Quantum Technology and Quantum Phenomena in Macroscopic Systems

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Lecture - 30
Introduction to Cavity Optomechanics; Fabry-Perot Cavity.

Hello, welcome to the 3rd module of the course. This is the first lecture in the Module 3. And, it is overall the Lecture number 22 of the course. In this lecture, we will briefly discuss about cavity optomechanics. This is going to be an introductory lecture. I will give you a brief overview of what cavity optomechanics is. And, as the so called Fabry-Perot cavity kind of devices or setup are at the backbone of cavity optomechanical system we are going to discuss in some more details about Fabry-Perot cavity because that will help you to appreciate the concepts later. So, let us begin.

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In this lecture, I am going to give you a brief overview of the area of cavity optomechanics. Cavity optomechanics is one of the most useful platforms for quantum technologies. In fact, this platform is useful even in the classical regime. So, the question is what is cavity optomechanics? As the name itself suggests, it is a topic relating interaction between light and mechanics.

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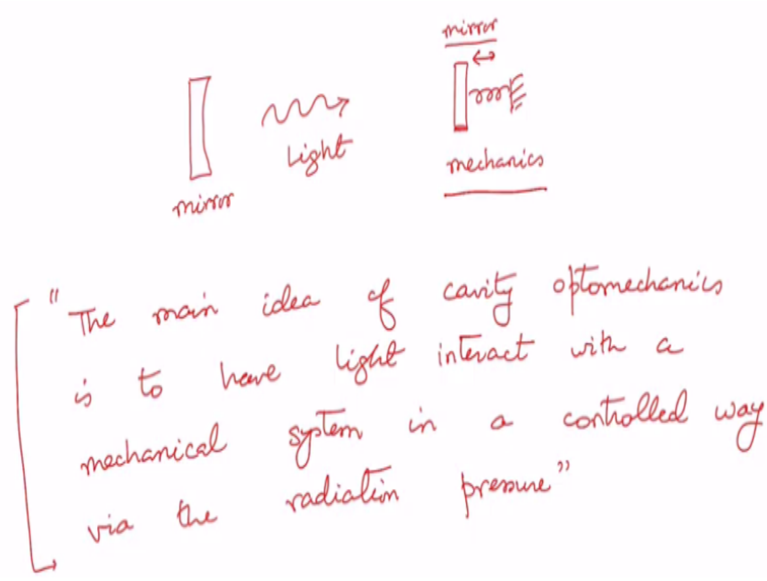

So, suppose this is a mechanical oscillator and they are interacting with each other. And, this is mediated by a cavity. That is why it is called cavity optomechanics. So, if you have a mirror here and another mirror, but the mirror is attached to a spring or say the mirror is allowed to vibrate. As you know, you have a so called Fabry-Perot cavity where both the mirrors are fixed, but here one of the mirrors is allowed to vibrate or oscillate.

So, this is the domain of cavity optomechanics. In fact, the main idea in cavity optomechanics as put by Markus Aspelmeyer a pioneer researcher in the area. He says, let me write here, the main idea of cavity optomechanics is to have light interact with a mechanical system in a controlled way via the radiation pressure force. We will discuss about radiation pressure in some more details later on.

So, this is essentially the idea. This way one can manipulate light actually both the state of light and the state of the mechanical system. And since this mechanical system comes in all shapes and sizes, cavity optomechanics gives us a new way to manipulate light and matter on nano, micro or even macro scale. And, our discussion here will primarily rely on these review articles.

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REVIEWS OF MODERN PHYSICS, VOLUME 86, OCTOBER–DECEMBER 2014

Cavity optomechanics

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First review article is by Markus Aspelmeyer, Tobias J Kippenberg and Florian Marquardt. This is the title of the review article is Cavity Optomechanics. This was published in Reviews of Modern Physics. And, this is an extremely beautiful article.

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annalen der physik

A short walk through quantum optomechanics

Pierre Meystre*

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And, another article is A Short of Walk Through Quantum Optomechanics by Pierre Meystre. It was published in Annalen der Physik.

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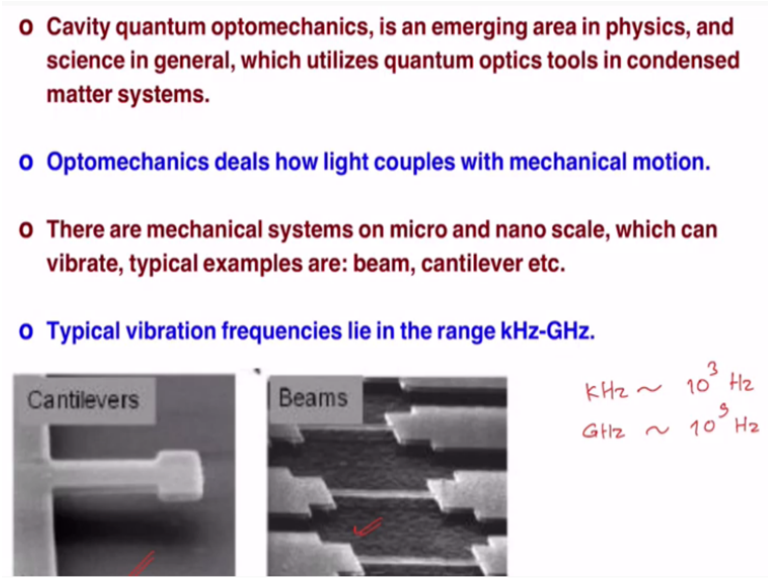

Now, so, cavity optomechanics is an emerging area in physics and science in general which utilizes quantum optics tools in condensed matter system. And, you know that optomechanics deals how light couples with mechanical system. And, in fact, there are mechanical systems on micro or nano scale which can vibrate and typical examples are say cantilevers like this. Then, beams and all these systems they vibrate with a frequency in the range kilohertz to gigahertz.

Kilohertz means 10 to the power 3 hertz. On the other hand, gigahertz refers to 10 to the power 9 hertz. And radiation pressure force is the one which responsible for coupling between light and mechanical motion.

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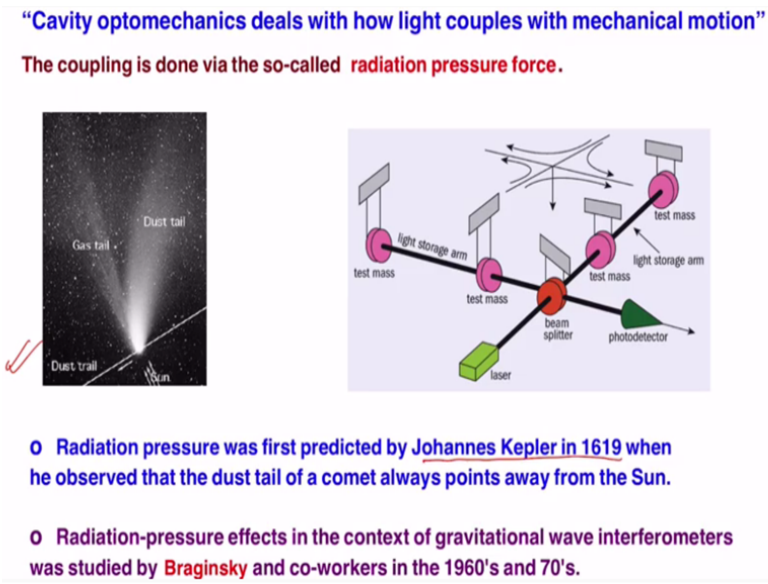

In fact, radiation pressure was first predicted by Johannes Kepler in the year 1619. When he observed that the dust tail as you can see from this plot here, he observed that the dust tail of a comet always points away from the sun. In fact, way back in the early 1960s and 1970s Vladimir Braginsky and his coworkers studied radiation pressure effects in the context of gravitational wave interferometers.

Because, at that time, people were looking various ways to detect gravitational waves and Braginsky and his coworkers came up with many proposals and eventually you know, ultimately gravitational wave got detected. And, in a way, gravitational wave detectors or the whole system is a kind of a huge cavity optomechanical system, of course, in the very large scale or macro scale.

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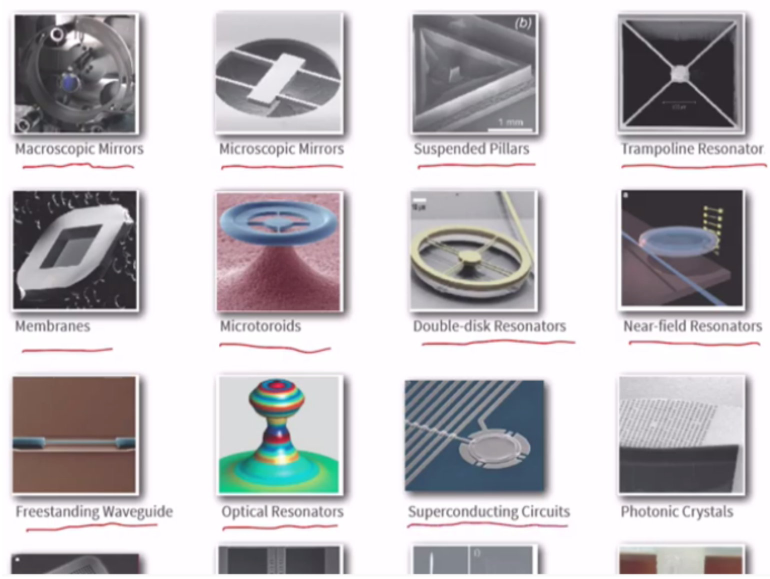

Now, today, there are a variety of optomechanical systems as is illustrated in this diagram in this photograph here. And, the mass of this system ranges from gram to zeptogram. Zeptogram means very small. That is 10 to the power minus 21 gram. And the systems like these are all optomechanical systems. For example, we have these macroscopic mirrors, microscopic mirrors, suspended pillars, trampoline resonator, membranes, microtoroids, double-disk resonators, near-field resonators, freestanding waveguide, optical resonators, even the superconducting circuits.

This is also a, kind of a, it is basically a mechanical oscillator. And, it can also be used in optomechanical setup.

