Quantum Technology and Quantum Phenomena in Macroscopic Systems
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Lecture – 26
ransmon;Introduction to Dissipation in Quantum Systems

Welcome to lecture 19 of the course it is lecture 9 of the second module in this lecture we will continue our discussion on transmon and then we will start discussing a very important topic called quantum dissipation or noise. Quantum dissipation or noise is extremely relevant for all practical quantum circuits or quantum devices. So, let us begin.

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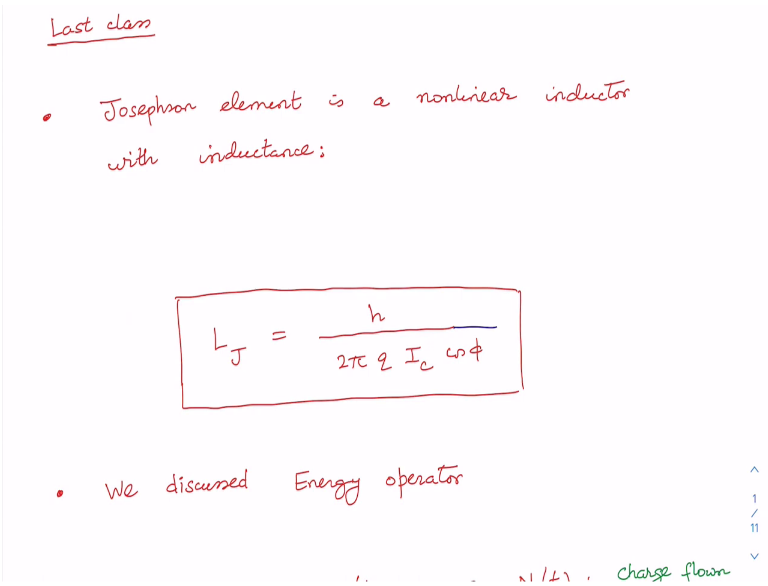

In the last class we continued our discussion on the Josephson element from the previous one where we learned that the Josephson element is a non-linear inductor with inductance given by this expression. Here h is the Planck's constant I C is the critical current and cos phi is the term where phi is the phase difference across the junction. And then we went on to discuss the energy operator and energy operator we expressed in terms of the number state basis or the Fock state basis.

And where this operator N cap it is an operator whose eigen states correspond to macroscopic state of the circuit with a well-defined cooper pair number, N refers to the cooper pair numbers and E J is the Josephson energy. This Hamiltonian this form of the Hamiltonian is similar to the one that we discussed in the context of cooper pair box Hamiltonian in a in a previous class in this module.

And this Hamiltonian can be written in terms of phase difference across the junction phi where this function phi can be treated as an operator and we can define the basis state phi cap which can be expressed in the in terms of the number state basis or inversely we can express this number state basis in terms of this phi basis.

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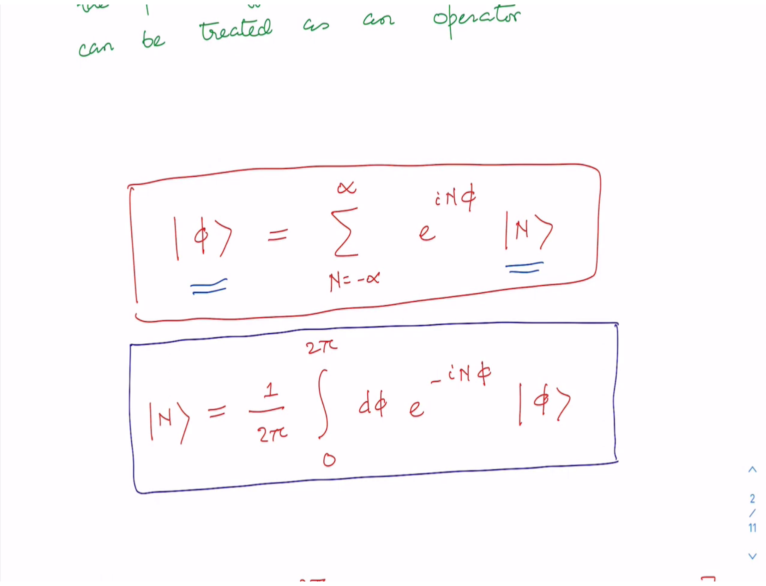

And using all these things it turns out that if we define the operator e to the power i phi cap to have to have this particular form then this Hamiltonian can be written in a very compact form that is H J is equal to - E J cos phi cap. Now phi cap is the phase conjugate of the number operator. So, naturally they satisfy this commutation relation. Now there are 2 things I mentioned one is that the Josephson element this term is actually referred to generally for to understand nonlinear by this we actually mean that we are talking about non-linear inductance.

On the other hand when we say Josephson junction this basically refers to physical realization of the junction. Now the Josephson junction is a nonlinear inductor in parallel with a capacitor and this is represented in this symbol by this symbol. Whenever you come across any symbol like this you should understand that this is referring to the Josephson junction.

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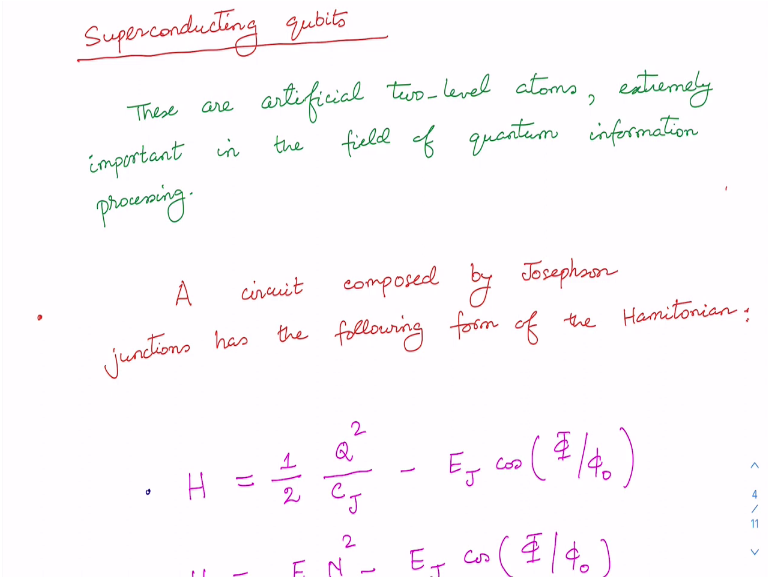

Then we went on to discuss superconducting qubits and these are extremely important because they are artificial 2 level atoms very relevant in the field of quantum information processing. Now a circuit composed by Josephson junction has this form of the Hamiltonian where this one is the charging energy and this is the tunneling part of the energy and this we can express in the in terms of the number of cooper pairs also.

