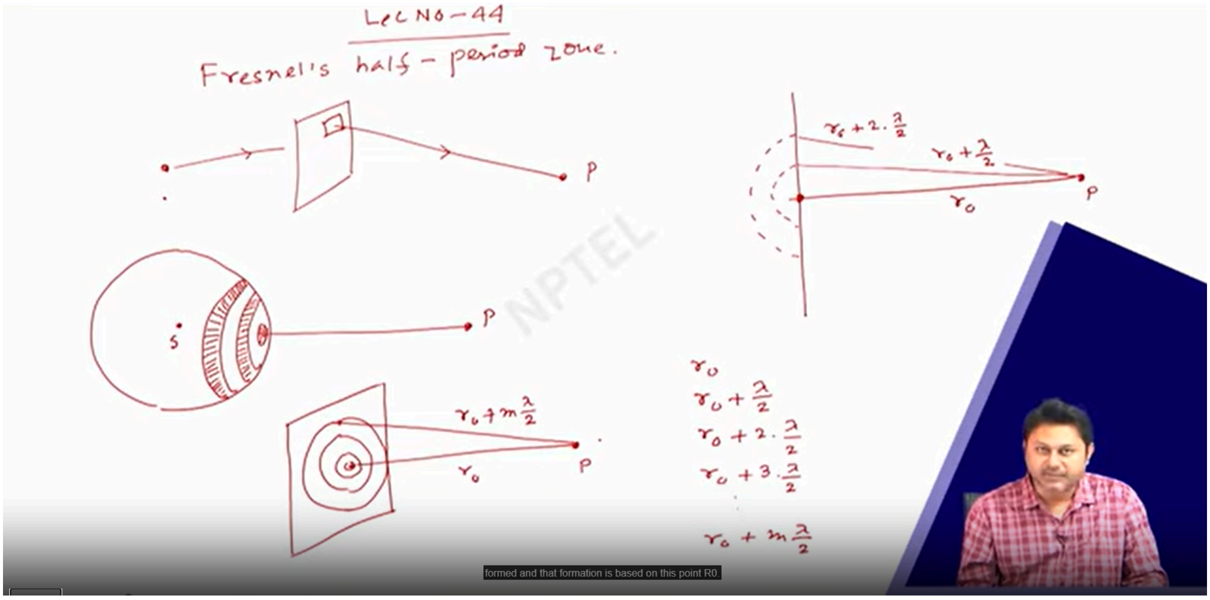
WAVE OPTICS

Prof. Samudra Roy
Department of Physics
Indian Institute of Technology Kharagpur

Lecture - 44: Fresnel's half period zone

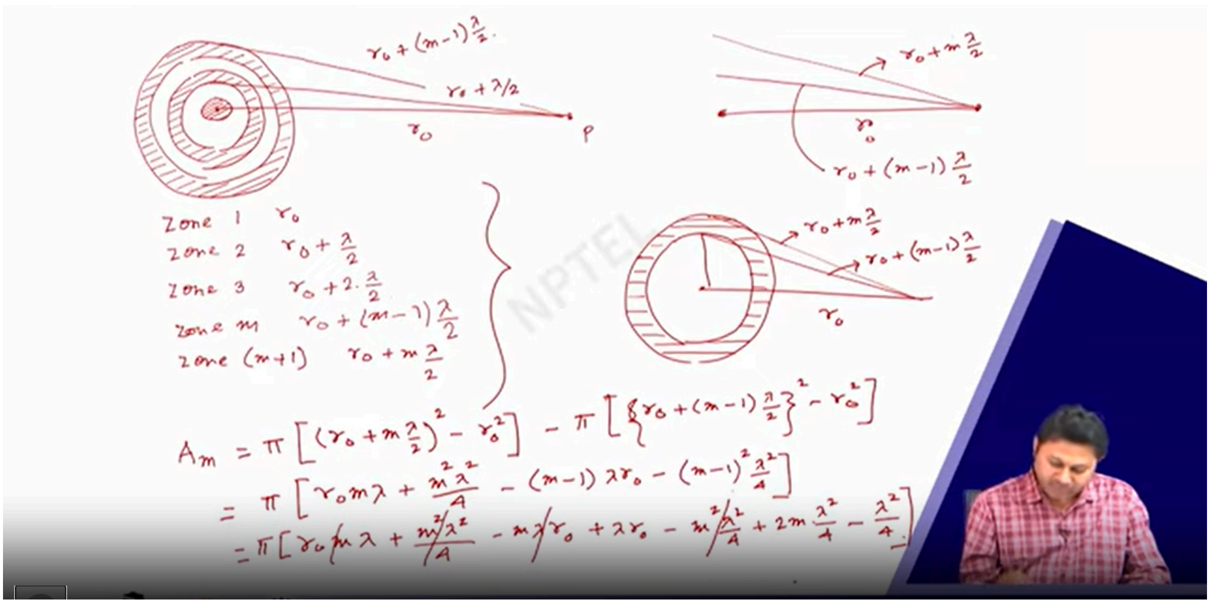
Hello, student, welcome to the wave optics course. Today, we have lecture number 44 and in this lecture, we will discuss Fresnel's half-period zone. So we have lecture number 44. Today's topic, as I mentioned in the earlier class, is Fresnel's half-period zone. So, on the last day, if you remember, we try to calculate that, if we have a point source here, light is emitting, and then we have an aperture here. Then we try to find out what the intensity was at this point p and we have a generalized expression by integrating over this aperture, and at the end of the day, we find what is the value of the intensity of the electric field at this point p. With the knowledge that what is the intensity at this source point, before going to that rigorous calculation there is another method, that is the Fresnel sub-period zone. So Fresnel, what did he do? He did an interesting thing, suppose we have a spherical wavefront let me try to draw in a 3d manner, and in this spherical wavefront is emerging from a source S sitting at the center, and the wavefront area is divided by these zones, and this is the point at P, where we have our observation and from point P, we can see in this wavefront, we can see these zones that are placed. This is one zone, this is the second zone, this is the third zone. I am just separating these zones by drawing these annular strips, making individual zones. I'm going to draw that in detail. So the contribution of the intensity at that point P coming out through these individual zones is an interesting idea by Fresnel, but there is a formulation for these zones. So, let me now draw this cross-section. So this is a cross-section and if it emerges essentially this spherical wavefront will be a plane wave, and over this plane wavefront, I can have concentric rings at different zones, etcetera.

(Refer slide time: 08:02)



