WAVE OPTICS

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Lecture - 35 : Double–Slit Diffraction

Hello, students, welcome to the wave optics course. So today we have lecture number 35 and in this lecture, we are going to discuss the double slit diffraction problem. But before that let me remind you what we had so far, if we have an aperture in the form of a single slit and allow the light to fall, these are the wavefront that is coming and these are the plane wavefront because we are dealing with Fraunhofer diffraction pattern. Then at some point, if I place a screen we are going to get a diffraction pattern like this, we have a maximum intensity here, and then in the geometric shadow region, we have a distribution of the intensity in a specific form. So this is the intensity distribution I which is essentially a function of theta and that is I naught sine square beta divided by beta square. This is the form we derived where beta is a quantity which is pi of d divided by lambda and sine theta, where theta is an angular distance. For example, if I want to find out the intensity at angle theta, so this angle is the theta angle and this is the point where this intensity is there. So, if I put the value of the theta, then I am going to get the value of the beta, and based on that I can calculate what is the intensity of that particular theta point, where d is the separation, this is the d. the separation of the width and the fringe if practically the fringe will look like this kind of lines so I have a central fringe like this and then we have another fringe here and so on This is roughly the fringe pattern. Difficult to draw in principle but I am trying my best. So if you do the experiment then you are going to see patterns like these. And if I draw the intensity distribution, it should be bright, then dark, then bright, dark, then bright with reducing intensity like this, and so on. So this peak corresponds to this bright, this peak corresponds to this bright and this dark region is this region, and that distribution is over theta.

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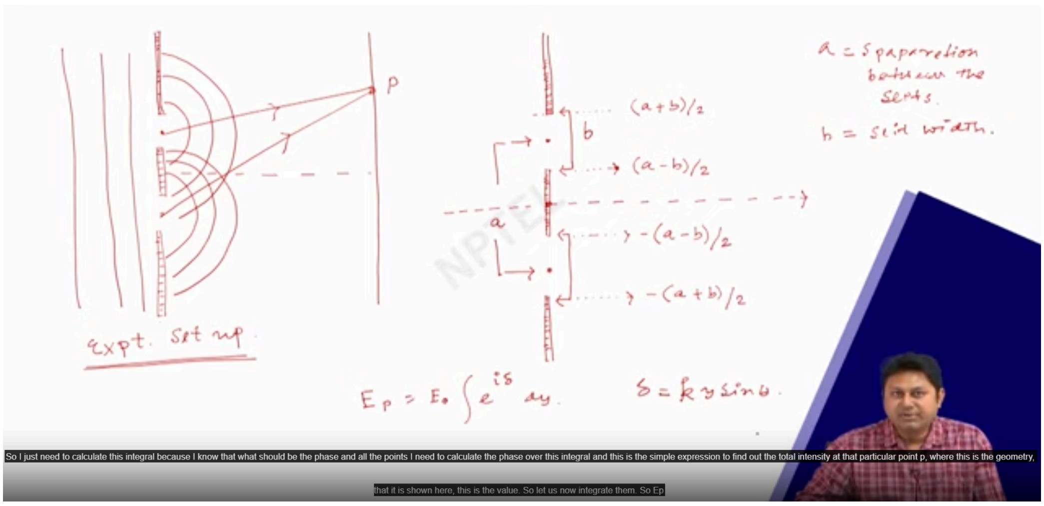

So we calculate from the first principle everything and then derive this expression. So today we are going to extend this calculation not only for a single slit but a double slit. So double slit is a very very important experiment in physics and what happened let me describe. So this is a double slit set up and plane wavefronts are coming and then it reaches to this point and from here we have wavefronts, that is emitting and these two wavefronts that are emitting here will going to interfere over the log distance, and if I want to find out some point at p the intensity we are going to see some intensity pattern. So diffraction and interference both will take place together here and we are going to see a pattern here on the screen. So that calculation we will do today. From the first principle the way we calculate for a single slit, we will continue with this and then try to understand what is going on. So, let us now draw again the setup. This is the experimental setup and schematic diagram to show how this experiment is happening. We have a light source and then we place this aperture double slit and then at some point, we are going to see some distribution and here we like to draw the overall structure. So this is basically the double slit, this is the opaque region, this is slit number one and this is slit number two and we need to define the coordinates properly to do the calculation. So I am defining this dotted line as my x and along this direction perpendicular to the dotted line is y and these slits are exactly on the y axis. So this point is, so let us first describe what is the separation between two slits, A is a separation between two slits. So, this is A center to center, and individual slit width from here to here, it is B. So, that is our definition that A will be the separation between two slits and that we define by this, that the central point to the central point of this slit is A and B is individual separation in the width of the individual slit.

