WAVE OPTICS
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Lecture -16: Two beam interference

Hi, students in the wave optics course in today's course we will discuss how two beams will interfere with each other and because of that what happens on the screen. So today we have lecture number 16 and today's topic is two beam interference. So beforehand we have the idea of the coherence of two-beam temporal coherence as well as spatial coherence. So we are going to use this concept here as well to understand what happened. Before that let me go back to our topic, the major topic that is superposition of waves. In the superposition of waves, we have two conditions, one is constructive interference and the other is destructive interference. Okay, so in constructive and destructive interference, what happened? Alternative bright and dark fringes will be generated. That we are going to discuss later. We are going to see mathematically how it is happening. But before doing that, first, we need to understand what is the meaning of two-beam interference. When two beams interfere, how do we tackle the problem, let us do that. So, two beams or wave interference; okay, so let us consider two sources S1 and S2 emitting light as a wave and they superimpose or interfere at some point P. So, these are the light waves that are coming from the source S1 and S2 and they superimpose on the point P. So, if I want to write down the electric field of these propagating waves that are coming out from the source S1 and S2 as E1 and E2, it has to be like this.

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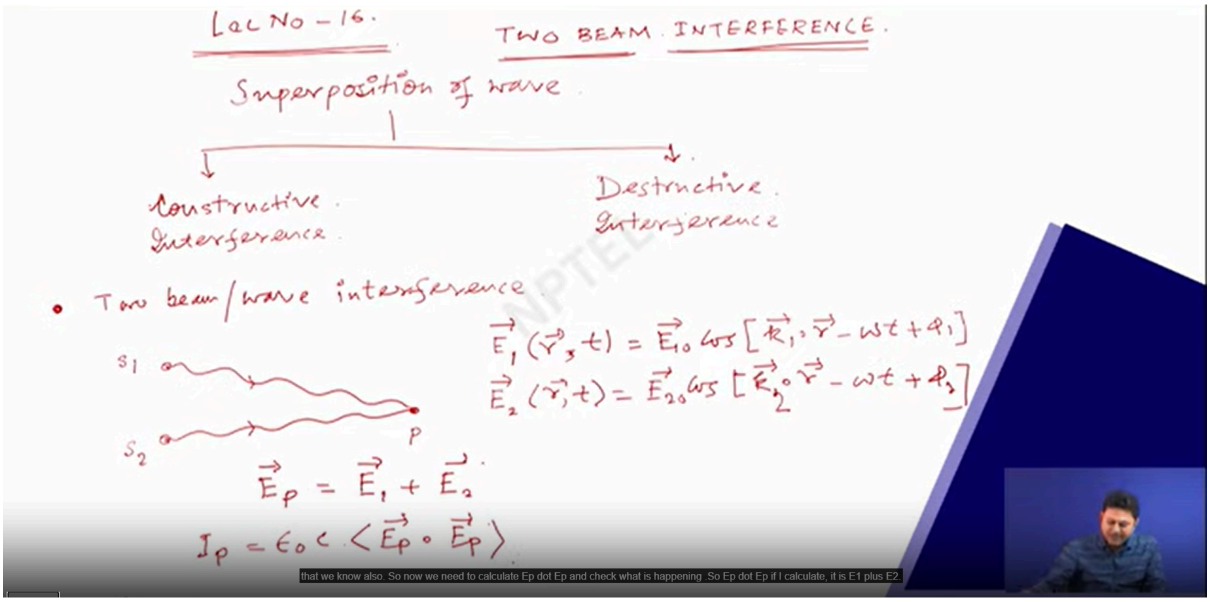

E1 vector is equal to, okay, it is a function of, so let me write it down in a complete manner.

So, the E1 vector, vector r t is equal to the E10 vector, cos of k1 dot r minus omega t, which is the form of the propagating wave. And say, having some phase phi 1 time-dependent phase for instead now we have E2 in a similar way it should be a function of r and t as well it should be E20 and cos of k2 dot r, because it should have a different wave vector k 2, but same frequency omega t plus phi 2. So, this calculation we had done earlier, but the approach will be slightly different here. So, Ep is the field that we get at the point P because of the superposition of these two waves. So, let me write it down Ep is also a vector quantity should be E1 plus E2 and the intensity at point P should be epsilon naught c, time average of Ep dot Ep that we know also. So now we need to calculate Ep dot Ep and check what is happening. So Ep dot Ep if I calculate, it is E1 plus E2. E So, this quantity is E1 square, plus E2 square, plus 2 of E1 star E1 dot E2. Now, Ip is this quantity it is epsilon naught c Ep star Ep dot Ep time average. So, it should be essentially epsilon naught c and then E1 square, time average, plus epsilon naught c and then E2 square time average, plus 2 of epsilon naught c, this. So I can write it as I1, I can write it as I2, and this quantity I can write I12 and that is our interference term, which is coming because of the interference of the coupling between these two. So let us concentrate on what happened in this interference term. So I12 is equal to 2 of epsilon naught c, E1 dot E2 and I know the explicit form of E1 and E2, so E1 dot E2 I can calculate, so this is essentially E10 dot E20. Then we have cos of k1 dot r minus omega t, plus phi1 into cos of k2 dot r, minus omega t plus phi2. This is the way I can calculate E1 and E2. To make life simple, let us consider two variables alpha1 which is equal to k1 dot r, minus omega t. Let, alpha2 be equal to k2 dot r minus omega t.

