WAVE OPTICS
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Lecture 13: Concept of Coherence (Cont.)

Hello, students, so welcome to the course on wave optics today we have lecture number 13 and we are going to continue the concept of coherence that we started in our last class. So, today we have lecture number 13. So, in the last class we started the concept of coherence and the concept was something like that in a system: suppose this is a system where the atoms are distributed randomly and they radiate in a random manner like this. Then the corresponding radiation can be given suppose they are emitting and the emission can be given represented by these discrete waves, where up to a certain time we have pure harmonic radiation and after that, there is no phase relationship and this particular time length at which we have a pure harmonic wave we call the coherent length and we also mentioned in the last class that this coherent this phase relationship depends on that how monochromatic the light is for a finite harmonic wave. We have a function that has a finite width that calculation we do. So let us go back to this concept that this time length at which this wave is emitting, having a phase relationship after that, there is a sudden phase jump you can see, this dotted line shows that these are the locations where there is a sudden phase jump, we do not have any phase relationship after that.

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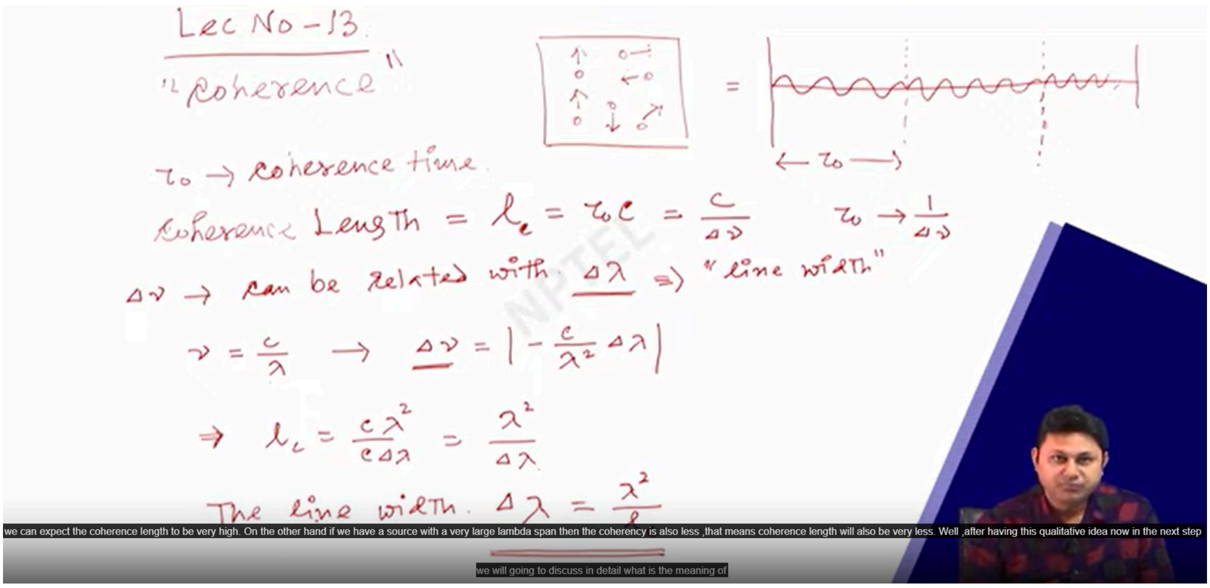