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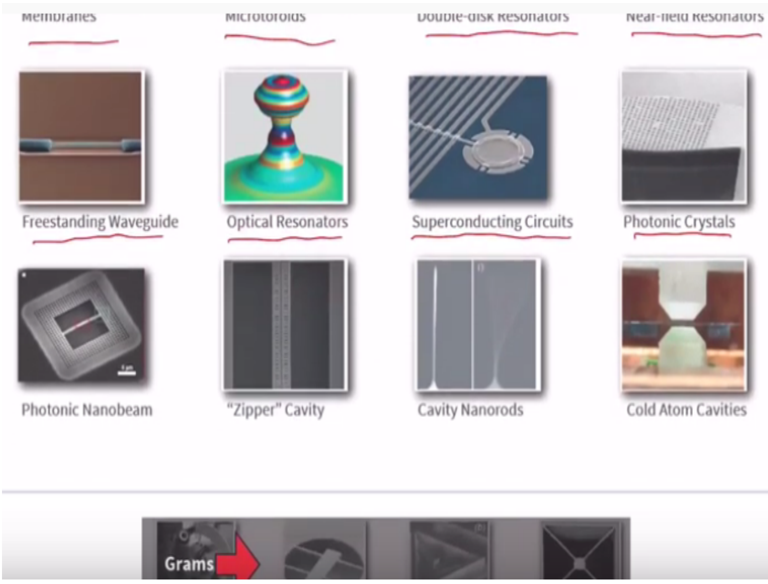

Then, you have this photonic crystals, photonic nanobeam, zipper cavity, cavity nanorods and cold atom cavities.

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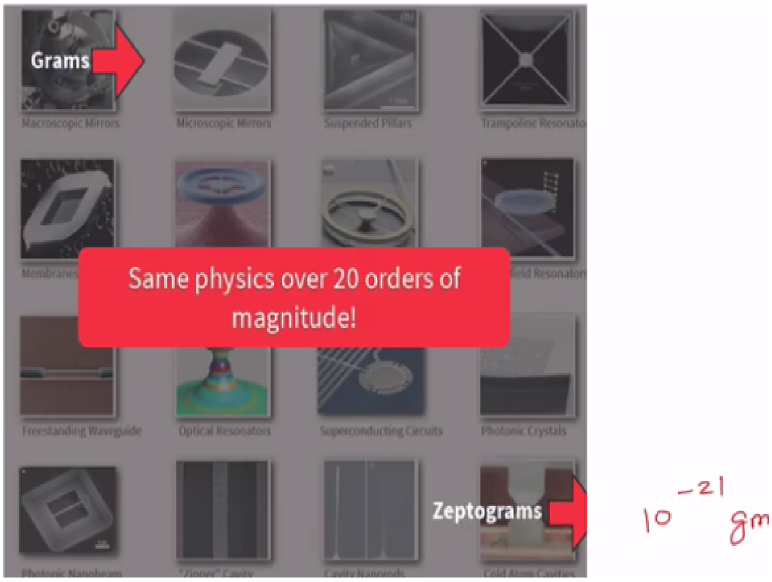

Interestingly, the mass of the system go from, vary from say gram to very small say zeptogram. That is on the order of 10 to the power minus 21 grams. So, you see that we have these 20 orders of magnitude in terms of masses. But striking an interesting fact is that the same physics is there over these 20 orders of magnitude and is very interesting that all these various looking optomechanical devices or system can be modeled by this simple setup which you have a Fabry-Perot kind of a cavity.

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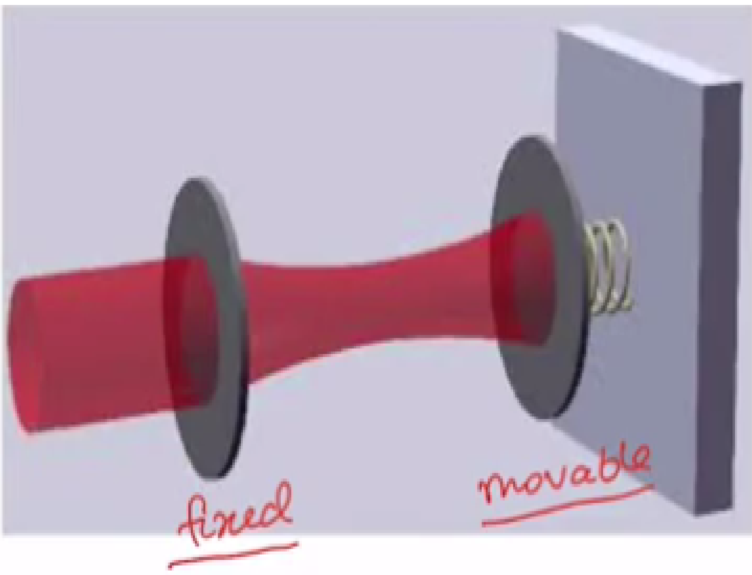

With one mirror fixed and the other mirror is movable. And this model, as we will see its potentiality later on. This model can theoretically explain a lot of the features that is displayed by these various kinds of mechanical or optomechanical devices.

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And, goals of cavity optomechanics. Primarily, cavity optomechanics has 2 goals.

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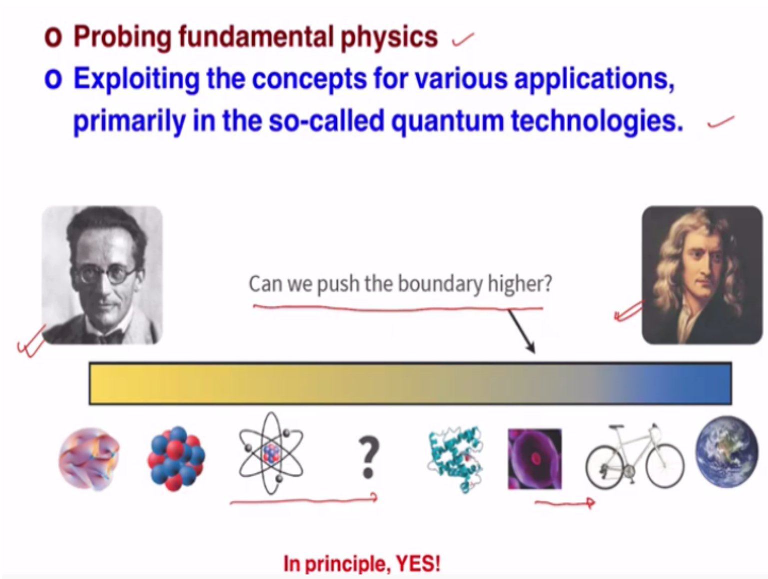

One is to probe fundamental physics. Another one is to exploit the platform optomechanical platform for various technology and applications. In fact, its role in the so called 2nd quantum revolution is expected to be pretty impactful. Optomechanics can be used to test quantum mechanics in an entirely new domain. That is the domain of macroscopic objects. Because, so far, we know that quantum mechanics is always mainly used to understand lighter objects like atoms, molecules and so on.

In fact, as regards fundamental physics, one pressing issue is to know the boundary between the classical world and quantum world. Nobody knows exactly when to stop using Newtonian formalism and start using the so-called Schrodinger formalism. In fact, one of the goals is that or question is that can we push the boundary higher? That means whether we can use quantum mechanics for higher objects or heavier objects, I mean to say. The trouble is that as we go to larger and larger objects, they coupled to the unavoidable fluctuations of noisy environment resulting in the washing out of quantum features. But in principle, yes, we can push the boundary higher, but because of this trouble, as I say it, which is actually also known as the issue of decoherence. Lot of technological advancement is still to be made to beat this decoherence issue.

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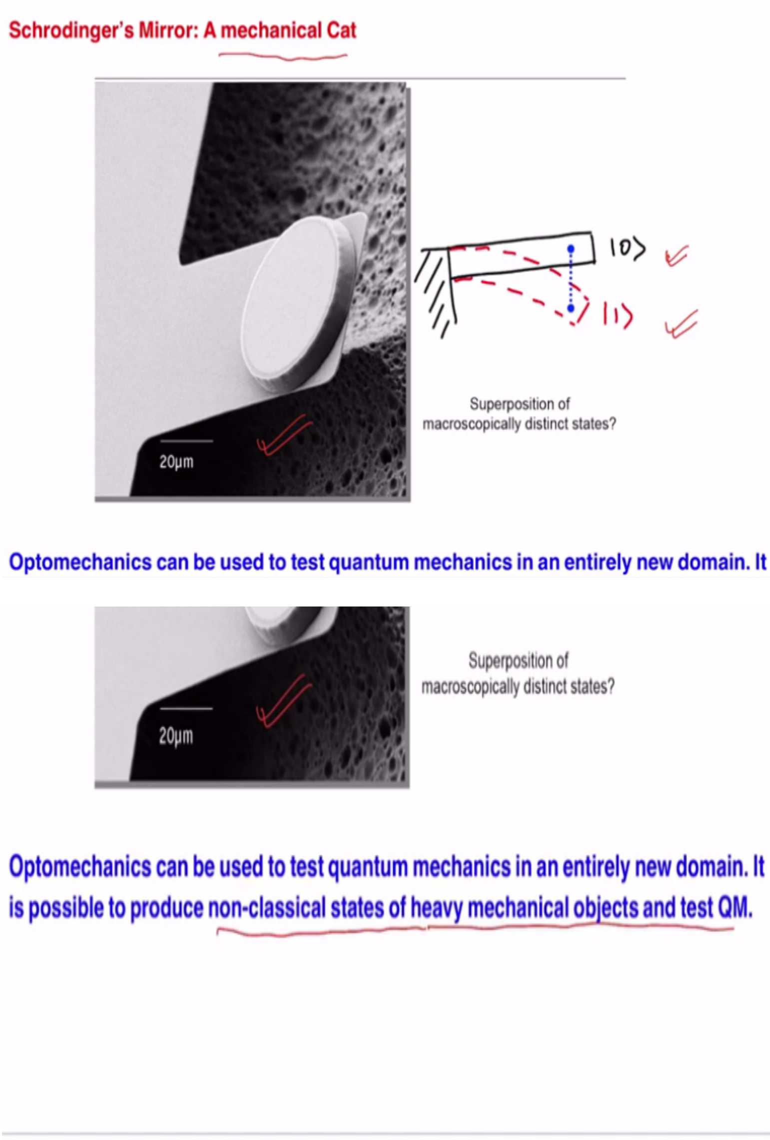

One example, as we all of us know, is the so-called Schrodinger ket problem, which we briefly discussed about in Lecture 1 of this course using actually mechanical systems. For example, as shown here in this figure, using a mechanical oscillator a cantilever, we may create a superposition states of the oscillator at 2 different location denoted by ket 0 and ket

1. Before making a measurement, we have no idea if the mechanical oscillator is in ket 0, or in ket 1.

This is equivalent to this Schrodinger ket problem. And, it is termed as Schrodinger’s mechanical cat. Also, it is known as the Schrodinger’s mirror problem. Fortunately, thanks to optomechanics, we are today in a position to create such superposition states. And, they may be tested in the laboratory. So, that is the reason why you know understanding quantum mechanics in this new domain is getting lot of interest nowadays.

