WAVE OPTICS
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Lecture - 29: Resolving power of Fabry-Perot interferometer

Hello, student, welcome to the wave optics course. Today we have lecture number 29 and today we are going to discuss an important topic called resolving power of the Fabry-Perot interferometer. So we have lecture number 29 here. And we are going to discuss the resolving power of the Fabry-Perot interferometer. So, quickly describe again what our system was. So, we have a cavity here in the Fabry-Perot interferometer system. This is the cavity where this side is a highly polished side made by two glass plates, quartz or mirror, and then the light falls here and then we have this pattern that is light is reflected back and forth inside this cavity and whatever the outcome we have this is essentially the transmitted light and this is a source extended source plane s. And if I place a lens here, it will converge the entire light to some point and they will superimpose all these light rays or waves essentially superimpose on this point P. This is the structure. So if I have a source with a circular pattern then obviously here we will see this kind of structure that is the typical pattern for Fabry-Perot, if you have a circular kind of structure. Now if I calculate what is the intensity of this transmitted light that we had already calculated in our previous calculation, I transmitted the intensity of the incident light divided by some factor and that factor contains something called F, which is the coefficient of finesse. This is sine square delta by 2, where delta is the consecutive path difference, the path difference between the consecutive rays. So, delta is the phase difference, it will be multiplied by k 0 into the path difference delta where k0 is 2 pi divided by lambda. And this phase difference was 2 of nf multiplied by thickness t and cos of theta t. So, where was the cos of theta t was this angle?
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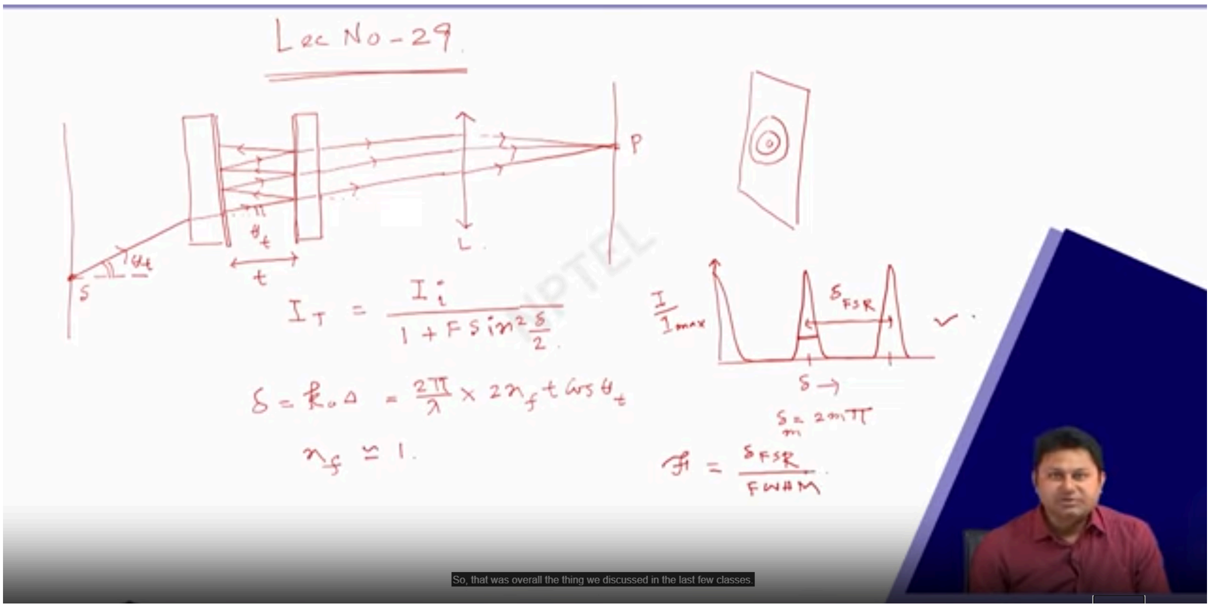

