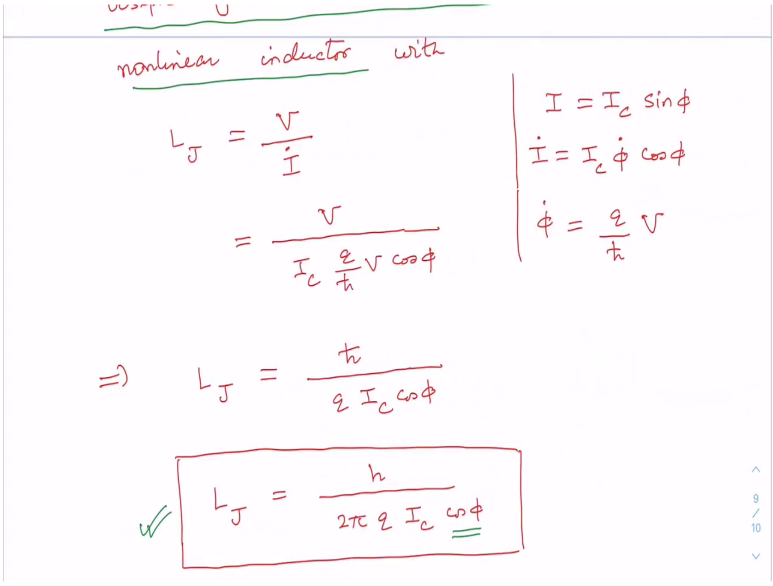
Quantum Technology and Quantum Phenomena in Macroscopic Systems
Prof. Amarendra Kumar Sarma
Department of Physics
Indian Institute of Technology, Guwahati

Lecture – 24
Josephson Junctions-II.

Hello, welcome to lecture eight of module 2 and this is lecture number 18 of the course. In this lecture we will continue our discussion on Josephson junctions. We will see how the Josephson junctions could be classified. And we will discuss a few Josephson junctions, but in particular we will discuss the so-called transmon qubit. It's a Josephson junction in somewhat great detail because of its immense applications and relevance in circuit quantum electrodynamics. So, let us begin.

(Refer Slide Time: 01:06)



In the last class we started discussing Josephson junctions. Josephson junction is basically two superconductors separated by a thin insulating layer. Earlier we discussed the so-called Cooper pair box which is a charge qubit. Apart from Cooper pair box there are other kinds of Josephson junctions like flux qubit and transmon and the development in Josephson junctions comes from the Ginsberg Landau theory which we discussed actually the main features in the last class.

Previously we learned about the BCS theory and one of the main results of BCS theory was that the band gap in a superconductor is related to the critical temperature of the superconductor by this expression. And in the Ginsberg Landau theory however it was they assumed that the all these superconducting electrons are described by many-particle wave function say psi of rt which is given by this expression.

And here psi 0 this quantity is related to the local density of the superconducting electrons and theta is the phase and this theta the phase is the same for all the Cooper pair electrons. Now the to understand the physical meaning of phase this phase one can use the so-called continuity equation for the probability of charged particle in an electromagnetic field.

And we discussed that we discussed that the continuity equation for the probability of a charged particle in an electromagnetic field is given by this expression, where rho is the probability density and J rho, this is the probability current density and also we showed you that how one can actually get this continuity equation starting from the Schrodinger equation, just I reminded you to how to get it.

And the same strategy can be actually applied to get this particular expression that is written here which is actually in the presence of a charged particle in the presence of a constant magnetic field. And the similar strategy applied then we can get the continuity equation there where this probability current density is given by this particular expression.

Now as Ginsberg Landau theory says that the local density of the superconducting electrons is given by the modulus of these wave functions many-particle wave functions where this J rho refers to the number of condensate or superconducting electrons per unit area part unit time. And from here we can write down the current density you see there is a difference between probability current density and current density.

Current density one can get just by multiplying by the charge and which is again related to the velocity of the superconducting electrons and the local density of the superconducting electrons, q is the charge here and we can use the wave function as part of Ginsberg Landau theory and put it in the expression here. And then we will get a couple of equations.

And thereby finally we saw that this phase actually is related to the so-called we can term it as we can give it a physical meaning in terms of we can say that h cross grade theta where theta is the phase is acting like a canonical momentum for these superconducting electrons.

Now then what we did we went on to do the flux quantization and show that the flux enclosed by a superconductor must be quantized in the unit of flux quantum phi 0 is equal to h by 2 e.

Which we proved. To prove that we have taken a superconducting ring pierced by magnetic flux and we have taken a contour there, a curve there which is a multiply connected region okay and far from the surface and in this curve both the current density the magnetic field is 0 here along this curve C. And using this fact that the current density and the magnetic field is 0 we have utilized this integral and then put the expression for current density and went on to obtain finally that indeed the flux is quantized and this flux quantum is given by h by 2e.

