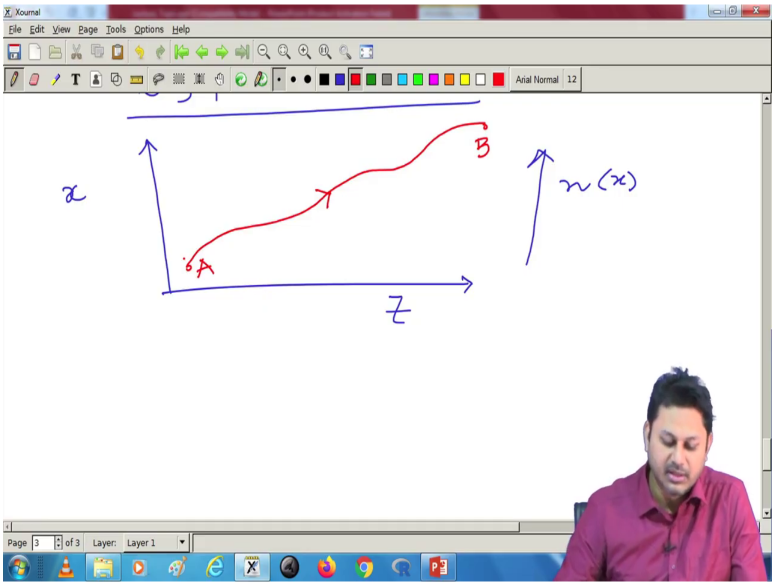
Physics of Linear and Non-Linear Optical Waveguides
Prof. Samudra Roy
Department of Physics
Indian Institute of Technology, Kharagpur

Module - 02
Basic Fiber Optics
Lecture - 10
Ray Path Constant, Ray Equation

Welcome student to the Physics of Linear and Non-Linear Optical Waveguides. So, today we have lecture-10, and today we will going to cover the Ray Path Constant and Ray Equation; which is a continuation of the previous lecture. So, we have today lecture number-10. And in today's lecture, we have the ray path constant. What is ray path constant? We will discuss soon.

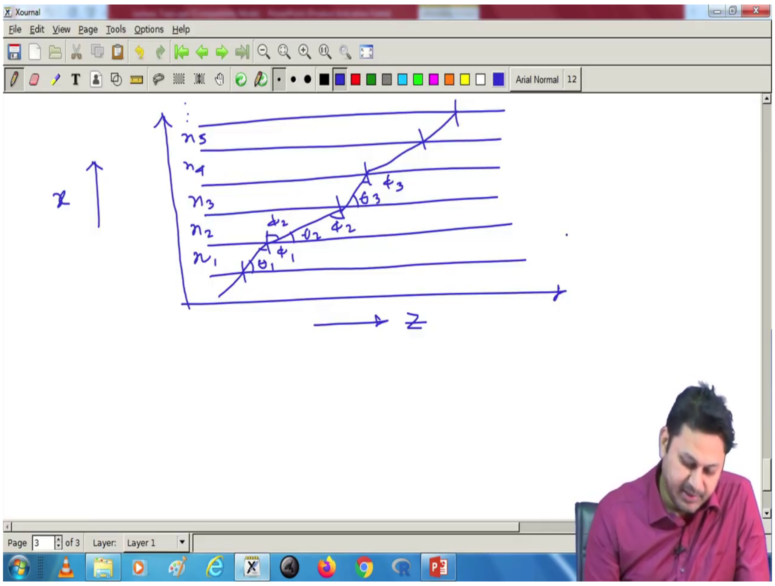
(Refer Slide Time: 01:10)



Suppose, we have a arbitrary refractive index. If we have a arbitrary refractive index over say x-direction and this is the z-direction. And along this direction, I have a change of refractive index as a function of x. If I have a change of refractive index along this direction, so what happened, if I launch a ray it can follow certain paths; depending on the value of the refractive index.

This is the ray following certain paths. With certain initial condition, it can go to some point A to B. During the, during this path, certain thing remain constant. And this constant thing will going to calculate today which we called the ray path constant.

