WAVE OPTICS

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Lecture - 25: Michelson Interferometer

Hello, students, welcome to our course wave optics. Today we have lecture number 25. Where we will discuss in detail the Michelson interferometer. So today we have lecture number 25 and today our topic is the Michelson interferometer. We started this in the last class but today we are going to discuss it in detail. So let us draw the setup once again. This is an instrument, a very important instrument where we have an extended light source like this, and then light is coming in this path and then here we have something called beam splitter, which splits the incident beam into two paths and one is reflected. The angle of this splitter is 45 degrees with respect to this horizontal line and then one path will be transmitted and it hits another mirror here, I call it M1 and then it also reflects from this mirror and comes back. Here also have another mirror sitting, we call it M2, this is an M2 mirror and then it goes up but here you may remember that we put a similar structure: a glass block with the same material as this beam splitter having the same width. So this is a beam splitter, which splits the beam and this is a compensator. So light goes in this direction, hits this portion, and one portion is reflected passing through this compensator because this ray is passing only one time to this beam splitter, where the other beam is passing twice. So that we described in the last class? I am not going to do this once again here and then both the ray is collected here. So that is ray number 1 here, it is reflected from this point. So this is ray number 2, which is reflected and then from this beam splitter and then again reflected by this mirror M2 and then comes back in the same path. In this case, also it is happening like that, and at the end of the day, these two rays will merge together and come to our eye or the detector. So, this is ray 3, which is transmitted followed by a reflection through M1 and then this is ray 4, ray 4 is a combination of ray 2 and ray 3. These two rays are traveling the same path, if the distance between M1 and M2 is the same with this beam splitter. So, that was the structure we described earlier. We have already discussed this structure in detail in the last class. So, I am not going to do that. So, that is the basic structure. So, today we try to understand what is the equivalent setup. So, what actually how these things work that we try to understand?

So, from this figure, we can see that somebody who is sitting here and looking at what is the interference of these two lights can see the two lights that are coming from M1 and M2. Because one ray is coming directly from M2 it is coming here and another is reflected by this then this beam splitter and coming here and they are going to interfere through this ray 4 and we can see the interference pattern. So, let us draw an equivalent picture to make life easy ah and equivalent picture. So, here is what we do. This is the equivalent optics for the Michelson interferometer, MI stands for Michelson interferometer. So, what is the equivalent optics? So, let me do that here. So, we have an extended source here, suppose, and then 2 mirrors are sitting here, they are parallel to each other. These are the mirrors because from this side, okay let me draw first and then I will explain. So, this is the source point, say O, and then it is emitting the light like this. So here the ray will reflect and another is also reflected from another mirror. These two say this is the angle theta, that is going here and coming back like this and here we can place our instrument which is capturing this interference pattern but virtually one can see because this mirror will virtually be placed like this, this dotted line okay and if I extend this, so this is one virtual source, this is another virtual source. So, these two are in the same line as their virtual sources because if we have say mirror 2 here and in the original case mirror 1 was perpendicular to mirror 2. So I just rotate this mirror 90 degrees and assume that the placement of the mirror is here in the same direction, then these two mirrors have the reflection, and the reflection image will be here it is here in this point. So, it should be an S2 source and this will be S1 prime, whatever the objective we have from here and here these are the images of that. Now, easily we can show that if the separation of these two mirrors is d then the separation between these two planes which are the image will be 2d ok. Now, this angle is theta. So, if we say this is the point O2 prime and this is O1 prime, that is the image of this original point O then we can see that these 2 virtual sources are not at the same point, but there is a path difference between these two because they are in two different planes. So, this path difference delta p will be simply if this length is 2d it is very easy to show it is 2d cos of theta. So, the path difference is simply 2d cos of theta, where the separation between these 2 mirrors m1 and m2 is d, and this is essentially the equivalent structure. So, let me write here 1 by 1. So, the mirror separation m2 m1 prime is equal to d then the image separation of the source plane S is a source plane, which is S1 prime S2 prime that has to be 2d. So their separation will be 2d. Now the optical path difference between these two points, as I mentioned this virtual source point is of O prime and O. I have already mentioned that this is delta P equal to 2d cos theta and the fringe. So, this is if the source has an extended source, and if if we have a spot for example, then the fringe pattern that one can have is simply concentric circles. C So, that is the equivalent optics for the Michelson interferometer, that I have already mentioned in the last class or in this class. Now, we need to check in detail the net optical path difference.