So, only for this length this time length we have the phase relationship of the wave, and this tau 0 is the time length at which we have the phase relationship for the time window, for which we have the phase relationship we call this coherence time. Now from this coherence time, we can have something called coherence length. For this coherence length l, I put a suffix c here to make it coherent. This c stands for coherence. This coherence length can be defined as tau 0 multiplied by c, c is the velocity of the light medium in a vacuum, and then if I multiplied with this coherence time the length during which the light travels with which time period t 0 the length is called the coherence length. Now this is c divided by delta nu because we know that tau 0 is related to 1 divided by delta nu. Here this delta nu can be related to something called delta lambda. Delta lambda is the wavelength distribution that the source has and this is called line width. T So, nu we know that it is c divided by lambda. From that, we can find out delta nu to be, if I put if I make a derivative on both sides. So, we should get the minus of c divided by lambda square delta lambda. If I put the mod sign this minus sign will no longer be there. So, this quantity I can with this relation I can correlate delta nu with the line width delta lambda, and then my coherence length lc will be c divided by delta nu. So c divided by c delta lambda multiplied by lambda square, neglecting the negative sign, taking the mod sign. So it should be simply a lambda square divided by delta lambda. So the line width delta lambda is simply lambda square divided by the coherence length. So if the coherence length is known then I can calculate what is the delta lambda, that is the line width. If the line width is known I can calculate what is the coherence length. Now if the delta lambda that is line width is small, then from this expression it is clear that the coherence length will also be very very high.

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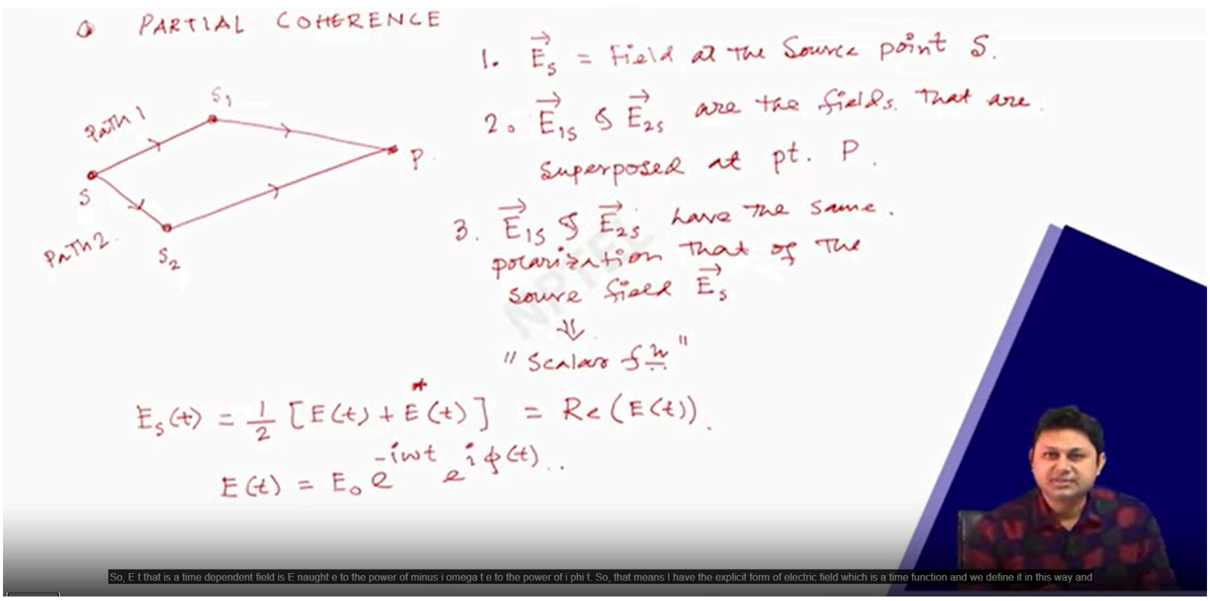