Because you know that Q is equal to twice of the elementary charge into the number of cooper pairs. So, we can express it as a 2 energy coefficient are appearing here one is the Josephson energy and another one is the charging energy. By the way here this actually phi refers to the magnetic flux and phi 0 is the flux quantum this particular thing I did not mention in the last class that this phase difference.

You see initially we express this Hamiltonian in terms of the phase difference across the junction and it can be shown that this phase difference is related to the magnetic flux via this expression.

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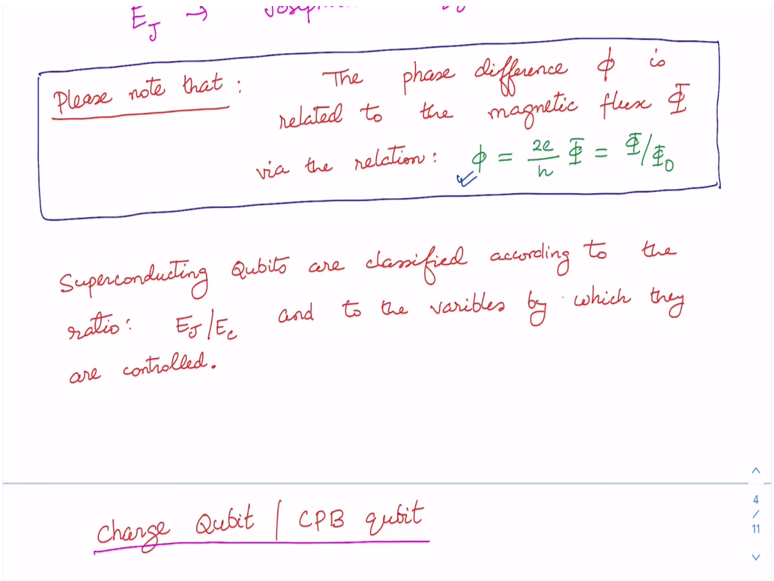

Where phi is equal to 2 e by h which is related to the flux quantum you know the flux quantum phi 0 is equal to h by 2 e and phi is the this is the magnetic flux. So, and maybe we'll do this in a problem solving session and this maybe I will explicitly show it. Now the superconducting qubits are classified according to the ratio between this Josephson energy versus the charging energy and to the variables by which they are control.

In this context we briefly discussed about the charge qubit and the cooper pair box qubit where naturally because it is charge qubit the charging part of the energy dominates over the Josephson part of the energy that is the flux energy. And symbolically or the circuit diagram for a charge qubit is of this type and as you can see that this kind of qubit can be controlled by electric field because ultimately charge dominates here.

On the other hand the flux qubit case. So, we have this Josephson energy E J is much greater is greater than the charging part of the energy and this is basically a superconducting ring interrupted by one or three Josephson junctions and the this flux actually induces 2 circulating super conducting currents one going in the anti-clockwise direction other going in the clockwise direction.

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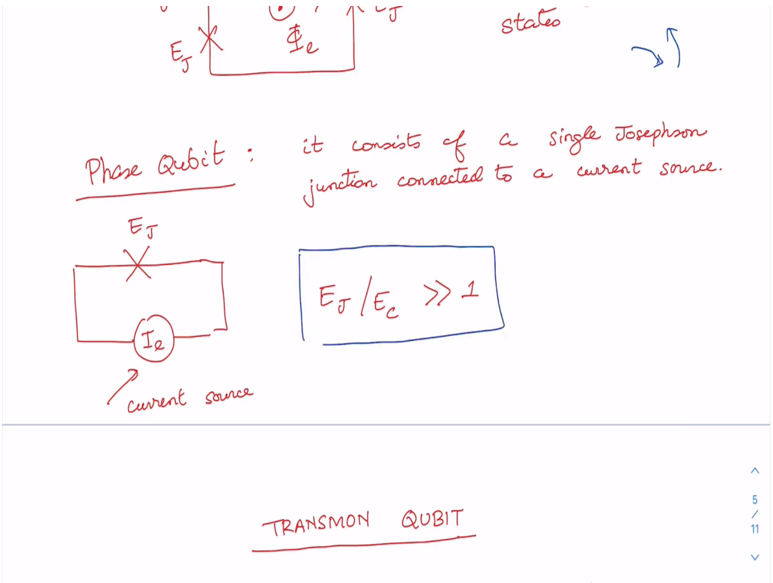

So, this forms the 2 states and then we briefly talked about phase qubit where uh a single Josephson junction is connected to a current source. And in this case the Josephson part of the energy is much much greater than the charging part of the energy.

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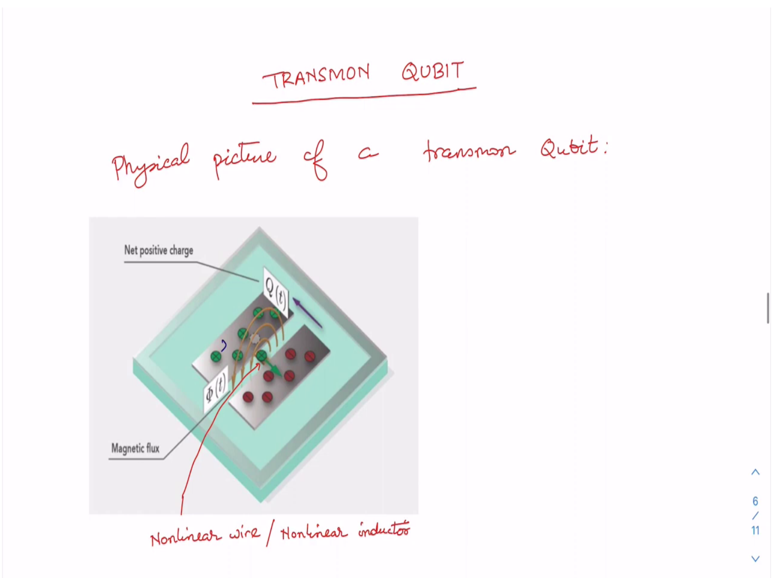

Then finally we discussed in somewhat details about the transmon qubit physically speaking a physical picture was given we discussed about it that it basically consists of 2 metal plates metal plate one and say metal plate 2 connected by a non-linear wire or a non-linear inductance and to discuss it we briefly revised what we learned about the linear quantum linear LC harmonic oscillator.