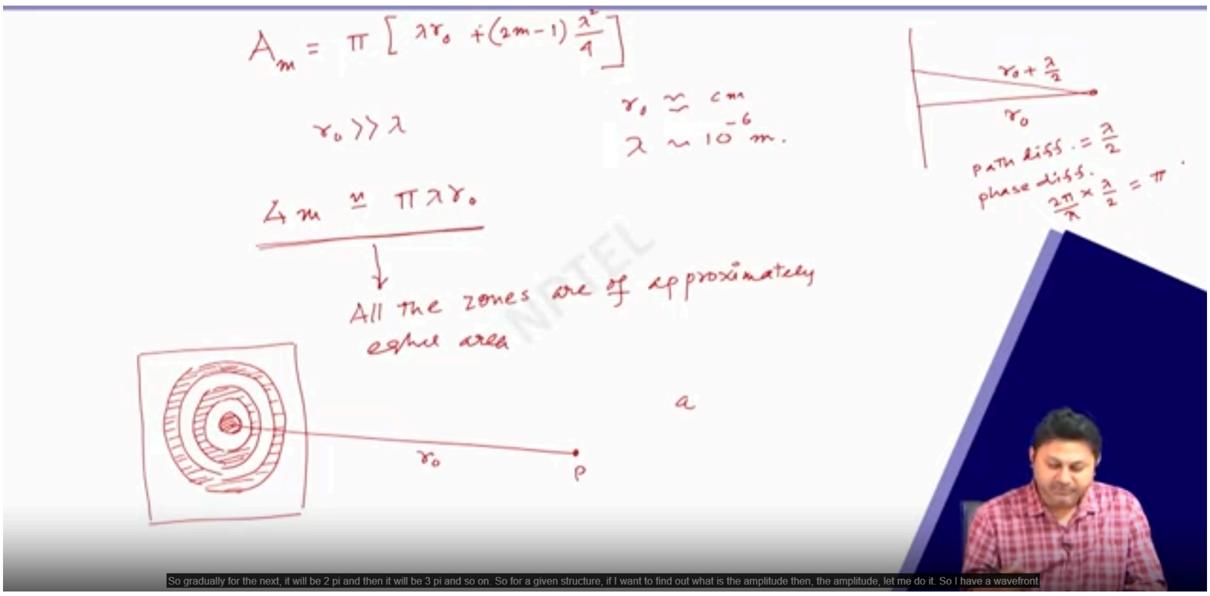
So the zone is calculated in such a way that if I have p here, which is a distance, and if r0 is a distance from the center point to p then this is the first zone, this is the second zone, this is the third and so on the distance of the nth zone will be r naught plus mth zone I am talking about, lambda by 2. So, the zones are formed in such a way that the distance from the center to P is r naught. If I draw here more precisely, So this is in 2D. This is P, and this is the center point that is this point, and this length is R naught. The second zone is the annular part and I am drawing this annular part on the top of the annular part. So I have one circle here like this. So, this value will be r naught plus lambda by 2. So, there is a lambda by 2 extra path for this zone that is interesting. Now, I have another zone, and if I draw these things here, it should be r naught plus 2 into lambda by 2 and so on. So, r naught, then I have r naught plus lambda by 2 is the contribution, that is the length from this point to r, the zones are formed in such a way that it is r naught plus 2 into lambda by 2, then it is r naught plus 3 into lambda by 2 and so on. And essentially we have r naught plus m into lambda by 2; this is the way the individual zones are formed and that formation is based on this point R0. So from P to this point if it is R0 then rest of the thing we can imagine in this way. Now before going to the detailed calculation, why is this thing taken, etcetera? Let us try to calculate that if this is the way the zones are formed, then what are the areas of these zones? So let me draw it. So we have zones like this. This is one circle, this is another one, a concentric circle I am drawing, and each circle encompasses some area. So that area is interesting here. So I have one area here, another area there, another area is this. So the contribution of the entire wavefront is subdivided by these zones. So whatever the intensity that we find at point P, which is away, is calculated by the cumulative contribution of each zone. And the zones are formed in such a way that from this point to here P this is r0 then first this is r0 plus lambda by 2, for the second one, for the third one, it should be r 0 plus 2 into lambda by 2 and so on. So, if this is mth one, note that it should be r naught plus m minus 1 lambda by 2. Okay, that is the mth one, m plus 1 to 1 will be r 0 plus m into lambda by 2 m plus 2 1, in a similar way. So if I consider these two in between m plus 2 and m plus 1 that is my r 0, and these two lines I am drawing one is for the first one, this is the 0th one, then this is the first one, this is a and this is a minus nth one, so another is M plus nth one and so on. So if I consider this is zone 1, then if I consider this is zone 2, then zone 2 is r0 plus lambda by 2 so zone 3 will be r0. So, in zone 1, if I nomenclate in that way, zone 1, it is simply r0. Zone 2, it is r 0 plus lambda by 2 zone 3 it is r 0 plus 2 lambda by 2, zone m is r0 plus m minus 1 lambda by 2 and zone m plus 1 is r0 plus m lambda by 2, this is the way these distances are defined. So suppose this is the mth zone and we have r 0 plus m minus 1 lambda by 2 and this is m plus 1 zone, so, this is r 0 plus m lambda by 2. So, if I want to find out what is the area for the mth zone, then a m will be simply the