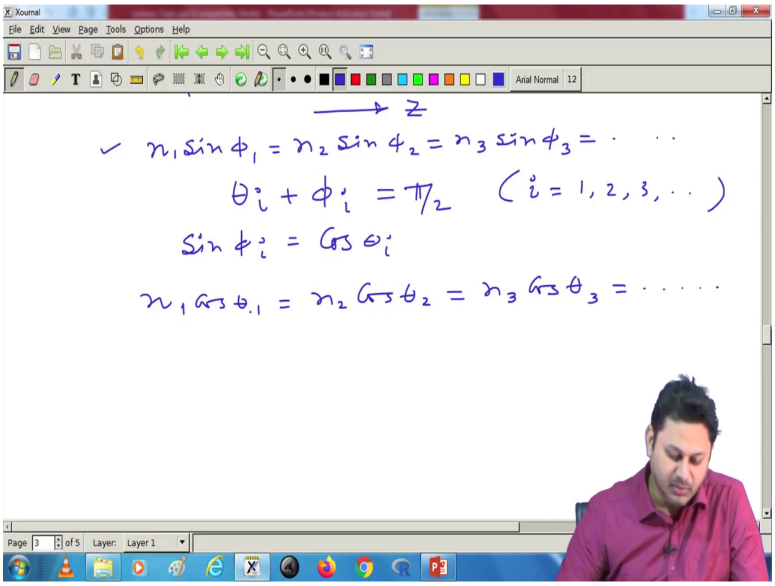
(Refer Slide Time: 02:07)



To make this calculation systematic, let us divide this refractive index profile into small parts. So, I have along this direction I have x; along this direction, I have z. And suppose the refractive index, there is a change of refractive index. And this change of refractive index, I make these sections small sections to make. There is a continuous refractive index change, but in order to calculate something, I make it like this. So, this in this way in this small blocks, the refractive index are changing.

Now, I launch a light here. So, as soon as is facing some interface, so there will be a change, and then there will be a change, there will be a change, every time it follows the Snell's law and propagates the distance along x direction it goes. Suppose, this angle is theta, I write theta 1 here. This angle is phi 1. In the similar way, this angle is theta 2; this angle is phi 2. If this is phi 2, please note this angle is also phi 2. This angle is theta 3; this angle is phi 3 and so on.

(Refer Slide Time: 04:22)

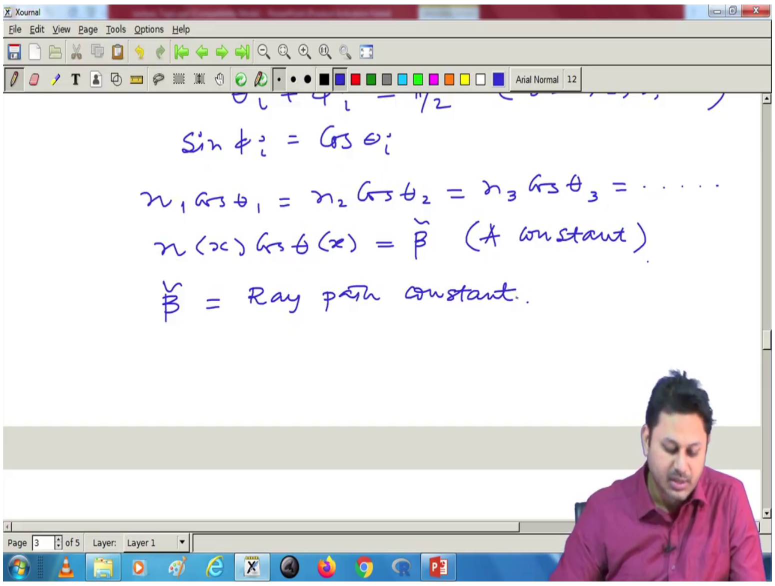


So, at every interface, I can write the Snell's law. And I can have straight wave equation n 1 sin phi 1 is equal to n 2 sin phi 2 is equal to n 3 sin phi 3 and so on. Now, from this geometry, it is easy to check that theta i plus phi i is equal to pi by 2 when i is 1, 2, 3. The layers in each layer, for example, this one, in this layer, theta 1 plus phi 1 is pi by 2; in this layer, theta 2 plus phi 2 again pi by 2 and so on. So, this condition holds for all the layers. So, this is a general expression I have in our hand.

So, from here, I can simply write sin phi i is equal to cos of theta 1 theta i. So, the first equation is modified in terms of if I write in terms of theta, it should be like n 1 cos theta 1 equal to n 2 cos of theta 2 equal to n 3 cos of theta 3 and so on. Now, you can see that this quantity n 1 into n 3 this quantity a refractive index, and theta 1 theta 2 theta 3 is the angle here with respect to I mean angle that the ray is making with respect to the z-axis.

So, when I multiply n 1 multiplied by n 1 with the cos theta 1 and n 2 with the cos theta 2 and so on, then this quantity are equal. So, I can write this quantity as a constant.