This angle was theta t, if it is air and this is a thickness we had, if this is air then n effective is basically equal to 1, and based on that we do all the calculations. Another issue that we also discuss is the transmittance of the system and that is the transmittance spectra. This sharpness depends on the value of f that we describe this is over delta, this peak arises when delta m and that is 2 m pi this is the point where this peak arises and this is basically I divided by I max, which we plot and the separation we called the free spectral range. We define something like this and also this width we calculate full-width half maxima and then we define another parameter we call simply finesse, which is f and that is delta free spectral range divided by the ratio between these two, then the full-width half maximum of this structure. So, that was overall the thing we discussed in the last few classes. So, today we will extend this idea to understand what is called the resolving power or chromatic resulting power of the Fabry-Perot interferometer. So, before going to the detailed calculation. So, by definition actually. So, this chromatic resolving power by definition, is simply lambda divided by delta lambda, where delta lambda is a deviation, and is a minimum wavelength difference. So, delta lambda is the minimum wavelength difference that can be just resolved at the wavelength lambda. So, the point is suppose we have 2 lambda very closely separated one is a lambda and another is a lambda plus, say, the delta lambda that is the two wavelengths we have and we use this Fabry-Perot interferometer to find out what is the minimum separation. So what will be the outcome here? If we have this lambda separation in the transmittance plot we have a peak here at some Delta. And this delta, if you remember, this delta is, this is the delta m at which we have the peak. And this delta m depends on the wavelength that we are going to use. Now, for one particular wavelength, we have this transmittance. On the other hand, if we have delta plus lambda plus delta lambda, that is another wavelength. Say this is I write lambda 1 for simplicity and this is lambda 2. So, 2 wavelengths are there where lambda 2 minus lambda 1 is my delta lambda. So, in principle what we have is I have another delta prime m where we will get the transmission of that data. My drawing was not good, but this is a very closely separated peak. Now the point is what will be the condition for which this separation can be calculated.

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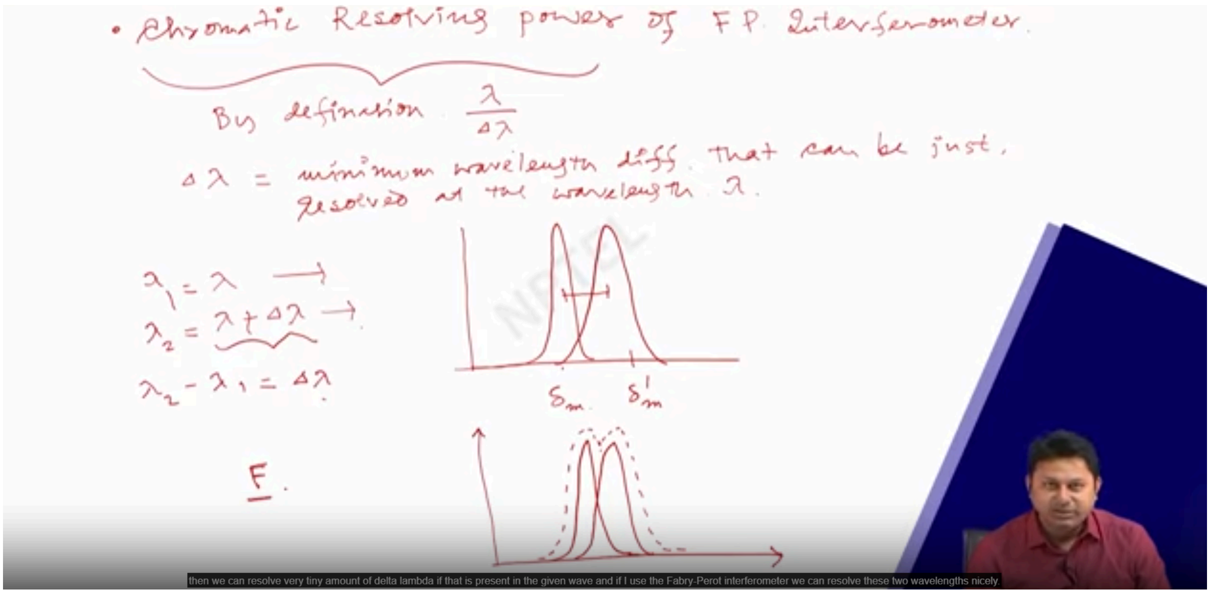