area. So, mth zone do I find? So, I have this annular region if I want to find the length from here to here and here to here, this length I calculate. So I want to find out the area of this region. This is r naught, this is r naught plus m minus 1 lambda by 2 and this is r naught plus m lambda by 2. So, that we know. Once this is known, I can find out this annular region and the area will be pi r0 plus m lambda by 2, whole square minus r naught square, that is the area of this mth region, and I subtract with pi. Let me put a second bracket here, square minus r naught square. So, these are the straightforward calculations. So, if I do, so you can see pi r naught square, pi r naught square, they will cancel out and we will go to get this pi if I common, then essentially we have r naught square again, pi r naught square that will cancel out. So we essentially have two of so r naught then m lambda plus m square, lambda square by 4 minus, that contribution we have. So that is m minus 1 lambda r naught and then we have minus m minus 1 square lambda square by 4. So, these things I can have simply m square lambda square by 4 minus m square, lambda square by 4 that will cancel out and we eventually have. So, r 0 okay so let me expand this pi r naught m lambda plus m square lambda square by 4 minus m lambda r naught, plus lambda r naught minus, we have a square of these things, so we have m square lambda square by 4, then plus 2 m lambda square by 4 and then minus we have lambda square by 4. I think that we have. So 1, 2, 3, 4, 5, 6, 7 terms, 1 few terms will cancel out, for example, these will no longer be here, this will no longer be here, m zero this will cancel out, and we have simply lambda R naught and then 2 m lambda square by 4, and then we have the order of lambda square. So, essentially A m will be the area of mth zone half-period zone. Why is it a half-period? Because the lengths are lambda by 2, it is pi. Then we have lambda lambda r0 and we have lambda square term. So, we have a lambda square by 4, if I take common then 2m minus 1, then, this is plus. So, plus 2m minus 1 lambda square by 4 that is the area we calculate. Now note that so, in this case r0 actually is much much greater than the lambda, which is the wavelength of the light. So r0 is the physical length, that is the order of a few centimeters or even one meter, but lambda is the wavelength which is much much less of the order, set into the power. So if r is r0 of the order of a few centimeters then, lambda is a wavelength of the order of 10 to the power micron or nanometer, so it is of the order of 10 to the power minus 6 meters. Okay so that means r0 is much much larger than lambda and it is here, it is lambda square. So we can assume a m is nearly equal to, we can neglect this compared to the first term, the second term can be neglected and we have pi lambda r naught. Now that is an interesting result because here you can see that the area of the mth zone no longer depends on the integer m which means this is a constant quantity. So we can say that all the zones that are constructed are almost of equal area. So all the zones are of approximately equal area. So whatever the construction we are making here. So we have one zone, like this. So whatever the area we have for the first zone, the next zone that is this annular region will almost have an equal area.