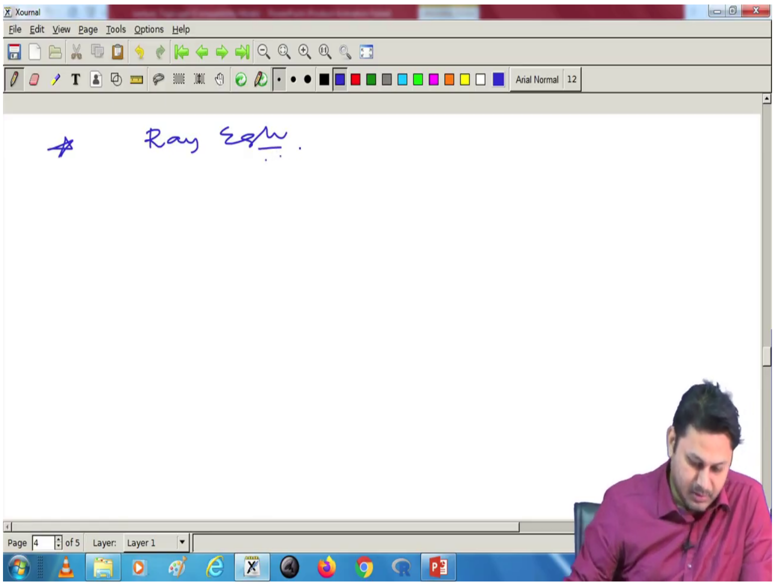
(Refer Slide Time: 07:03)



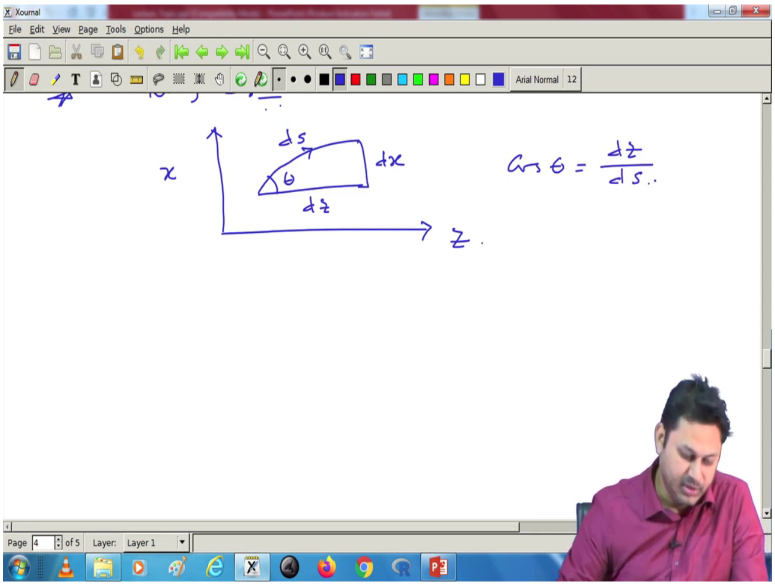
So, in general n x cos theta which is again a function of x is equal to some constant I write as beta tilde, a constant. This quantity will remain constant. So, this beta tilde is called which is a constant quantity is called the ray path constant, which is not changing whatever the ray it follows, whatever the ray I launch then for that ray, for example this arbitrary ray that is coming going from A point to B point, every every at every point over this ray beta tilde is constant.

That is really interesting concept we have that I launch a ray with a varying refractive index, the refractive index is continuously varying over x over x-axis where x is this direction. And if I launch a ray it is following certain part, but over the path what happened that this quantity beta tilde is remain constant. So, over this particular ray, for this particular ray, beta tilde is always remain constant or conserve a quantity.

(Refer Slide Time: 08:43)

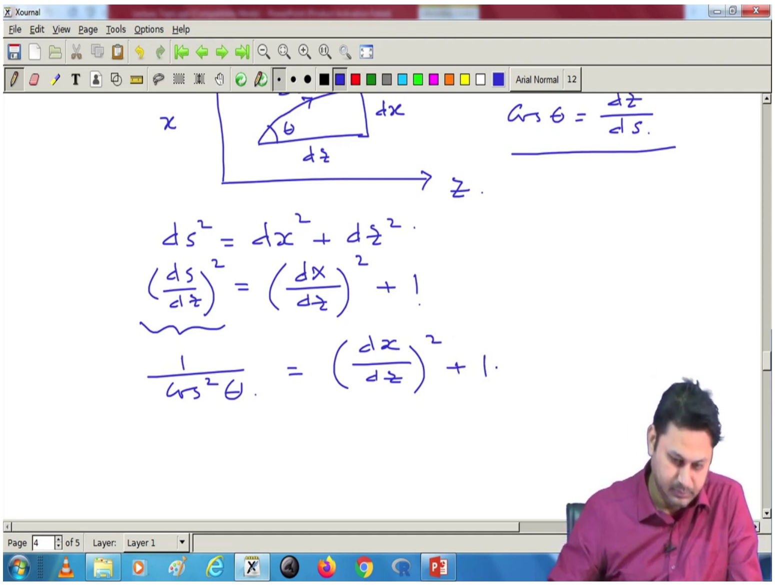


(Refer Slide Time: 08:57)



