WAVE OPTICS
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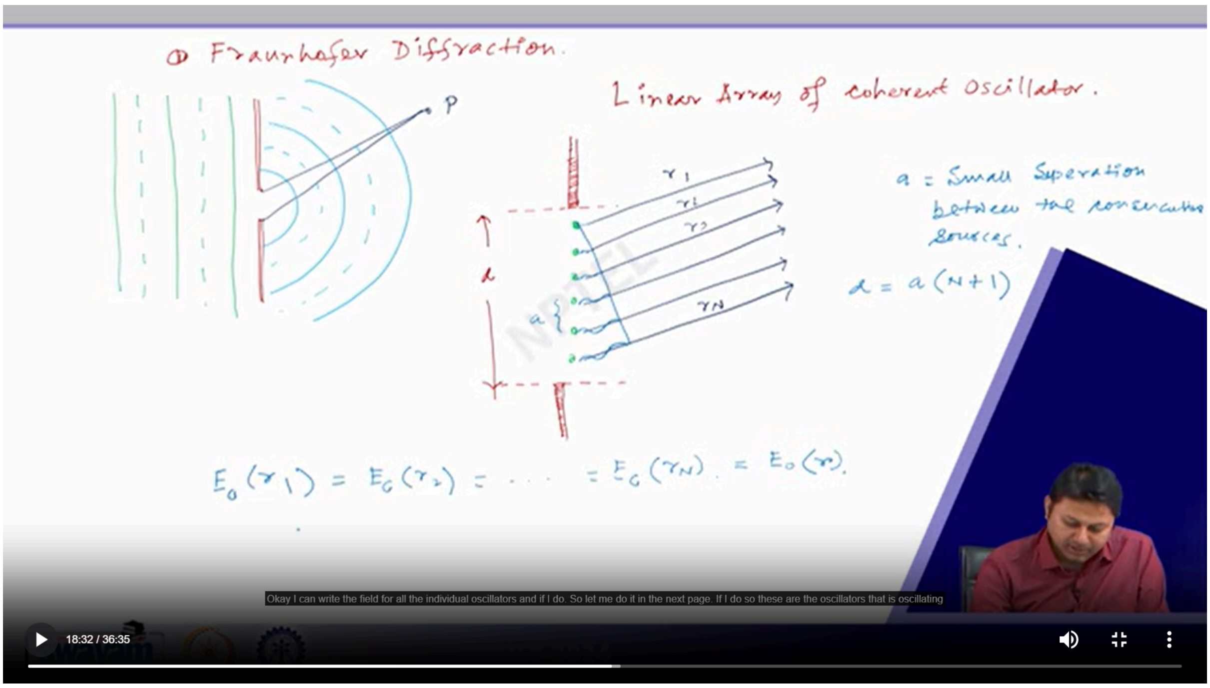
Lecture - 32 : Fraunhofer Diffraction

Hello, a student in our wave optics course. Today we have lecture number 32 and today we will start Fraunhofer diffraction. So we have lecture number 32 and we are going to discuss the Fraunhofer diffraction. As I mentioned in the last class, the diffraction phenomena can be in principle defined into two broad classes. So, diffraction of the light or the diffraction phenomena can be divided mainly into two classes one is called the Fraunhofer diffraction and another is called the Fresnel's diffraction. So in the Fraunhofer class diffraction and the Fresnel diffraction, there are differences here. In Fraunhofer class diffraction what happened? The light ray that is coming is assumed to be infinity. So suppose we have a light source where this light is coming and we place a lens here such that this light can be considered to be a parallel ray and then it falls on some aperture and then again we place a lens here and then we have a screen where we see the pattern. So this is the focal length of the lens and this is the screen normally defined by Sigma and this is the light source and again light source is placed at the focal point of this length and this is the aperture defined by Big Sigma. So, this is the structure we have for Fraunhofer diffraction. So, both the source and here at infinity source and screen are placed at infinity. This is the structure of Fraunhofer. So structure, setup-wise is a bit complicated we can see, compared to Fresnel's diffraction but calculation the Fraunhofer diffraction calculations are easier. Here, what we considered the light ray that is coming from the source is a plane wave, not a spherical wave because we considered the light to be coming from an infinite distance. So everything we calculate is a plane wave. So that makes lights much simpler. On the other hand in Fresnel's diffraction type, the structure is this. So I have a source here, so we have a demarcation. So here we have a source at some point say S and we have an aperture here and light is simply allowed to pass through this aperture without any specific setup and we have a screen that is sigma and this aperture that is big sigma and this is simply the structure. So when either source or screen or both are at a finite distance from the aperture then the diffraction pattern corresponds to the Fresnel's class. So this is the Fresnel's class diffraction and this is the Fresnel's class. So here, instead of having plane waves, we have spherical waves. That is the major difference we have. In this case, in the Fraunhofer diffraction case, we have a plane wave that is incident on that. And in Fresnel's case, we have the spherical wave that is hitting this aperture. So if I write in terms of, draw in terms of you know the wavefront, so this is the wavefront that is coming here and hitting the structure and here this is the wavefront, this is the way the wavefront will hit to the aperture and then move. So here we have a circular wave front that is hitting here. We have a plane wavefront that is hitting the aperture. These are the major differences and because of that the mathematical calculation will be different here. So now concentrate on the first, we concentrated on the Fraunhofer kind of diffraction because it is easier to calculate. So let us concentrate first on the Fraunhofer diffraction, F, what we try to calculate is what happens when the plane wavefront hits a simple aperture. So the structure is this. So I have a plane wavefront that is coming. So let me draw the first structure. So this is the aperture we are talking about. And a light ray that is coming as a plane waves up and down. This is the wavefront that is hitting here and then what happens I try to find out the intensity at some point P. So this is the point at P where we try to long-distance P where we try to find out the intensity. Now from here essentially the wavelet that will come this way. So, if I want to find out the intensity at any point, then according to Huygens Fresnel's principle it should be the superposition of all