(Refer slide time: 21:07)



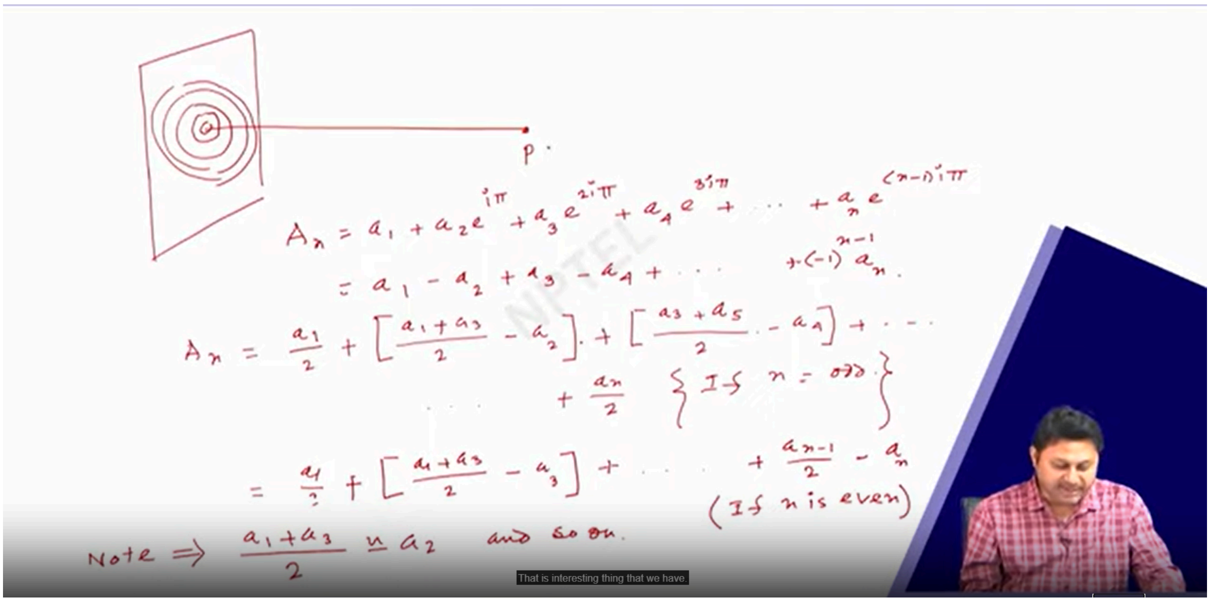
My drawing is not perfect here, but I believe you understand the concept here. Another zone icon I drew and the shaded area whatever the area I am getting here which is done by this shading has almost the same area as this one. So three zones are here. So three zones are almost equal. Now I draw another one, this has an almost equal area. Again I am saying my drawing is not perfect, so if you understand what zones will look like in practice. So this is another area and this area is again roughly equal to pi r naught multiplied by lambda. And note that that is interesting. So this area, whatever the zones we are talking about, mainly depends on in which distance I am talking about. So these zones are constructed, and this area is constructed based on this R naught, that means if I move from this point to another point, if my R0 reduces, then I can have another zone construction with a different amount of area or if I change the wavelength also this area will change. So that means the area here, whatever the area is constant and that zone formation solely depends on how far we are. So this is the point of reference and from the point of reference, the zones' constructions are calculated. So you should remember that this structure is not a unique structure, it is a structure over a plane wavefront that is moving and when the plane wavefront is moving this R0 is changing. So the reference point P the zone construction, is constantly changing if it is moving because r0 is changing for a fixed point, however, we can see that these values are like this. Now if I increase my length, and increase my point of reference then r0 is going to change, and the zone construction will also accordingly change such that their area becomes pi lambda r naught. So that important point we should note now, If I want to find out what is the resultant amplitude here. So, the resultant amplitude will have, so, we know that is one. So, from here, we have one amplitude from the center. If we have one amplitude, say A, then for other contributions, there is a path difference of lambda by 2. So because the construction is such that there is a path difference of lambda by 2. So let me draw it here once again about the construction, and that is the beauty of these things. So if this is r naught from the next point, if the field that is coming from this point is traveling a path r 0 plus lambda by 2. So, what is the path difference?

(Refer slide time: 30:31)



