WAVE OPTICS

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Lecture - 54: Matrix treatment of polarization

Welcome, students to our wave optics course. Today we have lecture number 54 and today we will be going to understand how to represent this polarization in matrix form. It is a very interesting topic, today we will discuss. So, today we have lecture number 54 and today's topic is matrix method for polarization problem is a very very interesting topic and I request the student to carefully understand what is going on here . So, the electric field is x, y and z coordinate and the electric field is situated on x y coordinate, x y plane rather and this value is Ex and this is Ey the component. So, total electric field E, I write in terms of component at Ex x unit vector plus Ey y unit vector. So, we write this complex field in this way. So, E x tilde this is E 0 x and previously we wrote cos of ,so that was the previous expression we used that E x is E 0 x cos k z minus omega t, say one phase phi x and E y was E 0 y cos of k z minus omega t plus phi y, phi y and phi x are 2 arbitrary phase that is associated with E x and E y component in general. So, this I can write in this way instead of writing cos, I write e to the power of i k z minus omega t plus phi x and e y tilde I write e 0 y e to the power of i k z minus omega t plus phi y okay. Now, E x, if I write this, is a real part of E x tilde similarly E y is a real part of this as per our notation. So, I can write the total electric field in terms of this tilde. So, total electric field, So, I can write E tilde is equal to e0 x e to the power of i kz minus omega t plus phi x, x unit vector plus e0y e to the power of i and then kz minus omega t plus phi y with y unit vector. Now these I can write as E0x, e to the power of i phi x, x unit vector plus E0 y, e to the power of i phi y, y unit vector and e to the power i k z omega t, I take it common and I put it outside k z minus omega t.

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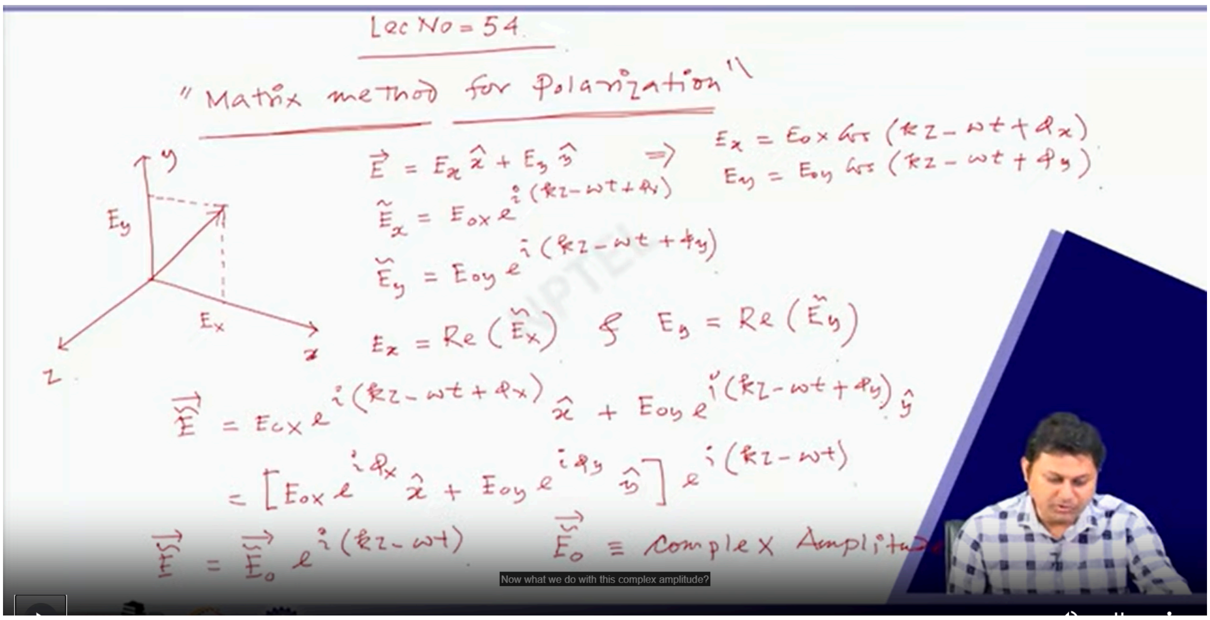