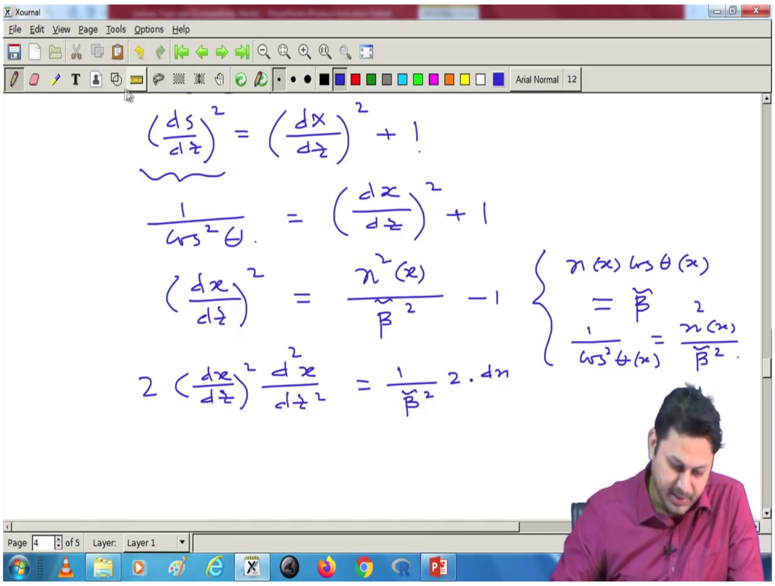
With this note, now I can derive an equation which we call the ray equation. So, I have a structure like this. This is x, this is z. And some arbitrary path, I am drawing this path is followed by the ray. This is the length I write ds, and this angle is theta. So, this is dz, and this is dx. So, the ray is suppose the ray is moving in this way.

(Refer Slide Time: 09:51)



So, readily I have cos theta is dz divided by ds into d. Now, ds square is equal to dx square plus dz square. I can have from here ds dz from this equation, I can write it as ds dz square dx divide everything by dz square plus 1. Now, this quantity what we have in the left hand side is nothing but 1 divided by cos square theta which I already defined here, already find. These is equal to d of x d of z square plus 1.

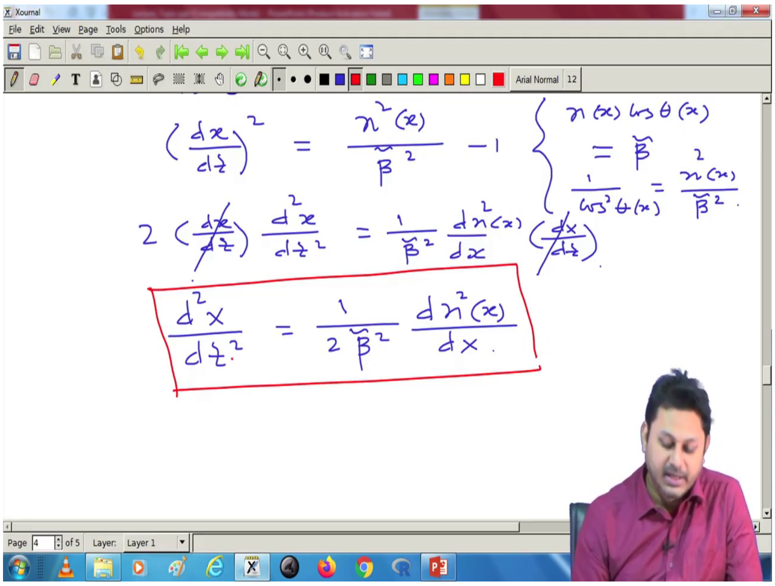
(Refer Slide Time: 11:16)



Now, I from here what I do? I write dx square d x divided by dz square is equal to; is equal to cos theta. Now, I write in different way. So, let me write it in terms of n and the constant beta. Please note n x cos theta x is equal to beta tilde. So, 1 by cos square theta, 1 by cos square theta which is a function of x by the way should be equal to; should be equal to n square x divided by beta tilde square. So, that value, I put it here.

After that, we make a derivative with both the side with respect to x because this is a function of x. And if I do I can write in the right hand side as 2 dx dz square of that d 2 x dz square. So, this side, I derive with respect to z. Right hand side also, I have to do that. So, I write 2 using the chain rule or I can directly put it like this. I do not need to evaluate this.