In fact, it is possible to and some experiments has already been done to show that it is possible to produce non-classical states of heavy mechanical objects and test quantum mechanics.

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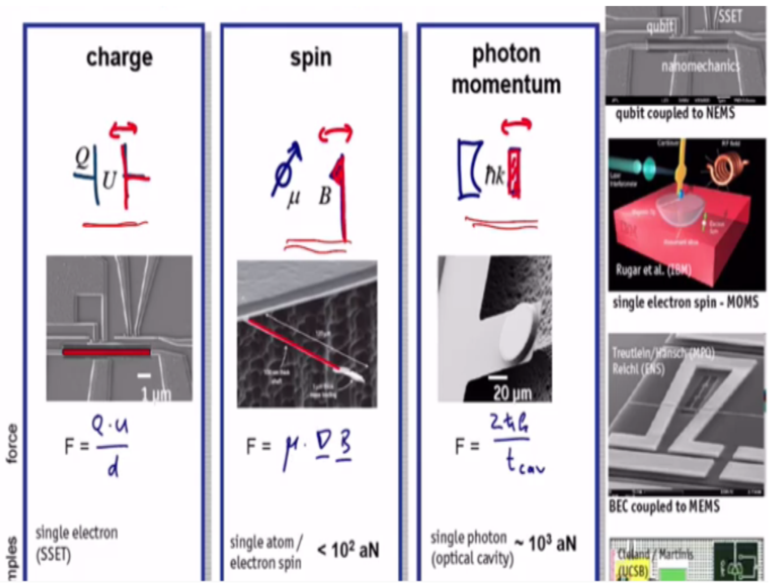

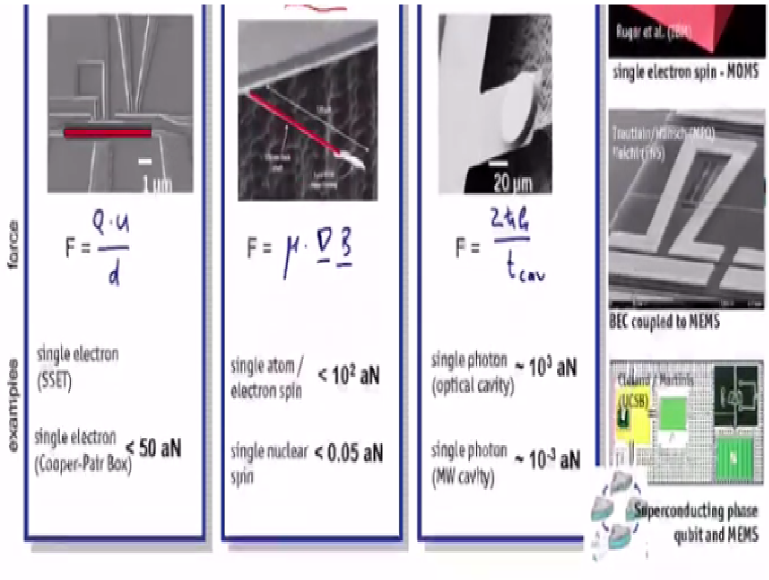

One of the biggest advantages of mechanical solid-state system is its functionality. Say, if we put a conductor on the top of a mechanical system, just like here, it gets coupled to a charge. If we put something magnetic on the top, just like here, it gets coupled to spin. Or, if we put a mirror, just like here, in the mechanical system, it gets coupled to photons and so on. So, the mechanical system acts like a bus system.

This way, we have a mechanical transducer that allows us to make different quantum systems interact, which otherwise would not have interacted.

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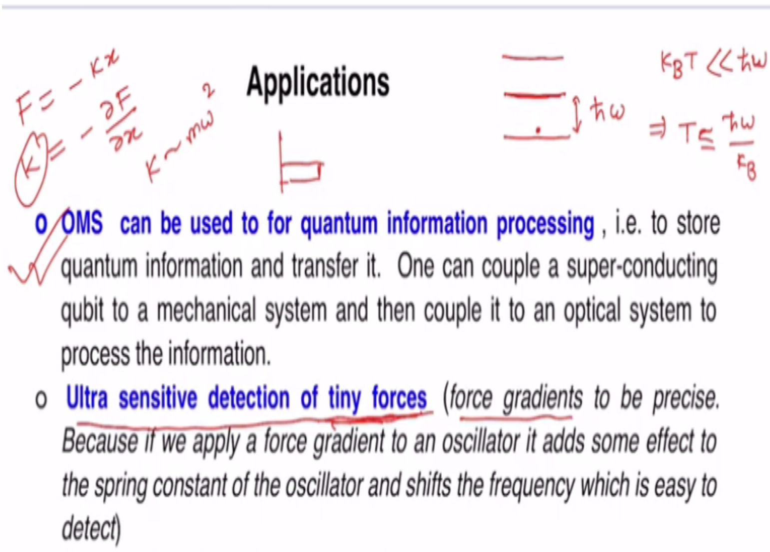

Maybe we will come to this particular topic later on as well. Now, as regards other applications, and there are many. In fact, optomechanical systems may be used for quantum information processing. That is to store quantum information and transfer it. One can couple, as an example, a superconducting qubit to a mechanical system and then couple it to an optical system to process the information.

Now, it is important to note that to study quantum states of such systems, we need to cool these mechanical systems because they are why I am saying that because you know that these mechanical systems are basically harmonic oscillators and they are equally spaced say spacing is h cross omega. And, if the thermal energy is greater than h cross omega, then the system will go from the ground state to the excited state.

And then actually, you will not be able to study the whole thing as a discretized system. This discrete energy levels will not be exploit. So, the cavity this thermal energy should be much smaller than h cross omega. So, as you can see, to have this condition, the temperature should be below than h cross omega by K B we can just make a rough estimate. And, it turns out that because these mechanical oscillators are oscillating in the frequency range of kilohertz to gigahertz, which amounts to temperature far below 20 millikelvin or so.

So, we need to actually look for some clever methods to cool this mechanical system. In fact, this is going to be one of the topics in this course when we were discussing optomechanics. How to cool a mechanical oscillator to its ground state? And, there are other applications. For example, one can have ultra-sensitive detection of tiny forces. As you actually know that you know, if a mechanical oscillator is there, then this Hooke's law you know force is equal to say minus k x.

From here, you can find out the spring constant. That would be say this is the force gradient rather than actually tiny forces. We should better say force gradient to be precise and if we apply a force gradient to the mechanical oscillator, it adds some effect to the spring constant of the oscillator and because the spring constant is associated with the frequency of the oscillator.

So, it shifts the frequency and this shift is very easy to measure. So, that is how we can make ultra-sensitive detection of very tiny forces.

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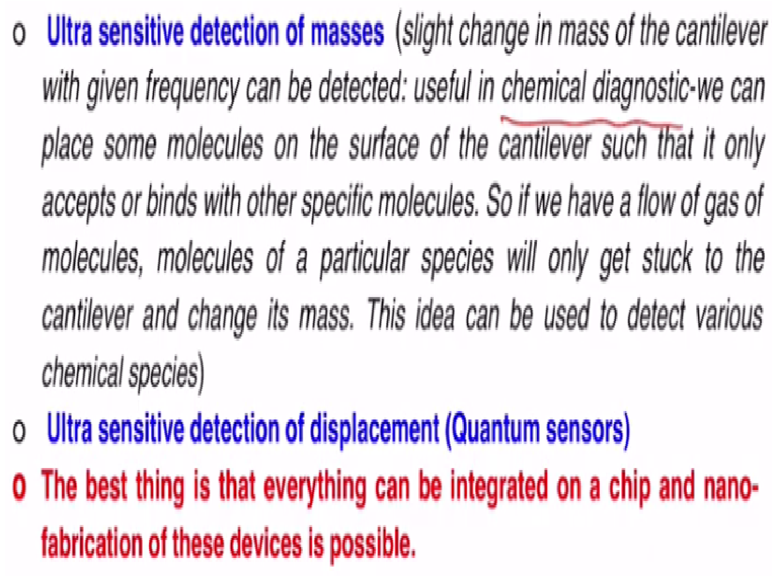

On the other hand, there is we can in the similar way we can have ultra-sensitive detection of masses because again if on the oscillator because it is this the cantilever if some mass is added to the oscillator, then its spring constant gets changed and because of that, there is a shift in frequency which can further be detected. And, this way ultra-sensitive detection of masses is also possible.

In fact, this is already, for that you need not have to cool the mechanical oscillator to the ground state. This kind of applications are already in place in chemical diagnostic. And, ultra-sensitive detection of displacement is also another very important application and, so, what I want to say now is that the primary advantages of optomechanical platforms over that of the others are due to the small size high quality factor and because it has high quality factor.

So, information can be stored for very long amount of time. And, it is integrability to various other system as I explained earlier that we can integrate this system to various other system. And the best thing is that everything can be as I have put it here, everything can be integrated on a chip and nano-fabrication of these devices are very much possible.