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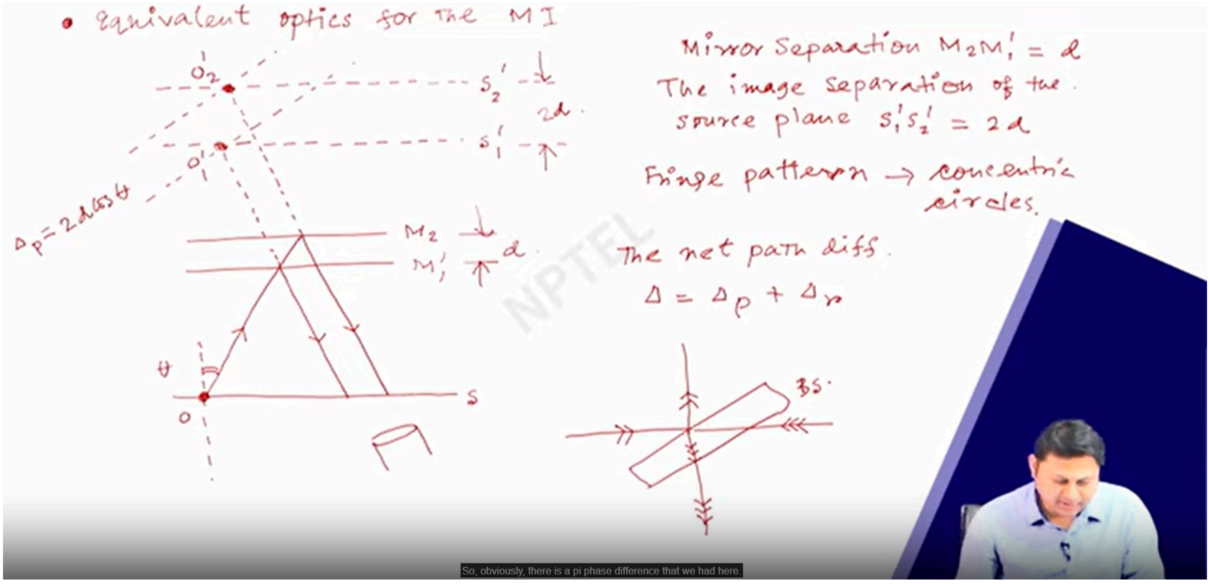

This is the path difference we have, but if you look carefully at the previous figure that the beam splitter, ok so that the lines are removed somehow, so one can see that in one case the ray is reflected from this front surface, that is the reflection from highest refractive index region to lowest refractive index. In other cases, however, it is reflected from the back surface. In that case, the reflection from the lower refractive index to the boundary is like that on this side is a lower refractive index and this side is a higher refractive index. So that means due to this we need to have an additional path difference and that will give us the net path difference. So, the net path difference will be delta p which is this path difference 2 d cos theta, that is the geometrical path difference we had, plus the path difference due to reflection and as I mentioned the relative path difference. So let me draw that once again, that is how this relative path difference is there. So that was the structure of the beam splitter and in one case the ray is falling here in one case it is reflecting like this and in other cases it is transmitted and reflected back. So in one case this two arrow line, I am just drawing it is going like this direction, and in another case, these three arrows are coming and then get reflected from this region and then it goes here. So in this way, there is a pi so this is a beam splitter. So, obviously, there is a pi-phase difference that we had here. So, if this pi phase difference we consider then our condition for dark fringe. So for dark fringe total path difference delta is equal to delta p plus delta r, which is equal to 2d cos of theta plus lambda by 2, which is due to this reflection that will be equal to the m plus half lambda odd multiple of lambda by 2 in other word. So, from this expression we can simply have for dark ring 2d cos theta d sorry 2d cos theta is equal to m of lambda because this lambda by 2, lambda pi 2 should cancel out, where m can take the value 0, 1, 2 etcetera these are the values m can take. Now, for theta equal to 0, theta equal to 0 means, we are talking about the central spot because the fringe pattern will be something like this. Let me draw concentric circles and this is our central spot. So, for the central spot, we simply have 2d equal to m lambda. So, for a given lambda and d usually this m value should be very large. So, for the central dark spot or fringe, I write m max because this is a very high number that will be 2d divided by lambda. Now, for another case if I go from this spot to another spot or the dark ring then there will be a value of cos theta, cos theta will not be 0. And in that case, what happens is that d and lambda are the same and we have a cos theta value.

