WAVE OPTICS
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Lecture - 17: Young's double slit experiment

Hello, student in the wave optics course. Today we have lecture number 17 and today we are going to discuss a very important experiment in wave optics, which is called Young's double slit experiment. So we have lecture number 17 today. So we have already started the basic concept that if, we're going to formally define the young slit experiment. Let me do what we have done so far that is if we have two source points defined by S1 and S2, say, this is my S1 and this is S2, the wave that is coming out from these two, will superimpose to some point p and this is a two-beam interference problem . We know what was my E1 which is the field that is coming from S1. It was E10 vector then cos of k1 dot r vector minus omega t, plus phi1 that can be a function of time, that was the complete form of the propagating wave, that is coming from the source S1. Similarly, E2 was E2 naught cause of k2 dot r minus omega t and then phi 2t, a different phase. But since the frequency of these two waves is the same, then the electric field at p was simply the summation of these two E1 and E2. We consider both the fields to be the same polarization and that's why we just drop this vector psi considering these waves are scalar waves and then the intensity at p, which is epsilon naught c, and then the time average of Ep square rather. Then that quantity we calculated and at the end of the day we found that Ip was I1, plus I2, plus 2 of root over of I1, root over of I2, and then cos of delta when I1 was epsilon naught c time average of I1 square.

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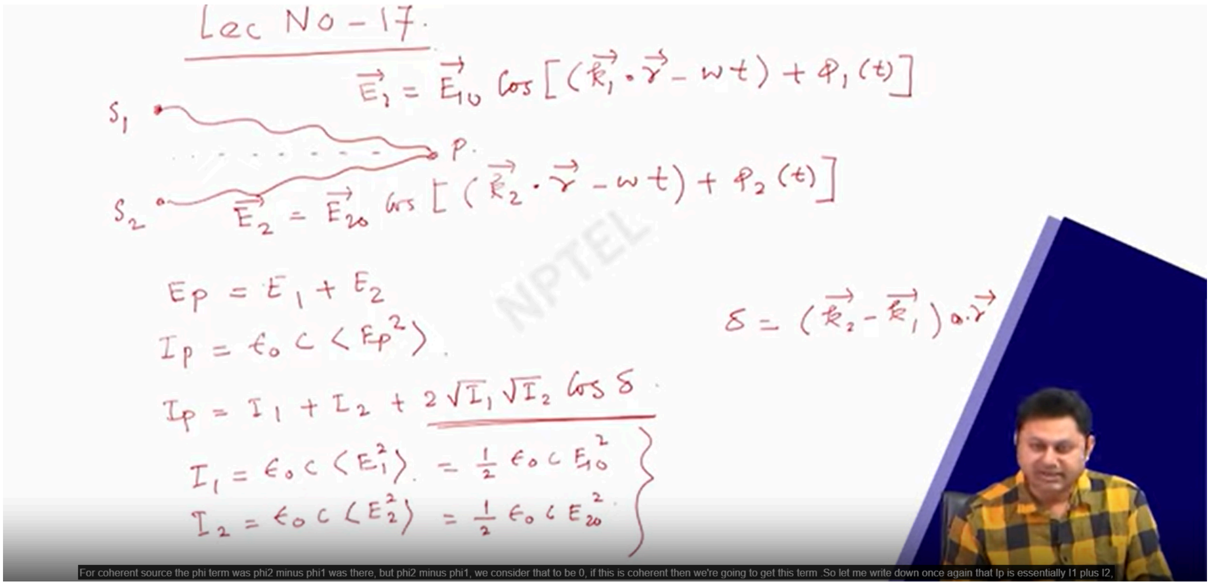