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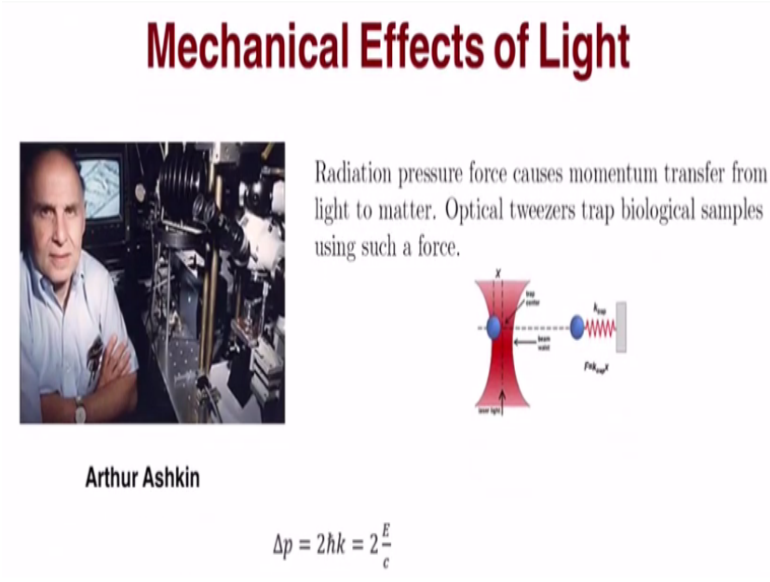

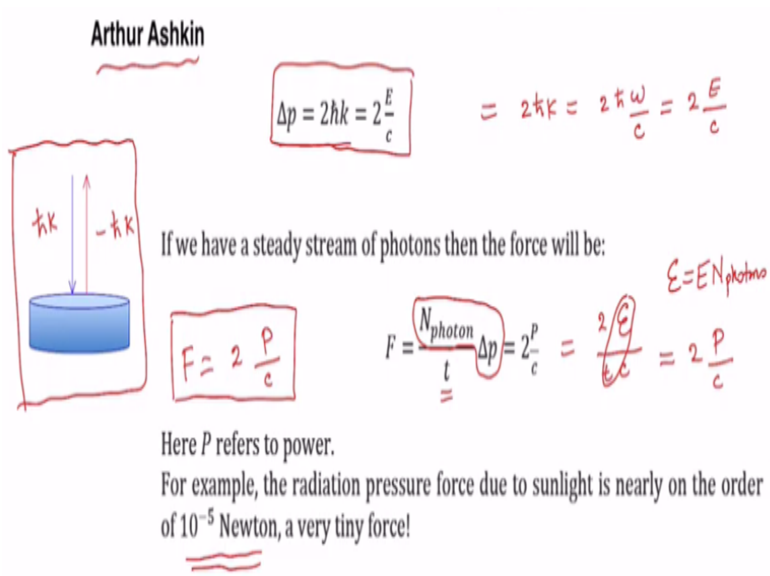

Now, let us briefly discuss about mechanical effects of light a bit qualitatively. Mechanical effects of light, in particular, the so-called radiation pressure force is at the root of all optomechanical phenomena. In fact, radiation pressure force is behind the most well-known optical tools called optical tweezer. Frequently used by biologist. Arthur Ashkin of Bell Labs contributed significantly towards understanding of the radiation pressure force and for which he eventually got the Nobel Prize also.

Now, to understand the radiation pressure force, let us consider a typical setup like this. We have a mirror. And, on this mirror, a photon is getting incident. Say, the photon has momentum h cross k. It is getting incident on this perfectly reflecting mirror. And as a result of, as it is perfectly reflecting the reflected photon has momentum minus h cross k. So, therefore, as you see that the total change in momentum would be twice h cross k.

And, thus, it is going to experience an overall momentum change given by this particular equation where let me explain, we have twice h cross k and which we can write it as twice h cross k is equal to omega by c and h cross omega is the energy of a photon. So, that is why E is the energy of the photon, c is the speed of light. Now, if we have an, you know a steady stream of photons, suppose N number of photons are getting incident, then the total force would be we know that the rate of change of momentum is the force.

And, total momentum would be number of photons into the momentum change due to 1 photon divided by the time. This is going to give us the force. And this, we can write it as 2P by c because energy in fact this I can also write it is from here this is total energy. Here, E, this curly E is equal to E into N number of N photons. And then, we have twice E divided by c. And then, we have also time is also there from here.

And, energy per unit time is nothing but power. So, therefore, we can write it in this by this equation also. So, the force is equal to twice that of the power divided by speed of light. This is a useful formula and we can apply it. For example, the radiation pressure force due to sunlight is nearly on the order of 10 to the power minus 5 Newtons which is a very tiny force. It is very tiny force.

Let us analyze the, this radiation pressure force a little bit more, because we will see this analysis is going to help us in understanding in the optomechanical system.

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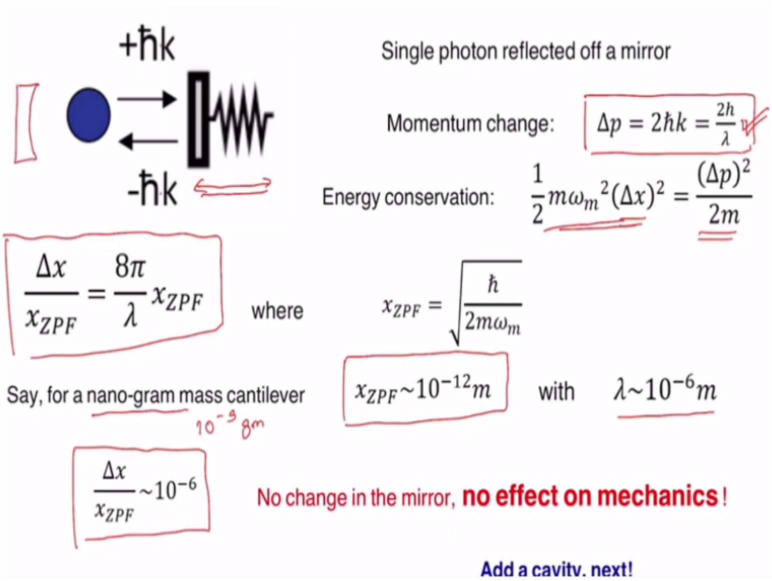

So, as I already said that when a photon is getting incident on this mirror suppose this now mirror is movable, it is a mechanical oscillator. And, this is perfectly reflecting mirrors. So, the momentum change as I explained would be twice h cross k which I can write it as 2h by lambda because twice, let me do this things here. Let me do the calculations here.

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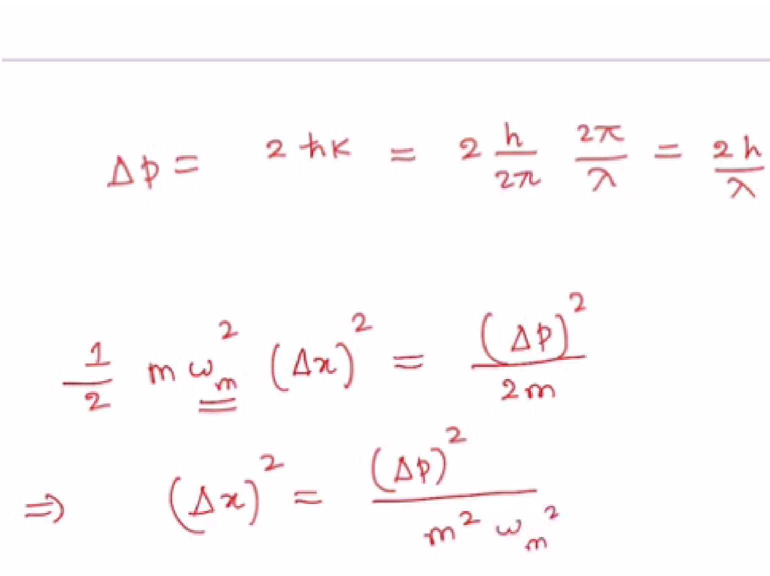

Del p is equal to when 1 photon is getting incident that would be twice h cross k. And, this I can write it as k is equal to or h cross is equal to h by 2 pi and k is equal to 2 pi by lambda. So, therefore, I can simply write it as 2h by lambda. So, this is what is written here. Now, this as a result, when the photon is getting incident, the mirror is say getting displaced by an amount delta x.

So, this mass of potential energy is getting stored in the mirror which is coming due to the kinetic energy of the photon here. So, due to energy conservation, I can write this equation and from this equation, we can get this particular equation. Let me explain it again. So, what we have here is that the potential energy is half k x square or is I have half m omega m square. And so, delta x square is the displacement of the mirror.

And, this is equal to the kinetic energy of the photon. That is getting incident delta P square by twice m. So, this omega m is the oscillation frequency of the mirror. So, from here, I can write delta x square is equal to I have here m square if I take it that side and I have here del P square and I also have omega m square.

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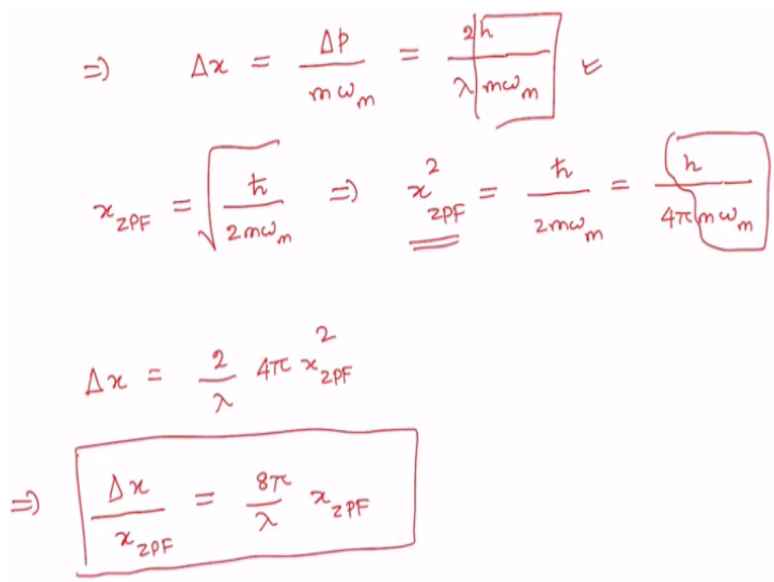

So, this will give me the displacement of the mirror delta x is equal to del P divided by m omega m and del P is equal to h by 2h by, let me use this formula 2h by lambda. So, this is 2h by lambda m omega m. Now, you know that the so called zero-point fluctuations which we discussed. When we discussed about mechanical harmonic oscillator, the, this zero-point fluctuation is h cross divided by twice m omega m square root.