So these two things will when we see together what we are going to see let me draw it once again in a nice way. So I have one transmission and I have another transmission. These two wavelengths are placed so that these two transmittances are there in this way for two different wavelengths. So, the resultant will be given by this dotted line where we have a dip here and we will see the resultant distribution of this intensity in this way. Now, you can see that if these two peaks are very close to each other it is very difficult to resolve the two transmittance lines or these two transmittance spectra. So, that means if they are a bit apart then this dip which is here will be a little bit lower. And, then there is a condition called Rayleigh's criteria, and based on this Rayleigh's criteria one can calculate what should be the value, and one can calculate what should be the condition for resolving those two different lambdas. So, that resolving power we will show that in fact proportional to that specific parameter F, it is proportional to this specific parameter F. So, that means if the finesse is very high then we can resolve a very tiny amount of delta lambda if that is present in the given wave and if I use the Fabry-Perot interferometer we can resolve these two wavelengths nicely. So, let's see how one can resolve that. So, the starting point should be this. So, the transmitted intensity which obviously would be a function of delta will be the intensity that we have incident intensity plus 1 divided by 1 plus F sin square delta by 2. So, the delta was k naught, this k naught was 2 pi by lambda and delta was 2 of nf then thickness and cos of theta t nf is normally 1 and theta t is small. So, cos theta t is roughly equal to 1. So, this is the phase difference for the successive transmitted ray. This delta is the phase difference between transmitted successively transmitted rays. So, as I mentioned, nf is nearly equal to 1 here and theta t is small. So, that makes cos of theta t of the order of 1. So, nf is equal to 1 cos theta is equal to 1. So delta is simply 2 pi divided by lambda and then this quantity will be simply 2 of t where t is a thickness. Now, if I write delta lambda here the deviation of these changes of phase with respect to lambda then we should have minus of 2 pi divided by lambda square and then 2 of t multiplied by delta lambda or in other words delta lambda divided by delta will be equal to minus of I can write 2 pi t. So, delta is 2 pi multiplied by 2 t divided by lambda. So, that I can use here and that becomes delta here.

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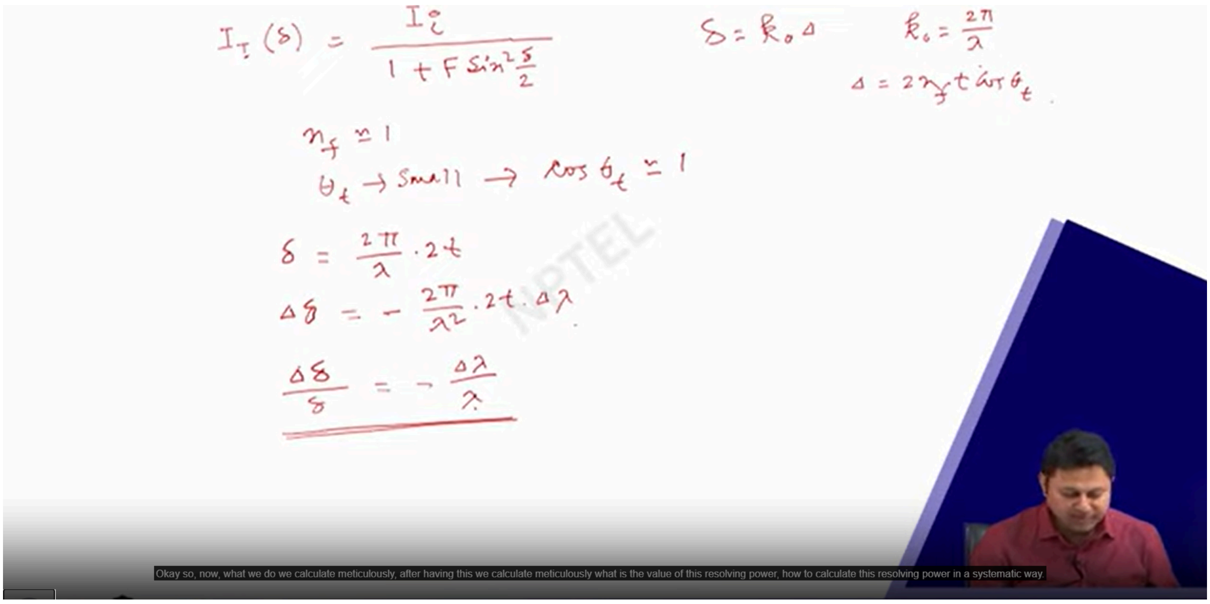