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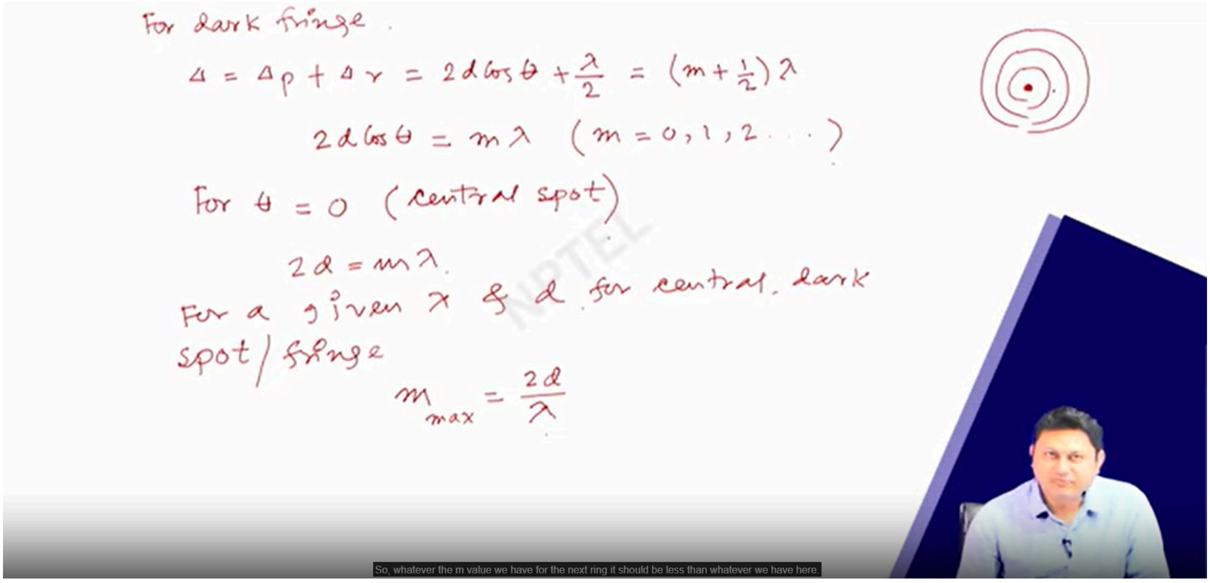

So, whatever the m value we have for the next ring it should be less than whatever we have here. So, m max that is why I write m max is 2 d divided by lambda. Well, with this condition let me now go back to the ring structure and try to understand. So, this is the fringe pattern we have. So, let me draw it. So, this is the central spot and then we have rings like this and so on. Now this is a central spot so for the central spot say m max say this is 100 and then what happens that if I go to the next ring it will be some less value and this less value I can calculate like this is the next lowest integer of m something like that. So, let me write down the condition first. So, for a dark ring we have 2 d cos theta is equal to m of lambda right and we also have 2d is equal to m max into lambda. So, from these two expressions one can get something like 2d 1 minus cos theta subtracting lower 1 to the upper 1 then that is equal to m max minus m lambda or simply p lambda where p is a new integer. So, p is now a new integer. So, now, if I go back to this picture, suppose I have the central one in the central one the value of m max is 100. If I go to the next one So, my m max, my m value rather will be 99 where I have a P value which is 1. So, that I can have because here in this equation you can see that my m max will be m plus p. So, m plus p will always be 100 and if I go 1 by 1 to the outer rings the m value will reduce and p value will increase. So, if I add these 2 we are going to get back m max. In a similar way, for example, in this ring my m will be say 98 and p will be 2, and so on. For the third ring my m will be 97 and p will be 3 and so on. So, another thing as d varies now that is important as d varies. So, the fringe of what happened suppose these two mirrors, whatever the mirror we have, we change the relative difference between these two mirrors that means, now d varies. So, as d varies the fringe the fringes appear to move towards the center, where they disappear or else move outward from the center, where it seems it is where it seems it originated. So, what is the meaning of that? Suppose I change the relative separation between these two mirrors. So, I have a mirror here. So, this is my beam splitter, light is falling here going up this is the location of one mirror M2 and this is M1, this is a source point light is coming here it hits back going this side and then again coming back it goes here and then again coming back both the ray will merge together and come so that is the structure.