Then we discussed the so-called Josephson effect and I forgot to tell you last time that it was discovered by Brian D Josephson in the year 1962 and Josephson effect refers to the fact that it is a coherent phenomenon which predicts a superconducting current flow between 2 superconductors separated by very thin insulating layer. And we then try to discuss this effect very simply by using a very simple model based on the Schrodinger equation.

We have written this couple Schrodinger equation. We have taken 2 superconductors made up of the same material and they are homogeneous and they are separated by a thin insulating layer and psi 1 refers to the wave function, psi 1 refers to the wave function describing the superconductor 1, psi 2 refers to the wave function describing the superconductor 2 and U 1 and U 2 are the ground state energies of each superconductor and a is a parameter which characterizes the overlap between the wave functions psi 1 and psi 2.

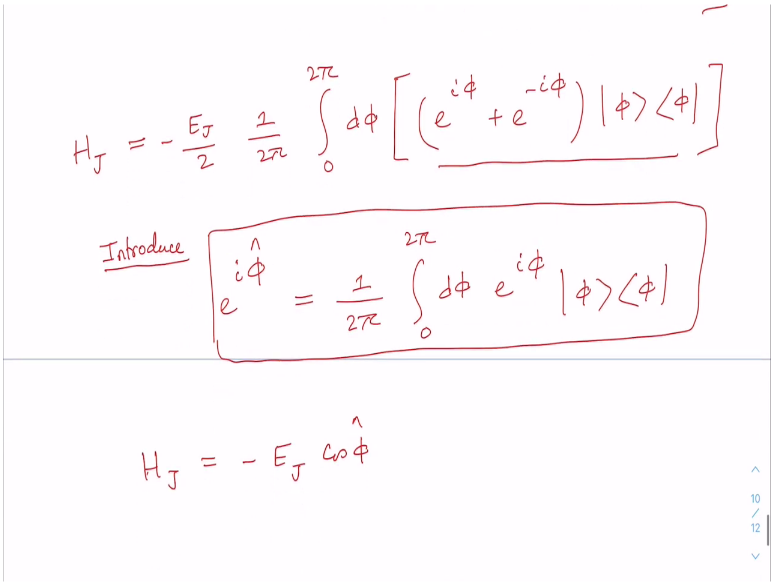
And in fact, you can term a as the coupling parameter between the 2 superconductors and then using the Ginsberg Landau this wave function psi 1 is equal to square root of n 1 e to the power i theta 1 n 1 is the local density of super electrons in conductor 1 n 2 is the super this local density of super electrons in conductor superconductor 2 theta 1 and theta 2 are the respective phases in the 2 conductors, then we have put equation 2 in equation 1.

And then after analyzing it turned out that the current density is basically is a non-linear function of the phase difference between the 2 super superconductors and J c here is the critical Josephson current density. And another expression we got which relate how the temporal evolution of the phase is dependent on the difference in the ground state potential or the ground state energy.

And if there is a potential difference between the 2 superconductors is there then we saw that this phase difference between the 2 superconductors is given by this expression where phi 0 refers to the DC phase because if say you can see that if there is no potential difference V then this phase is simply related to phi 0 which is the phase at time t is equal to 0 and current density is J s is equal to J c sine phi which I already told.

So, if the potential is 0, there is no potential difference is there between the 2 superconductor no external potential difference then there is a DC super current across the junction and in the presence of constant voltage and AC current appears which is quite evident from this expression. This expression we can further express in terms of the flux quantum. And finally, we saw that this Josephson junction is actually equivalent to a nonlinear inductor with inductance L J given by this expression.

(Refer Slide Time: 10:33)



Now let me discuss about the energy operator for a Josephson element energy operator. The energy operator can be deduced from the discreteness of the charge that tunnels to the barrier across this barrier that charge getting tunneled from this side to that side or from the other side to this one. And the charge Q J that is tunneling through the barrier is an integer N times, N is a function of time, N times the charge q and you know the q is basically twice that of the electron charge q twice e into N of t.

We can consider this N as an operator whose eigenstates correspond to the macroscopic state of a circuit with a well-defined number of Cooper pairs such that we can define it in this form. So, ket N N and N is the eigenvalue, here sum over all these things. So, this is the operator we have. The tunneling Cooper pairs through the barrier can be translated into in a coupling between the eigenstates of the operator N which can be expressed in this form, the Hamiltonian can be written in this form, we have H J is equal to - E J by 2 and now this going from N is equal to - infinity to + infinity and Cooper pair say is tunneling from one N to another one and it has to be Hermitian.

So, we have to have a term like this as well. I think this should already remind you about what we have discussed about the tunneling part of the Hamiltonian in the context of Cooper pair box. And here this E J is the Josephson energy and it is basically given by this flux quantum into the current critical current divided by 2 pi and this refers to the so-called Josephson energy. This is Josephson energy okay.

Now this Hamiltonian here, this Hamiltonian can be written in terms of phase difference across the junction phi as well. So, if we introduce a new basis, say, basis state in terms of this phi is the phase difference here. Now we are constructing a basis which is say it is in the basis of this number state i N phi we defined a basis like this N is going from - infinity to + infinity and you can see that under this change, if phi goes from say phi to 2 pi phi + 2 pi.