So, it is simply delta lambda divided by lambda. So, the negative sign suggests that, the negative increment. So, we have this quantity that is the deviation. So, the resolving power is lambda divided by delta lambda. So, the delta divided by d delta will be equivalent to the resolving power. So the negative sign suggests that if we increase the lambda in the positive direction, on the positive note, then there will be a change of delta in the negative side. So that is the only thing that we have here in this expression. Okay so, now, what we do is we calculate meticulously. After having this we calculate meticulously what is the value of this resolving power, and how to calculate this resolving power in a systematic way. So, let us calculate that. So I need to draw that same figure in a nicer way. So we have this transmittance curve, this is one and I have another one. So this is the central point and we have another somewhere here to say.

Okay let me draw it, so this is the point where they will cut, so this point should be in the same line. My drawing is not good. That is why it shows a different place but anyway, the resulting one will be, let me try to draw in a different color, maybe blue. So the resultant line will be like this, the resultant intensity will go up because of these two intensities, where the intensity of 1 and intensity of 2 will add up because these two incoherent lines when superimposed will be simply the addition one can expect and we are doing that only. So, let me draw first then I will explain once again. What we are doing here is, we are drawing the resultant intensity distribution. This is the resultant intensity distribution drawn in the blue line now noted here. So, for this resultant line, this is the minima and that line on top of that is the maximum I write Imax okay and now these two maybe I can define as A and B, so this is A and this is B, AB defines these two spectral lines. No, this AB maybe I can defined as a peak point, then it will be meaningful. One by one I need to do it. So let us define this point and this point as A and B. The cutting point here lets us define C, this point lets us define D and this point lets us define this as E. Okay now, we know that this plot is a i delta plot, i is a function of the delta that I am plotting and this side it is delta. So, this value is delta m, and this value is where we have another maximum for another wavelength that should deviate slightly. So, I write delta m plus this. This deviation should be there because the condition will change for different wavelengths for this maximum. Now, here we can write that the intensity at point A will be equal to the intensity at point B because these two are of the same height. What is I max? Then let me write it, I max let me do it here, I max will be the maximum intensity for the left-hand distribution which is A IA plus this amount of intensity which is here plus ID or this is the same thing as IB plus IE because ID intensity at the point of D and the intensity of point of E they are again same, these two points. My drawing is bad, that is why it looks like they are not the same, but they are in the same horizontal line. So, these and these are in the same horizontal line. So, what is IA? Then similarly if I want to find out I min, this line is I min, which is 2 of IC because IC for one wave, one spectral line intensity of one spectral line and another spectral line they superimpose and that is why we have a large intensity and that is where I min. So this is the condition we have, this is now what Rayleigh's condition says or Rayleigh's criteria of the resolution of two closely spaced lines is that this is the resultant intensity we calculate. So, this minima in the plot, say, this is the plot. So, this minima, should be at least the highest value of this minima should be 8 by pi square I max. Whatever the maximum value we have, if this is I max then I min should be 8 by pi square of that quantity. So, that is the condition Rayleigh's condition one can have. So, exploiting that condition I can write. So, in general, we have I at some point delta is I naught

1 plus F sin square delta by 2 where delta m is 2m pi that we know. So, what is IA here? IA is calculated at delta m and that is my I0. Similarly, IE or ID whatever is I0 divided by 1 plus F sin square. Here, this is calculated at d delta divided by 2 because I am shifting here by delta. So, as per the formula, it should be delta by 2 And IC is in between these two points. So, IC will be I naught divided by 1 plus f sin square d delta divided by 4. Okay so, now I put the condition I know everything. So, I max, sorry I min the Rayleigh's condition 8 by pi square I max that is the condition and that condition I now calculate. So, what is I min? I min is 2 of IC and 2 of IC I already calculate here. So, it is 2 I naught divided by 1 plus F sin square, which is equal to 8 by pi square into I max, I max again it is. What is I max? Imax is IA plus ID or IA plus IE. So I calculate and it should be I naught because IA is I naught 1 plus, 1 plus F sine square delta by 2. So, that is the equation we have and our goal is to find out what is this big delta, this deviation delta. So, this is a transcendental kind of equation. So, on the left-hand side and right-hand side if you plot there will be a cutting point, and that cutting point basically gives you the solution you can simplify also one can. So, like sin square d delta divided by 4 for small delta one can simplify this and sine square. One can simplify these two and put it back in the equation and then they get a little bit simpler expression and simple algebraic expression. So if we do some algebra then it simply comes out to be like this. Let me just write it. I suggest the student please do that on your own to check whether what I am writing here is correct or not. So as I mentioned a simple graphical solution one can have. And if one does the graphical solution, he will get some value like 4.147 divided by root over of F. Now, you can see that delta has become a function of S. Now, as per our calculation, the resolving power R that is d by delta and it is essentially 2 m pi because delta is 2 m pi divided by 4.147 and root over of F. So now, one can further simplify it to get something like this. So, like 1.515 this 2 pi divided by 4.147 further one can resolve, but the point is here we can see the expression of the resolving power and this expression of the resolving power suggests that it depends on 2 parameters. One major parameter is F.