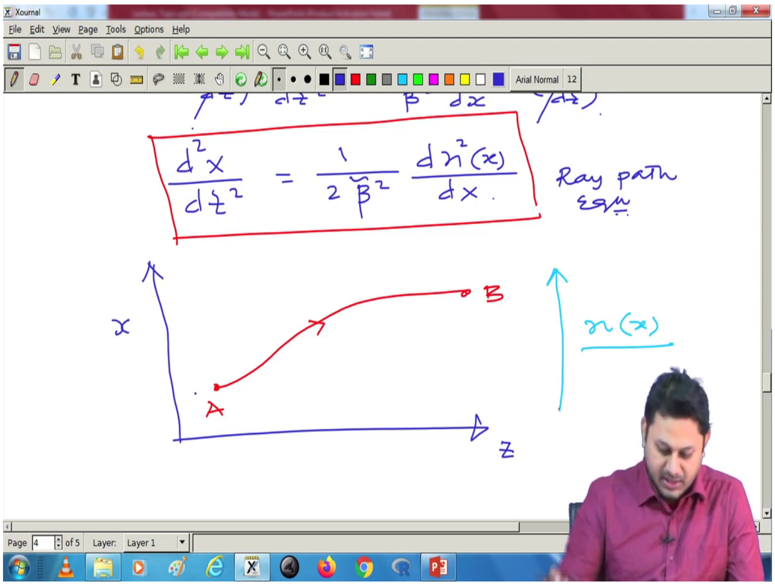
(Refer Slide Time: 13:18)



I can write it as simply d of d x n square which is a function of x. And then since I am deriving both side with z, I need to get back I need to put back this one ok. So, here I am making a mistake, this square will not be there, because I am making a derivative. So, these two term is outside, so this and then the derivative with respect to z.

So, this quantity delta z, delta z will cancel out. And I will have an equation like this. A very very interesting equation that we have in our hand, this equation tells us what is the relationship between x and z. If a n square z is given in the right hand side. So, I will have a differential equation in my hand which tells us how the path.

(Refer Slide Time: 14:48)

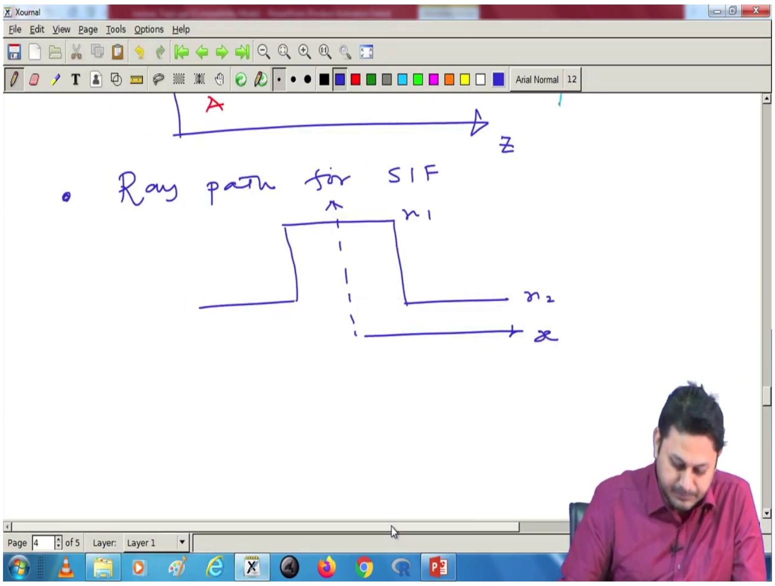


So, you should remember that what we have done earlier that this is my z-direction and this is my x-direction. And a ray is moving from this to this path suppose this is point A and some point B, ray is moving. But along this direction, we have a change of refractive index; along this direction, we have a change of refractive index n is a function of x.

Now, if n is a function of x given to us, then I can map this path because in the left hand side it gives me how the x will going to change with respect to z. So, every point what should be the value of x along z, I can figure out. If in the right hand side, the distribution of the n is given; the distribution of the n is given, then I can map that I find out what is the path this is the path.