And we utilize these things in later part and also I talked about I have discussed about a scanning electron microscope image of a real Josephson junction there is no need to go into the great details. But thing is that there are two metal say metal 1 and metal 2 and they are overlapping each other and they form a kind of a gap or junction is there and they are widthwise each of the metal is around 200 nanometer in size.

Details does not matter, only thing what matters is that we have these 2 metal pieces and they form a gap between them very tiny gap and then this can be represented by this diagram here this terminal 1 for metal 1 and terminal 2 for metal 2 and then they form a junction. And magnetic flux is changing magnetic flux is the important factor that we have to keep in mind.

And this element can be its energy function can be described by this particular expression here you see what matters is only the change in the magnetic flux. And this when we expand this cosine function it can be shown that this energy function of this Josephson element can be written into 2 parts one is the linear part and one is the non-linear part. And it turns out that the linear part basically refers to the energy of a linear inductor and this refers to the nonlinear part and which is extremely important.

And recalling that the linear inductor we know the energy in the linear inductor is magnetic flux square divided by twice L. So, this allows us to make a correspondence and thereby we express the Josephson energy as a magnetic flux quantum square divided by L J and L J is the inductance.

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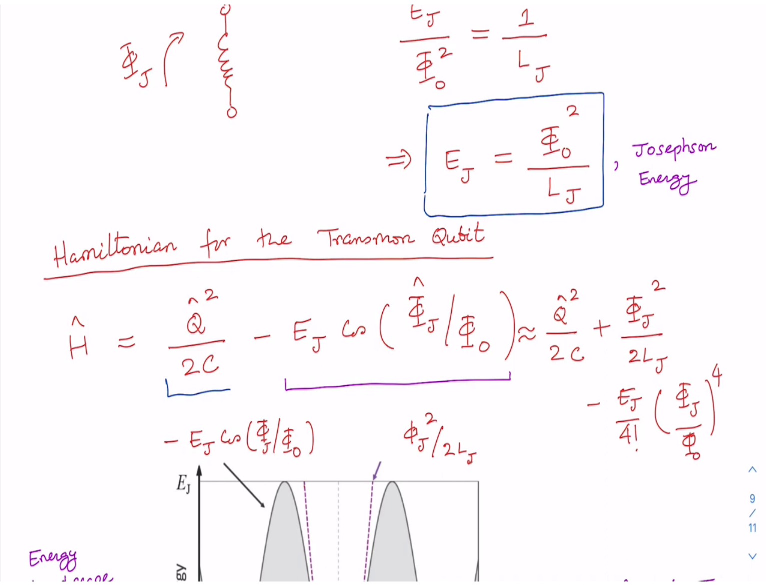

And then we discussed we then we wrote the total full Hamiltonian for the transmon qubit. So, we have this charging energy part and this the potential energy part and if we actually plot the potential energy part here this cosine function if we plot as a function of reduced magnetic flux that is phi J by phi zero across the Josephson junction then we get this particular plot here.

You see this particular one gray one refers to the cosine potential and in fact if we break it into linear and non-linear part here as you see this is the linear part and this is the nonlinear part. And linear part is drawn here this refers to the this is the linear harmonic potential phi J square by twice L J and this cosine function is periodic in pi. So, if our magnetic flux value is close to zero or very small then this whole potential can be termed as the linear.

So, if we deviate much from it then the non-linearity has to be taken into account. And then we discussed that as we after dividing it to linear and nonlinear part linear part already we studied in great details throughout the course. So, linear part if we write in terms of the annihilation and creation operator that is simply h cross omega 0 a dagger a. On the other hand nonlinear part can be expressed in this particular form in terms of creation and annihilation operator.

Then taking this phi zero point fluctuation okay this is a reduced fluctuation quantum zero point fluctuation to be much much smaller which is a realistic approximation then under rotating wave approximation where we have dropped all the rotating terms and only keep the non-rotating terms. This particular Hamiltonian under rotating wave approximation can be written into this very very simple form.

And this is what we are going to discuss today in our initial part of this course class. And as you see this first term is particularly interesting where it basically shows that the harmonic oscillator frequency there is a shift by delta q and this is the so-called lamb shift and this happens due to the quantum zero point fluctuation. On the other hand this term is coming due to the non-linearity and this is what now we are going to discuss.

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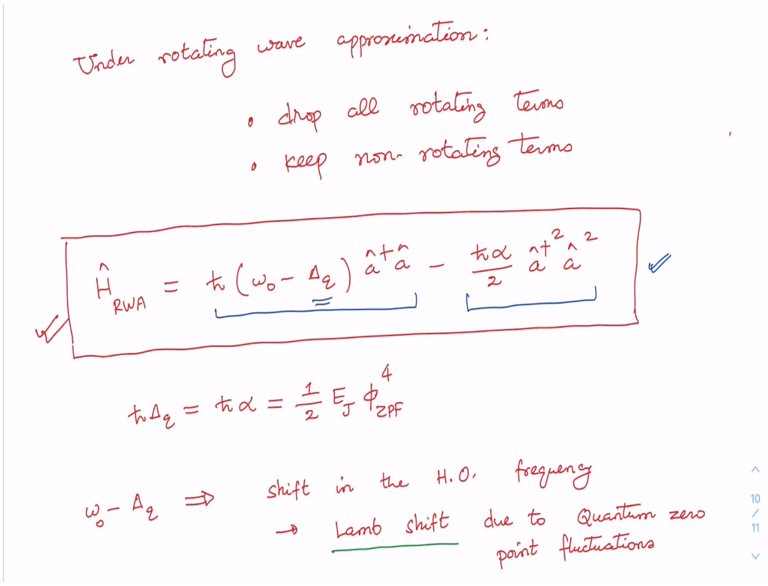

We can rewrite this Hamiltonian in terms of number operator N is equal to a dagger a as follows: we can write H RWA is equal to h cross omega q N cap - h cross alpha by 2 N N - 1. So, this is an alternative form of the same Hamiltonian that we have worked out here and here this omega q is equal to omega 0 - delta q. Let me explain the first term is obvious because simply a dagger a is nothing but the number operator and omega 0 - a delta q is omega q.

Now as regards the second term by the way the first term denotes the number of Fock state this term basically denotes the number of Fock state we have and it is the linear part of the total Hamiltonian on the other hand to get the second term let me analyze the term a dagger square a square let us analyze it then you will understand how I got the second term. So, a dagger square a square this I can write as a dagger a dagger a a.

