WAVE OPTICS

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Lecture - 11: Material Dispersion (Cont.)

Hello, student to our course wave optics. Today we have lecture number 11 and we will describe the material dispersion. However, we have already started this in the early class but today we are going to extend it a bit more. So today we have our lecture number 11 and in the last class if you remember that we started the concept of material dispersion where we mentioned that if you have a time varying electric field e which is a function of t which is e naught, e to the power of i omega t, if the electric field is something like this then what happened because of this electric time varying electric field there is the response in the material and like a spring mass system electron will be vibrating based on this electric field. Now inside the material, due to this vibration of the electron, what happens is the polarization we're going to generate, especially in the dielectric medium we have polarization which is represented by this expression. So the material response can be considered through this term which is our susceptibility. So when the electron vibrates under the presence of this time dependent electric field, which is sinusoidal in nature then what happens, we can write down the expression for the equation of motion and this expression of the motion will be simply this. If I write ft that was the total force that we are having in this case, it should be electric field and few other forces like damping forces and then the restoring forces etcetera, where m is the mass of the electron. Now in the last class putting everything together we find that the electron will vibrate with this particular form which is e equal to e by m e or divided by omega naught squared minus, omega squared and then plus, i of gamma but gamma was the damping term and omega 0 was the natural frequency, e will force the electron to vibrate in this particular form.

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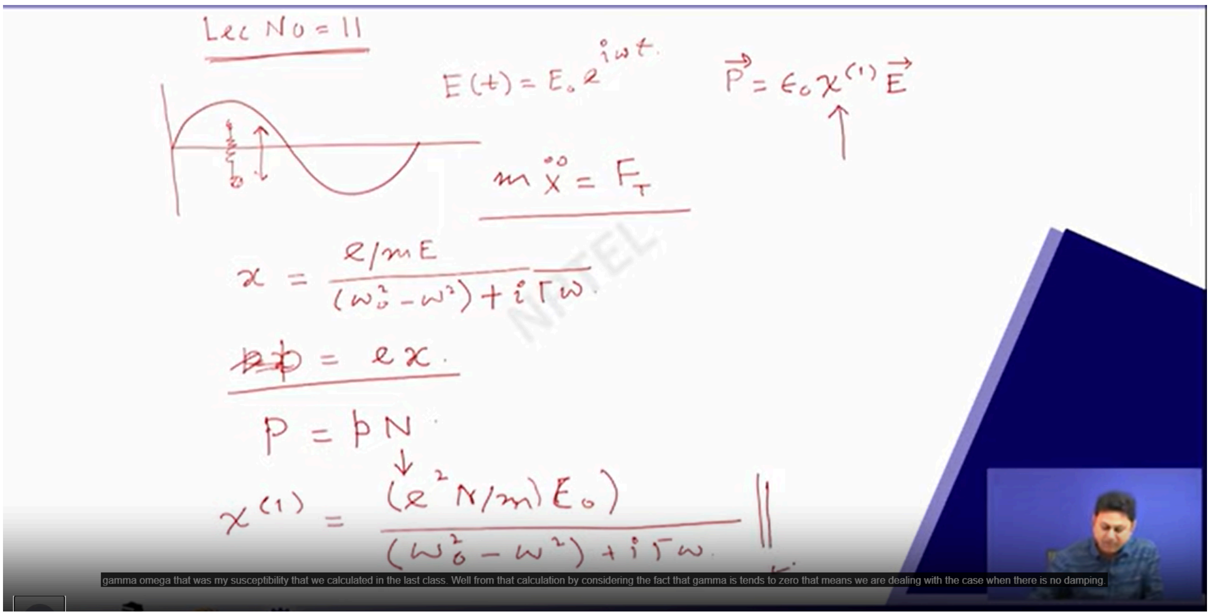