Then this state phi remains unchanged or remains unaltered it does not change it is very easy to see or conversely what we can define we can define the number ket here as 1 by 2 pi 0 to 2 pi d phi e to the power - i N phi phi, okay. This is also in terms of this phase basis we can write this number state basis as well. Now if we utilize this expression in our Hamiltonian that we have written H J is equal to - E J by 2 sum over N goes from - infinity to + infinity.

And we have these terms here okay this whole Hamiltonian that I have written here can be rewritten. So, let me do that actually okay this is not difficult we can some two or three steps I can take to show you explicitly it is easier by 2. Now let me utilize this definition okay if you look at this definition here I can write this guy as this summation sign I can utilize it only I have to multiply it by e to the power - i N phi because if I put e to the power i okay let me just show you e to the power i N phi N.

And this okay this summation sign is also there. Now summation of this guy with this one is already we said this is ket phi. So, hope you can see it. So, therefore I can write utilizing that expression I can simply write this expression as e to the power - i N phi and this I can write as ket phi I have utilized e to the + i N phi here okay and then I am left out with this one N + 1 the bra part.

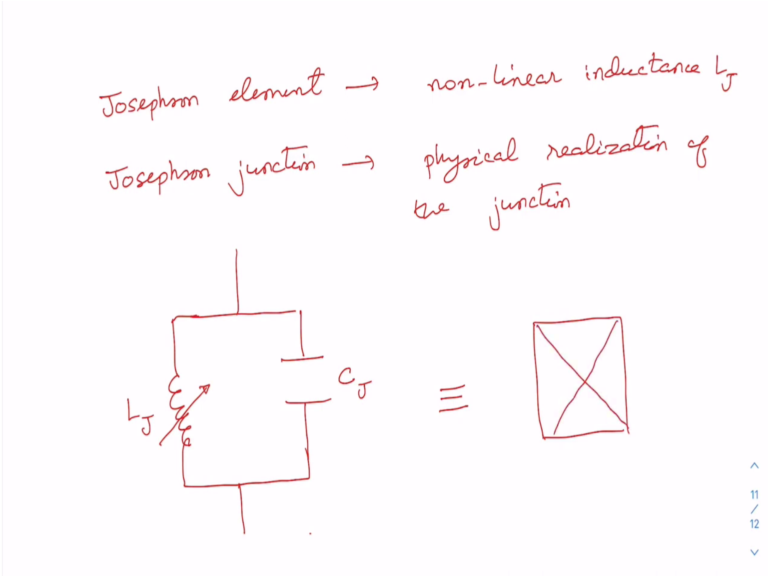
And similarly, here I can write it as e to the power - i N + 1 phi and this guy I can write it as ket phi and then I have this bra N here. And now let me utilize this one E J by 2 and as per my, let me introduce this one now, that would be 1 by 2 pi 0 to 2 pi d phi e to the power i N + 1 phi because now I am taking the bra of this one. So, here it is minus, now it will become plus. I am just dealing with the bra part here and I already have e to the power - i N phi and then I have here this ket phi and this bra phi.

I hope you are getting it. Similarly in the next term I can write + e to the power - i N + 1 phi that is already there and I have this ket phi and for the bra N I can write this one I write 1 by 2 pi 0 to 2 pi d phi e to the power i N phi this is bra phi, this is what I get and you can easily see that whole thing I can write it as - E J by 2 1 by 2 pi and then I have 0 to 2 pi, this is d phi and I have terms e to the power i phi + e to the power - i phi.

And then I have this ket phi and bra phi and so, this is what I get, now if I introduce a new operator let me introduce a short notation or say because you see this is an operator if I introduce the operator say e to the power i phi cap that if I defined this quantity as 1 by 2 pi 0 to 2 pi d phi e to the power i phi, this ket phi bra phi and this I write it define it as this operator e to the power i phi cap.

Then you can easily see that this Hamiltonian I can write in a more simpler form a compact form that would be - E J cos phi cap, all right.

(Refer Slide Time: 19:38)



So, this is another way to express this tunneling part of the Hamiltonian or the Hamiltonian product, Josephson element. Now this operator phi cap the phase operator is the phase conjugate this is the phase conjugate of the number operator N cap. And in fact, this is reflected in the commutation relation between phi cap and N cap and they do not commute they are uncertain. They are conjugate variables.

And now this can be used to obtain, this commutation relation can be used to along with another fact that suppose this N couples linearly with the voltage operator through the Cooper pair charge 2e then using these 2 facts one can actually obtain the quantum version of the Josephson equation which we wrote earlier J s is equal to J c sine phi and where this phase phi is equal to phi 0 and you recall that this is actually at phase at time t.