the contributions that we have over this play plate. So, if that is the case, then we can consider this problem simply as a linear array of coherent oscillators. What is the meaning of that? That means let me draw again, so we have this aperture. I magnify this portion this is my aperture and suppose d is the separation of this aperture. So the plane wave that is hitting here will produce the independent source here and each source. So these are the source points. Each source behaves like a coherent oscillator like this. So these are some sort of linear array of coherent oscillators and at some point r if I try to find out. So here it is emitting this oscillator, will emit the waves and it will go like this and if it is r1 and r2 r3 rn, n oscillators are there. So, this is the distance that is I mean this is the way it is going to emit. Now there will be a superposition. So what I need to do to find out what is the intensity pattern at some point P? I need to superimpose all the contributions as per Huygens Fresnel's law. So I need to consider the relative phase difference also because when I calculate the superposition then that is the way we did. So these things, so if I draw this, here this is the way it moves, the waves are moving and the separation between the two consecutive sources is A. So, A is a small separation between the consecutive sources. So, this is the structure we have. Now, d is the aperture that we mentioned. So, d should be if there are n numbers of oscillators, d should be multiplied by n plus 1, where n is a number of sources. Okay, I need to write it properly n plus 1. So the amplitude of the separate waves arriving at the point P is almost the same because of all the cases. (Refer to slide time: 18:32)



So I can write that E0r1 is equal to E0r2, E0rn which we call in general E0R. Okay, I can write the field for all the individual oscillators, and if I do. So let me do it on the next page. If I do so these are the oscillators that are oscillating and I can write it as E1 which is E0 which should be a function of r e to the power of so all of them are emitting plane waves. So I write ek r1 minus omega t, this omega is the frequency. Similarly, E2 will be E0 r e to the power i kr2 minus omega t, E3 will be E0 r e to the power of i kr3 minus Omega t and so on, and essentially I get P in as a0 is e to the power of i krn minus omega t. So these are the plane waves that are emitting from this individual oscillator one two and up to n and now I need to find out the relative phase difference between all because that creates the issue. So these consecutive points have a separation a, if it is r1 and r2, if this angle is theta. So I have r2 minus r1 as a sine theta which we call as delta. Similarly, r3 minus r1 that is if I add another point here it should be 2 of a of sine theta and that is 2 delta. In a similar way if I go up to n then rn minus r1 that should be n minus 1 a sine theta, which is n minus 1 delta. So, now the total field at some point P the total field at P that is E and that is the superposition of all the plane waves, j goes to 1 to n that is essentially the Fresnel's size in principle, that whatever the field you want to find that should be the contribution of superposition of all the waves that is emitting. So if I do the sum, let me go to the next page. Then E will be E naught r, I can take e to the power of i kr1 minus omega t common then I have the rest of the term. So I have one term I have, It will start from day 1, then I have e to the power of i of k then r2 minus r1, then I have another term e to the power of i k r3 minus r1, and so on up to e to the power of i k r n minus r1. So I just add all the terms there and after that I get this so, I can write it as for this time being E0, e to the power of i k r1 minus omega t and then I have 1 plus e to the power of i k delta plus e to the power of i 2 k delta and so on e to the power of i n minus 1 k delta. So, if I take x. as e to the power of ik delta, then this is simply 1 plus x plus x squared plus x cube, up to x to the power of n minus 1. The sum of these things is simple. So it is simply x to the power of n minus 1 divided by x minus 1.

