

WAVE OPTICS
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Lecture - 20 : Interference by division of amplitude (cont.)

Hello, student in the wave optics course. So today we have lecture number 20 and in this lecture, we will continue the study of interference by division of amplitude. So today we have lecture number 20 and we will continue with the topic of interference due to division of amplitude. So, what we had let me quickly describe in the last class if we have a film like this light ray will fall here and reflect and one portion is transmitted then it is reflected from this surface and again it is transmitted. So this is ray 1 and this is ray 2, these two rays will interfere and also we calculate this is the way the ray will propagate, also we calculate that if the thickness of the film is t , the refractive index is n_f and the refracted angle is θ_t , sorry θ_t , θ_i is this angle incident angle. Then what happened was that I could find out the path difference between ray 1 and ray 2. And this path difference is simply $2 n_f t \cos \theta_t$. That is the path difference we have. Now also we mentioned that due to the reflection from the medium n_1 to n_2 where n_2 is greater than n_1 , we have an additional π phase shift that leads to a path difference of $\lambda/2$. So, the final condition for maxima and minima, here if I write, so for maxima, that is for constructive interference, we have the path difference $2 n_f t \cos \theta_t$, then $\lambda/2$, plus this additional path that is due to this reflection which is $\lambda/2$ is equal to $m \lambda$ or we can write $2 n_f t \cos \theta_t + \lambda/2 = m \lambda$, just putting $\lambda/2$ this side. Now we can have t into, sorry this is not $n_f t$, this is n_f , according to our notation the refractive index of the film is n_f . So this is n_f and then t and then \cos of, not is already there so it is $2 n_f t \cos \theta_t$ it is coming so let me write $n_f t \cos \theta_t$ is equal to, it is simply $(2m - 1) \lambda/4$, where m is 1, 2, 3, etc integer numbers.

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Diagram 1: Shows a light ray incident at angle θ_i on a film of thickness t and refractive index n_f . Ray 1 is reflected from the top surface, and Ray 2 is reflected from the bottom surface after a path difference of $2 n_f t \cos \theta_t$.

Diagram 2: Shows a ray reflecting from a boundary where $n_2 > n_1$, resulting in a π phase shift.

For maxima

$$2 n_f t \cos \theta_t + \frac{\lambda}{2} = m \lambda$$

$$2 n_f t \cos \theta_t = (m - \frac{1}{2}) \lambda$$

$$2 n_f t \cos \theta_t = (2m - 1) \frac{\lambda}{4}$$

$m = 1, 2, 3, \dots$

For minima

$$2 n_f t \cos \theta_t + \frac{\lambda}{2} = (m + \frac{1}{2}) \lambda$$

$$n_f t \cos \theta_t = 2m \frac{\lambda}{4}$$

$m = 0, 1, 2, \dots$

$\lambda/4$, where m can have the value 0, 1, 2, etc. here m can have the value in m equal to 0. So this is the expression for the condition for minimum.

So that is the condition for maxima, for minima, we have $2 n_f t \cos \theta_t$, that is the path

difference plus $\lambda/2$ is equal to $m\lambda$. So, this $\lambda/2$ will cancel out. So, essentially we get $2m\lambda = 2t \cos \theta$ that will be equal to $2m\lambda/4$, where m can have the value 0, 1, 2, etc. Here I can be allowed to put the value in m equal to 0. So this is the expression for the condition for minimum. Okay so next quickly I like to show that if we have interference by extended source then what should be the setup? So this is interference by an extended source. So what is the meaning of extended source? Extended source means instead of one point source I have more point sources and say this is S_1 , this is S_2 , this is S_3 and the ray is coming from this S_1, S_2, S_3 like this parallel rays are coming and obviously these two rays will go to reflect like this and we may have an arrangement here where we can converge all the rays to some point. But here instead of these rays, we have also the reflected ray from this surface's lower surface which is like this and it goes like this. So these are the rays that will fall and it will reflect from the surface like this then the transmitted ray will also get reflected by the lower surface and it will again come here and suppose this is a lens I can put. So here we will want to see the interference pattern so I can converge all the rays here at some point, this is the point where all the rays converge by using the lens and we will go to see a pattern. So here is the point P and maybe we can see a pattern here like this, a fringe pattern like this. This is the fringe pattern one can expect and this is my film. So this is the way one can find out or one can understand how for extended sources the interference patterns are formed. In general, we have in the experiments, we have extended sources, and we don't have a point source. So it is important that you understand the setup through which the fringe patterns are formed in this way. Okay, so after that, after having this setup for extended source, we will understand another thing and that is called the anti-reflective coating. So we will make use of this property and try to find out what are the applications and how we can apply them. So our next topic which is very much related to this concept is called anti-reflective coating. So what is the meaning of anti-reflective coating? That let me describe quickly, this is case one, when we have a glass placed in air bare glass this is air and this is glass and we have an interface here between air and glass. So the light falls here. It will simply reflect and you can't do anything with this reflection, this will happen, this is simply a light reflection. Now we can make this reflection small or we can make this reflection destructive by putting a coating over it. So this is case one. In case two what we can do we have a glass but on top of that we put a thin layer and then allow the light to allow the light to pass over this system what happens the light will fall like before here at this point. This is the incident light ray then it will reflect from the surface of this coating and one portion will go back here and it will reflect again from the glass boundary it's here and then it go in this direction. So we now have two rays instead of one this is one and this is two rays because of the placement of this coating. So this is air, this is my film or coating material which behaves like a film and this is my glass. What is the advantage here compared to the previous case? So in this case, since we have two rays there is a possibility that we can make these two rays interfere destructively and if that is the case then there will be no reflection out of this system and that is called the anti-reflective coating. So the goal here is to put a coat over the glass plate. So that there should not be any undesired reflection from this glass block by just putting a coating we can do that So now if we can say that if the reflectivity of this. So okay let us put a name here A and this is A' and this is B , this is B' . So the rate of the reflectivity of AB the ray AB which is ray 1 and another ray $A' B'$ which is