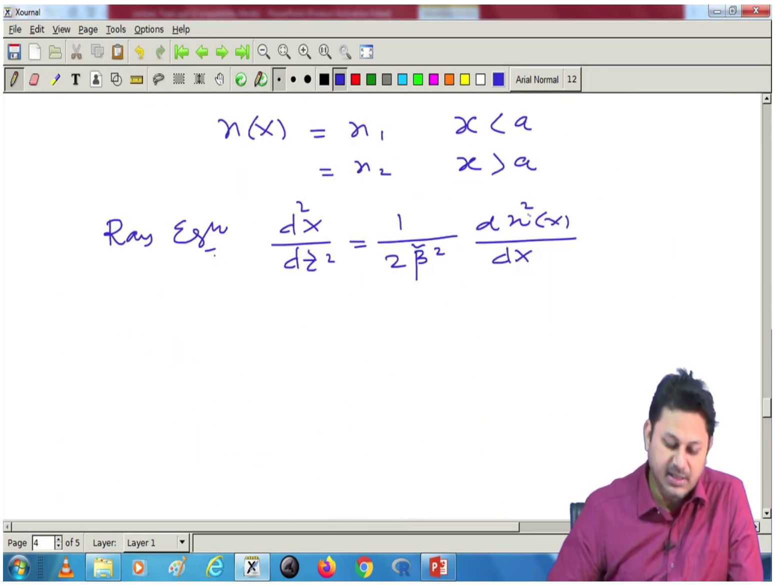
So, eventually that is why it is called the Ray path equation, or simply the Ray equation. This tells us how the ray will move for a given variation of the refractive index. So, after having a very important equation Ray path equation, we can simply test it with different refractive index profiles.

(Refer Slide Time: 16:22)



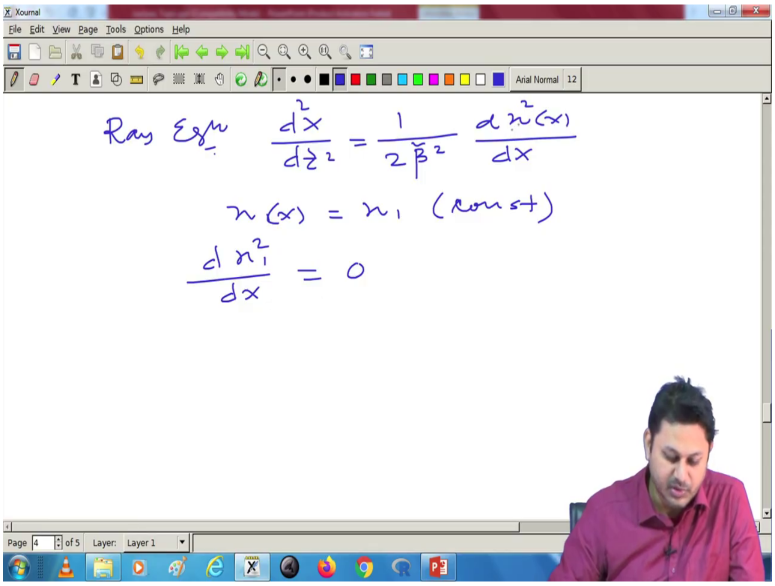
So, let us start with one profile. So, ray path for step index fiber. So, I know the refractive index profile for step index; which in the last class we mentioned, how the mathematical form will look like. We have used it earlier. So, this is n 2; this is n 1. So, n 1 and n 2 is constant here.

(Refer Slide Time: 17:05)



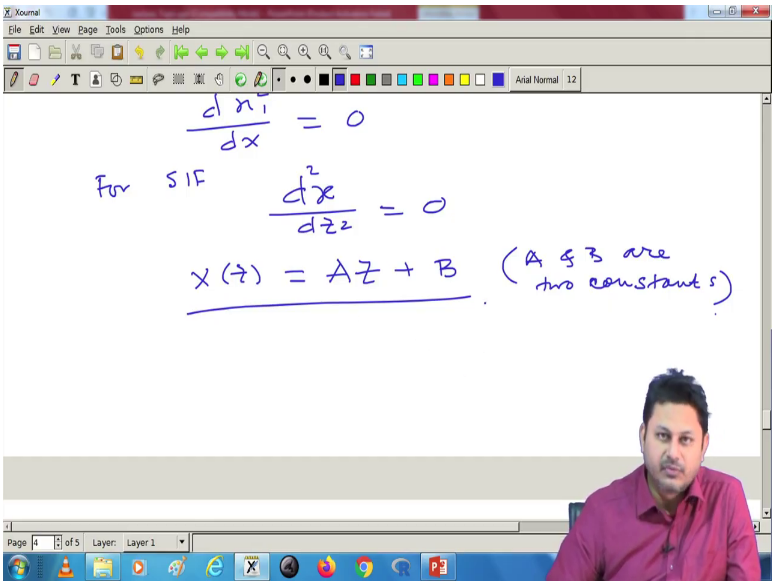
So, n of X is n 1 when x is less than a, equal to n 2 when x is greater than a. And for this given profile, now we need to calculate the ray path. So, ray equation gives us d 2 X d Z square is equal to 1 divided by 2 of beta tilde square d of n square x divided by d X square d X. So, this is my ray equation that I just write. And now you can see in this step index fiber, there is no variation of the n at all in the core. So, there is a certain jump; however, but there is no variation in the core.