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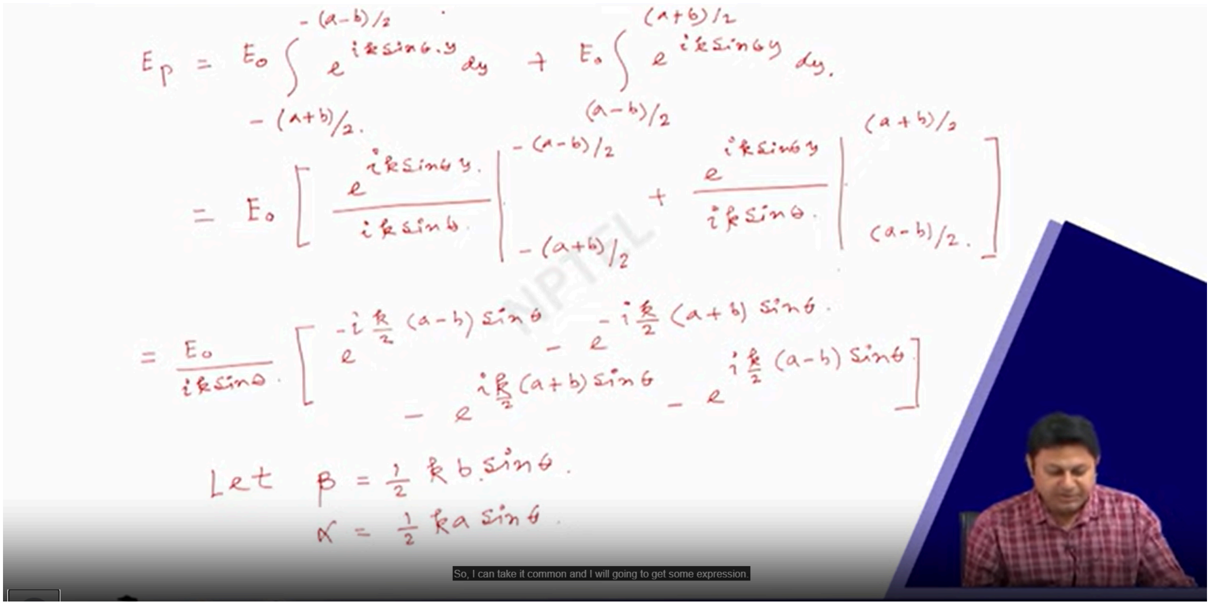

So now we need to just understand what the coordinates are and then we are done. So if this is, let me write down the coordinates of this point, so if this is y equal to 0 then the coordinate of this point. So from this to this is a by 2 and from here to here from this region is this is b by 2. So, this is a by 2 minus b by 2. So, the coordinate of this point is a minus b divided by 2. Similarly, the coordinate here is minus of this point, is minus of a minus b divided by 2. It is important that once we fix the coordinate, then I can integrate over, and then I will find out what the intensity is at some point. But before that the geometry we need to understand how these coordinates are there. Based on the definition we have a and b, this point here will be from here to this point is a by 2 and I add this which is b by 2. So it is simply a plus b divided by 2, this is symmetric. So the coordinate of this point will be minus of a plus b divided by 2 so all the points I define. Once we define all the points so I need to write a is the separation say between the slits and B is individual slit width. So, obviously, E p is the field at point P, if I write E p it is some amplitude E naught and then it is the integration of all this aperture to e to the power of i delta and dy. Note that here delta is a phase difference and that is k y of sine theta. So I just need to calculate this integral because I know what should be the phase and all the points I need to calculate the phase over this integral and this is the simple expression to find out the total intensity at that particular point p, where this is the geometry, that it is shown here, this is the value. So let us now integrate them. So Ep will be then simply E naught integral of minus lower point a plus b by 2, to other points a minus b by 2 e to the power of i k sin theta into y dy plus another contribution this is for slit 2 and for slit 1 it is E naught then lower limit, a minus b by 2, upper limit a plus b by 2 e to the power of i and then k sine theta y d y.