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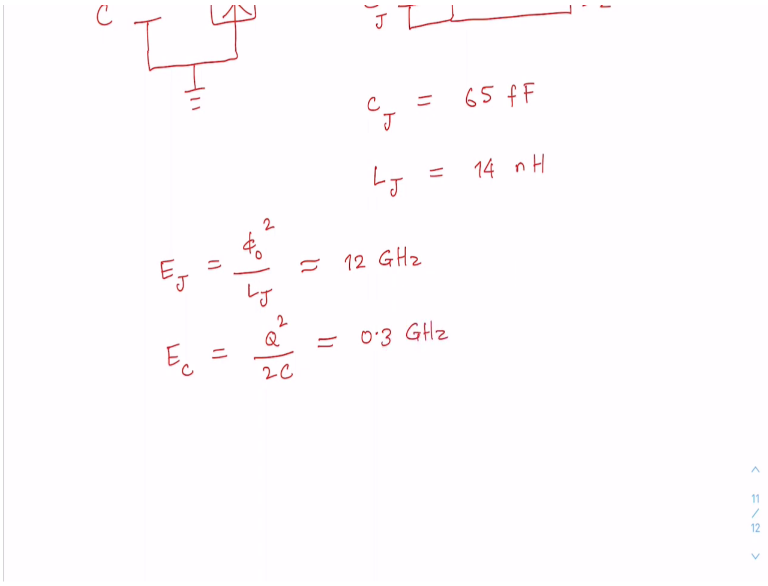

Now what I will do I will exploit this commutation relation a a dagger is equal to 1 which is I can write this a a dagger - a dagger a is equal to 1. So, from here you see that I can write a dagger a is equal to a a dagger - 1 right. So, therefore I will just replace this in this way a dagger and this would be this would be a a dagger. So, this would be a a dagger - 1 and then this is a.

So, you can easily see that I can write it as a dagger a a dagger a - a dagger a and obviously this is N this is N and this is N. So, therefore I have N square - N which I can write as N into N - 1 okay. So, clearly this second term is the nonlinear part of the total Hamiltonian. So, what we have is this, the linear part is equal to h cross omega q N and nonlinear part of the Hamiltonian is - h cross alpha by 2 N N - 1.

So, now let us look at some of the experimental parameters using which a transmon qubit was realized in laboratory. You see our transmon as you know the circuit diagram for the transmon we have this capacitance is there and this is the Josephson junction is there okay and this has a inductance say L J. So, this is the circuit diagram as we all know or another form is that we have basically 2 metal plate separated and this is connected by this non-linear inductor here.

And this is the capacitance between these two metal plates this is a metal plate number 1 this is your metal plate number 2 and the capacitance in experiment in real experiments are done, In real experiment the capacitance was taken to be around say 65 femto farad which is extremely small. And on the other hand, the inductance was taken out to be 14, nano henry which is again small.

And then if you do the calculation, it turns out that this corresponds to the Josephson energy is E J is phi 0 square by L J it was around 12 gigahertz and on the other hand the charging energy E C which is q square by twice C it turns out to be around 0.3 gigahertz. To first order the energy level diagram of this transmon could be actually drawn.

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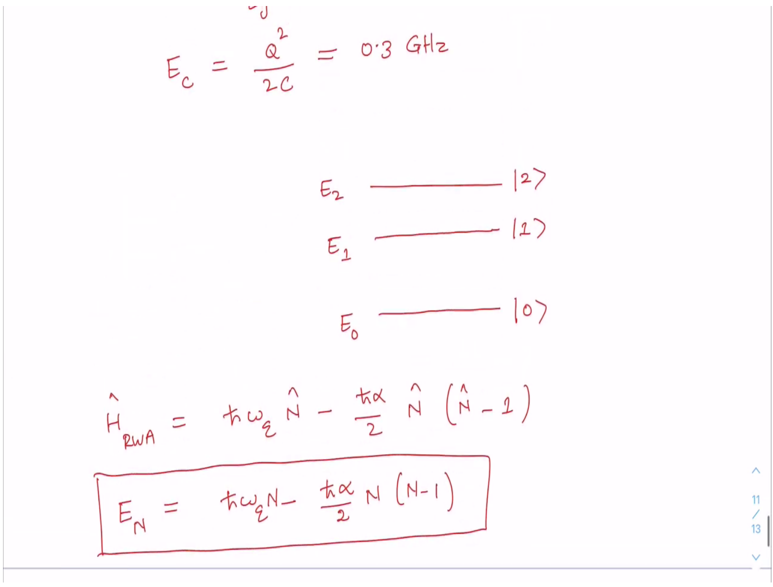

So, let me explain it. So, first energy level that is the ground state energy let me denote it by ket 0 the second energy level is say 1 and I will explain the this for the this is the first excited level then is the second excited level let me represent it by 2. Let us say this is energy let me denote it as E 0 this is say E 1 and E 2. Now from this Hamiltonian okay let me just utilize this from here you can see that the energy okay.

Let me write down the Hamiltonian once again here the RWA Hamiltonian that we have here is h cross omega q n - I have h cross alpha h cross alpha by 2 N N - 1. So, you can immediately see that the energy expression for the energy would be simply h cross omega q - h cross alpha by 2 actually let me okay if I have it if I operate on the number state it will be simply like this N N - 1 okay.

Taking this my energy nth energy level, so, let me find out the difference between the energy levels for example first excited state and the ground state energy level you can immediately see that this would be simply h cross omega q on the other hand say the second energy level minus the first energy level that would be from this expression itself you will see twice h cross omega q - you will have simply h cross alpha that is your E 2 minus the E 1 is simply h cross omega q and this is going to give you h cross omega q - h cross alpha.

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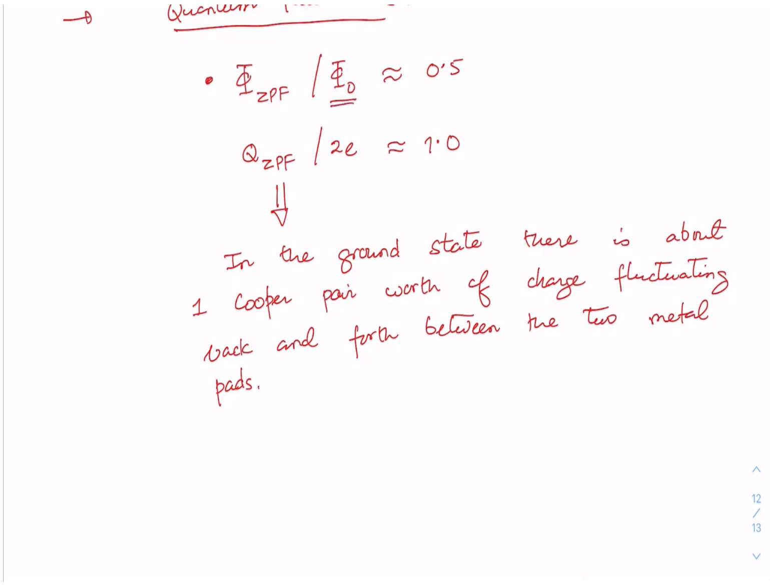

So, here you see that this energy difference is h cross omega q on the other hand this energy difference is h cross omega q - h cross alpha. So, what do you see from this energy level diagram for the transmon I am just considering this on the three energy levels. So, we see that the energy levels are no longer equally spaced right that is what happens in linear harmonic oscillator.