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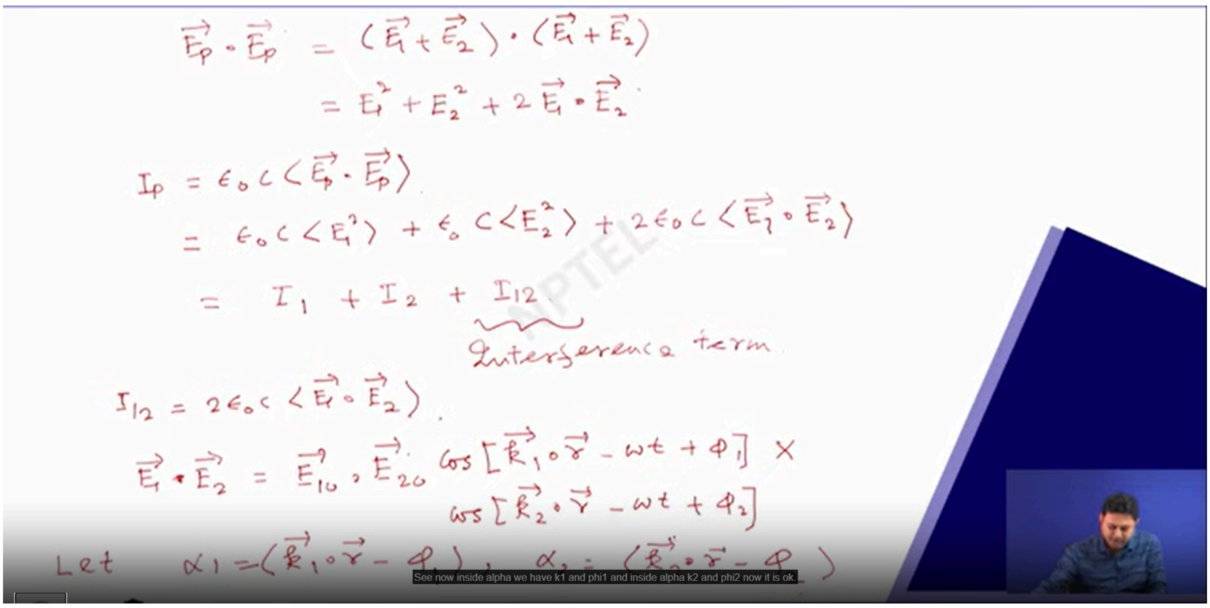

In terms of alpha 1 and alpha 2, if I write then the equation looks slightly compact and I can write I12, which is an interference term equal to 2 of epsilon naught c then we have E10 dot

E20. And then we have a time average of cos of alpha1 minus, okay I wrote wrong and then cos of alpha2 minus omega t, I made a slight mistake here in the previous line alpha in the definition of alpha. So, omega t I am not going to put, I will put phi here because omega is the same for both. So, I should not put it in alpha 1 and alpha 2 because omega t is the same for both. So, this will be phi1 and this will be phi2. See now inside alpha we have k1 and phi1 and inside alpha k2 and phi2 now it is ok. So, here we have cos ok. So, now I can simplify because we know that cos of 2 of cos alpha1 minus omega t and cos of alpha2 minus omega t I can write it as this. This is like cos of alpha1, plus alpha2, minus 2 omega t, plus cos of alpha2, minus alpha1. I can separate this term cos of alpha1, minus omega t, multiplied by cos of alpha2, minus omega t, by using this trigonometric identity. So, then my E1 dot E2 term that was the heart of the interference term is essentially E10, E20, and then these 2 terms. Now, so it is the cost of alpha1, plus alpha2, minus 2 omega t then plus the average of course. Now it is easy to show that the term cos alpha1, plus alpha2, minus 2 omega t.This average term will be 0 because of the random variation of the frequency. Cos of alpha2 plus alpha1, minus 2 of omega t, that will be 0, the average will be 0, that we are going to put here. Then we simply have E1, E2 is equal to E10, E20, and the average value of cos of Delta, where Delta is equal to alpha2, minus alpha1, the difference between these two. Now once we have this term then I can write once again. So my IP was I1, plus I2 plus I12, where I1 is epsilon naught c E1 square, I2 was epsilon naught c average E2 square and this value E in general j square is E j0 square, where j is 1 or 2 then cos square, minus omega t, an average of that. So that value is simply half. So we can have half of Ej0 square now if also these two are parallel, E10 is parallel to E20, which means they have the same polarization.