So, the e tilde vector is essentially the amplitude of e, then e to the power of i k z minus omega t, where this e 0 tilde vector is called the complex amplitude because it contains the phase complex amplitude. Now what do we do with this complex amplitude? So, E 0 tilde I can write it in this way, E 0 the amplitude part is equal to E 0 x tilde, E 0 y tilde. What is E0x, E0y tilde? So, in general, it is E0j tilde is E0j, j is x or y, e to the power of i phi j, j can be x or y, in a single notation, I use this. So, once we have this in our hand then we can write it in this form. So, this is essentially E0 x, e to the power of i phi x. and it is E0y, e to the power of i phi y including the phase I write this in a matrix form. So, I just write in a matrix form. Now, once we formulate that, then we try to understand how to represent the 3 different polarizations that we discussed in our previous classes. So, first we will like to understand linear polarization, the simplest one. So linear polarization, how do we deal with that? Suppose I have linear polarization along my axis. So this is my axis, so my polarization suppose it is along y axis, so this is the way we have the polarization. So this is my x axis, this is my y axis and along this direction we have the polarization. So in that case my general form is this. So, phi so, let me write down here e tilde 0 is E0 x, e to the power i phi x and E0 y, e to the power i phi y. So, I have some amplitude here because in x direction there the amplitude is 0, but y direction we have some amplitude. So, the matrix form will be something like amplitude and then one case. So, on the x axis there is nothing. So, we have 0 and in y axis it is 1. So, that will be the representation. What happens if the electric field which is a linear polarized light is vibrating along the x axis. So, that means, it is vibration along this direction where, this is my x axis, this is my y axis. So, again the same thing. So, e 0 tilde will be simply the some amplitude x axis is there. So, I have 1, but y axis there is nothing, so, I will put 0. Now, the question arises what happens if the electric field we are going to vibrate by making some angle is a linear polarization. So, an electric field can make an arbitrary angle with x axis and y axis and then vibrate. So, the picture is something like this. So, suppose the electric field is vibrating like this and if this angle is alpha then how to write this. So, my e tilde again is the same way as e tilde 0, that is the complex amplitude will be some amplitude a in the x axis the component is cos alpha.

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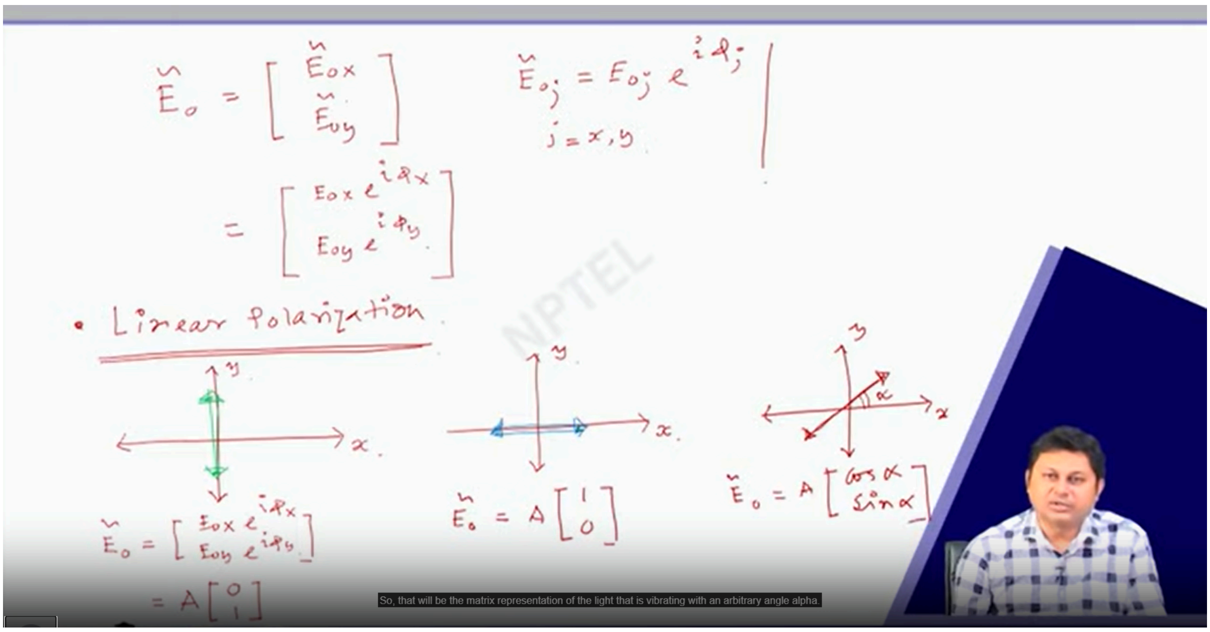