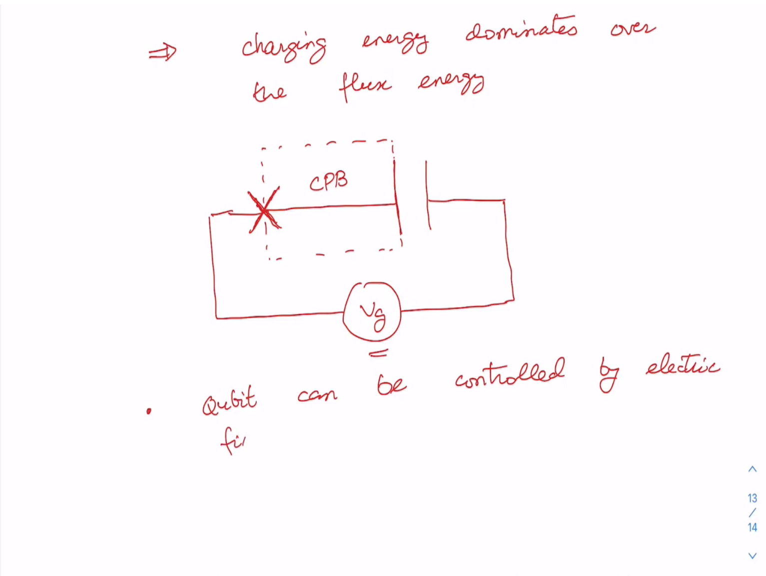
And it is associated with the phase at time t is equal to 0. It is charge divided by h cross and integration of V of t dt. Now all these quantities would become operators and this voltage operator voltage operator is going to be coupled linearly with this number operator and it would get coupled through the as I said through the Cooper pair charge 2e. Now generally the Josephson element that we discussed and I already told that this Josephson element is termed as the non-linear inductance L J right.

And the term, Josephson junction on the other hand these terms of Josephson junction is reserved for the physical realization physical realization of the junction of the junction and which is generally modeled by a capacitor say C because this Josephson junction, so, let us say C J this capacitor is connected to a non-linear inductor inductance L J in parallel. So, this is the model diagram for a Josephson junction for the physical realization and it is nonlinear inductor.

So, therefore this is the symbol that is used but actually symbolically it is represented in a simpler form and you just have to you this is denoted in literature simply by this cross. This cross actually refers to the non-linear inductance or the Josephson element. Now let us now briefly discuss about various superconducting circuits. Now we are going to discuss various superconducting circuits and all of us already know that the qubit is a fundamental element in the field of quantum information.

And it is basically nothing but a quantum 2 level system and one can take advantage of the Josephson junctions to build a 2 level system that will be called superconducting qubit actually rather let me say not circuit let me say I am going to discuss about superconducting qubits.

(Refer Slide Time: 24:48)



The Josephson junction as I said let me write again the Josephson junction the Josephson junction is a nonlinear inductor in parallel with a capacitor in parallel with a capacitor. The non-linearity here actually means that the energy levels are not regularly spaced because as you see that this is basically an LC oscillator this Josephson junction is an LC oscillator.

And you know the LC oscillator is a harmonic oscillator. If it is a linear harmonic oscillator we know that in the linear harmonic oscillator all the energy levels are equally spaced, this is linear harmonic oscillator all the energy levels are equally spaced. But when we have a nonlinear element like this reception element or nonlinear inductor what happens is that this harmonic leader becomes anharmonic and this we do not have equal energy spacing rather what we can do we can have a specific parameter circuit configuration.

We can take it in such a way that we can obtain these 2 low-lying energy states and that can be separated from all the higher energy levels by huge gap and in that case, we can consider this tool this whole structure is a kind of a 2-level system because then we can simply focus on only on these 2 energy levels. So, under the so called 2 level approximation this Josephson junction will behave like a qubit or a 2-level system.

In fact, when a circuit is composed by Josephson junction we know that the Hamiltonian which we discussed earlier the Hamiltonian has 2 parts one is the charging energy part. So, that is a half Q squared by 2C here it is C J and we have another part is because of the Josephson tunneling that is can be generally written as E J cos phi by phi 0 and or we can this is can also be written as H is equal to E C N square - E J cos phi by phi 0.

And you know here actually I have utilized the fact that q is equal to twice e the electron charge into the number operator number of Cooper pairs and here E C as you can see this would be simply twice e square divided by 2 C J. So, this is you can recall that similar kind of stuff and there is actually some convention is there some people write it in different way but essentially this E C basically refers to the charging energy refers to charging energy and E J refers to.

So, E C refers to charging energy and E J refers to the Josephson energy this is Josephson energy. And these Josephson junctions are our superconducting qubits are classified according to this ratio. So, superconducting let me just write here that superconducting circuits or qubits are classified according to according to the ratio E J by E C. I will give some example then it will be clearer.