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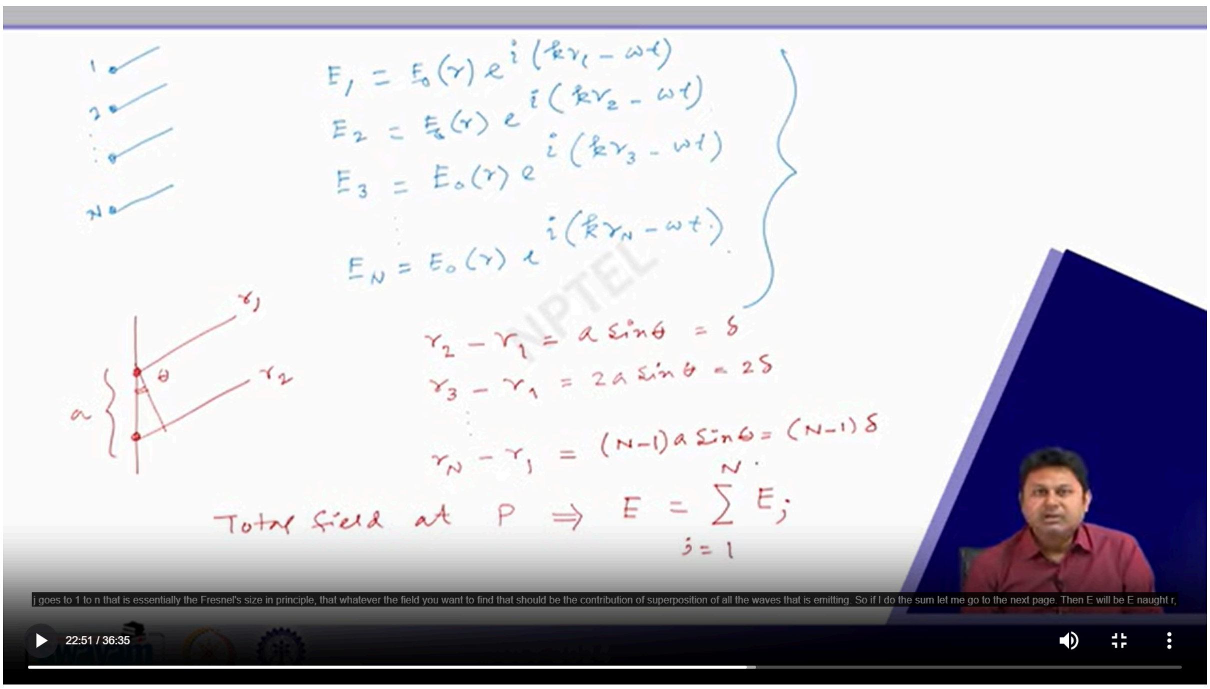

So I can have E as E naught e to the power of i k r 1 minus omega t then multiply by e to the power of i n k delta, minus 1 divided by e to the power of i k delta minus 1, this is the term we have in by adding everything. So that we can write in a more compact way. So it is nothing but E0 e to the power of i k r1 minus omega t, then we have e to the power of i, then n by 2 k delta and here I have e to the power of i k by 2 delta, by doing so I can write it as e to the power of n by 2 k delta minus, e to the power of minus n by k 2 delta. So, that leads to 2 i sin, and here also in the denominator, I have 2 i sin. So, essentially this quantity will be sin of n by 2 k of delta divided by sin of k by 2 delta. So, I have a very compact, kind-looking term. So let me write del k by 2 delta, it is k by 2 and delta we know this is a of sine theta and so that value will be 2 pi divided by 2 into lambda multiplied by A sine theta. So, that quantity is pi A by lambda and sine theta. So, let phi be equal to k delta by 2, which is this quantity I write it as pi which is equivalent to pi a divided by lambda then sine theta. If that is the case then what we get is E, total E is equal to E naught e to the power of i k r1 minus omega t. Then we have e to the power of i n phi k divided by e to the power of i phi multiplied by sine of n phi, whole divided by sine