So, I write cos alpha and in y axis it is sin alpha. So, that will be the matrix representation of the light that is vibrating with an arbitrary angle alpha. So, how do we write it? For example, now if alpha says 60 degrees, that is if I have x axis, y axis and electric field is vibrating with an angle 60 degree like this. So setting A equal to 1, that we can do, can normalize this value A equal to 1, what we get is E 0 tilde is equal to cos of 60 degree and sine of 60 degree which is essentially equal to half of 1 and root over of 3. So, this matrix element can give you the same representation of a linearly polarized light making an angle 60 degree. So, how do you find out the angle? It is simply alpha, is simply tan inverse of E0 y, E0 x. So, that is tan inverse of this quantity root over of 3, which is equivalent to 60 degrees. So, in general we can write E is simply A and B if you have this matrix with A and B, where A and B are real numbers, then that represents linearly polarized light and what should be the value of these angles the tan alpha will be simply b by a. So, alpha one can calculate by this number that will be tan inverse b by a. So, if you know b by a, then we can calculate what is the angle that is making this linearly polarized light. Note that this linearly polarized light can orient in any angle and then based on this orientation this value of the a and b will change. So if A and B are real and in this column matrix if you get something in A and B then that basically represents that it is a linearly polarized light that is the outcome we have. Now, we go to more general cases and another important case and that is the, how we represent circularly polarized light in matrix form. So, circularly polarized light, we are going to represent in terms of matrix. So, e x tilde it should be E0 x, e to the power of i k z minus omega t and e y tilde that is the general representation E0 y, e to the power of i k z minus omega t plus, phi that representation we used earlier that is a phi is a relative phase difference between these two making phi x equal to 0 because we also introduce phi x here the previous case. So, making phi x equal to 0 and phi y equal to phi, I can write it without loss of generality. So if phi equal to pi by 2 and E0 x equal to E0 y is E0 that we put for getting the circularly polarized light earlier also in our classical treatment.

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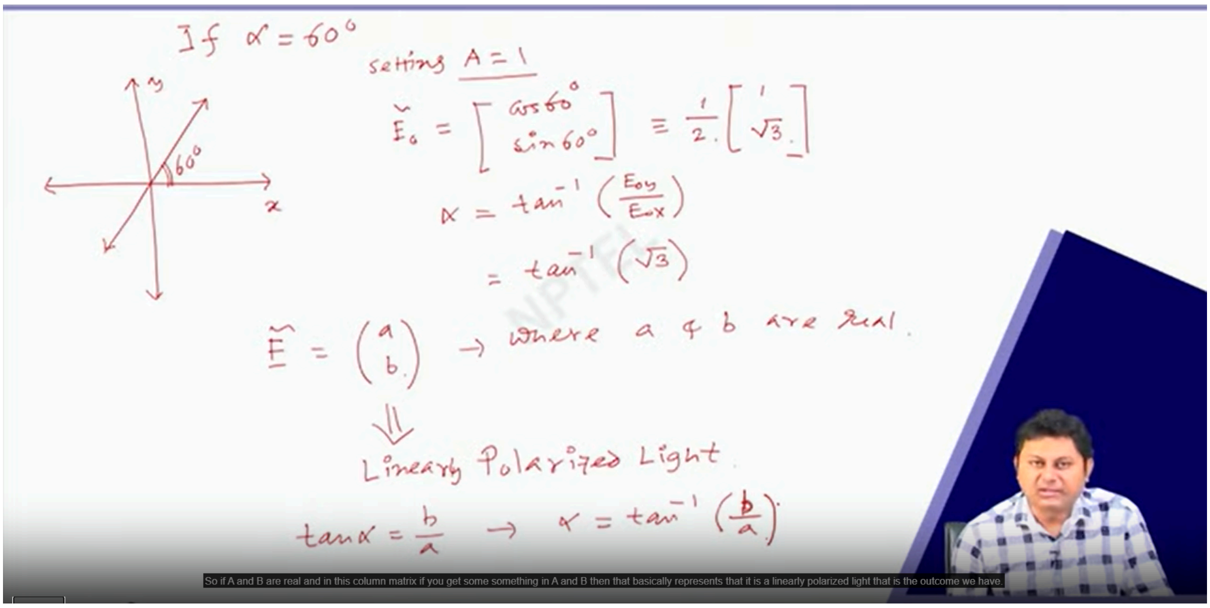