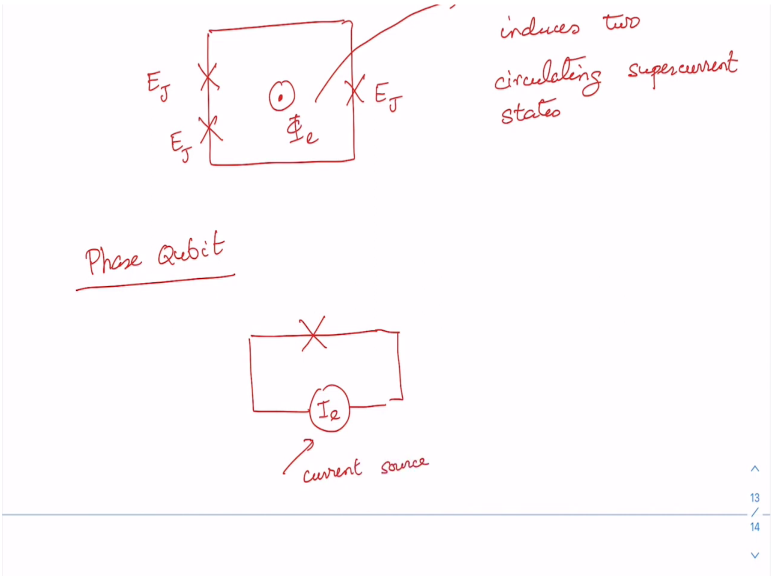
For example, we have the so-called charge qubit charge qubit and which is also known as CPB qubit Cooper pair box qubit. In this case this ratio E J by E C is less than 1 and this clearly shows that this actually implies that the charging part of the Hamiltonian charging energy dominates over the Josephson energy which is actually called flux energy you can say or you can simply say Josephson energy.

And it actually consists of a small superconducting island placed between the barriers of the Josephson junction and a plate of a capacitor. So, this is what we have. This is one of the plates of the capacitor, this is the other plate of the capacitor and this is the Josephson junction and then they are connected by gate voltage say V g. So, this is the symbolic representation or diagrammatic representation of a charge qubit here.

And sometimes some people also represent it in this form also and this is the box, it is CPB or it is called a charge qubit because in this case as I said this is more charge contribution to the energy dominates over the flux. And in fact, you can recall that when we discuss Cooper pair box there we said that the tunneling part of the Hamiltonian acts like a perturbation.

Now applying this voltage a charge difference between the 2 sides of the Josephson junction right is induced and this charge qubit the charge cubit can be control can be controlled by electric fields. There is another qubit and that is called a flux qubit and you can guess that in the case of the flux qubit the ratio is the opposite E J by E C is greater than one here.

(Refer Slide Time: 31:48)



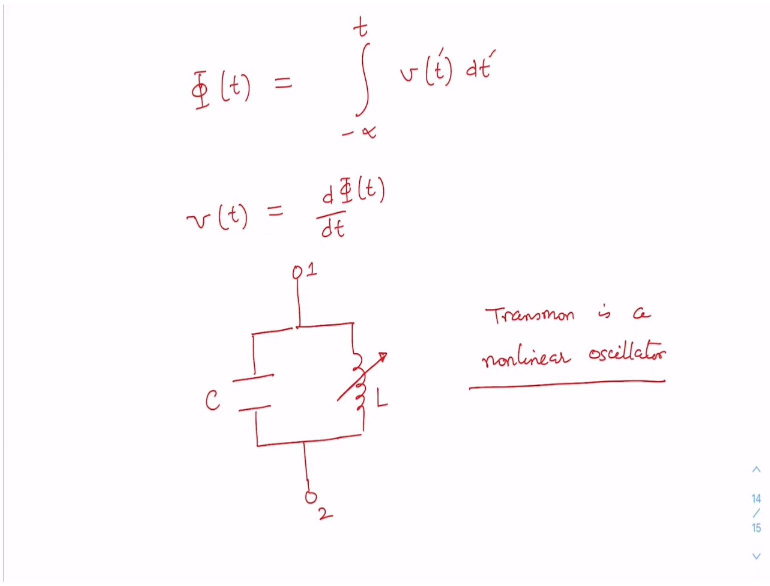
And in fact, this is basically a superconducting ring like this and it is interrupted by one or say three Josephson junctions like this. So, this is what the flux qubit and here E J is greater than E C. So, clearly the Josephson energy dominates over the charging energy and this is getting coupled to an external magnetic field this is getting coupled to an external magnetic field that is the magnetic flux phi e that actually flow across the closed loop and this flux induces 2 circulating super currents.

One may be in this direction one current may be in this direction the other may be in the other direction. These 2 superconducting currents either clockwise or anticlockwise and this is going to define a qubit. So, it actually let me write here the flux these flux this external flux. Let me write it more clearly this flux induces 2 circulating super currents 2 circulating super currents super current states rather super current states.

So, super current states and this may be either as I said either in the clockwise or anti-clockwise direction and this is going to define the qubit. Another one is the so-called phase qubit and in the phase qubit it basically consists of a single Josephson junction. So, you have a Josephson junction like this and this is connected to a current source. This is connected to external current source this is current source and this is the Josephson junction.