of phi, where phi is k delta by 2, this is just a, so, intensity will be proportional to mod of E square. So, that is the thing I want to find intensity and then I can write that the total intensity at point P should be equal to some intensity I0 and then say I0 tilde some amplitude and sine square n phi divided by sine square phi. So note that d is equal to n plus 1 into a that is constant and when n tends to a very large number say infinity then a almost goes to 0, that is the separation becomes very very tiny and d will be nearly equal to n a, when a tends to 0, that is. d tends to, so phi is equal to k delta and inside the delta, we have sine theta and if a is small, then what happened is that phi is small, so, that makes sin phi equal to phi. So, then the expression if I write should be, i equal to i at some point theta equal to I naught tilde sin square n phi, divided by Phi square, where n Phi is pi into a lambda, multiplied by n sine theta, which is essentially pi d by lambda sin theta.

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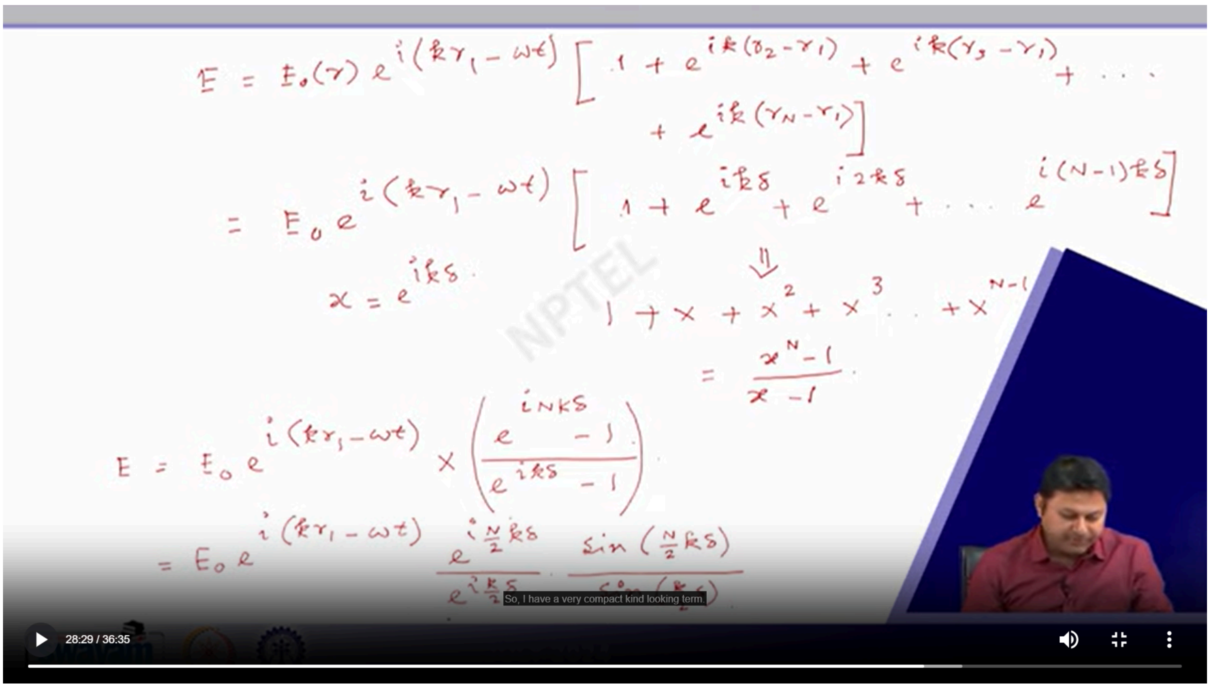

So that is the expression one can have and we will stop here because that is an interesting kind of expression of the intensity distribution that one can have. So what do we have? So if I just for the timing we draw that if this is the obstacle or this is the aperture and if the light that is coming from is trying to fall on that and using some lens