I2 was epsilon naught c and then the time average of E2 square. This time average contains a cos square term and we find that these values are essentially half of epsilon naught c and then E10 square. Similarly, this value was half of epsilon naught c, E2 naught square, that was the value we calculated in the last class. But interestingly we have an interference term this one which contains this cos Delta, where Delta we find it as K2 minus K1 dot r. For a coherent source, the phi term was phi2 minus phi1 was there, but phi2 minus phi1, we consider that to be 0, if this is coherent then we're going to get this term. So let me write down once again that Ip is essentially I1 plus I2, plus 2 of root over of I1, root over of I2 cos of delta. Now this cos of delta, this value will vary between plus, and minus one. So this is a term that varies between one plus minus one. If that is the case, then we can readily find out what is the maximum value at P. So, I write Imax which is actually I1, plus I2, plus 2 of root over of I1, root over of I2, and this value one can have when cos of delta is equal to plus of 1. What is the meaning of cos of delta equal to plus of 1 ? We're going to understand but let us just try to find out what the maximum and minimum value of I1 can have because of this interference term. So similarly" I mean "should be I1 plus I2, minus 2 of root over of 1 I1, I2, this value one can get by considering cos delta equal to minus 1, for plus and minus 1 we get the maximum and minimum. So Imax, if I look carefully at the terms Imax is essentially root over I1 plus root over I2, the whole square of that. Similarly "I mean" is root over of I1 minus root over of the I2 square of that. Now let us consider a special case if, I1 and I2 are of the same intensity, which I write, say, I0 that is two sources having the same electric field, the amplitude of the electric field is the same. So the intensity that we calculate from the amplitude should be the same and if that is the case then I1 and I2 are the same and I write I1 equal to I2 which is equal to something called I naught.

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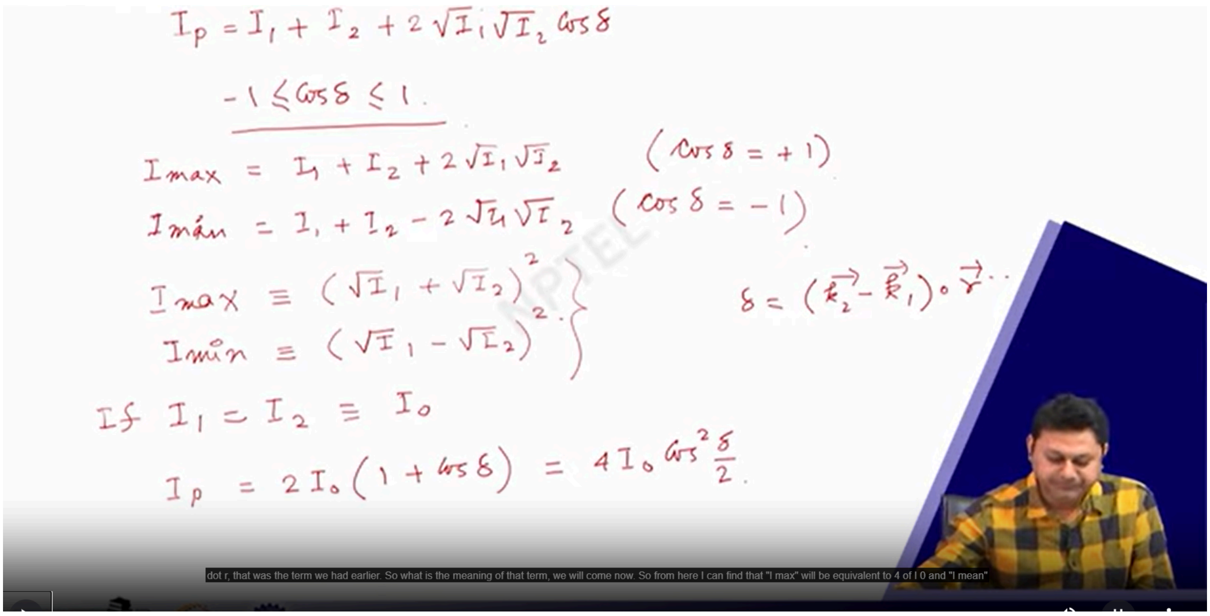