(Refer Slide Time: 18:23)



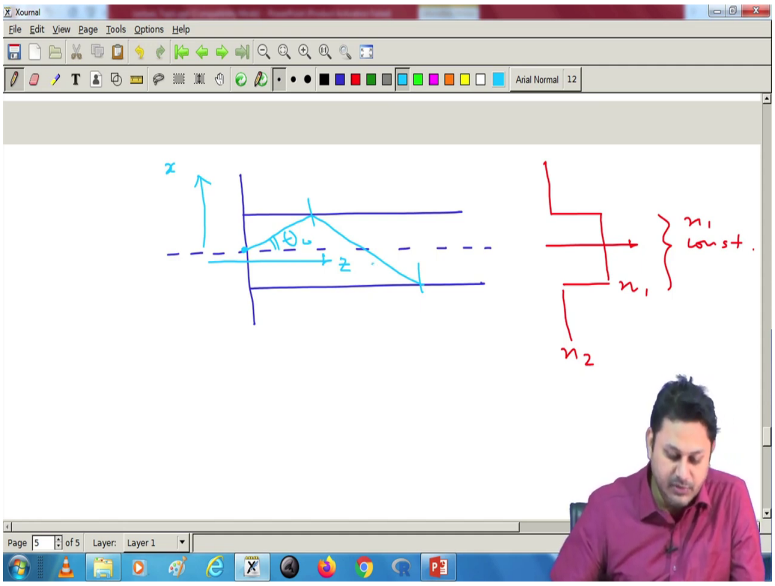
If I want to find out what happened in the core, then n 1 in x equal to n 1 which is a constant. So, definitely this value will be 0, sorry this is so I should write at n square, because this is the original equation. So, this equation this portion the right hand side is simply 0.

(Refer Slide Time: 18:59)



So, I have a very, very straight forward. So, for step index fiber, I have a very, very straight forward differential equation. And this equation says d 2 x d z square is equal to 0. And the solution readily I can write as A x plus B, when A and B are two constants. So, quickly, we can check that whether I mean from this equation it is obvious this should be I have a mistake. So, it should be Z. So, the ray is basically following, the ray is basically following a straight line. The ray is basically following a straight line.

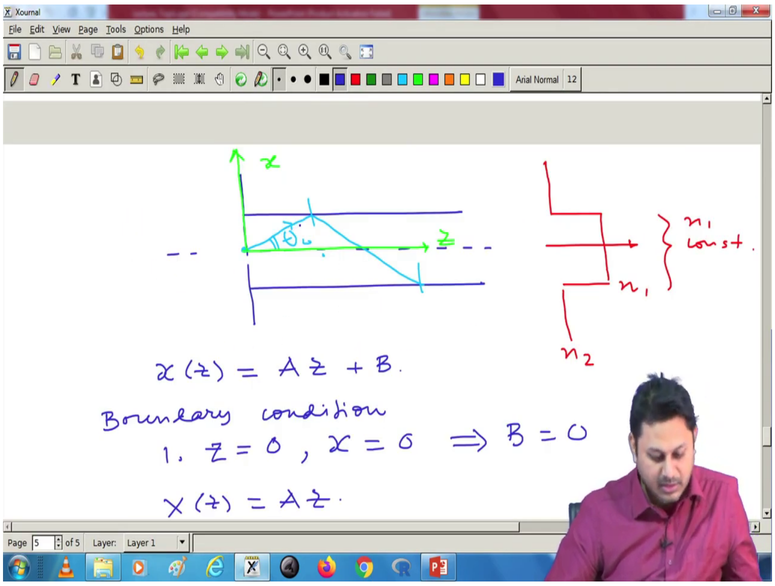
(Refer Slide Time: 20:19)



So, that means, if I have a step, step index fiber, this is the structure of the fiber or waveguide. And the index profile if I draw which is something like this. So, this is n 1, this is n 2. And in this region, n 1 constant, it is constant; there is no change in n 1. And now if I launch a light here, then it should move with the straight line. And then there is a reflection that is happening total internal reflection.

But this path is a straight line. Now, if this initial angle is given and if this is my x, and from here to this direction if my z, then I have an idea how it is changing. If this is my origin, then I can use certain boundary condition to evaluate this constant A and B.