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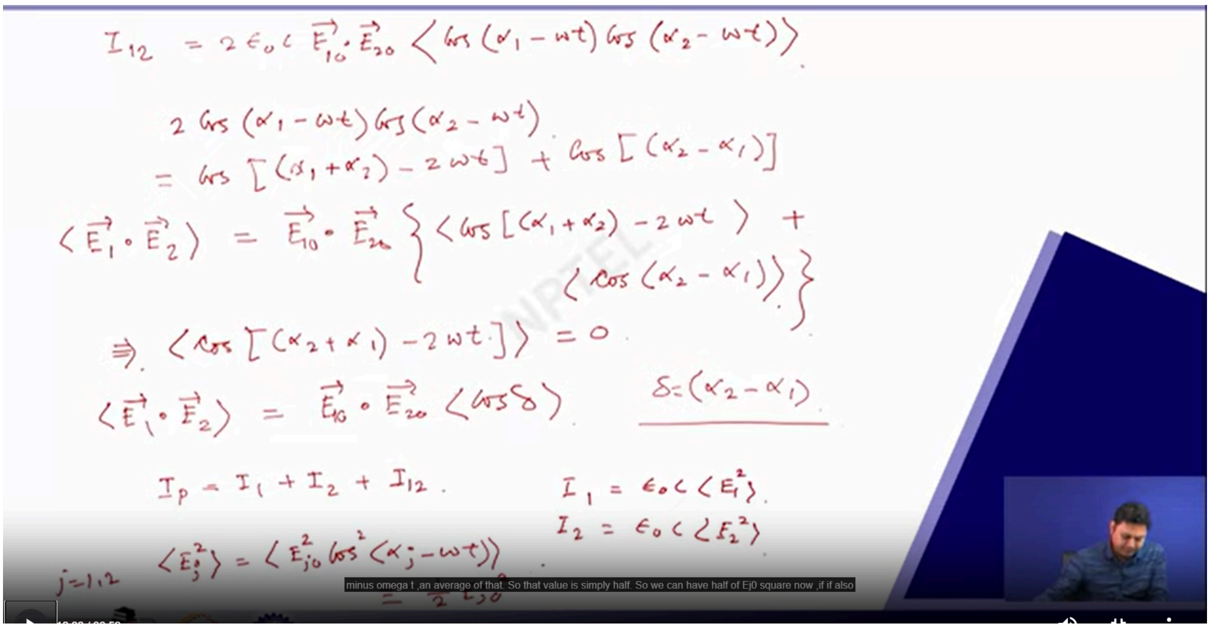

Then what happened? This dot product will simply become, so once you calculate this quantity it should be simply the multiplication of the amplitude 10 dot product will simply become the multiplication and the average value of cos delta as usual. So, after having all this we can have I12 to be simply epsilon naught c, E10, E20, and cos of delta, which I can write, by the way in terms of I1 and I2 as 2 of root over of I1 root over of I2 and cos of delta average of that. Note that I1, I already calculated this value is half of the epsilon naught then C, and then E10 square. I2 is half epsilon naught cE20 square. So, replacing E10 and E20 in terms of I1 and I2 one can have this. So, finally, Ip I can write in terms of I1 and I2 as I1, plus I2, plus 2 root over of I1, root over of I2, and then the average value of time average value of cos delta. That is the interference term we have in terms of I1 and I2. What is delta? Okay let me write it here, delta which can be a function of time is equal to k1. So, delta is phi2, alpha2, minus alpha1. So, that means k2 minus k1 dot r and then minus of phi2. If it is a time-dependent phase minus phi1 t this quantity. Now, it is obvious that for a mutually coherent beam, this average value of cos delta should not vanish. So this is not equal to 0. However, for incoherent beams, this is average because we have the random difference between phi2, minus phi1 and this quantity cos of delta will simply vanish. But in this case, it will not go into 0. For example, if we have a condition that phi2 t, minus phi1 t is equal to 0. That is if two waves are mutually coherent to each other they have a definite phase relationship and the time change of phi2, minus phi1. We should be 0 always over all the time and if the beam is traveling for the same length, equal distance then that always be the case for purely coherent beams. Then what happened? The intensity will be I1 plus I2 plus 2 of root over of I1 root over of I2 cos of k2 minus k1 dot r because phi 2, minus phi 1 vanished here. So, that is now my delta, but this delta is now time-independent because the time part is no longer there.