ray 2 is the same then we can write that r_1 is equal to r_2 , which is the coefficient of this reflectivity r_1 r_2 . So for this is for normal this happens for normal incidence. So suppose the ray falls normally. So for normal incidence. Then what we have if we put the refractive index of the air as in A, the coating as n_c , and the refractive index of the glass as n_g , we can find a relationship between n_a , n_c , and n_g that basically allows us to find out the refractive index of the coating for our desired application. So here for normal incidence we can write n_c minus n_a divided by n_c plus n_a is equal to n_g minus n_c divided by n_g plus n_c So that is the condition one can get for normal incidence when we will come to this again maybe in the future lecture. For the time being my goal is to find out what should be the refractive index n_c in terms of n_a and n_g . By putting certain conditions for normal incidence, if the reflectivity is the same then it is possible that I can write this equation and this equation is valid here in this case. Now if that is the case we can manipulate this equation. Let me write it down once again because this is another page. So n_c minus n_a divided by n_c plus n_a square is equal to n_g minus n_c whole divided by n_g plus n_c . For normal incidence, we can use this expression, which is an expression of reflectivity. So reflectivity from coating to air and reflectivity from glass to coating, we consider this to be the same. So in that case, I can write it. Now, we know that if I write 2 ratios a by b is equal to c by d, then that is equivalent to, this thing is equivalent to a minus b by a plus b is equal to c minus d, divided by c plus d that we know exploiting this expression we can find an equation here from this, which is 4 of n_c n_a divided by 2 of n_c squared plus n_a squared and then equal to 4 of n_g n_c divided by 2 of n_g squared plus n_c squared just using this identity we have something like this. So, n_c , n_c will cancel out from both sides. And then if I multiply and then 2, 4, 4, 2, it will cancel out. So, we have something like n_a multiplied by n_c squared plus n_a squared. Sorry, n_c squared plus n_g squared and n_g multiplied by n_c squared plus n_g squared. So that gives us an expression if I manipulate this it should be n_c squared. If I take n_c squared common and put it here then it is n_a minus n_g which is equal to n_g n_c n_a minus n_g . So, from here, I can find the refractive index n_c , which should be n_g root over n_g multiplied by n_a .

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Anti-reflecting coating

Diagram (1) shows a ray incident on an interface between Air and Glass, resulting in "Light reflection".

Diagram (2) shows a ray incident on an interface between Air (refractive index n_a), a thin coating (refractive index n_c), and Glass (refractive index n_g). The ray is labeled "ray 1" (AB) and "ray 2" (A'B').

If the reflectivity of AB (ray 1) & A'B' (ray 2) is same.
 $r_1 = r_2$ (Normal incidence)

$$\left(\frac{n_c - n_a}{n_c + n_a}\right)^2 = \left(\frac{n_g - n_c}{n_g + n_c}\right)^2$$

another page. So n_c minus n_a

So this condition allows us to write down the refractive index of the coating. So that should