So, this qubit actually profits from the phase difference phi of the Josephson junction in the regime here E J by E C is much larger than one that and it is affected by only flux fluctuation. Now one particularly important qubit is the so-called transmon qubit and which is heavily used in circuit quantum electrodynamics transmon qubit and this is what we are going to discuss somewhat in details.

(Refer Slide Time: 35:12)



Now we can represent a physical picture of a transmon qubit physical picture of a transmon qubit by this diagram. So, this is one representation of transmon qubit as you can see there are 2 metallic plates or islands say metallic island number one and metallic island number 2 actually they are superconductors. And they are connected by a junction and this is basically a non-linear inductor.

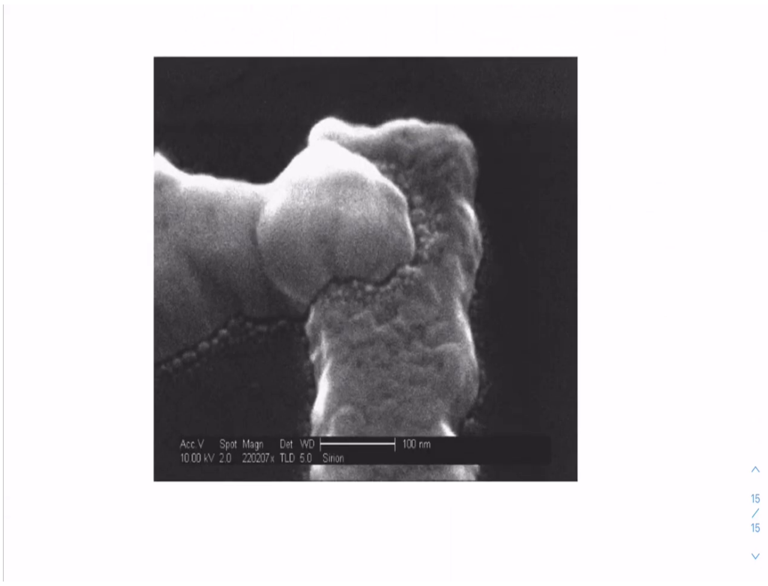
Actually, it is a Josephson tunnel junction. You can imagine just like in Cooper pair box Cooper pair to tunnel from the first island to the second island which make the first island to have a net positive charge Q of t and as you know the whole structure could be described in terms of charge Q of t and current I of t. So, the whole structure would have these quantities. You can represent them by Q of t the charge, current I of t and there is a voltage also this is V of t and also the magnetic flux phi of t.

So, these are the four quantities by which you can define this whole structure. In fact you know that some of these quantities are related to each other for example the magnetic flux phi of t is nothing but the time integral of the voltage that is V of t dt integration is from say - infinity to some time t. So, in that case let me write it like this and or we can express the voltage as the time derivative of the magnetic flux.

From our previous classes we know that this structure could also be represented by a circuit comprising of a capacitance and an inductor and an inductor but here the as I said the inductor is a nonlinear inductor. So, generally this is as you know it is represented by this symbol and we have these 2 terminals this terminal 1 and 2 you can consider it to be one as the metallic island one or the superconducting island one at the 2 as the superconducting island number 2.

And this nonlinear inductor the Josephson junction is described by a magnetic flux phi. In fact, you know that LC circuit is a harmonic oscillator if inductance is a linear inductor but because the inductor is now a nonlinear inductor. So, this becomes a nonlinear oscillator. So, in other words we can say that the transmon, transmon is a non-linear oscillator all right.

(Refer Slide Time: 38:59)



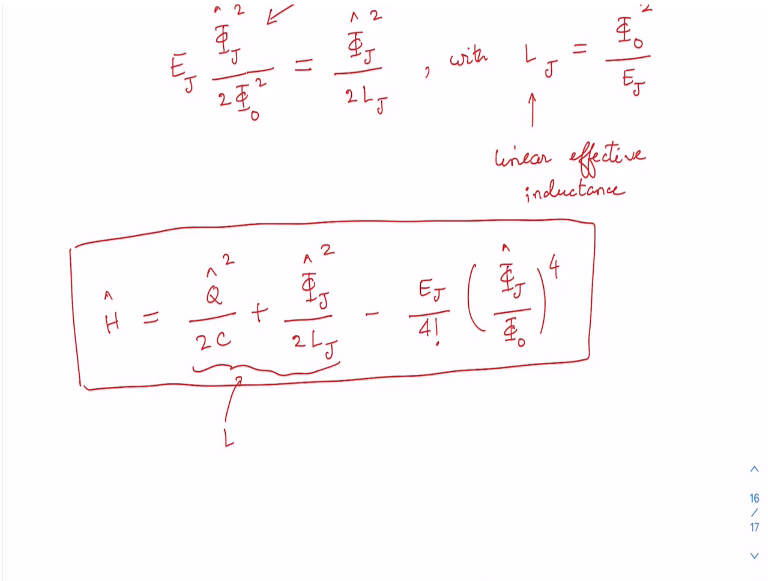
Before I proceed further let me show you a picture of a real Josephson tunnel junction. So, here it is a picture of a real Josephson junction, Josephson tunnel junction and it is a picture of a scanning electron microscope or an SCM image if you look at this image carefully you will see that there are 2 pieces of metals say metal number one actually superconductors superconductor number one and superconductor number 2 and it is having a width of approximately 200 nanometer.