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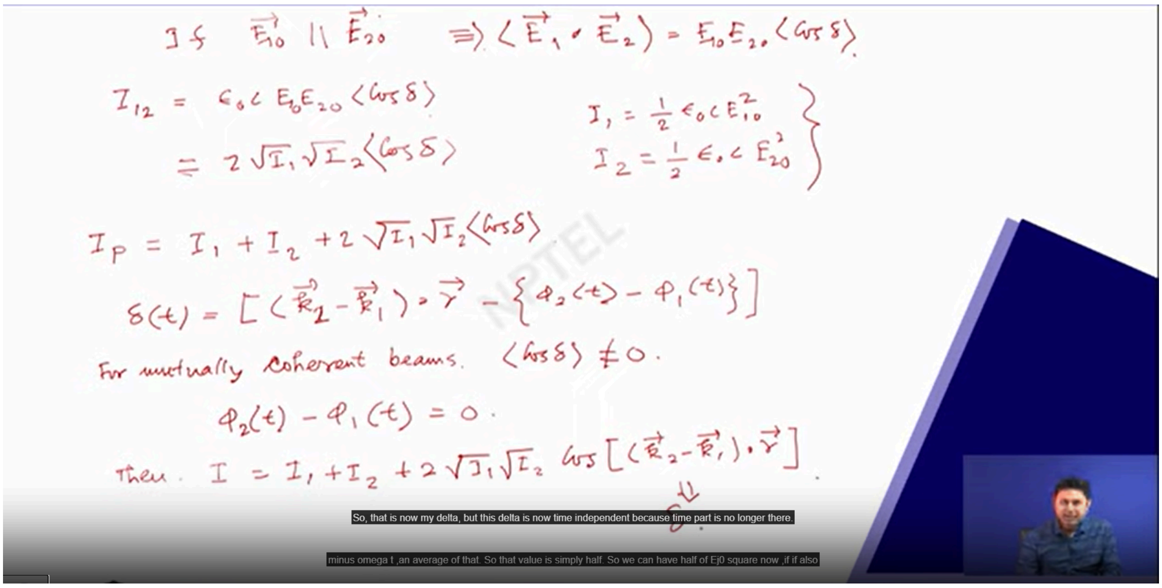

So, that means that the interference 2 beam is going to interfere. So, let me draw this once again when the 2 beams interfere, say, here to here and this is the point, say, this point is equally separated from this source to source S1 and S2. So, if the part difference is the same and these two are coherent, then it is P . Then I will say, the intensity here is I1 and the intensity is I2. Then I1 plus I2 plus 2 root over of I1 root over of I2 cos of delta, where delta is equal to k2 minus k1 dot r. If there is no path difference the phi will simply have the value. Let me erase it and write it clearly. So, the delta will be k2 minus k1 dot r, this will be my delta. So, if we have, let me give an idea about what happened for interference by two beams coming from source S1 and S2. S From another point, there are few conditions that need to be fulfilled. So this is r1 and this is r2, note that if they have a path difference like this then the time difference between these and these is delta tau and that value is r2 minus r1 divided by c. Now, this delta t value is the time difference of the two rays that are coming in these two paths and making an interference pattern. So that delta t should be much much less than the coherence time of the source. That means the condition is r2 minus r1 divided by c for two-beam interference. Especially when we have two slit experiments r2 minus r1 divided by c, which should be much much less than tau 0. If this condition we have then we should get rather a proper interference pattern. Once this condition is there in the system for two-slit interference then we can get a proper interference pattern in the screen. So in the next class, I'm going to stop here. In the next class, we're going to learn more about what happens when to be going to interfere with each other how the interference term, is related to the cos delta, and in Young's last experiment, how the fringes are formed and how one can find out the fringe with and other things out of that. So, in interference, the double-slit experiment is a very very important phenomenon for understanding, and in the next class, we are going to cover more about these two beam interferences in the light of the double-slit experiment. With that note, I would like to conclude here, Thank you very much for your attention and see you in the next class.

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