Or, I can write it as x square zero-point fluctuation is equal to h cross divided by twice m omega m. Or, I can write it as h divided by 4 pi m omega m. So, this equation this one I can utilize it here. So, I can write delta x is equal to I can write it as 2 by lambda. This part I can replace by h cross. This one I can replace by 4 pi x zero-point fluctuations square here. Or, I can write the ratio delta x divided by which is the displacement with respect to the zero-point fluctuation.

That would be equal to 2 by lambda. In fact, better I can write it as 8 pi by lambda x zero-point fluctuation. So, this is the formula we get when we have this setup. Now, if this mechanical mirror, this movable mirror has mass, 1 nanogram that means 10 to the power minus 9 gram. This is or we can consider the cantilever also. This cantilever is 10 to the power minus 9 gram as its mass and the wavelength of the radiation is say 1 micrometer.

That is 10 to the power minus 6 meter. Then, the zero-point fluctuation and the displacement due to the zero-point fluctuation would be on the order of 10 to the power minus 12. And, if we put all these things, then you see the ratio would turn out to be only 10 to the power minus 6 which is actually not detectable. What does it mean physically? It means that when a photon is getting incident on a movable mirror, a free photon is incident on a movable mirror.

Effectively there is no change in the mechanics. There is hardly any displacement that is we can detect. So, to have a sizable impact on the displacement of the mirror, or, in order to study useful physics, the displacement of the mirror must have to be greater than the zero-point fluctuations. And in order to do that, what we have to, what we can try? We can try by putting another mirror here. And then, we will see what happens if we put another mirror there.

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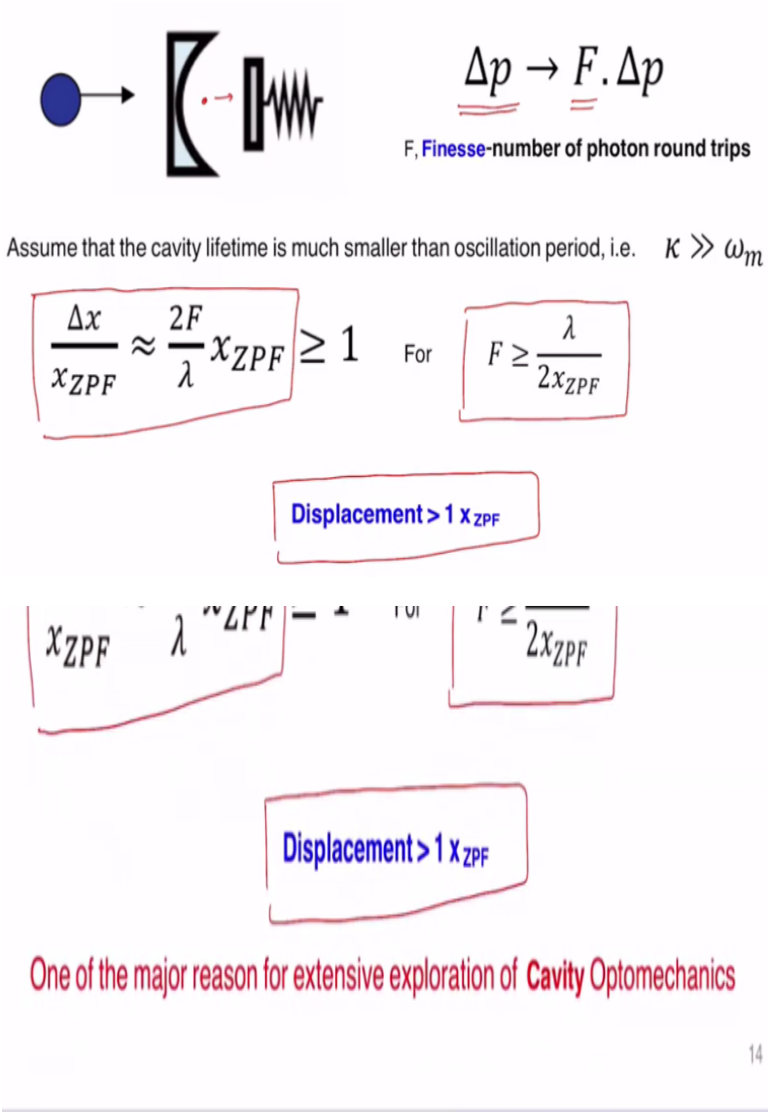

So, this will become basically a cavity. Now, when a cavity, it becomes a cavity, the photon will oscillate between these 2 mirrors back and forth depending on the quality factor of the mirror or the so-called finesse of the mirror. Finesse of the mirror gives the number of round trips of the photon inside the cavity before the photon eventually leaks away from the cavity. So, what it happens is addition of the another mirror basically increases the change in momentum.

Del p, F into del p, where F is the, as I said, it is called the finesse of the cavity. Now, if the cavity because the finesse you know, it depends on the cavity decay time also the whole thing. So, if the cavity lifetime is smaller than the mechanical oscillation period then it can be shown that this ratio would be given by this particular formula. And, it turns out that this displacement of the mirror.

The movable mirror would be greater than the zero-point fluctuation provided the finesse is greater than or equal to lambda by 2 into twice of the zero-point fluctuation displacement due to the zero point fluctuation. So, in that case, depending on the cavity, if we put a cavity then we can always construct such a cavity with appropriate finesse and then we can have a displacement greater than the zero-point fluctuation.

Displacement due to zero-point fluctuation and which is the reason of the popularity of cavity optomechanics because, now, we can actually study quantum effects using such kind of a setup.

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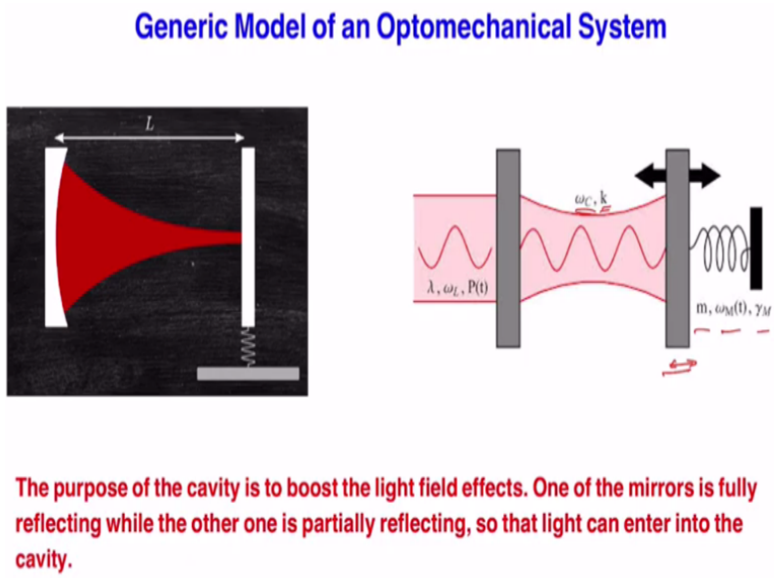

So, now, the generic model of an optomechanical system is shown here. It is a system with 2 mirrors with one fixed and other movable. Movable mirror is characterized by mass m, oscillation period omega m and a phonon decay rate gamma M. As you know, quantum of vibrate this mirror is movable that means, it is vibrating and you know the quantum vibration is called phonon.

The cavity is characterized by the cavity resonance frequency say omega c and, it's a cavity decayed kappa or k. On the other hand, incident laser light is characterized by its wavelength lambda its frequency omega L and the power P. The purpose of the cavity is to boost the light field effects. And, one of the mirrors is fully reflecting this one say and while the other one is partially reflecting so that light can enter into the cavity.

Now, there is a basically certain condition has to be satisfied for that light can enter into the cavity and that, to understand that we need to revisit the Fabry-Perot cavity. As you can see from this diagram that the system is primarily a kind of a Fabry-Perot cavity and it may be now useful to digress a bit and discuss about Fabry-Perot cavity in some details.

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So, let us discuss Fabry-Perot cavity now. We will start with the simplest case of a planar cavity of length say L.

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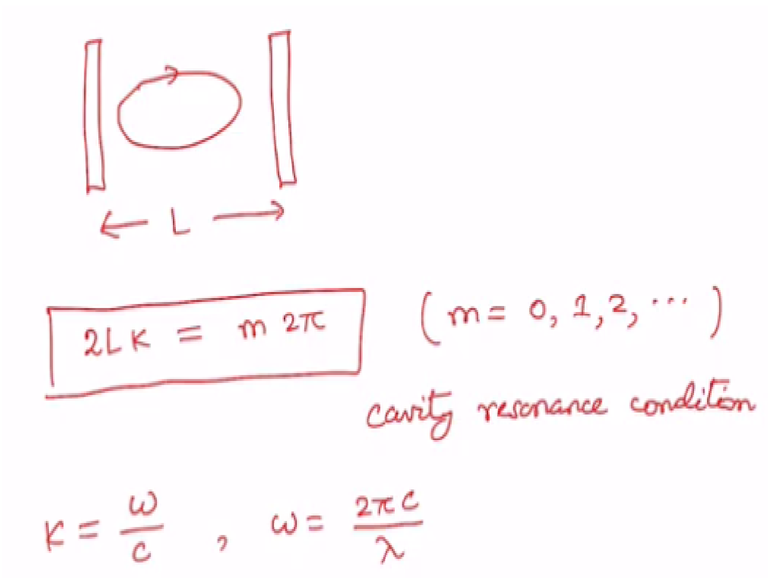

So, planar cavity means we have 2 plane mirrors separated by a length L. And, let us assume that these 2 mirrors are perfect reflectors. And, in order for a plane wave to exist in the cavity, it must return exactly the same phase after 1 round trip through the cavity. So, that has to be the condition. And, in this case, it will constructively interfere with itself. Otherwise, the phase will precess on each successive round trip.

And eventually, it is going to lead to destructive interference. So, the round trip accumulated phase must be some integral multiple of 2 pi. Now, because in 1 round trip the total length covered would be 2L. And therefore, the phase would be 2L into k, k is the wave vector and that has to be integral multiple of 2 pi. So, m is an integer, say m is equal to 0, 1, 2. So, it is an integer. This is the condition. This is called cavity resonance condition.