Then E0 y not E 0 y it is just simply E0, let me erase this part. It is e according to our general notation E0x, e to the power of i phi x and E0y, e to the power of i phi y that is A because the amplitude is same for both the cases and then we have 1 here because phi is 0 and phi y is pi by 2, so I write e to the power i pi by 2 which is essentially i. Let me write phi x here is 0 and phi y is pi by 2 and which gives e to the power i pi by 2 is essentially i. So these basically give us the left circularly polarized light where you know over the time the electric field will move from this point to this point, it goes here to back here, initially when time is 0, so you will have. So if this is x axis and this is y axis and at t equal to 0 we have the electric field is placed here over x axis and over the time it rotates to other points. So when, this is my x axis and it is my y axis. But from this, you can see that initially it is 1 and then it rotates, go to other points and it goes here at this point when t is equal to pi by 2 omega. So this is left circularly polarized light and this representation is for left circularly polarized light. In the same footing we can write that if phi is equal to minus of pi by 2, then simply the representation of the electric field complex amplitude will be A minus 1 minus i because here phi x will be 0 and phi y will be minus of pi by 2 and when phi y is minus pi by 2, we have e to the power of minus i pi by 2 which is essentially minus of i that we put here. So, that condition suggests that over time, we have a field here at t equal to 0 and that moves here at this point this is x, this is y, but it is the negative side. So, it is moving from here to here. So, I am having a rotation like this which is right circularly polarized okay. So, this is for t equal to later time say pi by 2 omega. So, in this way we can also have the general representation and the general representation is the elliptically polarized light that should be the general representation. So, in elliptical polarized light what happens is that we will get a general phase out of that and that general phase we will see how to do that. So, the most general form one can have is this. So, let me draw it. So let me draw first how we can define the elliptically polarized light. So first, let me draw what elliptically polarized light will look like if I fix a plane it will be something like this, this is one orientation and in this case this is X and it is Y and here the light is moving the tip of this electric field is moving in this direction. If that is the case, then the first condition that we need to remove is that E0x here is not equal to E0y. If that is the case then the complex amplitude that we have written so far,

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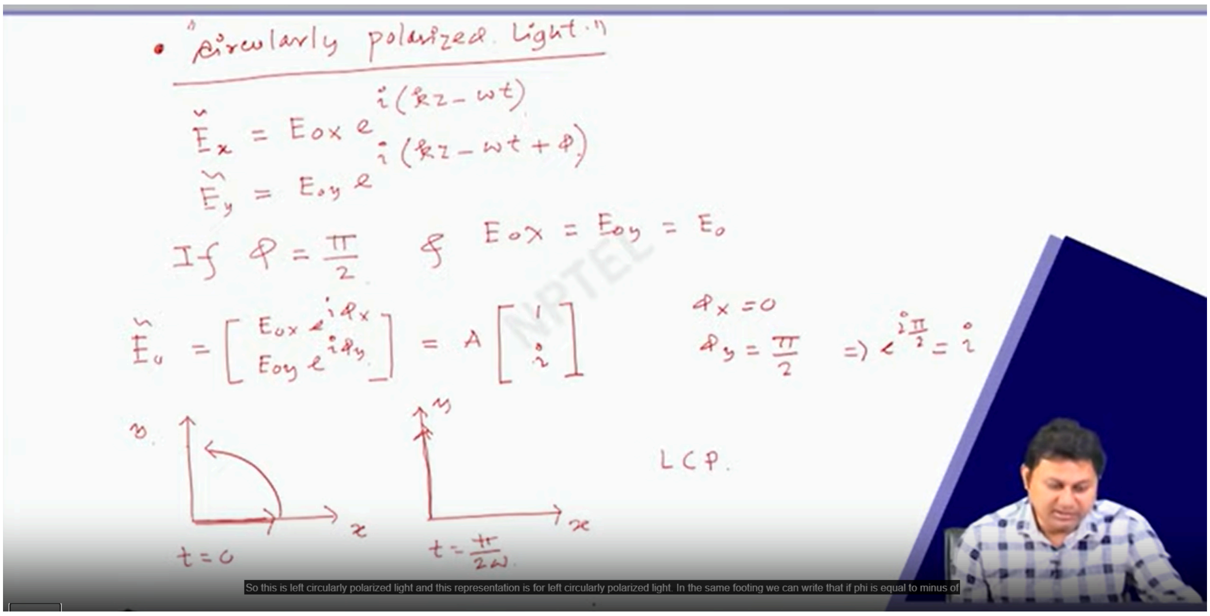