In that case, the general form of Ip will turn out to be something like 2 of I naught and then 1 plus cos of Delta, this is the general term when Delta varies between plus 2, minus 1, then this is the general term one can have. This is essentially 4 of I naught cos square delta by 2. So that means I can have the value of Ip as a function of delta, which is the, so if you remember that delta is equal to k2 minus k1, dot r, that was the term we had earlier. So what is the meaning of that term, we will come now. So from here, I can find that "I max" will be equivalent to 4 of I 0 and "I mean" will be simply zero, if I1 and I2 are the same which is I naught. So with this information, I can find out, I can plot in fact that, how these. So if I plot, let me plot how I will change as a function of delta. So this side I plot delta and on this side, I plot I. So, the maximum value here, if I1 equals I2 equal to I naught, then the maximum value of I reaches up to 4 I naught. That is the maximum value. And it varies, so the general form is I, which is a function of delta Ip, that is 4 I naught and then cos square delta by 2. So, if we plot this then we're going to get a distribution like this, this is simply a cos square function. So it reaches to at some point 0 and then it goes to maxima. This is the way it will vary like a cos square function and at this point it should have the minimum value at this point and this point and this point will be at pi and this will be at minus pi. Similarly, multiplying this pi will give the minimum. So let me now try to find out the condition explicitly. So, for maxima, we need to have a delta 2 by 2 that is essentially k by 2 r1 minus r2. So, r2 minus r1. Now, I put this condition r2 minus r1, considering that the k's are the same. So let me draw on how this condition can be. So here if I have a structure like this, at this point is far away so, this is my r1 and this is my r2. So these are the two points but the k value I can consider to be the same. So the value we get is k dot r.

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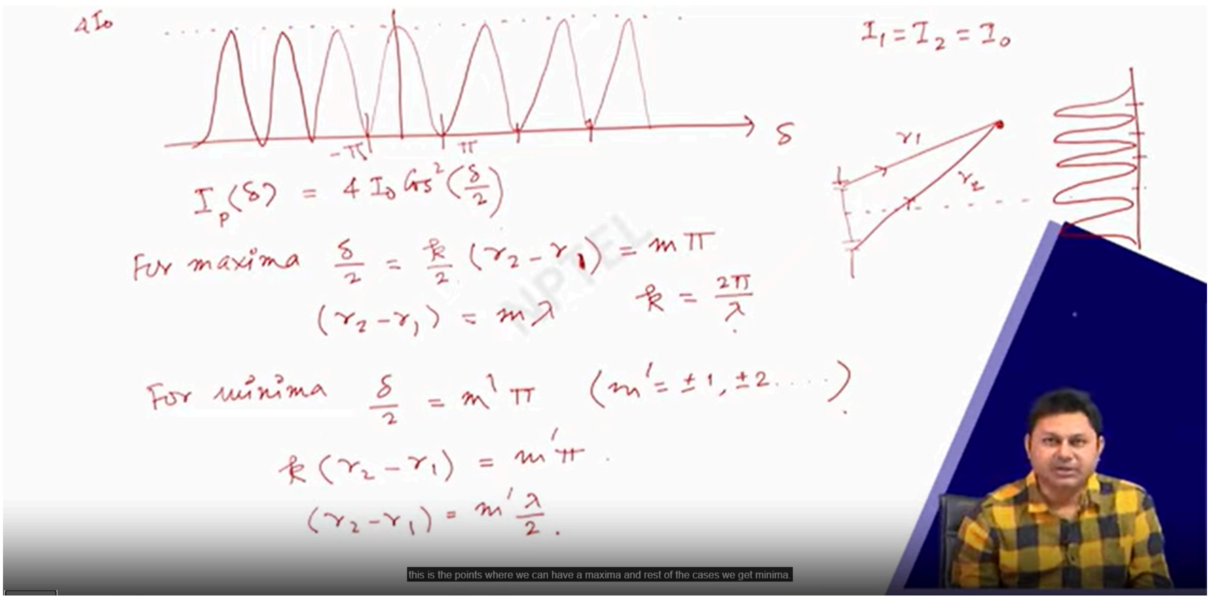

Now k is the same but r is not. So if I write in terms of k2 minus k1, dot r. So this value is now for this structure where we can consider that the k value for this wave and this wave is almost the same but the r value is different. Then that value will be simply reduced to k multiplied by r2 minus r1. So that value, which is basically the path difference, should be equal to m pi, for a maximum. So from here, you can see that r2 minus r1 has this value that needs to be m multiplied by lambda, because k, the propagation constant is 2 pi by lambda. So the path difference essentially comes out to be m lambda for maximum. Similarly, for minima, we can have delta by 2 equal to something, some other variable m prime, and pi, where m prime can take value plus minus 1, plus minus 2, and so on. So, here in a similar way, K r2 minus r1 will have m prime by pi or r2 minus r1 will have m prime lambda by 2. So, in this case, the path difference will be an integer multiple of lambda and we get a maximum and in this case, the path difference will be an integer multiple of lambda by 2 and we get a minimum and this is the way this field will be distributed. So, here and at these points we get a maxima that means the path difference becomes the integer multiple of lambda by 2. This is the point where we have maxima and in this case, we have the part difference of part difference of lambda. So the distribution here, if you see the distribution, distribution should be something like this, this is the distribution we are talking about should be like this, and this is the point where we can have a maxima and the rest of the cases we get minima. So now we are in a position to formally understand what is going on for Young's double slit experiment. So let us now go to the next page and formally define Young's double slit experiment. So after getting the idea that two beams are interfering, now things become easy. So our experiment is Young's very famous experiment Young's Double Slit Experiment. So, what do we have in this experiment? Let me draw. So let us have a light wave coming and we define this light wave as a plane wave and it hits to some aperture. We call it the point source S and then from here we are getting a circular spherical wavefront that is coming.