In fact, because you know that k the wave vector is associated with the angular frequency of the wave omega by this relation k is equal to omega by c. Again, you know that omega is equal to 2 pi c by lambda wavelength of the radiation. So, using this, we can write various resonance conditions.

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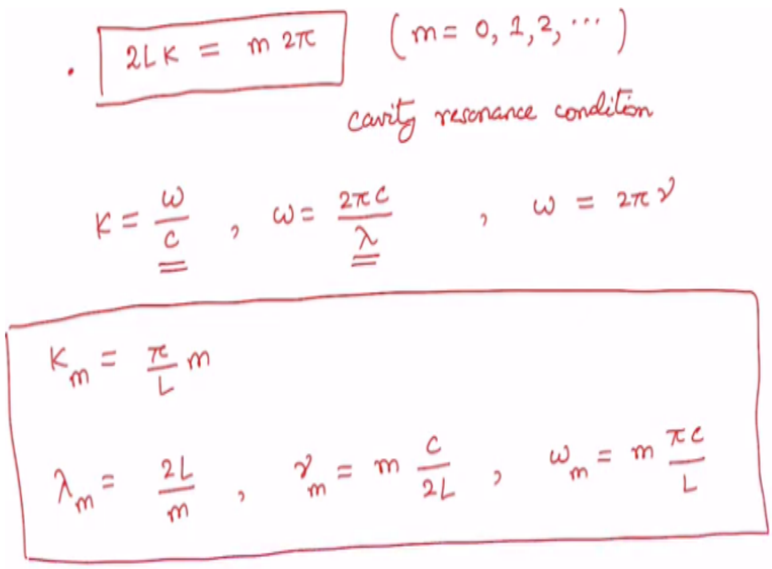

For example, cavity allowed wave number from this relation would be say k m is equal to it would be as you can see this would be simply pi by L into m. And then, using either omega or lambda, we can have cavity allowed wavelengths as lambda m is equal to 2L divided by m. And frequency, in terms of frequency also, we can write because you know that omega is related to frequency 2 pi as omega is equal to 2 pi nu.

So, therefore, nu m, this is the allowed frequency that would be m into c divided by 2L. Or, we can also write it as omega m is equal to m into pi c by L. So, these are the same conditions written in different form. And, all these forms are useful depending on the situations either of these forms can be used. And, we will use that.

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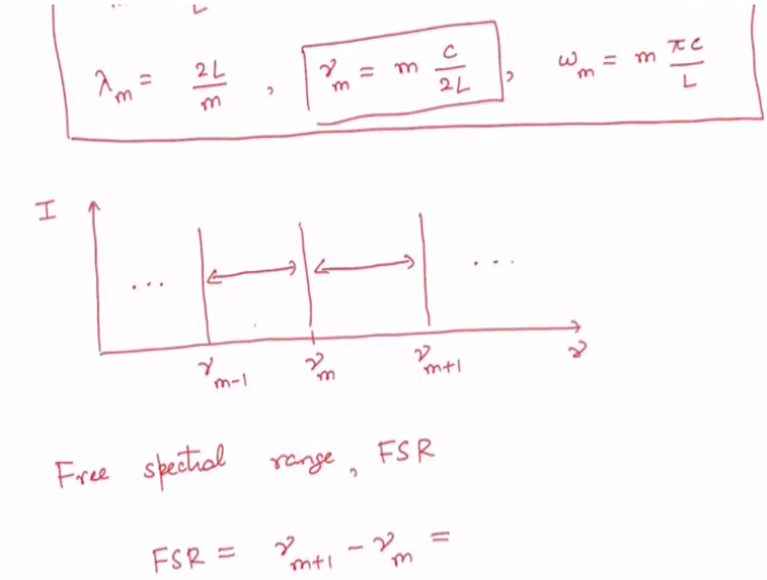

Now, the corresponding spectrum if I plot intensity inside the cavity versus the frequency, then only those frequencies or wavelengths would be allowed to enter into the cavity which is going to satisfy the resonance condition. So, therefore, for example, if it satisfy this conditions nu m, then you will have a sharp peak inside the cavity or if it satisfy say your m is equal to a nu m + 1 then also you will have a peak.

Otherwise, in between here, will not have any radiation because their resonance condition is not going to be satisfied there and here it will be say we will have, this would be nu m minus 1 and so on. You will get lot of peaks in between. Now, there is a very important quantity the frequency spacing between these peaks. This is a very important quantity. This frequency spacing is called free spectral range or also called FSR.

This quantity is of immense significance. And, this is simply the frequency spacing. So, therefore, nu m + 1 minus nu m. If you, from this relation, you can easily see that this would be simply c divided by 2L.

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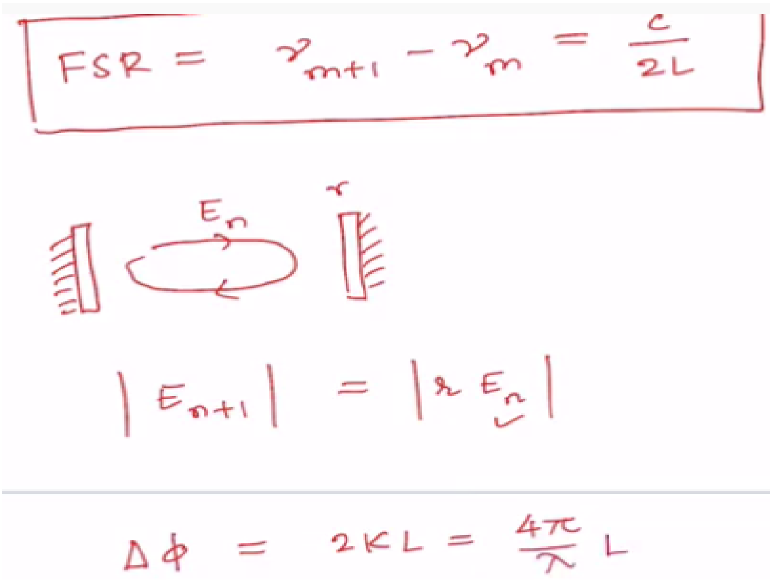

And, this is an important quantity. And, it is worth remembering FSR. Now, you know the real cavities involve some loss of light on each round trip. And, this loss then actually damps the amplitude of the cavity. So, let us consider again this cavity. If let us say suppose, light is circulating inside the cavity and after some round trip suppose its electric field amplitude is say E n and reflectivity of say this mirror is r.

Let us have both the mirrors has the same reflectivity for simplicity purposes. Then, the amplitude of the wave is going to reduce by a factor of r on each a round trip. So, after n + 1 at a round trip its amplitude would be reduced by a factor of r. Like this, the amplitude will reduce, r is less than 1. So, we are just writing about amplitude here. So, this is what is going to happen.

Now, here, E n is basically the electric field amplitude of the plane wave on the nth round trip, as I said, through the cavity. And, the loss due to r could be some transmission of the mirrors. Or, losses around the edges of the finite size of the mirror scattering from some suppose some gas is there inside the cavity or objects inside the cavity and so on. On 1 round trip, the face of the plane wave is going to change.

And, it is going to change by say phi delta phi. And, of course, the face would be in1 round trip, as we know, this would be 2k into L and which I also can write it as 4 pi k is equal to 2 pi by lambda. So, it would be 4 pi by lambda into L.

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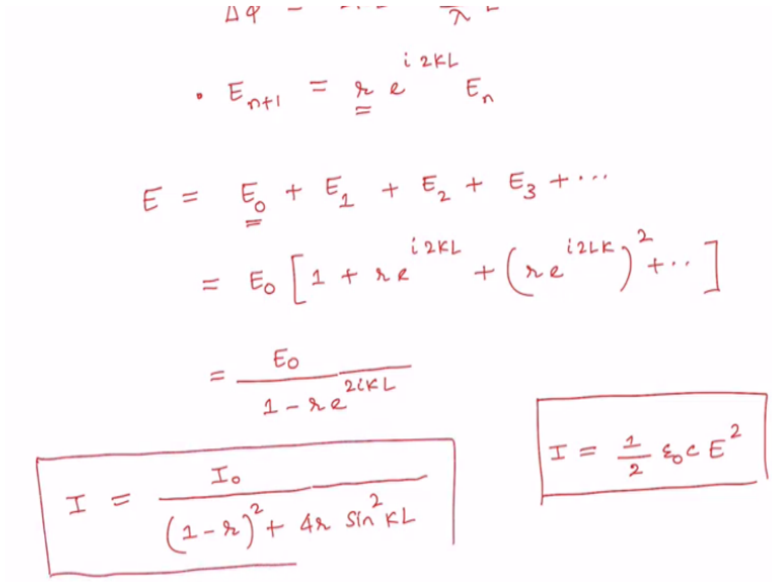

And therefore, the electric field after n + 1th round trip it would be now here I have written the magnitude. So, if I take phase also into account then it would be r. You will have e to the power phase is i 2kL. And, this would be E n. So, this is what we are going to get. Now, let us assume that this r this quantity is real and then the total wave if I consider all the round trips that total electric field say initially we had electric field is E 0.

Then, after 1 round trip, we have E 1 and then E 2, E 3 and so on. And, this is related to this quantity by means of this relation. So, immediately, you can see that I can write it as E 0 1 + E 1 would be E 0 E 2 r e to the power i2kL. Then, the second one, so, you can easily make it out. This would be r e to the power i2kL whole square and so on. So, you will get a geometric series. And, this is very simple to work out.

And, you will get this relation that would be E 0 divided by 1 minus r e to the power twice ikL. So, using this relation now, I can easily get the intensity inside the cavity just to remind you that intensity is related to the electric field by this expression. If I consider that the inside the cavity the refractive index is 1, then it would be intensity is half epsilon 0 c E square. So, this we know.

So, utilizing this relation in I can write the intensity inside the resonator would be equal to if you take the mod square you will get you can easily get this expression. It would be I 0 divided by 1 minus r whole square plus 4r sine square kL. So, this relation can be worked out very easily. So, this is the intensity we are going to obtain inside the cavity.