Now from that expression we can find out what is the polarization through the dipole moment and if we write down the dipole moment p, that should be electric charge multiplied by the position that was the value. So let me raise this part, it is my dipole moment and p is this quantity. It is not being used properly. So let me, this is the term we have p is a dipole moment. Now dipole moment per unit volume was big p is equal to p multiplied by n, where n deals with n accounts for the number of dipole moment per unit volume and at the end of the day we find that my susceptibility i, i is equal to e square n divided by m and multiplied by epsilon naught and then whole divided by omega naught square minus, omega square and then plus i gamma omega that was my susceptibility that we calculated in the last class. Well from that calculation by considering the fact that gamma tends to zero that means we are dealing with the case when there is no damping. We finally get the expression of the refractive index n, as a function of omega as 1 plus omega p square the most simplest form that we have is omega 0 square minus, omega square. Now the resonance frequency considered here is the same for all the cases. So we didn't put any summation sign in, however the resonance was different. If we consider that also we need to put a summation sign etcetera. But we are not going to go into that detail here but our aim is to just find out how the refractive index is a function of omega in the simplest way. So that should be the form. So let me continue what we did in the last class. So where omega p was e n, this is the plus we call the plasma frequency, e n hole divided by m cyber naught that was the plasma frequency we have. What happened is that in general as I mentioned a given molecule experiences different resonance frequencies. So, we had in the last class that n as a function of frequency is essentially 1 plus a summation sign and then we wrote that for different resonance frequencies we have this omega 0, which is a resonance frequency of this electron different and now it is a sum over i. So for all the cases when we have different resonance frequencies we had this expression in our hand. So that was the expression we derived and then we mentioned that if we replace my omega by the expression 2 pi c, divided by lambda, then if I replace this relation then one can have refractive index as a function of lambda and if I want to get an expression something like this a i lambda square,

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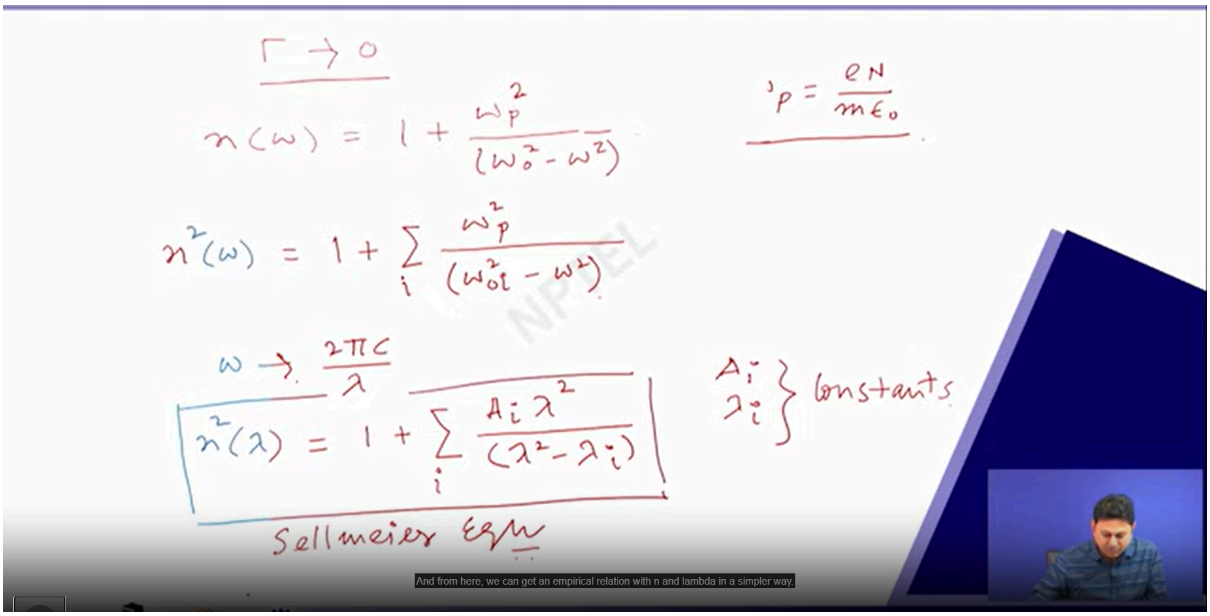

whole divided by lambda square minus, lambda i, where a i and lambda i these two are constant. This is over i, so we have an expression here after doing all these calculations that we did that we developed in the last class. We finally get an expression of n which is this and this suggests that the refractive index is a function of the lambda and this particular expression where the refractive index depends on the lambda in this way is called the Sellmeier equation. So this is the famous Sellmeier equation through which if you know what is the value of ai and lambda i. And expand this right hand side, you can find out how the refractive index varies as a function of lambda. Well, if somebody does that, then one can get this expression. And from here, we can get an empirical relation with n and lambda in a simpler way. And that one can do when we do our calculation far away from the resonance frequency here it should be done square. So far away from the resonance wavelength and that form is the most well-known form we have and that is called the cauchy's equation. And this is an empirical relationship between the refractive index of the material and the wavelength of the light. And what is the form of this expression? If I write in explicit way, it is n lambda is equal to a plus b, divided by lambda square plus, c divided by lambda to the power 4 and so on, W where a b c are the constants and we also have a simplified form of this by just taking two variables, n lambda is equal to a plus b by lambda square. So this a and b are constants and are called the Cauchy's constant. Now if we plot the refractive index as a function of lambda then according to this expression we can have a curve like this. When you do the experiments, there are nice experiments to which one can find out the value of a and b, where we put a prism and then find out the minimum and we expose this prism with white light and what we do that. Let me show it. So we put a prism, our n lambda is equal to a plus b by lambda square, that is the expression we have and in order to find out a and b we have a prism and we know that when the prism is exposed with white light this is a polychromatic light or some white light source then different lambdas we have. So we have a lambda span and different colors. We have all the colors, starting from violet, blue etcetera. So what do we do in this experiment? Let me erase this part once again.