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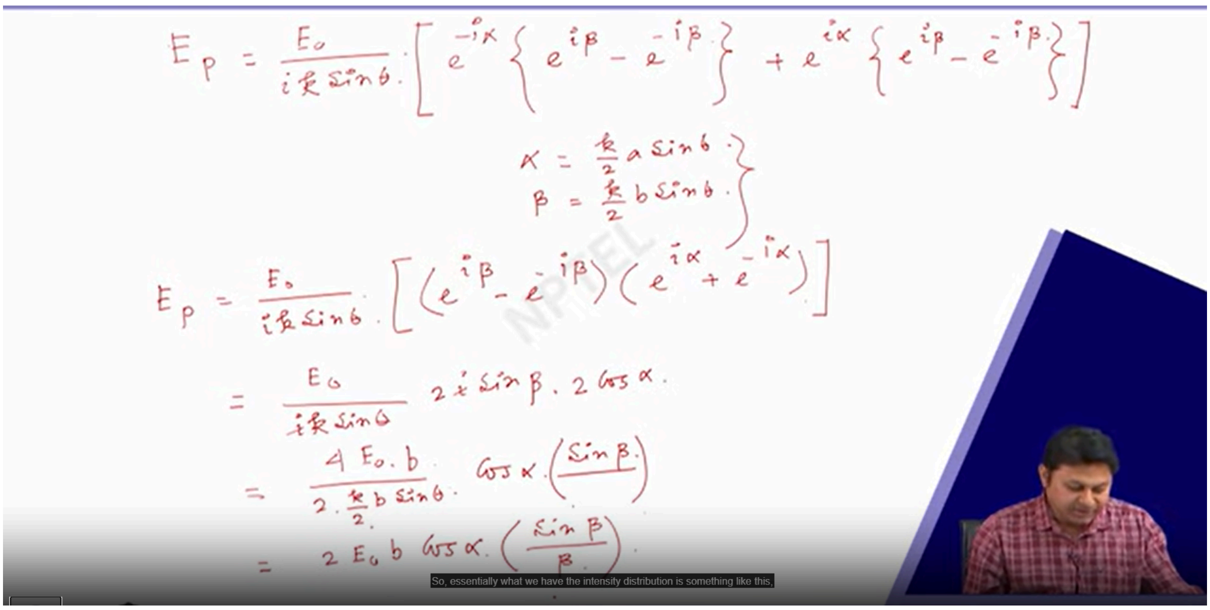

So, the contribution of the slit 1 and the contribution of the slit 2 are simply added by this integral, and whatever the result I get that should be the total field at this point according to