So for a very monochromatic kind of light where delta lambda is small, then we can expect the coherence length to be very high. On the other hand, if we have a source with a very large lambda span then the coherency is also less, which means the coherence length will also be very less. Well, after having this qualitative idea now in the next step we will discuss in detail what is the meaning of the superposition of two waves and based on the concept of the superposition of two waves, how one can calculate the coherency. The degree of coherency can be calculated and that calculation we will do here in today's class. So, normally it is very difficult to have a purely coherent course source or a purely incoherent source. However, all the sources are partially coherent. So, what is the meaning of partial coherence that we will be discussing now? So, let us write this as the concept of partial coherence. Okay, so let us have a source here that says S emits light via two points. This is one source and then it emits a light via two points S1 and S2 and these two lights merge at some point the superimpose and some point P. So the first light going through S2, S1 to P, I call it path 1. And another is path 2. Now we define two fields, say U ES, that is the field at the source point, the field at the source point S, this is the first definition. Similarly, the second definition is that E1S and E2S are the fields that are coming from s1 and s2 are the fields that are superposed at point P. Obviously, third points E1 and E2 maintain some polarization that is the same polarization that of E s. So, they have the same polarization, that of the source field Es. So since all the waves have the same polarization in all the fields, we can consider, this is the condition we can have, these all function as scalar functions because we consider the polarization to be the same for everybody. So, we can consider this as a scalar function. Now, the field at ES, the field S ES is a time-dependent field. I drop the vector sign because I consider that everything is a scalar function to make life easy. Then I can have, I can write it as half of some field E plus the complex conjugate of that one. So, that means this is the field that we defined, this is the real part of some field E which is time-varying.

Now, I define what my E is. So, E t, which is a time-dependent field, is E naught e to the power of minus i omega t e to the power of i phi t. So, that means I have the explicit form of the electric field which is a time function and we define it in this way and a time-dependent phase is there, this accounts for the departure from the monochromaticity of the source field, which means how the source field is departed from the monochromaticity. Then that will be defined by this phase, if the phase is zero then it is highly monochromatic. If we have some value then it is different from this monochromaticity. So this is a time-dependent phase. Okay, now after defining all these things I need to define the fields at the E1s and E2s that I want to define. So, before that, I say that this E the field that is coming here the field that is reaching here from here to here and here to here. I define these two fields as E1P that are reaching the point P as half of say some field E1 and E1 star. That is the real part of some field E1. I am going to define what is E1, but let me first define the fields that are there due to path 1 and path 2 at the point P, this is at the point P. Similarly, I can define E2 plus E2 start, this is again the real of E2t. Now E1t and E2t should be somehow related to the main field E that is coming from the source point S. So, I can write E1 which is a function of T is some fraction beta, this is a multiplicity factor beta 1 of the field that was there at S. At some time T minus big T1, because two waves are coming at two different paths. So I want to find out what is t and that t should be related to the the electric field at the point f sometime before t1 so that's why t minus t1 in a similar way E2 that is reaching the point related to the field at the point should be some B to E then t minus T2. Note it, you should note in the picture here that two fields are coming in two different paths. So, this is one field that is coming to this path and this is another field that is coming into this path and this is P and in the P these two E1 P and E2 P that are from the source S1 and S2 coming from the source S and these two paths are different this is path 1 and path 2 and that's why what happened, that if I defined the electric field at point P this should be some fraction of the source field multiplied by the source field at some earlier time T1 and T2 respectively. So, this beta 1, beta 2, as I mentioned, takes into account the change of the amplitude because it is moving some path and this T j i write at a stretch j can be taking the value 1 and 2 this is a technical this has a technical name and this is called the time of flight for the light field propagating along the path j. Okay, now if I want to find out what is the irradiance at point P, the total irradiance, the total intensity or irradiance at point P is IP, if I define it as IP. Then I p is as per our notation epsilon 0 c multiplied by the time average of the field that is there and that field is the E superposition of E1 and E2. So, E1 p plus E2 p square of that over time average.