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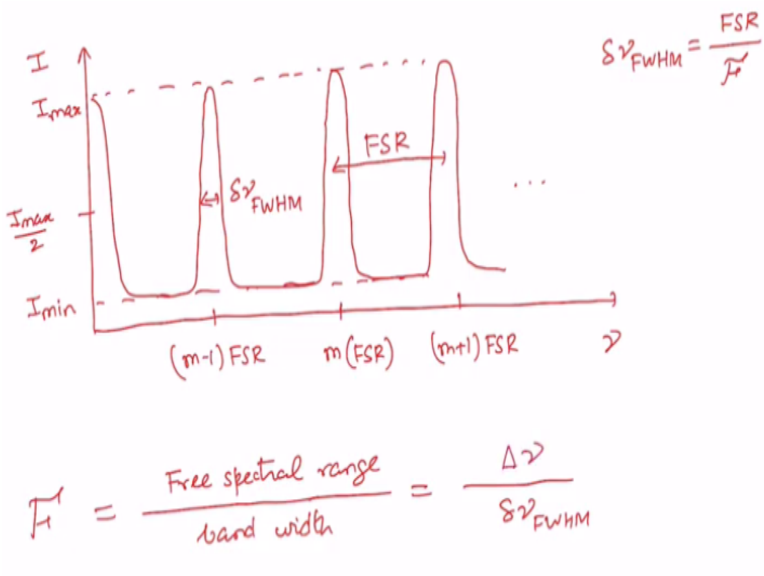

A typical intensity versus frequency for a Fabry-Perot cavity would look like this. Let us say we have in the y axis this intensity and the x axis frequency. Let me plot it. Let us say this is I maximum. I will explain these terms little bit later. Then, this is a I minimum. Intensity will oscillate between minimum and maximum. And, it will be like this. We will have several peaks like this and so on.

And, these peaks would be there provided the resonance condition is getting satisfied. For example, when the frequency is integral multiple of c by 2L, in fact c by 2L already you know this is nothing but the free spectral range, FSR. So, rather let me write it as m into FSR. And, this would be m + 1 into FSR. And, you know that distance between these two is basically free spectral range.

And, this would be m minus 1 FSR and so on. We will get lot of peaks like this. And, all this resonance peak would have width. That is, this generally this width is the full width half maximum. It will have full width half maximum. And, we will show that this full width at half maximum or this resonance width is would be actually it is the ratio between the free spectral range and the finesse of the cavity.

So, finesse of the cavity is a very important quantity. And, this is defined as the ratio between free spectral range divided by the bandwidth. In fact, this bandwidth is nothing but the full width at half maximum, delta nu FWHM. And, free spectral range let me denote it by delta nu.

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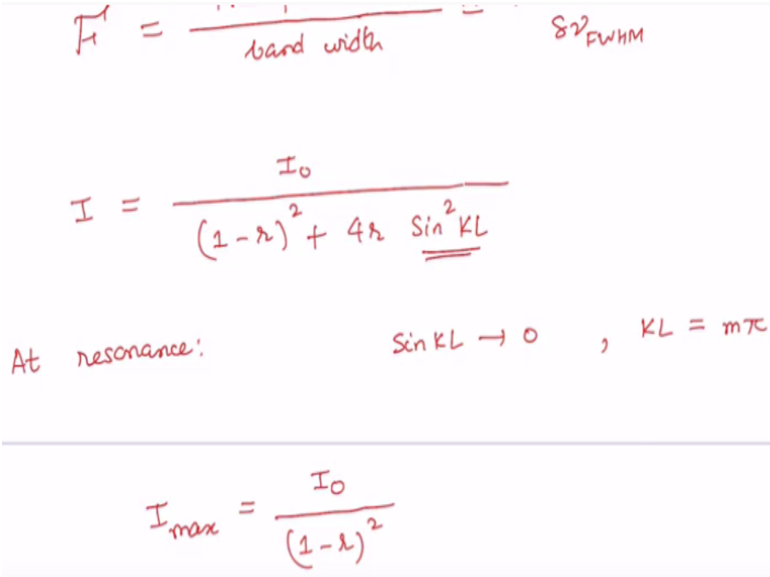

Now, from the expression that we obtained a little while back that is I is equal to I 0 divided by 1 minus r square plus 4r sine square kL. From this, we can work out the maximum intensity and the minimum intensity. Maximum intensity, we will get at resonance. You know this sine kL would tend to 0 because at resonance as you know the resonance condition this would be kL is equal to integral multiple of pi.

So, therefore, this term will vanish. So, you will have the maximum intensity. That is, I max is equal to I 0 divided by 1 minus r whole square.

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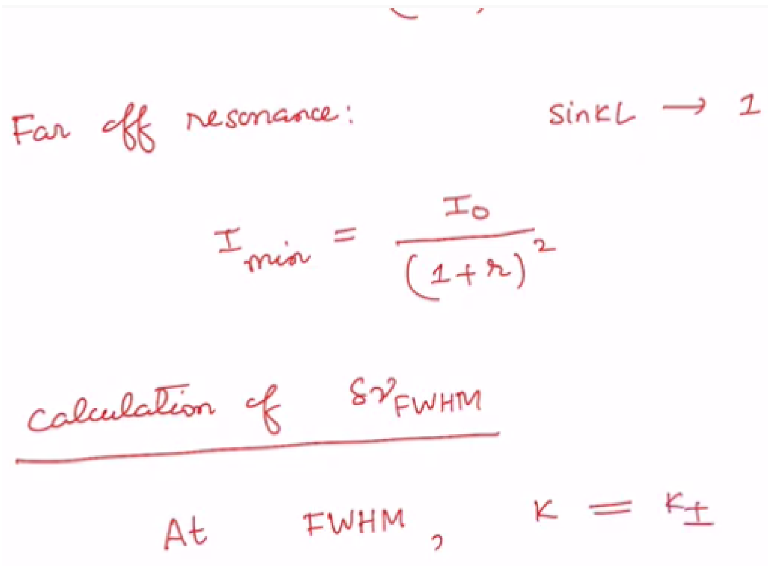

On the other hand, far off resonance will have sine kL will tend to 1. So, therefore, we will have as you can see from here when it would become maximum we will get minimum intensity. That is, I minimum would be equal to I 0. If it becomes equal to 1 then you immediately see that this would be I 0 divided by 1 + r whole square. Now, let us calculate the FWHM or width of the resonance delta nu.

Let me write, do the calculation of delta nu full width at half maximum. This is pretty easy, because you know that at full width at half maximum let us say k is taking the value k plus minus.

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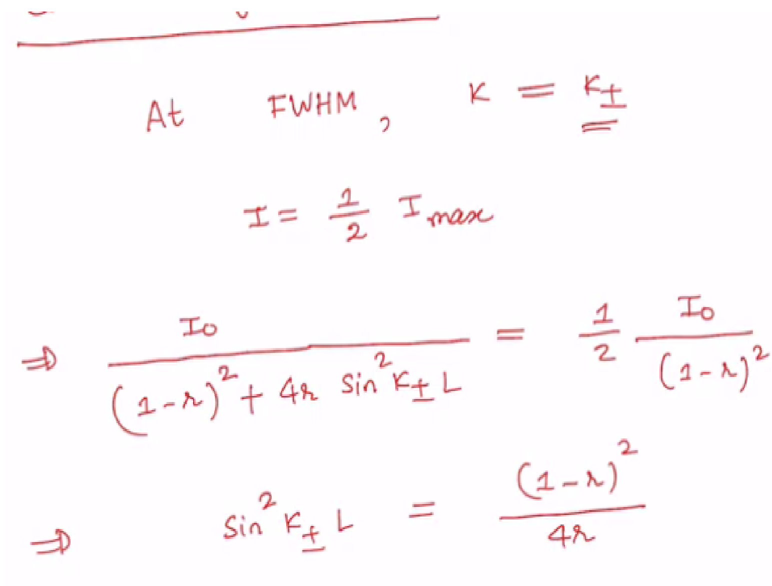

Then, the intensity because it is maximum half maximum, so, you have half I max. That is the half of the maximum. So, if I now put the expression of I from here, this one. And, k is equal to k plus minus. Then, I have I 0 divided by 1 minus r whole square plus 4r sine square k plus minus L. And, that would be equal to half I 0 1 minus r whole square. From here, you can immediately get sine square k plus minus L is equal to 1 minus r whole square divided by 4r.

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Now, $\omega_\pm = c k_\pm$

$$\sin^2\left(\frac{L\omega_\pm}{c}\right) = \frac{(1-r)^2}{4r}$$

$$\Rightarrow \quad \omega_\pm = \pm \frac{c}{L} \sin^{-1}\left(\frac{1-r}{2\sqrt{r}}\right)$$

Under small angle approximation:

$$\omega_\pm \approx \pm \frac{c}{L} \frac{(1-r)}{2\sqrt{r}}$$

Now, you see you have omega angular frequency plus minus I can write it as c into k plus minus. So, therefore, I have from here sine square omega plus minus, in fact, here, L into omega plus minus by c. This is equal to 1 minus r whole square divided by 4r. And from here, you can get omega plus minus is equal to, with some factor is there, but this is what you will get it would be c by L sine inverse 1 minus r divided by 2 into square root of r.

Now, under small angle approximation because this r reflectivity is usually 96 to 99%, 98%, so, for sine function, we can make the small angle approximation. So, under small angle approximation, I can, we can write omega plus minus. That would be nearly equal to plus minus C by L. And, we will just keep this term, 1 minus r 2 into square root of r.

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Under small angle approximation:

$$\omega_\pm \approx \pm \frac{c}{L} \frac{(1-r)}{2\sqrt{r}}$$

$$\delta\nu_{FWHM} = \frac{\omega_+ - \omega_-}{2\pi}$$

$$= \frac{c}{2L} \frac{(1-r)}{\pi\sqrt{r}}$$

$$= \Delta\nu \frac{(1-r)}{\pi\sqrt{r}}$$

$$\boxed{F = \frac{\Delta\nu}{\delta\nu_{FWHM}} = \frac{\pi\sqrt{r}}{1-r}}$$

Now, if I define this full width at half maximum that is basically delta nu FWHM. That is omega plus minus omega minus divided by 2 pi. And, if you work it out from here, you will get it as c by 2L 1 minus r pi into root r. And, c by 2L is nothing but the full free spectral range. That is, we are denoting it by delta nu. And, this is 1 minus r pi into root r. And, we know that the finesse is delta nu free spectral range divided by the full width at half maximum.

