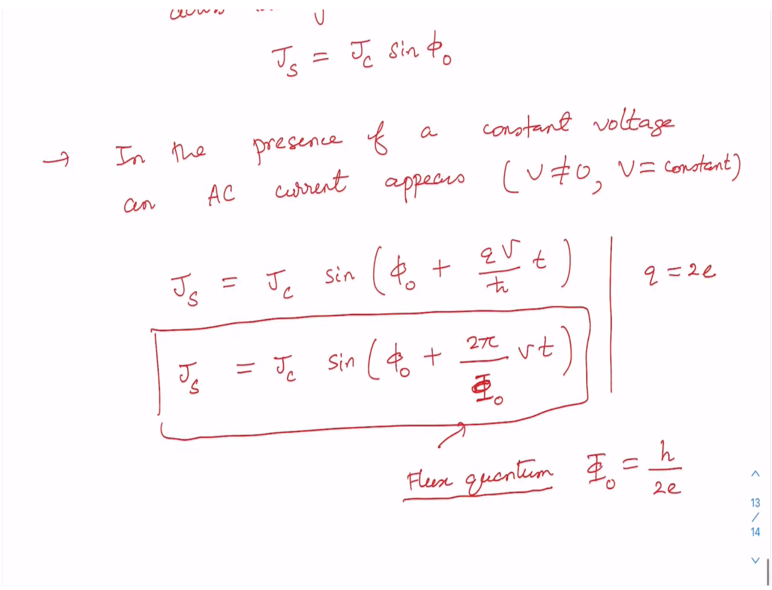

Now since superconductors are same on both sides and they are identical we can assume that n 1 is on the order of n 2 and let me write it as simply n and in this case and considering the absence of any scalar or vector potential. So, super current density. Now I am talking about current super current super current density as you can see it varies super current density varies sinusoidally.

So, what I mean to say is that if I multiply let us say this equation. So, if I take q n dot that is equal to twice q into a by h cross and this is simply n and then I have sine phi. And this is I can write as this is my super current density J s and I can write it as J c into sine phi. So, this is a very important expression that we obtain here and J c is the critical Josephson current density and which is here 2 q a by h cross into n right. So, this is what I have. So, as you can see that this J c is determined by the coupling, coupling between the superconductors and now again what we are left with we still to deal with equation number 2 and 4.

So, from equation number 2 and 4 if you can look at these equations carefully you will see that we will get let me write here from equation 2 and 4 we will get theta 2 dot minus theta 1 dot which is actually phi dot that is equal to 1 by h cross if you subtract 2 and 4, 2 from 4 you will get this expression U 1 – U 2. So, we consider now if we consider these equations this equation along with this one then if say a potential difference between the two superconductors are present.

That means say if a potential difference is present that means U 1 - U 2 is equal to say q V is the potential difference q into V this implies that the voltage the voltage related to the this implies that the voltage is related with the phase difference in the following way let me explain it as you can see from this expression what I can write from here phi let me first write phi dot is equal to 1 by h cross q into V.

From here you can see immediately that this phase difference if I integrate it phi of t would be equal to phi of 0 + q by h cross integration say V of t dt. So, where this phi of 0 is equal to the phase difference at time t is equal to 0. So, now the Josephson current as you see Josephson current J s is equal to J c into sine phi that implies that it depends because phi is now dependent on the potential.

Without any voltage there is now if V is equal to 0, there is a DC component there is a DC component because this term would still be there phi 0 would be there DC super current actually let me write DC super current across the junction would be there DC super current across the junction and it is given by J s is equal to J c sine phi 0.

Now in the in the presence of in the presence of a constant voltage in the presence of a constant voltage an AC current appears if V this voltage is no longer that means if V is non-zero and V is equal to constant then this ac current would be given by J s is equal to J c into sine. So, I have this phi 0 part is there phi 0 + q into V by h cross into t which could further be written in this form we can write it as J c sine phi 0 + 2 pi divided by this quantity here who is already we discussed earlier this is the so-called flux quantum.

So, this is flux quantum please do not get confused by the similarity in the symbol this is little bit different and this is phase and this is this is flux quantum that was defined as h divided by 2e and how we got it we know that this charge is equal to twice that of the electron charge and that is how you can you can very easily show that you can write this expression in this form in terms of the flux quantum.

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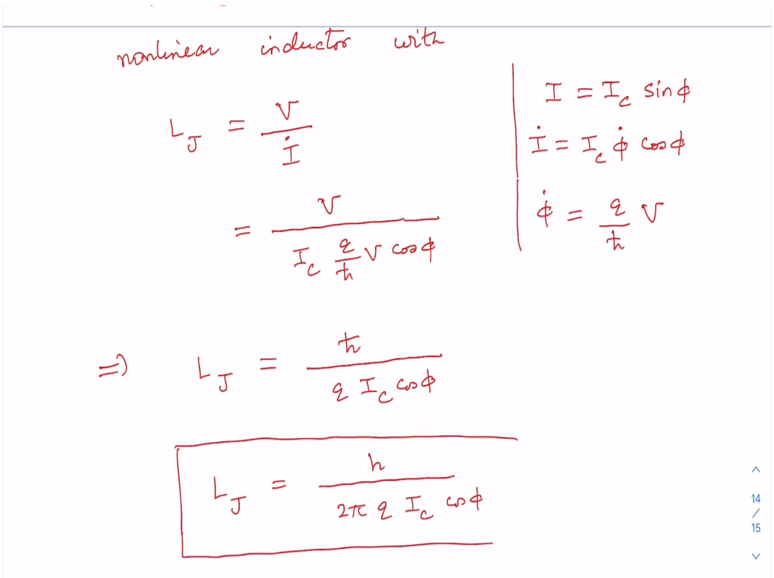

Now from the expression for this current density J s is equal to J c sine phi, phi is the phase and the phase is related to phase is now time dependent it is related to the voltage by this expression that we already discussed V of t dt. Also we know that the relation for the inductance in a circuit and it is related to the voltage by this expression L dI dt or simply L I dot, dot represents the time derivative.

Now from here we can deduce that the Josephson junction we can see that the Josephson junction is equivalent is equivalent to a non-linear to a non-linear inductor with inductance say L J is equal to that would be from this expression you see that that is V divided by i dot and from this expression here I can write I is equal to IC sine phi IC is the critical current through the junction.

And then I dot would be equal to IC phi dot cos phi and using this I can therefore write it as V divided by the by the way again phi dot from this expression here you can write phi dot to be equal to q by h cross V and if I put everything here I have here I C phi dot is equal to q by h cross V and I have here I have cos phi. So, this implies that the inductance of the Josephson junction is simply h cross divided by q into IC cos phi or I can write it as h divided by h is a Planck's constant it is divided by 2 pi q into IC cos phi.

And this is the reason why we say that Josephson junction is a non-linear element. Let me stop for today in this lecture we have discussed the primary feature of Ginsberg Landau theory which is quite instrumental in the development of various Josephson junctions. And we are going to continue our discussion on the Josephson junctions in the next class. So, see you in the next class, thank you.