The path difference is simply lambda by 2 and the phase difference between these consecutive zones will be how much 2 pi multiplied by path difference, that is, how much 2 pi by k 2 pi by lambda multiplied by lambda by 2. So, this is simply pi because the phase difference is k multiplied by the path difference and this phase difference comes out to be pi. So gradually for the next, it will be 2 pi and then it will be 3 pi, and so on. So for a given structure, if I want to find out what is the amplitude then, then the amplitude, let me do it. So I have a wavefront where the zones are there and this is the point P. So, the resultant amplitude P due to this zone is a n equal to a 1 plus a 2 e to the power, there is a phase difference between this. So, we know that this is i pi then a 3 e to the power of 2 i pi. Then we have a4 e to the power of 3 i pi and so on up to a n e to the power of n minus 1 i pi. So, for n zones, these are the total amount of amplitude that one can have. The only difference between these amplitudes is the associated phase because this is the way the zones are constructed. Now, if I see this is e to the power i pi, e to the power 2 i pi, it is nothing but the sign and if I write it is simply a1 minus a2, plus a3 minus a4 plus dot, dot, dot, then I have something minus i to the power n minus 1 a n right. So, let me see if n is 2 this is minus if n is 3. So, it is fine, this is the final term minus n to the power of this one. Now, if I rearrange these things slightly then we have a n is I can rearrange it in this way a 1 divided by 2 plus then I rewrite this as a 1 plus a 3, I coupled it minus a 2 plus a 3 plus a5 divided by 2 minus a4 and so on. And we have the last term a by 2, if n is odd because it depends on whether n is odd or even. So if n is odd then this is the case, however, if n is even then I can have a1 by 2, plus a1 plus a3 by 2 minus a3 plus all this term. I am looking forward to the last term. So, it should be plus a n minus 1 by 2 minus a n. So, that is if n is even, that is the number of zones we are talking about if it is even. But whatever the distribution we have, if you look carefully, then we can see an interesting fact that A1 plus A3, if I only concentrate, note, divided by 2, this term is nearly equal to A2 and so on. Because of the contribution of A and 3, if I add and divide by 2, it should be some average and that average must be close to the contribution of the other zone, which is A2 and it is there altogether. So, all the terms that are written in the bracket essentially cancel out. That is an interesting thing that we have.

(Refer slide time: 35:55)



So, A n eventually then will only have the contribution of the first zone divided by 2 plus the contribution of the last zone divided by 2, if n odd or a n will be the contribution of the first zone divided by 2 plus a n minus 1 divided by 2 minus a n, if n is even. Now if n is very very large then this obliquity factor, I mentioned that this obliquity factor. So suppose this is one ring, this is another ring, this is another ring, this is another ring, this is the ring that we are having over this plane wave. And this is my P and for a very large N, you can see this is the contribution we are talking about. But this obliquity factor that we are having is this angle. This is the angle theta. And the contribution, if you see that F theta, which was half of 1 plus I think 1 plus cos theta. So, for a very high value, this obliquity factor will be reduced. And that reduction and if you check whether this sign is plus or minus I think this sign, no, if it is 90 degrees then this factor is 0. So, the obliquity factor is going down and for a very large n value. So, for a large n value, this contribution of the amplitude or this is almost zero the contribution will not be that much. So we can approximate a very interesting result we can get with this that a n is then essentially the contribution of all the zones in point p is essentially a 1 divided by 2. So that means the amplitude contribution of all the zones is reduced by only the contribution of the first zone divided by two. So that is the interesting result in Fresnel's structure, where we have the zones, and if I want to find out what is the intensity or what is the amplitude that one can expect because of this zone construction this half plate zone construction. We can find out that the total amplitude contribution is nothing but whatever the zone we have, this is the first zone, we have the intensity of the first zone or intensity amplitude due to the first zone divided by two. So one and other will cancel out in such a way that only this term stands alone and that is the value one can have. So today my time is up. So I like to conclude here. So today we did is a very interesting concept we explored and that is Fresnel's half-period zone, and how the zones are constructed. I try to demonstrate by depicting a few figures and then what is the basic parameter to calculate it and that is the R0, the distance from the zone, and how the zone plates are constructed the zone regions are constructed, that is the path difference of lambda by 2. So, the next one is to say the path difference such that it is r 0 plus lambda by 2 r 0 plus 2 multiplied by lambda by 2 and so on. So, in the end, what we find is that if we want to find out what is the intensity due to these zones that is the cumulative effect of all the zones at the end of the day. We find that the results shrink to only one zone and this value is a 1 divided by 2. In the next class, we explore more about these zones, which is a very important concept in Fresnel's diffraction cases, where we just block one zone and try to find out what should be the resultant intensity and see how these things are going to change. If I move towards a zone and move backward forward or backward movement can lead to the intensity distribution. How the intensity is going to be distributed. We will calculate rigorously and also try to understand in a qualitative manner. With that note, I would like to conclude here. Thank you very much for your attention and see you in the next class.

(Refer slide time: 39:41)