Now because of the presence of the nonlinearity the harmonic oscillator now becomes an harmonic and this is what is needed to make the transmon to be used as an effective 2-level system or qubit. In fact, with the parameters that we have mentioned this omega q turns out to be around 5 gigahertz and on the other hand this alpha this turns out to be around 0.3 gigahertz.

Now as regards the quantum fluctuation if we look at the quantum fluctuation in the problem in this transmon with the given parameters quantum fluctuations what they were measured it turns out that this phi zero-point fluctuation divided by this flux quantum was around half or 0.5 and fluctuation in the charge that was with respect to the cooper pair that is twice of the charge of the electron was turns out to be 1.0

So, what it what it physically it means is that in the ground state there is so it means that in the ground state in the ground state there is about one cooper pair as you can see from this equation that q zero-point fluctuation divided by 2 is equal to 1. So, it means that in the ground state there is about one keep cooper pair worth of charge okay fluctuating fluctuating back and forth back and forth between the 2 metal pads this is what it mean between the 2 metal pads. So, okay 2 metal pads all right or the 2 islands.

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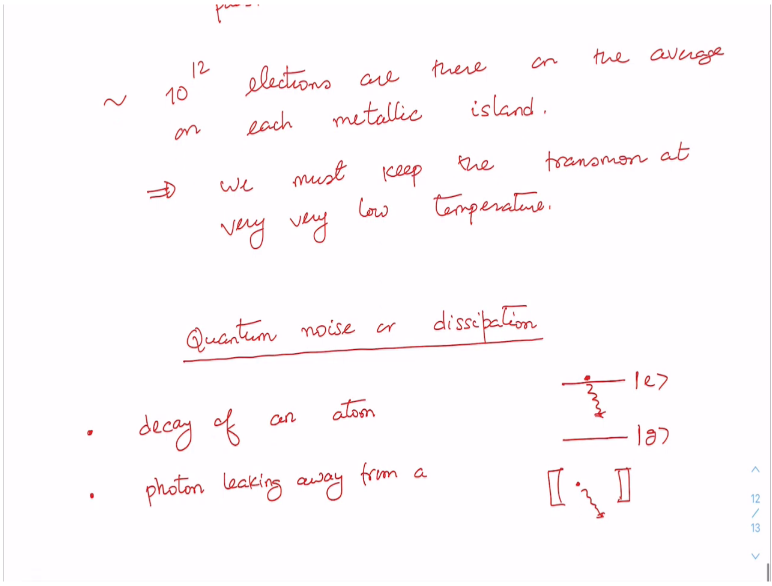

On the other hand the, if you look at this equation here the magnetic flux is oscillating on the level of about a half a magnetic flux quantum. So, this is magnetic flux quantum. So, the magnetic flux oscillates on the level of about half a magnetic flux quantum. Now typically there are on the average about you know on the average there is about 10 to the power 12 electrons are there, electrons are there on the average on each metal pads actually or metallic island.

On each metallic in fact here it is superconducting island metallic island. So, in the transmon qubit. So, this is actually it is absolutely clear and it clearly shows that why we must have to work at very cold temperature because our device must be isolated from noise because our quantum fluctuation is you see so sensitive one cooper pair as regards charge is concerned.

So, we have to keep our device at very very cool temperature at very low temperature this implies just to avoid too much of quantum fluctuation. So, we must keep the transmon transmon at very very low temperature. Nevertheless we cannot completely get rid of quantum noise or dissipation and we must know how to incorporate noise into our model.

So, that is why now what we are going to do we now have to see how quantum noise or dissipation could be taken into account in our mathematical model. So, this is what we are now going to discuss quantum noise or dissipation. This is a very important issue and we will try to discuss it in some more details now. Dissipation or quantum noise is encountered in many situations for example decay of an atom.

Suppose an atom a 2 level atom if you consider it has a ground state and an excited state even though these are eigen state ground state and the excited state we know that the atom cannot stay in the excited state for a long time. So, after some time it spontaneously decay into the ground state. Again, we can have a situation where suppose we have a cavity and inside the cavity the photon is oscillating back and forth but after some time the photon the cavity may leak away from the cavity.

So, photon leaking away from a cavity that is also a dissipation from a cavity and also there is something called decoherence or dephasing which typically means destruction of the superposition states say again we have a 2 level system.

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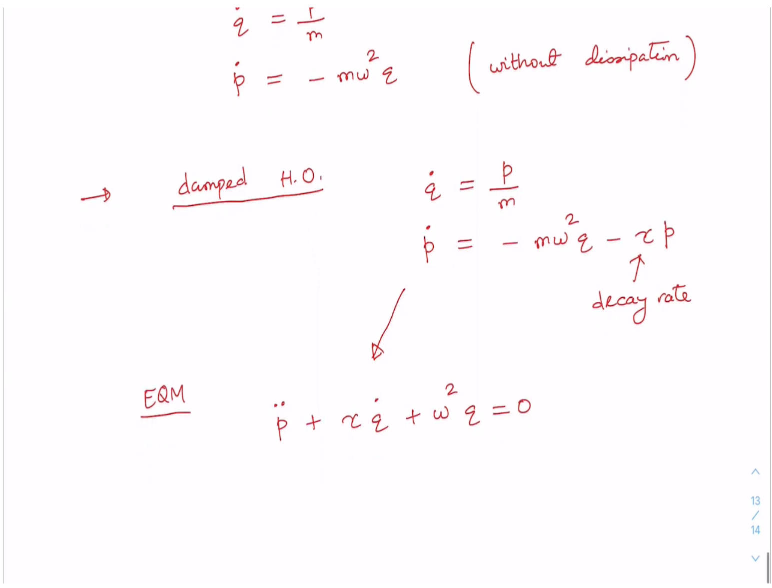

And we have a superposition state like say C g this ket g + C e this ket e this is a superposition state but we cannot it cannot be maintained for long time. So, this superposition state generally gets decayed this decay or get destroyed decay of superpositions quantum superposition state decay of superposition state this is also quantum dissipation or quantum noise it generally happens as soon as the system comes into contact with environment or any perturbation.