(Refer Slide Time: 21:40)

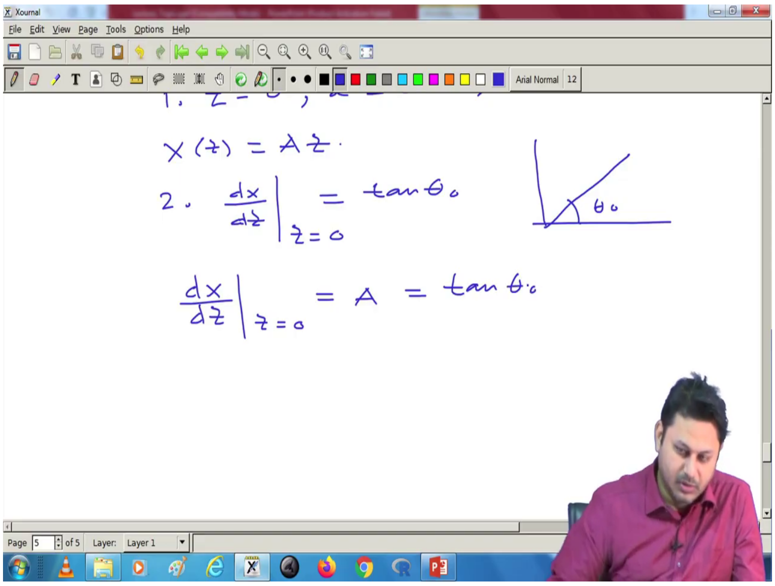


So, my expression is the ray will follow this path which is a straight line A z plus B, this is the path. And now if I put the boundary condition, so boundary condition, one boundary condition is if I look carefully to this ray, whatever the way I draw here, the boundary condition the one boundary condition is obvious that at z equal to 0, x is 0.

At z equal to 0, x is 0, because I launch this light here where x and z coordinate both are 0. So, please do not confuse the origin is here. So, let me, it is this path. It is this path.

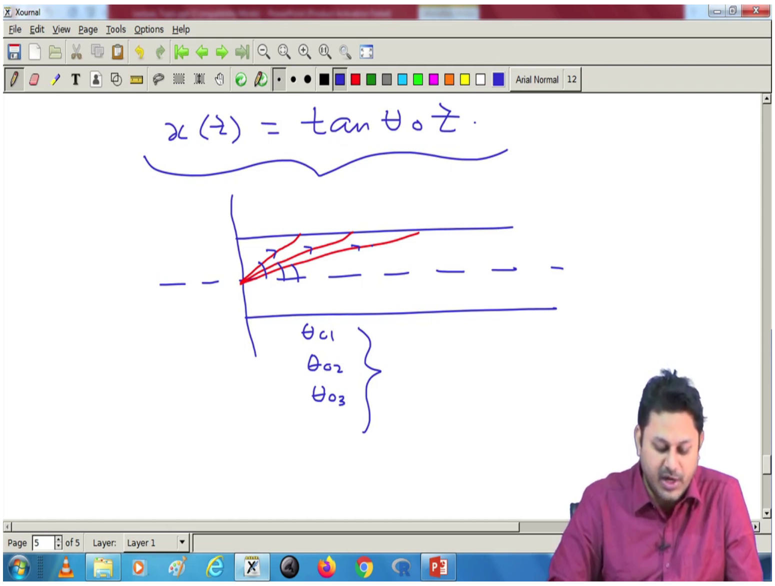
And draw once again with a different color that this is x, and along this we have z. So, with this boundary condition, I readily have my B equal to 0. So, my equation becomes simply A z. And now the initial angle is also given. Since the initial angle is given, I can eventually find out what is the value of A.

(Refer Slide Time: 23:55)



So, the boundary condition 2 gives me d x d z at z equal to 0 is tan theta 0, because initial angle is given. So, this angle is given. This is theta 0. So, the initial angle is given. An initial angle is nothing but the derivative with respect to derivative of x with respect to z at z equal to 0 point. And if I calculate that, I can find from this equation that d x d z at z equal to 0 is equal to A which is tan theta 0.

(Refer Slide Time: 25:05)



So, simply I have my x z is equal to tan theta 0 multiplied by z, so that should be the equation of the ray. And if I increase this theta, so let me again draw this structure. So, depending on the value of the x initial angle, I have different ray path like this. This three rays has three different initial angles, three different initial angles. In one case, suppose theta 0 1; in another case, theta 0 2; and another case theta 0 3. In these three cases, it both the in all three cases, it will be a straight line. Only thing is that due to this initial angle there is a change in initial angle. And as a result, I have different paths.

So, with this note, I like to conclude my class today here. In the next class, we will extrapolate this idea this equation path equation, and we try to find out what happened for a graded index fiber.

Thank you for your attention, and see you in the next class.