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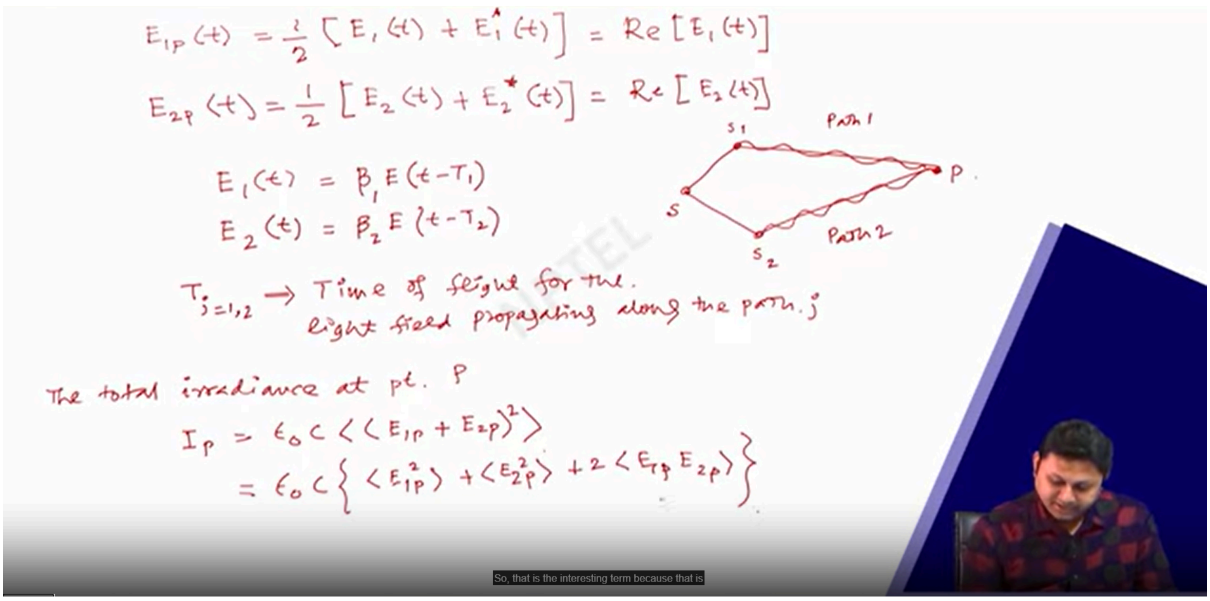

So, that is the total intensity one can have at this point E based on the structure. Whatever the structure is shown here is a very straightforward and simple structure. So, these I can expand like epsilon naught c and this is the time average of E1p square of that plus time average of E2p square of this time average plus 2 of E1p and E2p time average. So, that is an interesting term because that is basically the interference term that one can have there. So, I can now write IP as I1P plus I2P plus half of epsilon naught c and then E1. So if I go back to what that term was, let us see, that was E1p multiplied by E2p. Now I should write that E1p is E1 plus E1 star and E2p was E2 multiplied into plus E2 star. So, when I have E1p, let me write it here, E1p E2p, this is essentially 1 by 4, then E1 plus E1 star multiplied by E2 plus E2 star okay that multiplication I am writing here on this page. Okay to erase this. So, if I do then one term I will have E1 E2 then one term I have E1 star E2 then we have one term E1 star E2 and E1 E2 star. Okay, where this quantity is simply epsilon naught c E ip square this term is epsilon 2p squared and the rest of the term is just the expansion of the quantity E1p and then E2p this is the expansion by writing in terms of E1 and E2. So after having that, when we have this as I mentioned, this is the important interference term. Now it is easy to show that the E1 E2 average is time average should be 0 and E1 star E2 star this time average is 0. So, this term involves the time average of sine and cosine factors and that oscillates at 2 omega frequency. So, that is why this term will vanish. This is simply the term that will contain sine or cosine that will vibrate at the frequency 2 omega, that is why this is 0. On the other hand, the other two terms with star, not star, and star, not star will be still there. So, if I rewrite this Ip, then Ip will be I 1p, that is the intensity at point p is due to the intensity at p due to source 1, intensity at p due to source 2 and I have some interesting interference term because of the