So, taking dissipation into account in a in a mathematical model is a non-trivial affair in quantum mechanics to understand it let us see how dissipation is taken into account in classical mechanics by the example of a simple harmonic oscillator. So, let us say consider the case of damped simple harmonic oscillator and we will see actually we know how decays are taken into account in such simple system damped simple harmonic oscillator.

So, first let us see the classical case we know that the Hamiltonian for a one-dimensional simple harmonic oscillator in classical mechanics is given by this equation that is say h is equal to P square by 2 m where P is the momentum m is the mass of the harmonic oscillator this is the kinetic energy part. Then we have half m omega square q square where omega is the oscillation frequency q is the position there are many typical examples all of we know.

For example a mass attached to a spring. So, this is the example of a classical simple harmonic oscillator and from this equation we know now the equation of motion would be q dot is equal to P by m and P dot which is rate of change of momentum which is basically the force that is equal to m omega square q. So, at the moment we are not considering without dissipation or damping this is the equation of motion and we can get it easily from the Hamiltonian using Hamilton's equation of motion.

Now say the harmonic oscillator is damped it can be taken into account by the following prescription. So, let me now let us consider damp harmonic oscillator in this case we will have we can take this into account this way. So, q dot is equal to P by m and we can write P dot is equal to - m omega square q and now I will add another term minus gamma P gamma is the decay rate.

So, from these two equations this is going to lead us to the equation of motion you can just take the time derivative of the momentum. So, P double dot and you it is very trivial to show that you will be able to show that P double dot + gamma q dot + omega square q is equal to 0. So, this is the equation of motion and we know that this is the equation of a damped classical harmonic oscillator. So, this is the prescription for how the damping can be taken into account in the classical context

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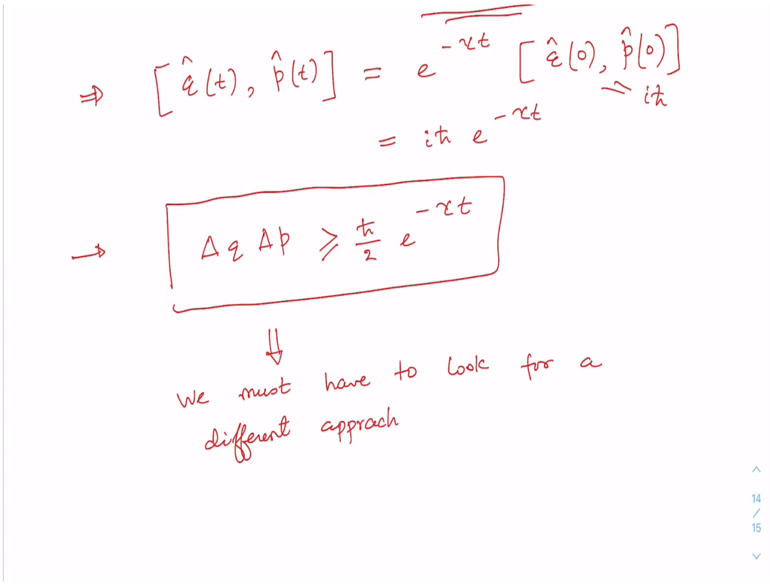

Now the question is the question is can this can the same approach can the same approach be applied to the quantum case to the quantum mechanical case. So, let us see well we know that in the quantum mechanical case we must have in the quantum mechanical case we must have the commutation relation because they will this variable. So, you already we know that they have to be operator this q P has to satisfy this commutation relation.

So, this has to be maintained. Now if I look at the time evolution of this commutation let us see what we will get q p, q P is going to give me this commutation okay. Let me take the time derivative of what is this commutation we have simply we have qp - pq. So, let me expand it. So, this will give me q dot P + q P dot - P dot q - P q dot right. So, this is what I will have.

Now in the analogy from if I take the quantum mechanical version of the classical equations. So, q dot is equal to P by m and I have P dot is equal to - m omega square q - gamma p. So, this is what we have taking the analogy from the classical mechanics case because now these variables will be operator. So, I can now write d of dt qp okay. So, let me put q dot is equal to P by m.

So, first term is going to give me P square by m and then the second term I will have q P dot P dot I will just write - m omega square q - gamma P and then again from this term is going to give me so I will just put P dot P dot is equal to - m omega square q - gamma P and here I have q and the last term is going to give me P, q dot is equal to P by m. So, it is going to get P square by m all right.

So, if I do the algebra. So, I will just have - gamma qp - pq or I can simply write it as - gamma commutation between q and p. Now very clear that this we can simply write q of t P of t okay this is a simple differential equation. So, using this I have the commutation relation between these variables as e to the power - gamma t q of 0 P of 0 and we know that this is has to be i h cross.

So, i h cross e to the power - gamma t this is i h cross. So, what we see something interesting here we see that as a consequence of this decay of decay of the commutator the Heisenberg uncertainty also decays okay and the Heisenberg uncertainty relation in this case would become del q del P is greater than or equal to h cross by 2 e to the power - gamma t. So, clearly so this is not the case as we know in quantum mechanics.

So, very clearly we need to look for a different approach. So, this is what we see that classical treatment cannot be extended to the quantum treatment. So, we must have to look for a different approach for a different approach.