I can write in a general way and that general way is a one amplitude and another amplitude for y and that is different, but with a phase, so, I can have i minus i B, this minus sign suggests that it is moving in the right circular manner exactly like we did in the previous case. So this is right circularly right, sorry right elliptically polarized light, the representation is like this. On the other hand I can also have another orientation, this is x, this is y and now the tip of the ellipse electric field follows the elliptical path and it is moving like this. So here it is moving like this. So it is right that elliptical polarization is moving like this. So left elliptically polarization light representation wise same condition that here also E0X should not equal to E0Y and rest of the thing will be same like circularly polarized light, here we have A and iB okay this. Now, this representation of the ellipse is straightforward because it is showing no tilt nature among themselves, but you may remember that we also produce this left elliptically polarized. So, the most general form we can figure out. So, let me do that here, in the next phase where the tilted ellipse can emerge. So, the most general form, so, I can write our complex amplitude as this: we have E0x phase, e to the power i phi x, E0y, e to the power i phi y. Now I can without loss of generality as I mentioned we can make phi x equal to 0, E0 x is equal to some value a phi y, I put some value delta that is the phase arbitrary phase and E0 y I put small b. So, these I write in this way a and then it is b e to the power of i e to the power of i delta. Now, the matrix form I can write it as a, then b of cos delta plus i b of sin delta just expanding e to the power i theta. Now, I put b cos delta as my big B and small b sin delta as my big C. If that is the case, then this representation will be like A B plus i c. So, that is the most general representation of elliptically polarized light in a matrix form, but note that this sign is plus. So, that gives me left elliptically polarized light. So, left elliptically polarized means this ellipse will give you counter clockwise rotation. Similarly, if delta is minus delta, then E will be simply A, then B minus I c, when phi y is minus delta and that should be the case and that represents right elliptically polarized light, that matrix will going to represents right elliptically polarized light that is clockwise rotations that is left elliptically polarization and this is right elliptically polarization, this will be the direction. So, this is the way we represent and in this most general representation.

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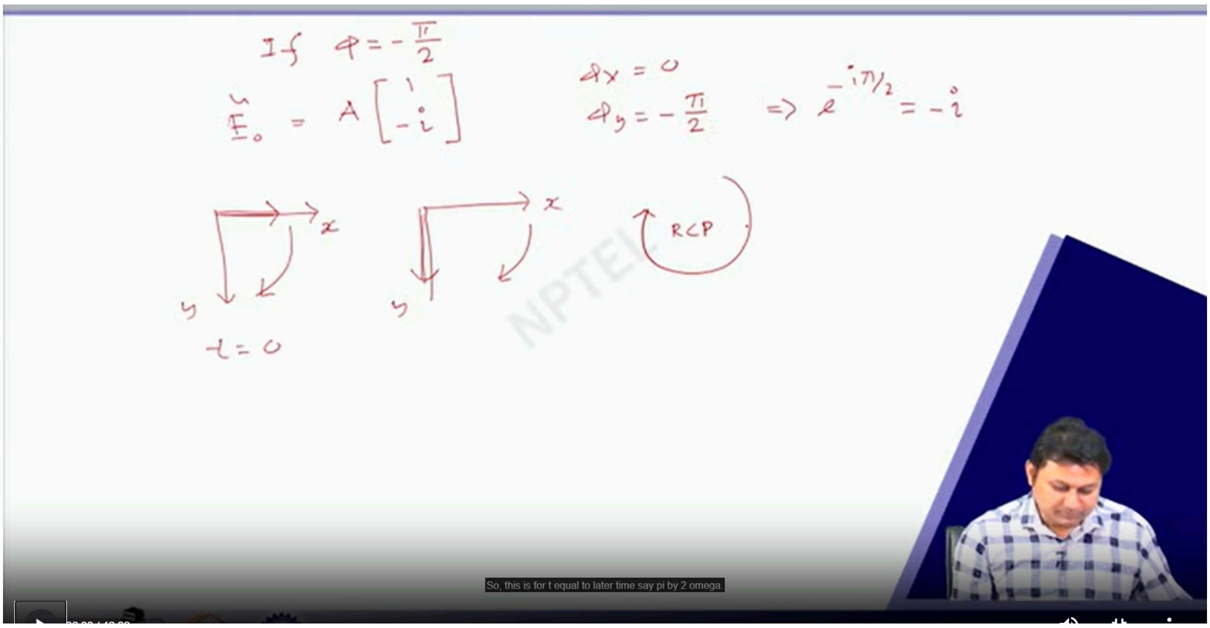