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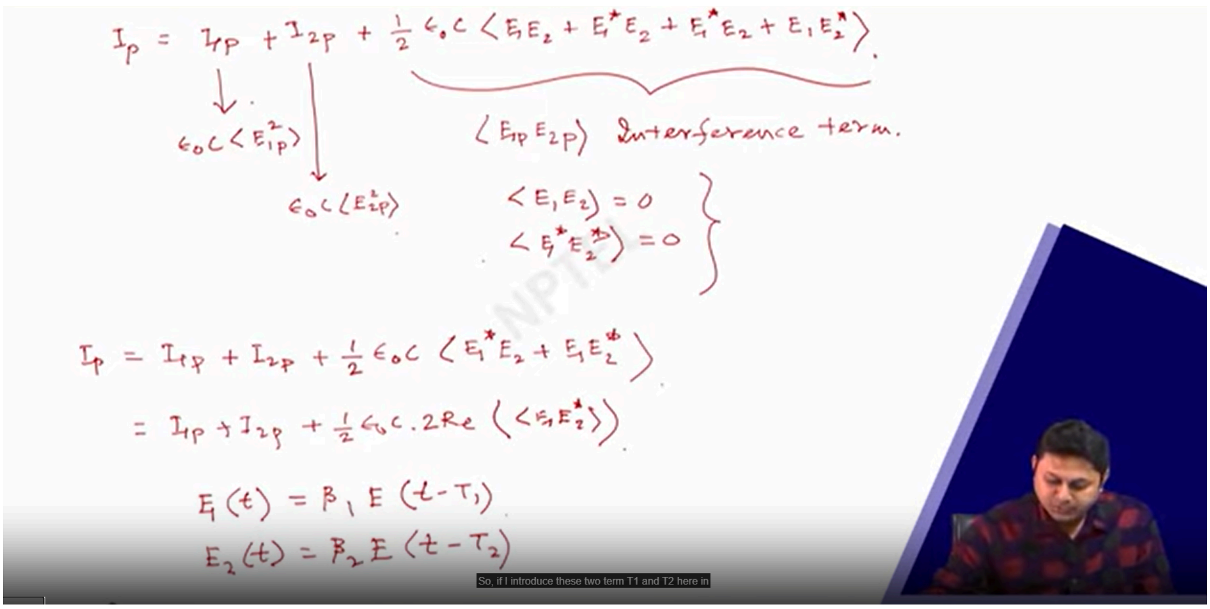

field at source 1 and source 2 which I can write as E1 E2 E1 star E2 and E1 E2 star. This will be a lengthy calculation. So, I am not going to cover this in a single class, but let me proceed with a few more lines and then maybe I can stop. So, I can simplify writing IP 2 plus half of epsilon naught c and then this is 2 of, I can write as real of the time average of anyone. So E1 says E2 star now, mind it we had the form of E1 and E2. So let me write it once again: this was beta 1 then E t minus T1 and this was 2 beta 2 E t minus T2 where T1 and T2 are time of flight. So, if I introduce these two terms T1 and T2 here in plaguing these interference terms then let us see what we get. Then I can get the intensity at point P due to the intensity of 0.1 plus intensity due to 0.2 and then I have epsilon naught C and then beta 1 beta 2 is the amplitude part nothing to do is the time average they can take it out and then we have a real part of this quantity which is e t minus T1. So now I enter into the source field. Now, the expression is written in terms of source field t minus T2. So, E t minus T1 and E t minus T2 star actually can be written now in the form that we had earlier, during the definition. T The first definition if you remember. I wrote it, E t equal to e 0 e to the power i omega t, E to the power i phi t. So that means when I write it in that particular form. Then, I equate E0 square e to the power of I minus omega, then I write t minus T1 and e to the power of I phi t minus T1. This is for 1 and in other cases it will be e to the power of i omega t minus T2 and e to the power of minus i phi t minus T2. So, I just replace whatever the form. So, the form that we had let me write E t was the form I considered as E naught e to the power of minus I omega t e to the power of I phi t that was the original form explicit form we consider and I put it. So, today I do not have much time to proceed with the calculation. So, that is why I need to stop here. So, in the next class again we will start from this particular point, then we extend our calculation and show how this interference term essentially leads to something called coherency. The degree of coherence can be calculated from this interference term itself. So, with that note I would like to conclude here, see you in the next class. Thank you for your attention.

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