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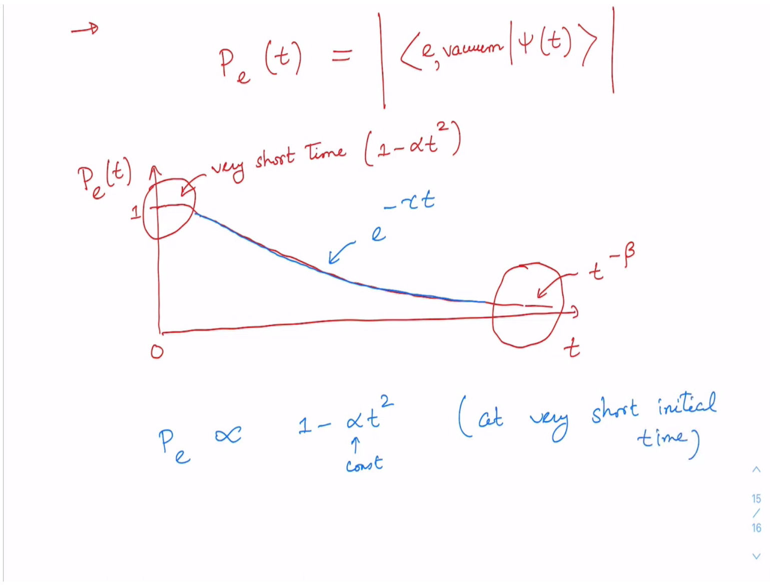

Now let us see how one can formulate a simple model for the so-called spontaneous decay of an atom. So, let us discuss spontaneous decay of an atom. For simplicity let us consider a 2 level atom say the atom has the ground state is represented by ket g and the excited state is represented by ket e and the energy difference is say h cross omega atom let me take the excited energy state to have energy h cross omega atom by 2 and this ground state has energy say h cross omega atom by 2 - h cross omega atom by 2 then we can easily write down the Hamiltonian for the atom and that would be simply h cross omega atom by 2 sigma z, z component of the Pauli matrix. Now what happens is that even it may appear that the atom is isolated but actually it is interacting with the so-called vacuum and due to the vacuum fluctuation if the atom is if it is in the excited state. So, ultimately it will decay to the ground state because of the interaction with the vacuum or due to the vacuum fluctuation. So, generally this can be very easily modeled. The atom is actually put in a we can consider the atom to be put in a bath of normal modes of electromagnetic field which could be thought of as plane waves in vacuum and it is modeled as a set of harmonic oscillators.

So, we have this is the atomic part of the Hamiltonian and then we have a collection of harmonic oscillators which is termed as bath oscillator or simply bath. So, we have a set of harmonic independent harmonic oscillators. So, this is the bath. Now then there is interaction between the atom and the bath which can be written as say sigma x because we know sigma x this the is responsible for interaction or transition between the excited state and the ground state or ground state and the excited state.

And it is and there is another operator F cap where as I said sigma x let me just write here sigma x induces it induces transition between transition between the ground state and the excited state or between the excited state and the ground state. On the other hand this operator F is it is actually proportional to the electric field at the point of the atom. So, it is proportional to the electric field at the point of the atom at the location of the atom and we can we know that electromagnetic field is quantized.

And so therefore this F operator can be expressed in terms of the normal modes and we can write it like this. So, say g k a k dagger a k that is a normal mode and of course it should have the energy. So, we have to write say h cross g k where g k is the coupling parameter this is the coupling parameter. Now using the so called RWA under the rotating wave approximation the Hamiltonian could be this Hamiltonian can be solved this Hamiltonian can be solved using rotating wave approximation.

And I think it should remind you about the so-called Jaynes Cummings model that we have already discussed. So, sigma x F under the so-called rotating wave approximation we can write it as say g k sigma + a k as you can see this is exactly the same way we are doing it we actually employed this technique in the case of Jaynes Cummings model. So, this is what we will have. Now assuming the atom to be in the ground in the excited state while the field to be in the vacuum at time t is equal to 0.

We can write the combined system as the atom is in the excited state or let me just write it up state or I can simply write it as e and the field is in the vacuum state. Then this problem can be solved exactly under this condition assuming it and it is known as it was solved exactly and this is actually the solution is called Weisskoff-Wigner solution. We will not go into the details of the calculation but I will just tell what they have done.

The probability of getting the atom in the excited state and the field modes at vacuum state could be calculated by at some probability say at some arbitrary time say t this is the probability P e t is the probability of getting the atom in the excited state at some arbitrary time t say it has the atom is in the arbitrary state psi of t and we want to find it in the excited state the atom in the excited state and the field in the vacuum.

So, let me do it this way. So, the field is in the vacuum. So, the probability would be given by simply the mod square of this, this Weisskoff-Wigner they have solved this problem rigorously and their solution can be represented by typical plot in this way suppose in the y-axis I have this probability of getting the atom in the excited state and the field in the vacuum and then this is time.

So, what happens that it will at very initial time suppose initially it has started from the excited state at initially at time t is equal to 0 the atom is in the excited state and so it is it will start from 1. Initially it will go like this and then it will fall exponentially like this and at larger and very large time this would not be exponential rather it will follow a power law kind of a behavior say t to the power - beta where beta is a constant.

On the other hand, at very very initial time very initial time very short time and initially this behaviour would be actually of the type say 1 minus it will be quadratic in time 1 - alpha t square but for the majority of the part majority of the time it will be exponential. So, largely it will behave this behaviour would be exponential decay. So, that is what they have found. So, as you see let me write here again at very short time or very short interval of time.

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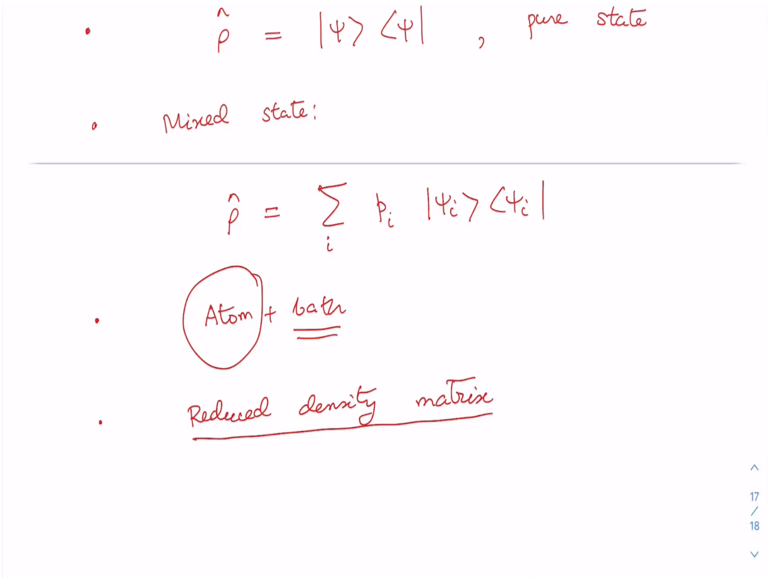

This probability will behave like this it will decay quadratically in time where say this is alpha is a constant and this is the case for at very short initial time sort initial time and this is actually, I am just what the rigorous solution will give us on the other hand this probability this probability at very large time or long time it will be it will go by this power law where this beta is a constant.