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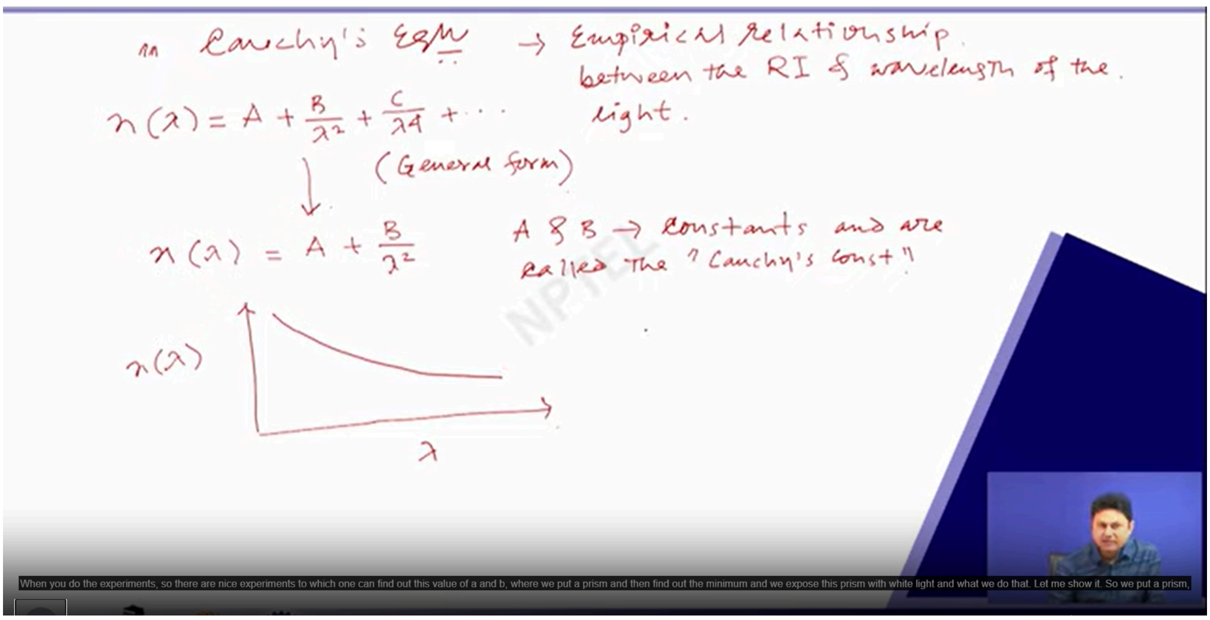

So I exposed the prism with white light and what we get is a spectrum here. My first expression was n as a function of lambda is equal to a plus b by lambda square. So we