Now this spherical wavefront hits another system where we have a couple of apertures and two holes. So it goes like this. This ensures that we have spatial coherence and then it hits the S1 and S2 holes and then the light will come from these two holes and the fringe pattern is captured here in the screen. We have a screen here, and here suppose at some point P, we want to find out what is the field pattern. So this is point P and say this length is D from the screen to the aperture. Aperture means, this double slit is the placement of the double slit and P is the point where we try to find out the intensity. So light will go from this to this point p and from here to here. So light will go from this point S1 and S2 and hit the point p and we will get the interference here that is two waves will superimpose here and that we need to figure out. So SS1, so this length from S to S1 so S1 and S2 this length says a, the separation between two slits is a. So this is the basic structure of the double slit experiment, and as I mentioned that I need to find out what is the field here, but we already know that when the two waves are superimposing at point P, the intensity distribution, if I write down the intensity distribution I, which should be a function of delta, is equal to 4 of I naught and then cos square delta by 2. That we already figured out and delta was the phase difference. Now the point is since the two waves that are coming from s1 and s2 will travel to different lengths and since they are traveling to different lengths there will be a path difference. If I calculate the part difference, I have to draw a perpendicular here and this is the part difference one can have. So, this delta is basically the part difference. If this is theta, then we can calculate the part difference in terms of theta and this separation between s1 and s2. Now if it is r1 and it is r2 and if I join this center point to the point p, then this angle will also be theta. So, let me calculate the path difference first. So, the path difference which we call delta is equal to r2 minus r1 and that is equal to A of sine of theta. From the geometry, it is very simple to check that the delta is equal to the A of the sine of theta. Now, if we are going to get a constructive interference in P, then this value should be an integer multiple of lambda. But before that, let me find out what is the phase difference that is coming out due to this path difference. So, the phase difference delta is K multiplied by r2 minus r1 which is K multiplied by delta. So, that value is equal to 2 pi by lambda and then a sine theta, which is the phase difference one can have. Once we know what is the phase difference delta then we can write down the intensity because delta is a function of theta. So that is a function of theta as well is equal to 4 of I naught, then cos square delta by 2. So delta by 2 here is pi a sine theta divided by lambda. Normally, d is very very large compared to a, so for small theta we can write sine theta is nearly equal to tan theta and that value is essentially, y by d . What is y? Y is a point that one can measure from here to here. So this is my y point from the center to this distance. So essentially the intensity I theta is equal to 4 of I 0 cos square pi a divided by lambda, multiplied by y by d.