here if I project the intensity distribution as a function of theta the distribution should be like this. So here we have a maxima but inside the geometrical shadow, we have some distribution like this. So in the next class, we are going to discuss these things in detail. So this is the region where we are supposed to have, so you can see that roughly this is the region, where by ray optics we can get intensity but this region we should not get any kind of intensity because this is a geometrical shadow but because of the phenomena of diffraction as the light can bend from this aperture we still get some kind of intensity here in this region. So we are going to discuss in detail in the next class what kind of intensity distribution one can see and how one can form this intensity pattern etcetera. So the intensity distribution equation we derived today. In the next class, what will we do? We will discuss more about the pattern and use a different expression, an easier expression to show the same equation the same thing. So with that note, I would like to conclude today. So hope you understand these effects and in the next class we will check this expression as I mentioned in a new way. So thank you very much for all of your attention and see you in the next class with more discussion. Thank you.

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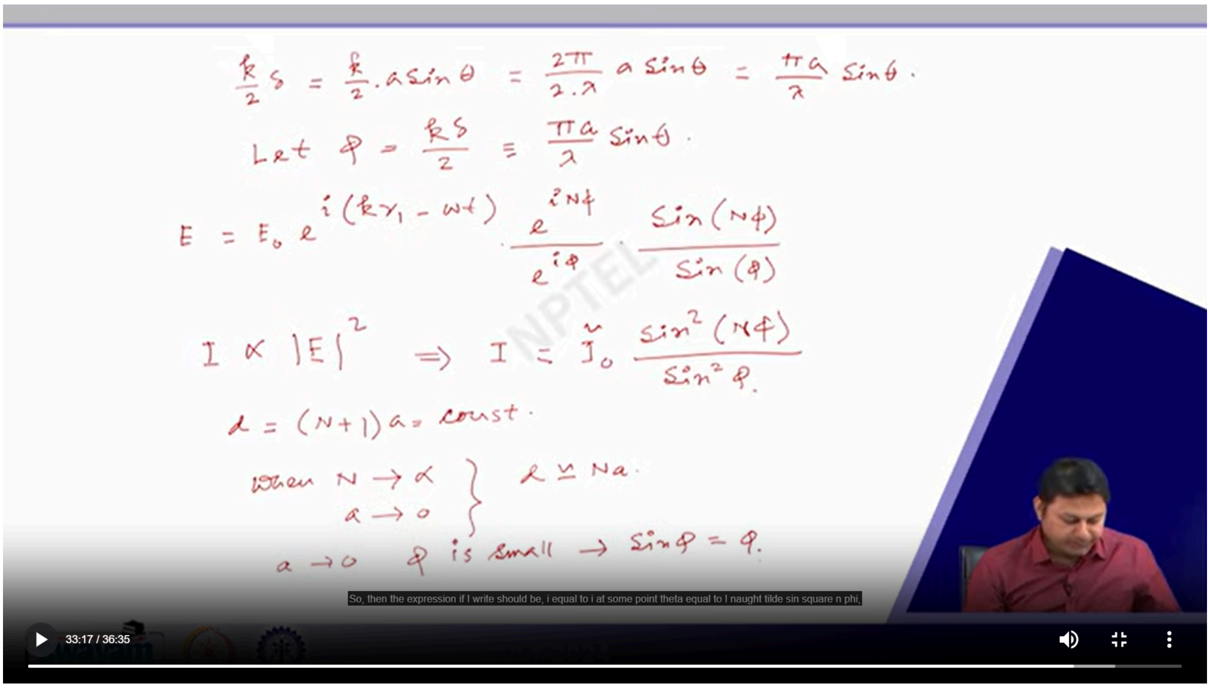

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