measure this angle of deviation and basically we measure the minimum angle of deviation delta m through which we find, then, n as a function of lambda for all the lights because here we have this spectrum. If we expose the prism in a white light and then we get different colours like blue, green etcetera. And for each light, we calculate the minimum deviation experimentally, which is very easy to find using the spectrometer. And we find each n and lambda. So for each given lambda, we find n. So say n1, n2, n3, n4, these are the values we have. By using this expression which is delta m, which is the minimum deviation and for each lambda value we're going to get 1 lambda, 2 lambda, 3 lambda 4, a value of refractive index n. So if we plot that we are going to get a curve like this where these are the values that one can determine using this experiment. However, in order to find A and B, what we essentially do is instead of, so this is the plot of the raw value that we can get from experiment, T n as a function of lambda and lambda that we get. But what we do, we plot n as a function of lambda. But instead of plotting lambda, we plot 1 divided by lambda squared here. When you plot 1 divided by a lambda square, it will become a straight line because we know our expression of n lambda, the Cauchy's expression is a plus b by lambda square. So, once we plot instead of lambda 1 by lambda square, so from that experimental value, we can draw the refractive index as a function of lambda. In this way by plotting instead of lambda 1 by lambda square as a function of 1 by lambda square and then we get a straight line and from this straight line what we can do, we can find out this cutting point and that is a and also we can find out what is the slope of this line and from this we can find out what is b because this expression n as a function of lambda a plus, b by lambda square is equivalent to y is equal to mx plus, c where where m is equivalent to b c is equivalent to a and x is equivalent to 1 by lambda square because this is the way we plot it. So using that it is possible one can find out the value of a and b and essentially if you find a and b then you can also know what should be the value of n in some unknown frequency in between these are the frequencies or these are the wavelengths that you find from your experimental data.
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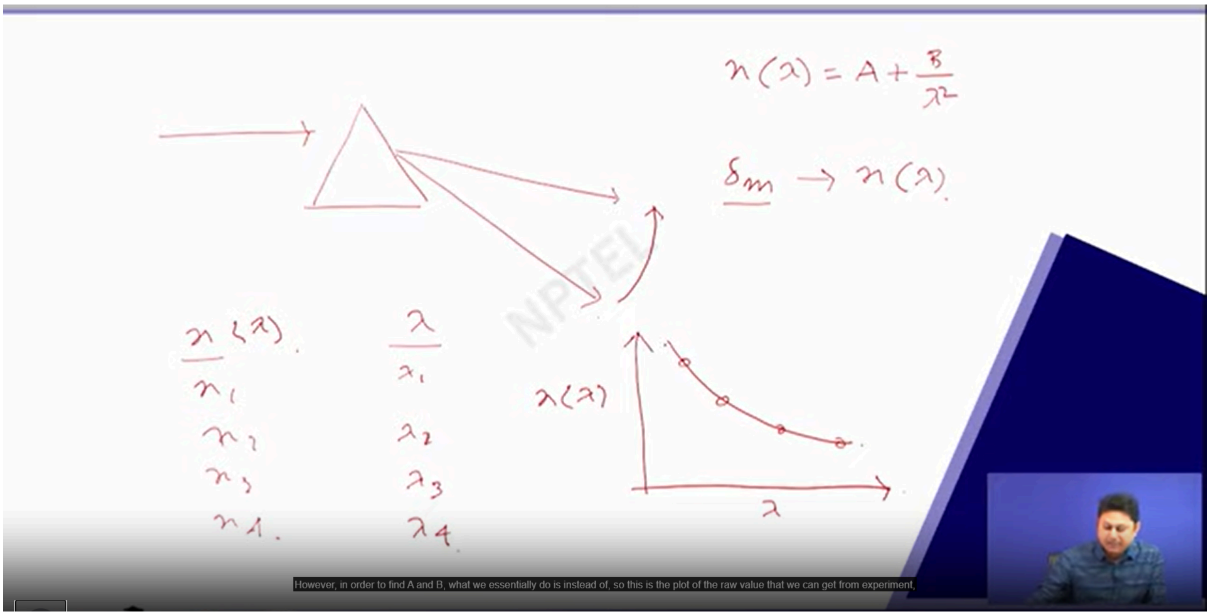

By exposing the prism to white light and then you get a spectrum here, different color and three or four strong color, for three and four strong color you can figure out what is the value of n for a given lambda and then draw it as a function of 1 by lambda square and you're going to get a straight line. From this straight line you can find out what is the value of a and b experimentally and once you know the value of a and b then any wavelength in between you can find out and that should be the value of the refractive index for that given a and b. Well, so this case when we have the variation of the refractive index we started, so these are the value of the refractive index n as a function of lambda and lambda that is the raw data we had and it is n as a function of lambda is equal to a plus the most simple expression we have is lambda square. So this region is called the normal expression because if I calculate dn d lambda then this quantity should always be less than 0 in this region. So, in that case if you remember the expression of the vg that we had earlier, vg is equal to vp into 1 plus lambda divided by n and then dn d lambda that was the expression we had earlier, that how the vg and vp is related and if we consider here the value of the vg and vp under the case of normal dispersion that is dn d lambda is negative, it is easy to show that for normal dispersion that is when dn d lambda is less than zero, then vg is less than vp normally in our usual optics experiments. So we had the material and we had the wavelength range always dn where dn d lambda is negative, however it is also possible that instead of moving down the refractive index can go up also with respect to lambda and that is the region where we get the anomalous dispersion region especially in the region of the absorption it happens and there will be a sudden jump of refractive index and if I want to draw the figure here I just quickly like to show that how one can have this. So it is like this and then there is a region. Suddenly there is a jump from here to here and this is the region, where we have dn d lambda greater than zero. So this is called the anomalous dispersion region, where we have the relationship between the n and lambda is differ from Cauchy's law and it is obvious this is because we have an absorption region here and the term that we initially had in the Sellmeier equation. That term we neglected here to find the simplest form which is the cautious relation but you cannot neglect this term.