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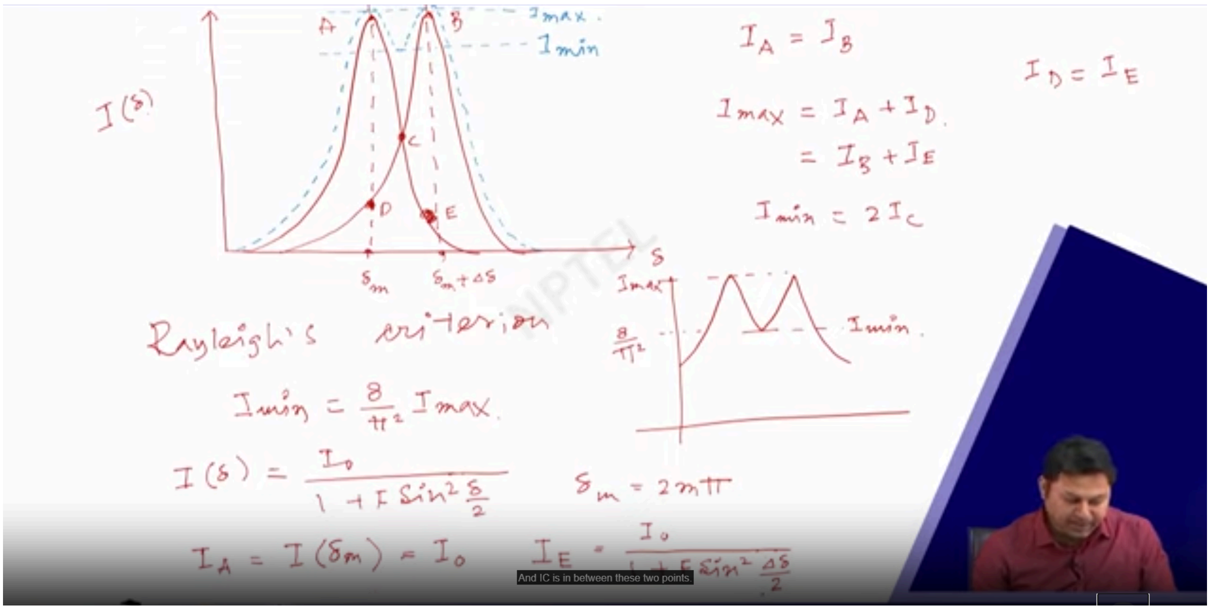

And, that is proportional to the root over of F, in the earlier we mentioned that it is proportional to F, but it is coming out to be root over of F. So, if the finesse is high, the resolving power is high and also the order if I calculate this for higher order then also there is a greater probability that we will get the better resolution of these 2 lambdas. So, that's all for today. We already discussed in detail the working principle of the Fabry-Perot interferometer. Now, we further discuss its resolving power and also I suggest that you please check the assignment problems, where several assignment problems will be given on how to calculate this Fabry-Perot interferometer by calculating the wavelength etcetera. That is the usual experiment that we do in the labs. I hope you understand that. So, today we have completed a major part of wave optics, which is the superposition of light that is interference phenomena. In the next class onward we start the new topic diffraction. So how the light will go to diffraction and the diffraction phenomena is what we are going to describe. So with that note, I would like to conclude here. So hope you enjoy the first part of this course where we have discussed interference. Now in the next part, we start another major topic which is diffraction. Thank you very much for your attention. See you in the next class.

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