So, from here, we immediately get an expression for the finesse for this planar cavity, when both the mirrors have reflectivity r. So, this is an important quantity.

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$$\mathcal{F} = \frac{\Delta\nu}{\delta\nu_{FWHM}} = \frac{\pi\sqrt{r}}{1-r}$$

$$I = \frac{I_0}{(1-r)^2 + 4r\sin^2 kL}$$

$$I = \frac{I_{max}}{1 + \left(\frac{2\mathcal{F}}{\pi}\right)^2 \sin^2 kL}$$

Now, in terms of finesse, we can rewrite the intensity expression. That is the expression that we had earlier was I am repeating it again. I is equal to I 0 1 minus r whole square plus 4r sine square kL. This, we can rewrite in the standard form. And, that would be I is equal to I max you can please verify. It would be 1 plus twice into finesse by pi whole square sine square kL.

So, this is the standard form for the expression for the intensity of a Fabry-Perot cavity. Now, let us discuss about photon lifetime inside the cavity.

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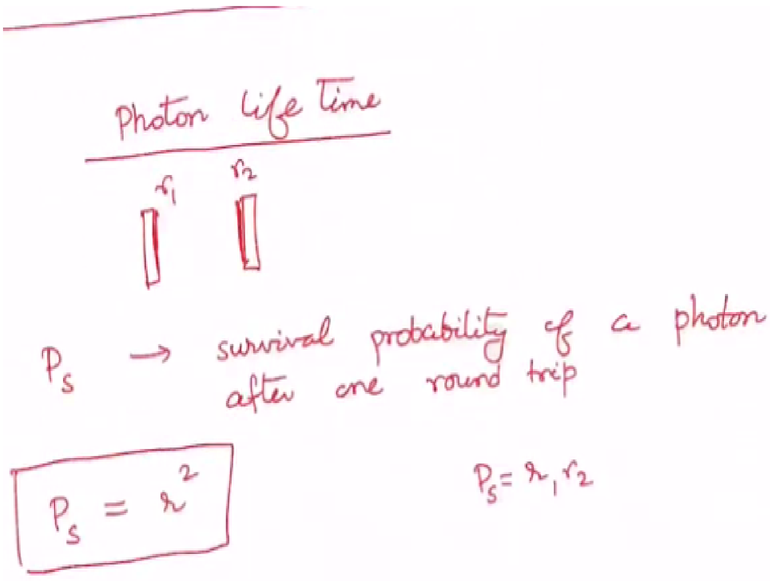

You know that losses in cavity can come from a number of sources, the mirrors can lead to intensity loss due to partial transmission and absorption of light in the reflective coating. There are coatings are there. So, because of that also light can be lost or photon can get lost, then scattering due to the surface roughness and so on. Let us say Ps is the survival probability of a photon inside the cavity after 1 round trip.

Actually, if the both the mirrors have reflectance or reflectivity r, then P s is equal to r square. Otherwise, if one mirror has reflectivity say r 1 the other one is r 2 then P s would be equal to r 1 into r 2. Anyway, we are assuming that both the mirrors are having the same reflectivity. Now, in terms of using this expression, we can rewrite the expression for finesse in terms of this survival probability because we have this expression for a finesse.

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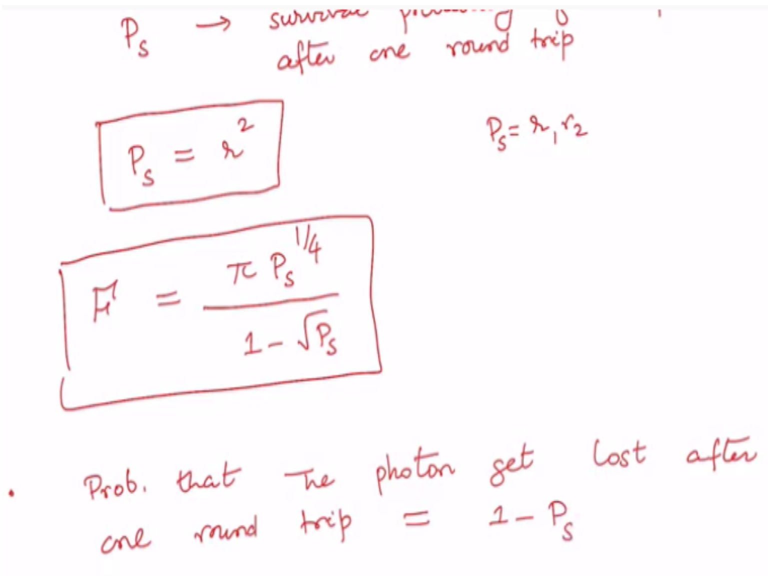

So, finesse in terms of survival probability would be pi into P s to the power 1 by 4 divided by 1 minus square root of P s. So, the probability, this is we are going to use. Now, the probability that the photon gets lost with probability actually rather let me write it as the probability that the photon gets lost after 1 round trip. So, it is clearly 1 minus P s because P s is this survival probability of the photon after 1 round trip.

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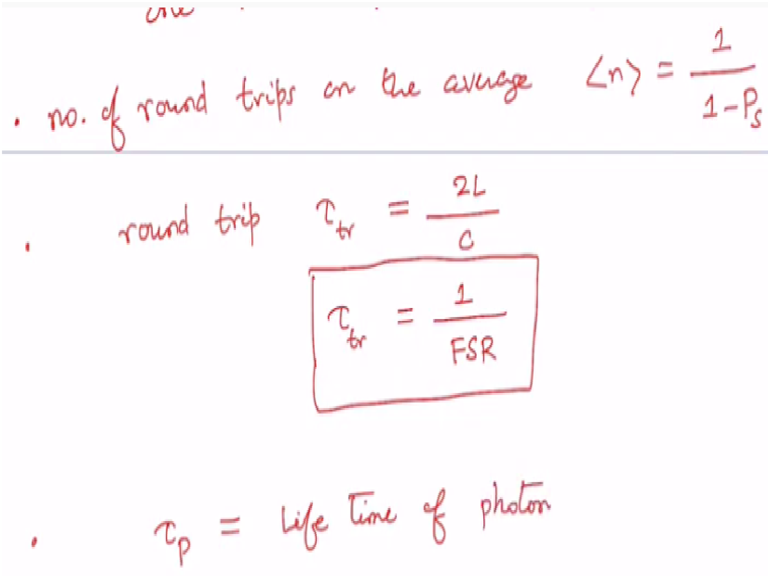

So, on the average, what would be the number of round trips would be, round trips on the average? Rather, let me write number of round trips on the average for the photon would be 1 divided by 1 minus P s. Now, the round trip time is easy to calculate because we know that the cavity has length 2L. So, the photon has to cover a distance of 2L and speed of the photon inside the cavity is c. Let us say.

Then, round trip time would be 2L by c which we can actually rather than t let me write is tau tr. And, in terms of free spectral range, we can write it simply as 1 divided by free spectral range because c by 2L is the free spectral range. So, this is an important quantity. This is the round trip time of the photon. So, lifetime of the photon inside the cavity let me denote the lifetime of the photon as tau P. This is the lifetime of photon inside the cavity.

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- τ_p = Life Time of photon

$$= \tau_{tr} \times \langle n \rangle$$

$$\boxed{\tau_p = \frac{1}{FSR\,(1-P_s)}}$$

- For good resonator $P_s \approx 1$, $\sqrt{P_s} \approx 1$

$$P_s^{1/4} \approx 1$$

$$1 - P_s = (1-\sqrt{P_s})(1+\sqrt{P_s}) \approx 2(1-\sqrt{P_s})$$

So, that is clearly it should be equal to the round trip time into the average number of round trips inside the cavity. So, using the expression that we derived here that would be photon lifetime. We will have as 1 divided by free spectral range into 1 minus P s. Now, actually for good resonator or for good cavity, this survival probability is nearly 1. So, therefore, we can guess, we can nearly assume that square root of P s is equal to 1.

Or, we can write P s to the power 1 by 4 to be nearly you can set it as 1. And, we can write 1 minus P s is equal to 1 minus square root of P s into 1 plus square root of P s. This I can write because of this approximation. I can write it as 2 into from here I will get 2 into 1 minus square root of P s.

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$$P_s^{1/4} \approx 1$$

$$1 - P_s = (1-\sqrt{P_s})(1+\sqrt{P_s}) \approx 2(1-\sqrt{P_s})$$

$$\mathcal{F} = \frac{\pi}{1-\sqrt{P_s}}$$

- $$\tau_p \approx \frac{1}{2\,(FSR)\,(1-\sqrt{P_s})} \approx \frac{\mathcal{F}}{2\pi\,(FSR)}$$

$$= \frac{1}{2\pi}\,\frac{1}{\delta\nu_{FWHM}}$$

$$\Rightarrow \quad \tau_p\,\delta\nu_{FWHM} = \frac{1}{2\pi}$$ (cavity uncertainty rela

So, using this, I can write down the expression for the finesse. I can simplify it as for a good resonator or a good cavity. It would be pi divided by 1 minus square root of P s. And, ultimately, what I can do? I can express the photon lifetime in terms of finesse. As this that would be 1 divided by 2 into free spectral range. And, this would be 1 minus square root of P s. And, this I can further write as finesse divided by 2 pi into free spectral range.

So, because I already know that free spectral range and finesse associated with the full width at half maximum. So, therefore, I can write it as 1 by 2 pi. And, this is nothing but full width at half maximum delta nu full width at half maximum. Or, I get a very interesting relation. That is photon lifetime into the width of the resonance peak. That is delta nu at full width half maximum is equal to 1 by 2 pi.

And, this is known as cavity uncertainty relation. Let me stop here for today. In this lecture after giving a brief introduction to cavity optomechanical systems, we discussed about Fabry-Perot cavity. In the next lecture, after completing our discussion on Fabry-Perot cavity, we will see how to guess the optomechanical Hamiltonian. And also, we will try to understand the basic physics based on this Hamiltonian. So, see you in the next class, thank you..