So, we can have the tilted one. So, that let me draw it once again. So, this is a tilted ellipse. So, Suppose this is the major axis of the ellipse and this is the way we have the ellipse. This is the central point, this is the x axis ,this is y axis the major and minor axis of this ellipse does not coincide with x and y. So, there is a rotation we can have with proper rotation we can show that it can be represented simply with this, the most familiar form the ellipse equation. But here the ellipse is tilted, that is the most general way to represent an ellipse, where the major axis and minor axis do not coincide with the x axis and y axis. So, here E is basically represented by numbers like a b plus minus i c that is the most general form of the representation of this plus and minus sign. However, it represents whether the ellipse is right circularly or left circularly that we discuss here tan 2 alpha. So, how do I get the numbers here? So, if you remember tan 2 alpha that is 2 of E 0 x, then E 0 y divided by E 0 x square minus E 0 y square and then we have cos delta where cos delta is a phase. So, if this numbers are given a, b, c etcetera then we can find out E0 x as A E 0 y as root over of this amplitude, root over of b squared plus c square and delta I can calculate this number which will be tan inverse of c by b, this is the way we can find the value of delta. So, the plus and minus sign represents left circularly, right circularly if a number is given. Suppose E is given the say 1 and 2 plus i 3, suppose a matrix is given. So, that matrix represents a left circular ellipse from the very first look one can say that this notation is equivalent to this a b plus minus i c. So, this definitely represents left elliptically polarized light. Now, we define what is the value of e x and e y, the value of e x will be 1, e 0 x will be 1, e 0 y will be root over 4 plus 9. So, it should be a root over of 13 that is the value of E 0 y and the delta the angle at which it is tilted is tan inverse of 3 by 2. So, if the numbers are given, that is important thing we can find out the state of the polarization. So, that is the relation we have and if the question is given how to define the state of the polarization. So, this is the way we can do it. So, I do not have much time to discuss more about these polarization representations. In the next class what will we do? We are going to discuss these matrix elements which are known as Jones. Using this Jones matrix we can identify the polarization and we can play with this Jones matrix to find how polarization problems are understood or how we can address different things and make it simple by using this Jones matrix. So with that note I would like to conclude here. Thank you very much for your attention and see you in the next class.