Huygens Fresnel's principle. So, let us do this integral which seems to be not that difficult. So, E naught I take common then e to the power i sine theta y dy it should be simply e to the power of i k sine theta and y divided by i k sine theta need to be calculated at these 2 points, this is the first integral, second integral similarly same thing, only the limits are going to change here. Y So, it should be E naught divided by this I k sin theta, I can take it outside and then I put all the values. So, here we have i minus k by 2. and then a minus b sine theta that is the first term minus e to the power of minus i k by 2, a plus b sine theta plus e to the power of i k, then a plus b k by 2 sine theta and finally minus e to the power of i k by 2, then we have a minus b sin theta bracket close. Okay now I simplify because there are many terms a plus b, a minus b sin theta k by 2, these terms are present several times. So I put a few names, so beta is half of K B sine theta and alpha is half of K A sine theta. So we have written this expression in terms of beta and alpha then it should be a little bit compact. So then Ep will be simply E naught divided by ik sine theta and then if I rearrange, if I look carefully at this expression. So, what I can do that, e to the power ik by 2 a sin theta if I take common from here. So, this term is present here also. So, I can take it in common and I will get some expressions. Similarly, e to the power ik a sin theta, if I take common then I will get some expression like that. So, if I do then, I simply get this e to the power of i alpha minus i alpha rather if I take common then I will get something like this e to the power of i beta minus e to the power of minus i beta. Then if I take e to the power of i alpha common, then I will get the same term back with the power of i beta minus e to the power of minus i beta, with the fact that alpha and beta and that let me write down the alpha and beta once again alpha was k by 2 a sin theta and beta was k by 2 b sin theta. So Ep, then it is E naught divided by i k sine theta, and that quantity I can simply take e to the power i beta, e to the power minus i beta again common so, let me do that actually, it should be e to the power of i beta minus, e to the power of minus i beta multiplied by e to the power of i alpha, plus e to the power of minus i alpha. So this gives e naught i k sine theta, this quantity e to the power i beta minus, e to the power minus of i beta simply turns out to be 2 i sine of beta, and the other term will be simply 2 of cos of alpha and I can have this four of i i can be canceled out here, this i this i and I have four of e naught divided by two of k by 2 and I put another b and sine theta here and I put this b here and then I get cos of alpha multiplied by sine of beta, divided by beta. I just want to put this sine of beta divided by beta term. So that I can write in a compact way. So this k by 2 b, these things are basically bit, okay, I made a mistake here so, I'll put these things later because still this is sin beta if I put this beta here then it will be like this. So, next line it will be 2 of E naught B, then it will be cos alpha and now I write sin of beta divided by beta because k by 2 b sin theta is beta which is already defined here. So, essentially what we have the intensity distribution is something like this, intensity distribution should be proportional to the mod of E p square. So, the intensity at point p will be this. So, from here I can write that the intensity at this point p should be I naught, then cos square alpha multiplied by sin square beta divided by beta square, that is the thing I have. So, that is the intensity distribution or intensity pattern we have, where alpha and beta are defined several times. So, let me write down once again because these are important quantities. This is k by 2 a sin theta or this k by 2 sin theta sometimes written because k is 2 pi by lambda. So, it is pi a by lambda sin theta and beta which is k by 2 b sin theta, it is again k is 2 pi by lambda. So, if I put this value 2 pi by lambda, then it should be pi B by lambda sin theta, so, that is alpha and beta. So, now it is time to draw what this distribution should look like. So, I believe it is proportional to cos square alpha, and also it is proportional to sin square beta divided by beta square. So, I like to show the distribution here. So, the sine square beta divided by beta square distribution is known because that is what we did for our single slit experiment and cos square alpha is simple. So, let me first draw the cos square distribution. So this will be the distribution of the cos square this side also something like this, this is theta, this is 0 point and that is the distribution and this is this value is 1 because cos square alpha should have a maximum value of 1 it should not have a negative value, so this value is 0 and I am plotting intensity here, which is a function of theta, but I am plotting individual functions. Now, what does the sin square beta will look like? Sin square beta on top of that. If I plot, let us plot in a different color. So, it should be like this. So, this blue line is sin square beta divided by beta square plot and the red line, this one is cos square alpha, is the plot. So the red line and blue line, now they are situated as a multiplier. So they are situated as a multiplier means, the original expression should be something like this. The original, the exact, or the actual distribution of the intensity will be this. We have an envelope here and so on this side also we have small things rapidly diminishing to 0 and inside this envelope we have this distribution. So this is the intensity pattern one can expect a few interesting aspects will come. Maybe we will discuss this in the next class. Let me draw the intensity distribution here. Inside the envelope, we have small peaks like this. So, in other words, there will be a modulation of the individual peaks and this modulation will be taken care of by sin square beta term sin square beta by beta square term and inside the term that we have is due to interference. So here the interesting thing that we need to note is there are two contributions one is cos square and another is sin square. The cos square contribution is coming due to interference of these two lights that are coming from two slits and the envelope that we are getting here is basically coming because of the contribution of the diffraction. So, this will be the final form of intensity over this theta and we will see in the next class that because of this overlapping of these two structures, there is a possibility that we are going to miss a few of the strict terms, which we will call the missing order. We will calculate this missing order. A few questions also may be expected in the exam or in the assignments due to this missing order problem. So, with that note, I would like to conclude here. We are going to discuss more about this spectrum of two-slit experiments in the next class and then we discuss in detail how the missing order can be calculated etc. So, thank you very much and see you in the next class.

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