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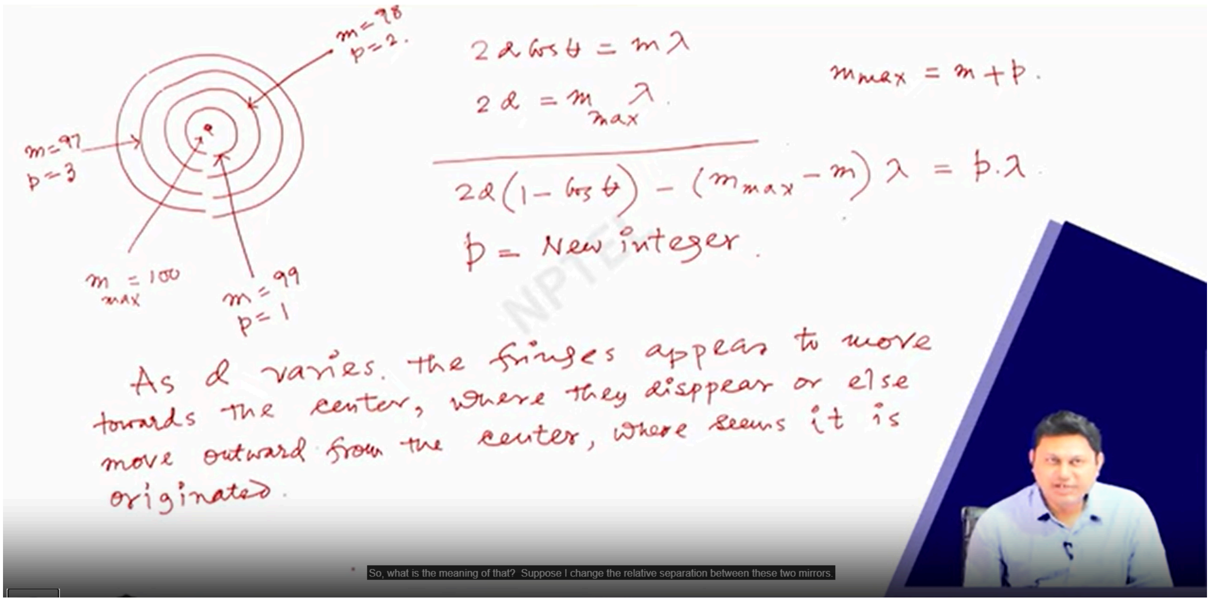

Now if I know the relative separation, you know the equivalent picture the m1 and m2 were sitting like this where this separation was d, this is M1 prime and M2 that was the equivalent picture, this is the equivalent picture and this is the Michelson interferometer original setup. So, if I change the separation between M1 and M2. So, d will change and when there d will change the path difference will change and we had the formula in our hand that 2d cos theta is equal to m lambda. So, this order of the, because we are changing d for a particular theta, and lambda m, has to change. So, that is why it seems that sometimes when we do the experiment by hand we will see that there will be a collapse of the fringes or it is evolving from the central part. So, if I now just concentrate on the central beam then what happens is that for the central spot what we had is 2d is equal to m lambda. Now, I am changing the d. So, if I change the d what happens m has to change ah simultaneously to you know to to maintain this equation to satisfy this equation. So, simply we can have that delta m will be simply equal to 2 of delta d divided by lambda simply we have. So, that means, if I calculate delta m, the number of fringe collapses, and delta d is the change in the mirror's relative location. So, if that is the case then we can have a delta m equal to 2 delta d lambda and if somebody during the experiment measures this delta d and delta m then one can find out what should be the lambda. So, lambda should be 2 of delta d divided by delta m. So, that is an important outcome of this Michelson interferometer setup that using this one can calculate the wavelength of a given light. So, during the experiment, we do this, a source is here and it is emitting some light with unknown wavelength. Using this Michelson interferometer setup by just changing the relative path difference between these virtual sources, we can find out how these two lights are going to interfere forming a pattern and then gradually change the relative location of the mirror. In order to avail the same condition, the fringes will be, for example, in the central fringes will be some way it is collapsed. So, if I find out the number of collapses with the change of this relative distance of M1 and M2, we can calculate through this simple formula, we can calculate the unknown wavelength. So, this is one of the important aspects of the Michelson interferometer and today we discuss it in detail. So, we do not have much time to discuss more about this. So, today what did we do? We discussed the basic setup of the Michelson interferometer and then we studied how equivalent optics can be generated through this setup. Through these equivalent optics, we can find out what are the path differences for this kind of setup. And, if we know this path difference then we can calculate what is the condition for central maxima or central minima. In this case, we find that in the center we always have a minimum, always have a dark spot, and by collapsing by looking at the collapsing of the beam. We can find out what should be the wavelength of the light that is used in this experiment. So, with that note, I would like to conclude here. Thank you very much for your attention and see you in the next class.

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