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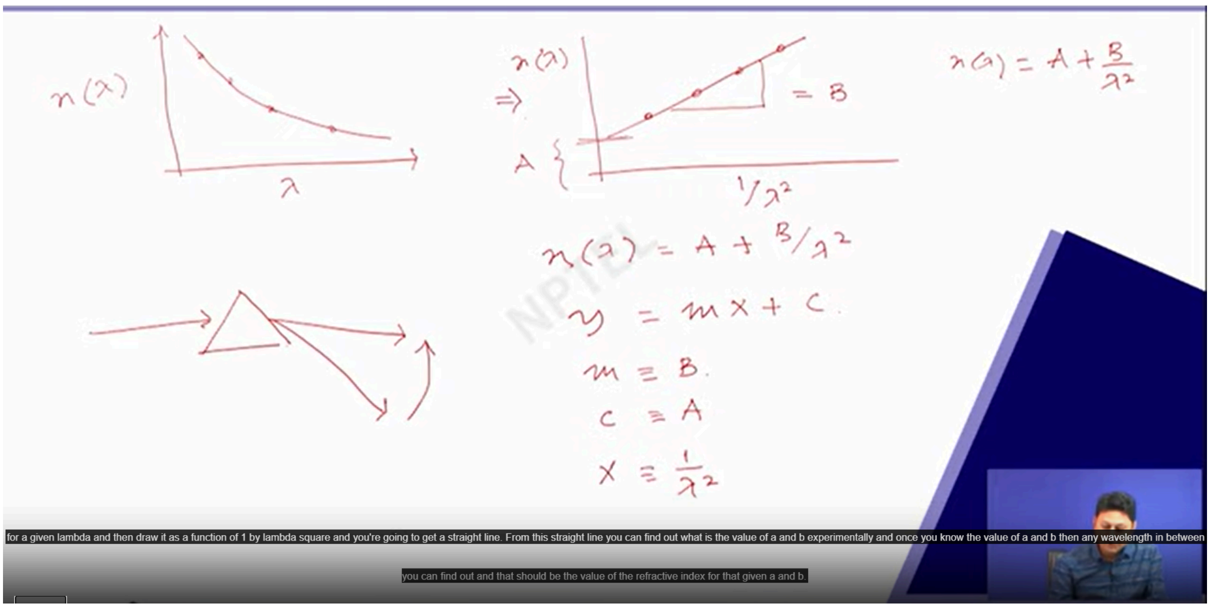

So in this dotted region, where there is a sudden jump of refractive index we need to take account those terms as well those absorb terms which are related to absorption and the dispersion relation is now in this region is opposite. Now the refractive index will going to change as a lambda in this way and we have d and d lambda positive instead of negative but usually we have the normal dispersion region, where dn d lambda is negative and use the Cauchy's relationship and based on that we find out the value of the refractive index and find out the dispersion when light enters in these kind of systems, it based on its wavelength, it behaves accordingly and if the group velocity, one supposed to calculate for this system where we have normal dispersion then that is the expression we have in our hand that vg equal to vp plus 1, plus lambda n and then dn d lambda for a given wavelength. If somebody wants to calculate what is the refractive index he can calculate this very easily by putting the dn d lambda value because of the Cauchy's equation in our hand. So they know what the value of b is and then they can calculate dn d lambda. So from dn d lambda they can find out a particular wavelength, what is the value and then from that they can calculate the vg, the value of n is also known for that particular wavelength. So all the values are known in the right hand side and they can calculate what is the value of the group velocity dispersion, mind it the group velocity by definition the phase velocity dispersion was omega divided by k and vg group velocity dispersion, coming to the picture if we have mixture of two wavelength, where the frequency is slightly different, then we find that the envelope may move in a different velocity and that is this. So vp is known for a given lambda and if you want to find out what is the value of the vg in a dispersive medium then the right hand side dn d lambda multiplied by lambda divided by n that one needs to calculate. From the Cauchy's expression if a and b is given one can calculate that part very easily and one can find that this value is negative. So Vg should be less than Vp but for every wavelength they can have the value. Well with this note, today we like to conclude our lecture here, so today we discussed what is material dispersion. In the last class we started the calculation and in this class what we did is we extend the idea and try to find out that by Cauchy's expression how we can expand the refractive index profile and from that how one can calculate the value of the group velocity dispersion and for a given wavelength and also the phase velocity dispersion which is straightforward. With that note I would like to conclude here. Thank you very much for your attention and see you in the next class.

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