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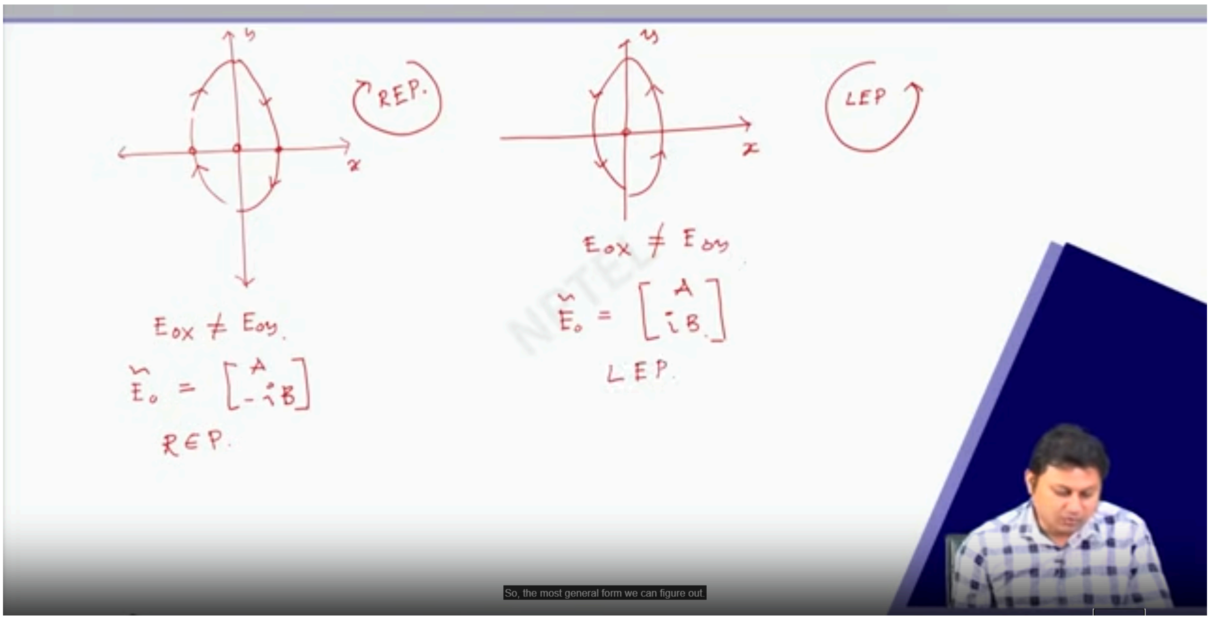

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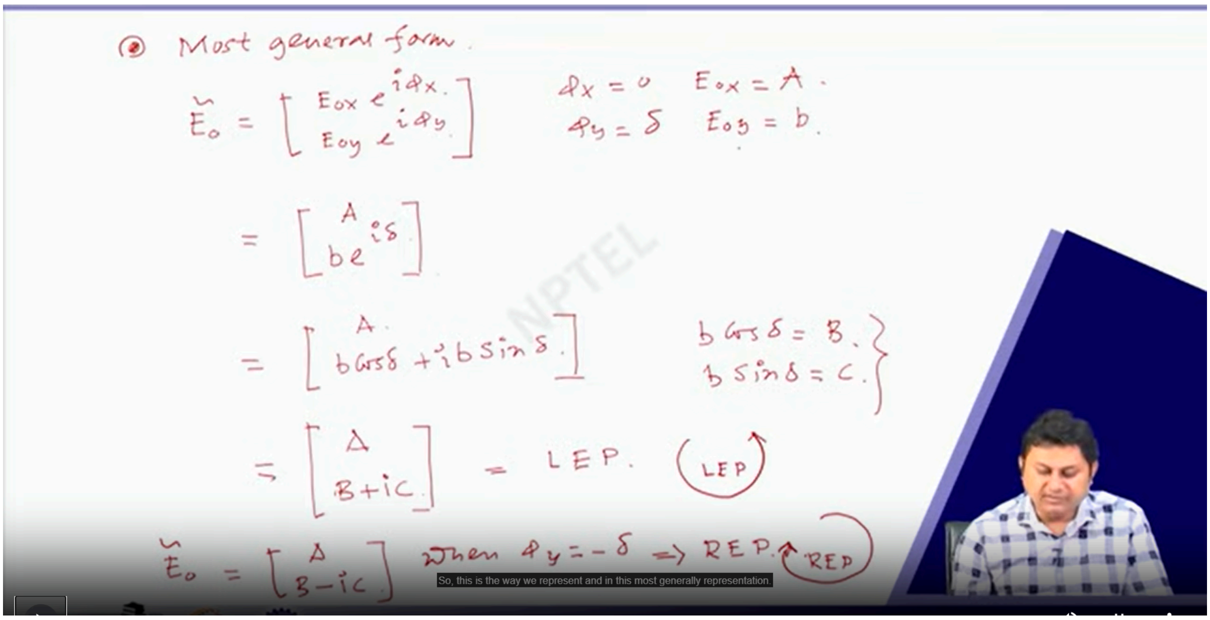

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