This one is its width is 200 nanometer here also the weight is around 200 nanometer and they are falling on top of each other between them there is some kind of a barrier if you look at you see there is some kind of a barrier this is an oxide barrier and it is an insulator. Details however does not matter the key point is that because of this barrier between the 2 superconductors which we can denote as terminal number one and terminal number 2.

And the Junction we can represent it by a cross sign like this. So, this is what this Josephson tunnel junction is, this element can be represented by an energy function that only depends on the change in the magnetic flux across the terminals across these 2 terminals and this energy function is given by E of phi of J that is the energy it is a function of the magnetic flux it is equal to - E J cos phi J divided by phi 0 this is the energy function.

Here E J is the so-called Josephson tunneling energy. This you already know. This is Josephson tunneling energy and phi J is the magnetic flux which is scaled phi J is scaled in terms of the flux quantum in terms of the flux quantum phi 0 which is we know that it is h by 2e.

(Refer Slide Time: 41:42)



Now after having this energy function we can write down the Hamiltonian for the transmon qubit the circuit diagram for the transmon qubit is this we have this capacitance and we have a nonlinear inductor now and the Hamiltonian would be H is equal to Q square by 2 C - E J cos of magnetic flux divided by the flux quantum and we have as you see we have a charging energy term and a tunneling energy term.

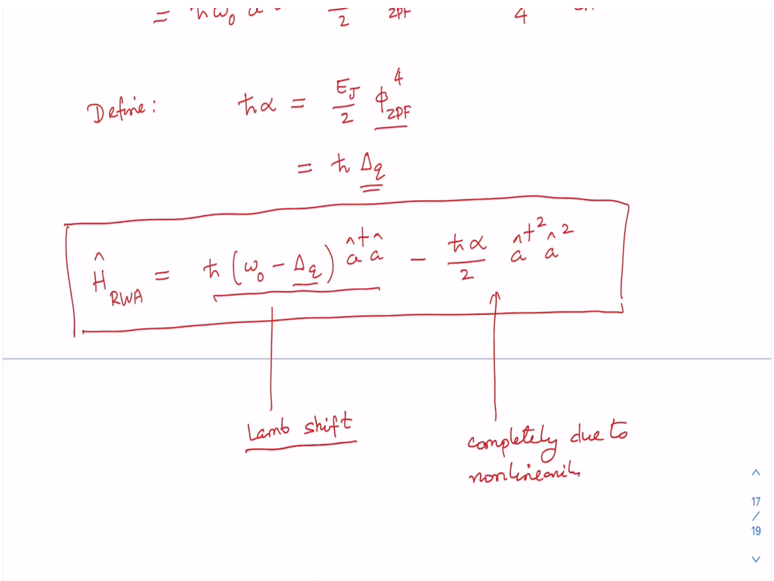
Let us look at the tunneling energy term which is due to the nonlinear inductor. We can break this energy function E J cos of phi J by phi 0 we can break it into a linear and then nonlinear part if I expand this function into a Taylor series we know the expansion of cosine function the expansion of the cos function would be 1 - phi J divided by phi J square divided by 2 phi 0 square then we have 1 by 4 factorial phi J by phi 0 to the power 4.

And then we will have higher order terms of the order of phi to the power 6 but we are going to ignore all the higher order terms beyond the phi J to the power 4 and then we can express the transform Hamiltonian as follows we will have H is equal to Q squared by 2 C - E J + E J all these are operators phi J squared by 2 phi 0 square - E J by 4 factorial phi J to the power 4 divided by phi 0 to the power 4.

We are ignoring all higher order terms as I already said and also we can ignore this term because this is simply an energy offset term. On the other hand the term if you look at this term we can rewrite is as follows this particular term E J into phi J square divided by 2 phi 0 square I can write it as phi J square divided by 2 L J with L J is equal to phi 0 square divided by E J.

In fact, this term should remind you about the energy stored in a linear inductor and we can name L J as an effective linear effective inductance. So, this is linear effective inductance. Therefore, we can write the Hamiltonian in this form. Now H is equal to Q square by 2 C + Phi J square by 2 L J - E J by 4 factorial phi J by phi 0 to the power 4. What you see here that there is a part in the Hamiltonian which resembles the linear harmonic oscillator it will remind you about linear harmonic oscillator and on the other hand there is another part that is due to the non-linearity.