be the refractive index. So first when you put the refractive coating on, this is the structure we have a coating, and then light is falling here reflected back transmitted light goes here and goes here and reflected back, transmitted back. So this is 1, 2. This is the way the light is going, this is in the air. This is n_c coating and this is n_g so what we find is that if the reflectivity of the n_g and n_c and n_c through n_c and n_a are same. Under that condition, we can find out what should be the refractive index of the coating and that is the root of $n_g n_c$. So that is $n_g n_a$, so that means if we know the refractive index of the glass we can find out the refractive index of the coating, such that these reflectivity will remain the same. Now from this nature, we can also find out. So once we know the refractive index of the coating we can also find out what the thickness of this coating should be. So next we need to find out the thickness of the anti-reflecting system. So again anti-reflective coating thickness, so for normal incidence, let me draw. So this is ray 1 and this is ray 2 these two are interfering, this is air, this is the coating, this is glass and this is the thickness I want to find and what is the condition? The condition is that ray 1 and ray 2 should be destructive. Then only we can say this is an anti-reflective coating. So in A, we figure out that it should be like this. So in C, the coating refractive index should be in between the refractive index of the air and glass. Now the path difference is Δ equal to 2 of n_c multiplied by t we already figure note that here ray 1 will be reflected from this point in this interface and ray 2 will reflect it from this interface but in both cases the ray 1 and ray 2 the reflection will happen from denser medium to rarer medium so that means both the cases there will be a π phase and these two basically cancel out and when it is canceling out so here we do not have any face change or face shift due to the reflection the effective phase shift due to the reflection will not be there as n_c follows this condition. So for destructive interference, because it is an anti-reflective coating, for destructive interference, we can have 2 of $n_c t$ equal to λ by 2 . So, the anti-reflective coating thickness t should be λ by $4 n_c$ where n_c we calculate that it should be root over $n_g n_a$.

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$$\Rightarrow \left(\frac{n_c - n_a}{n_c + n_a} \right)^2 = \left(\frac{n_g - n_c}{n_g + n_c} \right)^2$$

$$\frac{a}{b} = \frac{c}{d} \Rightarrow \frac{a-b}{a+b} = \frac{c-d}{c+d}$$

$$\frac{n_c - n_a}{n_c + n_a} = \frac{n_g - n_c}{n_g + n_c}$$

$$n_a(n_c^2 + n_g^2) = n_g(n_c^2 + n_a^2)$$

$$n_c^2(n_a - n_g) = n_g n_c(n_a - n_g)$$

$$n_c = \sqrt{n_g n_a}$$

Refractive index coating $\Rightarrow n_c = \sqrt{n_g n_a}$

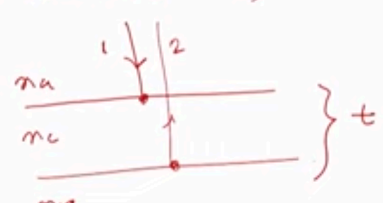
and that is root over of $n_g n_c$. So that is $n_g n_a$, so that means if we know the refractive index of the glass we can find out the refractive index of the coating, such that these reflectivity will remain same. Now

So here we think, we calculate that if I use an anti-reflective coating then what should be the refractive index of the anti-reflective coating and what should be the thickness of this

anti-reflective coating but at the end of the day what we are trying to find that, this is one of the uses that we have for interference problem in under amplitude division interference. So, this and due to amplitude division interference what happened that 2 rays will interfere and with this concept, it is possible to put an anti-reflective coating over different systems where we don't require any kind of reflection for different experiments or in daily life also we want that there should not be any reflection from the glass or any reflective material. Then what we do is that we need to put an anti-reflective coating in such a way that the desired wavelength for a particular wavelength happens. These rays will no longer be reflected because they will interfere in a destructive manner and in order to prevent this destructive interference. We need to put a condition and that condition we can find out from this interference condition itself that, the thickness of the coating should be λ divided by $4 n_c$ where n_c is the refractive index and also we find out that the refractive index of the coating can be figured out easily from the refractive index knowledge, of the refractive index, of the air or any other material which is where this glass is placed. So on that note, I would like to conclude today because I don't have much time to discuss more about these things. So, in the next class, we are going, further, we will going to discuss the other amplitude division problem for example, the formation of Newton rings which is a very very interesting phenomenon where we can see that whatever the problem we have here and exploiting this problem we can generate different kind of structure, a ring kind of structure we can produce. So with that note let me conclude today's class. Thank you very much for your attention and see you in the next class.

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• Anti-reflecting coating Thickness.



$n_a < n_c < n_g$

The path diff. $\Delta = 2n_c t$

Here we do not have any phase change due to the reflection

$\therefore n_a < n_c < n_g$

For destructive interference

$$2n_c t = \lambda/2 \Rightarrow t = \frac{\lambda}{4n_c} \quad (n_c = \sqrt{n_a n_g})$$

Then what we do that we need to put an anti-reflecting coating in such a way that the desired wavelength for a particular wavelength what happened that, these rays will no longer be reflected because they will interfere in a destructive manner