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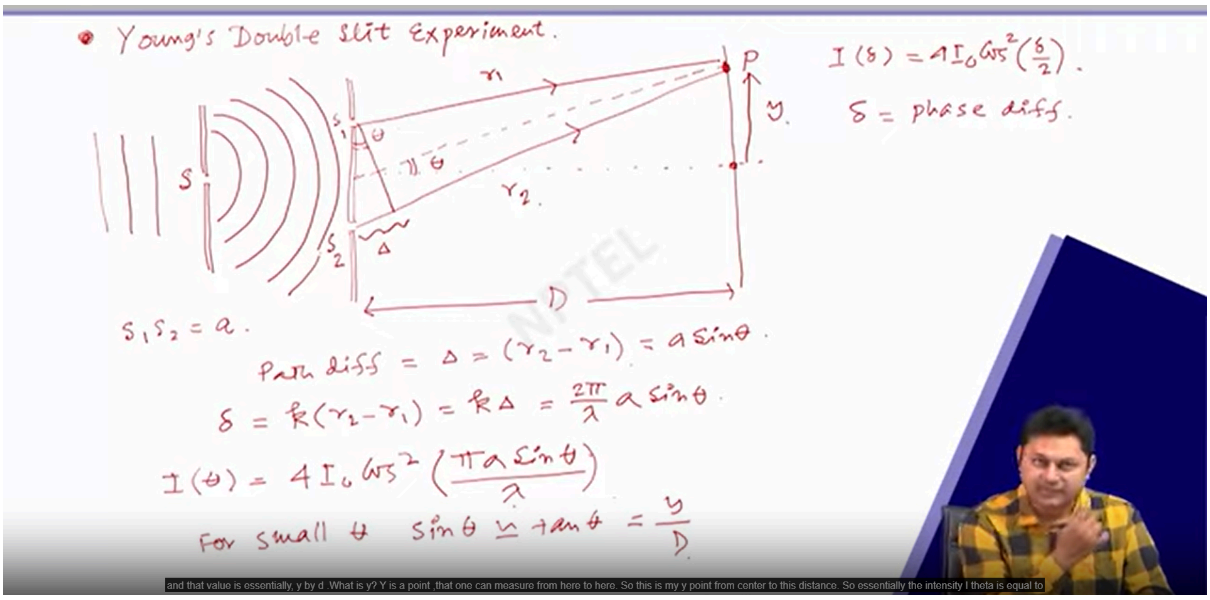

So that is the expression we have. Now if I want to find out, let me quickly draw the structure. So this is my point P, this is S1, this is S2 and this angle is theta, this is D, this is Y and this is A. That is the geometry we have. That is the entire geometry we have. I am drawing once again. Now for the bright fringe, if I want to find that there is a brightness in P, that means constructive interference is happening at P. So we can write for the bright fringe. So I can have pi a divided by lambda into y by d, that is the argument of cos square theta should be m pi when m can take the value 0 plus minus 1, plus minus 2, etc. So from here, we can find out what is the length. So the emit order bright fringe should have the expression, m multiplied by lambda d divided by a . Now if I draw that, the fringe pattern will be something like this that we already discussed. This is something like this. So this is the point where we have a bright fringe. So this is my say Y m one. In a similar way, I can have another one which is m minus 1 or m plus 1. So, by separating these two, m minus 1 to m or m plus 1 to m, we can find out what is the fringe width. So, the fringe width will be delta y and that is I can write m plus, 1 minus m, so the separation between these two gives us simply the fringe width and if we calculate we find this value is lambda d, divided by a, a fixed value, if a is fixed, d is fixed the fringe width depends on the distance d, the wavelength lambda and the separation between these two. So from here, we can see that, if I draw another one, this is y m plus 1. So y m plus, 1 minus y m is this length is my delta y, gives us the amount of fringe width. What is the fringe width? So from this expression, you can see that the fringe width that is generated due to the double slit depends inversely on the value of a, that is the separation. So if s1 or s2 are very close together then the value of the delta y will be large, which means the fringe width will be larger. However, if we increase the value of a by keeping d fixed then the fringe width will be more and more sharp by fixing lambda. And d in a similar way if I increase the value of d, then the fringe width will increase and if you decrease the d the fringe will go to decrease. So in today's class, we discuss in general how for a young double slit experiment, when we have two sources in terms of two pinholes and then allow the light to pass through these two pinholes or two slits and when the light coming from these two slits interact on the screen that is placed far away from the screen, then we're going to get a pattern and this pattern is equivalent to the cos square theta function. So that means we have maxima and minima and there will be no amplitude variation to that pattern. Later we will see that if you want to find out what is the pattern of a single slit diffraction then there will be a distribution in the amplitude as well. But this phenomenon is the pure interference phenomena that we are looking for at this stage and if we see only the interference pattern. If you just consider it, on the interference pattern it should be periodic dark and bright fringes and we find that these dark and bright fringes are placed equally and the width of these fringes are equal. And this value depends on the relative separation between the two slits as well as the distance from the slits to the screen where you are supposed to get these fringes. So, I hope you understand these things. In the next class, we will discuss more about what is the fringe distribution and try to find out the equation of motion, to find out the formation of the fringe and how it is distributed. So with that note, I would like to conclude today's class. Thank you very much for your attention and see you in the next class.

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