(Refer Slide Time: 46:30)



Now we can borrow our mathematical operators of creation and annihilation operators from our knowledge on quantized harmonic oscillator and in analogy with the mechanical harmonic oscillator we can see here that the magnetic flux phi J plays the role of coordinate. Therefore, we can define we can define phi J in terms of creation and annihilations operator in this form that would be phi ZPF that is the 0 point fluctuation of the magnetic flux into a + a dagger.

And then the linear part of the Hamiltonian one can easily write that would be H is equal to h cross omega 0 a dagger a and the nonlinear part is - E J by 4 factorial phi ZPF to the power 4 into a + a dagger to the power 4 where this omega 0 is equal to 1 by square root of L J C and this term phi without these symbols here you see without this desk sign this is normalized 0 point fluctuation and it is equal to phi ZPF divided by the flux quantum phi 0.

Now this normalized quantum fluctuation is generally mostly it is very very small and taking this into account let us now first focus on this particular term this nonlinear term a + a dagger to the power 4. We can expand it and because these are operators while expanding it one has to be very, very careful when we take the multiplication we have to multiply it as per the order.

And if we expand the whole thing please try to do that you will get all these terms. Let me write down you will get a to the power 4 + 6 a square + 12 a dagger a + 6 a dagger square + 3 + 4 a dagger square a square + 6 a dagger square a square in fact this is 4 a dagger square you have to be careful. Let me correct it this would be 4 into a dagger a cube please verify it yourself and you will get 4 a dagger cube a and a dagger to the power 4.

So, all these terms you will get but you did not have to worry because quite a number of terms can be dropped from this expansion invoking the so-called rotative wave approximation. For example, you see let us look at some of the terms say a square term you know that the annihilation operator it evolves in time as per e to the power – i omega t right. So, therefore a square this term is going to evolve in time as e to the power - twice i omega t.

So, therefore a square is a oscillating term it oscillates with frequency twice omega. Similarly, if you look at the term say a dagger a cube, a dagger oscillates as e to the power i omega t and a cube oscillates as e to the power - 3 i omega t. So, with frequency 3 omega, so, overall the whole term oscillates with frequency twice omega. So, this way you can analyze it and you will find that there will be lot of terms we should be rotating and there will be a couple of terms which are non-rotating terms.

And as per the rotating wave approximation you neglect all the rotating term and just let me here write down once again the whole thing rather than doing this let me show you here this term would rotate with frequency 4 omega. So, we can neglect that we can drop that term this term is going to rotate with frequency twice omega this term is a non-rotating term this term will rotate with frequency twice omega.

Again, we can drop that term this term we can also drop because this is simply an energy offset term this term is also going to rotate it frequency twice omega we can drop that this term is not going to rotate because a dagger square and a square they will nullify their rotations again this term is going to rotate. So, this we can neglect and this term is also going to rotate with frequency 4 omega.

So, that way we can draw all the rotating terms and we can just keep the non-rotating terms and here will be left out it only these 2 terms. So, therefore we can finally write down using that we can write down the RWA Hamiltonian the Hamiltonian for the transmon under rotating group approximation as h cross omega 0 a dagger a - E J by 4 factorial phi to the power 4 ZPF.

And here we have term 12 a dagger a + 6 a dagger square a square all right. Now okay let me simplify this further we have h cross omega 0 a dagger a - E J by 2 phi ZPF to the power 4a dagger a - E J by 4 phi ZPF to the power 4 a dagger square a square. Now if we define certain quantities we can simplify we can write the transmon Hamiltonian in a very simplified form if we define say h cross alpha is equal to E J by 2 phi ZPF to the power 4 which I can also if I equate it to a term h cross delta q with significance would be clear to you soon.

Then we can write the transmon Hamiltonian under RWA rotating wave approximation as h cross omega 0 - delta q a negative a - h cross alpha by 2 a dagger square a square. So, this is the form of the transmon we finally obtained. We will continue our discussion of this transform Hamiltonian in the next class. Before I stop let me make a couple of comments about this Hamiltonian.

You can see that from the first term in the Hamiltonian the frequency omega 0 of the linear harmonic oscillator is now modified by delta q and this can be attributed to the 0 point fluctuation of the magnetic flux as you can see the delta q is related to this normalized 0 point fluctuation of the magnetic flux. And many a times this phenomenon is referred to as lamb shift lamb shift.

On the other hand, the last term in the Hamiltonian is there completely due to non-linearity this term is completely due to non-linearity. Let me stop here for today. In this lecture we discuss various Josephson junctions very briefly and we learn how to classify a Josephson junction it is basically depends on the ratio of the Josephson energy and the charging energy in particular we studied in some more details about the transmon qubit it says Josephson junction.

And we in fact derive the Hamiltonian describing the transform qubit. In the next class also we will continue a bit of discussion on transmon qubit then it will be followed by a discussion on how to incorporate dissipations in our formalism because so, far we have not taken into account any kind of quantum dissipations or dissipative effects in our formalism and this is what we are going to do in our next class. So, see you in the next lecture, thank you so much..