So, this is the case at very long time okay these 2 times are actually does not matter much in real atoms and we can see easily keep the exponential decay part only so we can just focus on this part. So, that probability decays exponentially in this way. So, gamma is a is the so-called decay constant. So, this result obtained through mathematically rigorous calculation and it could be represented by a very it is very simple model and it is very easy to formulate it. So, let us do that quickly.

Now the situation is that we have this the atom is in the excited state and the we have a ground state like this. So, from the excited state it decays to the ground state by emitting a photon and the decay rate is as I said it is the gamma. So, we can set up an equation to see how the probability evolves in time. So, the probability that the atom is in the time rate, rate of change of the probability P dot e is equal to as you can see from this behavior that this is exponential decay.

So, we can just write it as - gamma of P of e. So, quite clearly from here you see that you will get P e of t would be equal to P e of 0 e to the power - gamma t. On the other hand, we can also get the probability of getting the atom in the ground state that would be simply can be found out from this expression because we know that probability of getting the atom in the excited state plus probability of getting the atom in the ground state should be equal to 1.

So, using this immediately you will see that this has to be simply equal to -P e dot and therefore it would be simply + gamma P e okay. So, and we can plot it very simply very trivially. So, this will decay exponentially on the other hand the it will from the ground state probability will increase like this. So, this is our P ground probability of referring to the ground state probability on the other hand this part refers to P e this is our time.

Now what happens if we start with the this was a very simple case because and we see that we have started with the atom to be in the initially to be in the excited state and then we analyze it. Now if we assume that case where say we started the superposition of the ground state and the excited state suppose initially our system is like this. So, the atom is prepared in a state which is a superposition state of the ground state and the excited state like this.

On the other hand the field is represented by a vacuum state initially the bath state or this is the bath state. So, this is in vacuum and this is the atom-atom and this is referring to the atomic state. Now can we apply this procedure that or this model the way we have formulated equations can we do the similar thing for that. Now you see the answer is actually no because of the fact that C g and C e this coefficient are complex quantities.

Because these are complex quantities and there are phase relations between them there are phase relations between C g and C e all right. On the other hand the model that just we discussed where the probability of getting the atom in the excited state is C e mod squared and the probability of getting the atom in the ground state is given by C g mod square.

So, this quantity is P e and P g they do not take phase relations into account. In other words, the wave function approach in other words the wave function approach cannot be actually applied wave function approach is inadequate because this approach that or the model that we discussed is insufficient it cannot take into account the phase relations. So, the appropriate approach would be so we need to take the density matrix formalism which we have actually discussed this formalism in our module 1 density matrix formalism where we can take into account the coherences as well. In module 1 already we have discussed density matrix formalism in somewhat details. So, I would not like to repeat it here but quickly very quickly

I can remind you about this density matrix formalism. Let us recall that if I consider a pure state psi and the density matrix is defined by this way right where this is for pure state on the other hand for mixed state the mixed state the density operator or the density matrix is defined as this is what we have this is p i is the probability classical probability and psi i psi i.

So, this I think if you have forgotten you can again go through that particular lecture and revise it. And in fact in many situations, we want to focus on the particular subsystem only out of the total system for example in this atom plus bath system atom + bath system we would like to focus on say atomic state only. And if we want to focus on the atomic state only then rather than the full density matrix we can do that by using the so-called reduced density matrix.

So, reduce density matrix we can utilize and using the reduced density matrix what we can do we can just trace out this bath and then we can focus on the atomic part only. And we have already discussed about reduced density matrix formalism as well.

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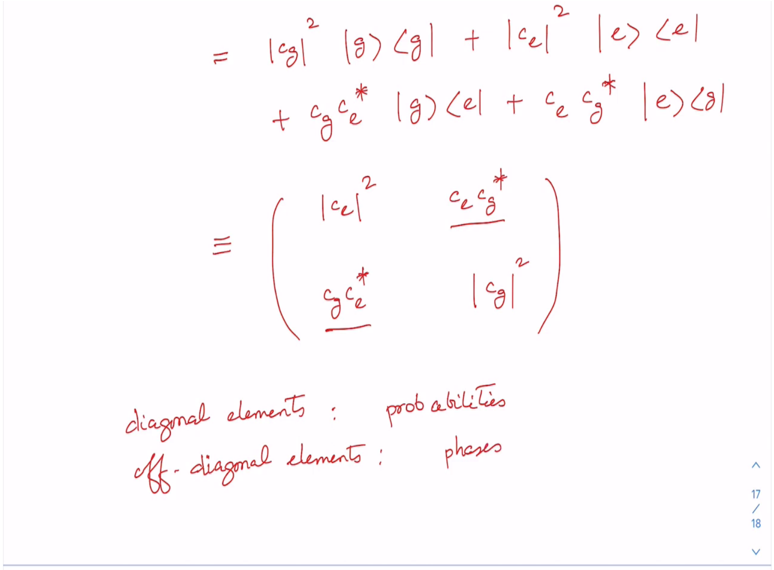

However, let me quickly refresh you about the reduced density matrix with a simple example let us again consider our 2 level system which at time t is equal to 0 is in the superposition of ground state and excited state superposition of ground state and excited state this is the atom and while the field is in the vacuum state. So, the density matrix of the combined atom density matrix for this combined atom + vacuum is of course it is given by psi of 0 ket psi of 0 and then this bra psi of 0.

Now to get the density matrix of the atom only we have to trace out the vacuum and this is basically the reduced density matrix and if we want to focus on the atom only then we will trace out vacuum and we just have to take out trace of this and if we do that in fact we did this kind of example in an earlier class in module one. So, if we work it out then we will be able to get this it would be mod C g square then this state and then you will have C e mod square e e and we will have these terms containing the information about the phases.

So, C e, C g star e g in fact this can be represented in the matrix form like this. So, C e mod square C e C g star and we have C g C e star and C g mod square. Now as you can see that the diagonal elements the diagonal elements represent the probabilities contains information about probabilities and on the other hand the off diagonal elements of diagonal elements like this they contained information about the phases and this is what is actually required.

And this was the this the model that we discussed earlier was unable to take into account but the if we take the density matrix formalism then we get both the information about the probability and the phases and this is going to be very important. Let me stop here for today. In this lecture we learned about the transmon Hamiltonian a little bit in more detail then we introduced the concept of quantum dissipation or quantum noise.

And there we learned that the wave function approach is inadequate. We have to go for the density matrix formalism also we found that the classical treatment cannot be extended to the quantum domain. In the next lecture we will see how the quantum master equation approach can be taken into account to understand quantum dissipation or quantum noise. So, see you in the next class.