WAVE OPTICS

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Lecture - 59: Jones matrix for polarization (cont.)

Hello student, welcome to the wave optics course. So, today we will discuss this Jones matrix formalism that we have been discussing for the last few classes. So, we will continue with this. So, we have lecture number 59 and we will continue our discussion on the Jones matrix which is important. So the Jones matrix we formulate for linear polarization. One thing we didn't do, so that's what I'm going to do today. So let me remind you what we had for linear polarizer, linear polarizer means if this is the optical system that can polarize the light. So this polarizer can have a certain pass axis, say this is the pass axis and if I launch an unpolarized light, light gets polarized here and this polarizer can be defined by a matrix and that matrix is our Jones matrix. Also the light that will produce the linear polarized light it can be represented also this matrix form which is forming a vector and whatever the light we get at the output if this is the E that is the light we are getting at the output this is a specific notation we call this is a but this is the output light that will be the polarizer operator, that is the matrix, that is defining, this the nature of the polarizer and the input light. So that is the recipe we have been using and in different cases we determine what is the matrix for m but one thing we didn't do yet and that was we supposed to do in earlier classes but somehow we missed that and so let me give the example of this m matrix for different cases. For example here m was 1-0-0-0 if you remember so that basically when the linear polarizer has Ta that is transfer axis or pass axis along x direction, similarly m was 0-0-0-1 when the linear polarizer having their Ta or pass axis along y direction okay and when this pass axis is making 45 degree angle, then it was simply 1-1-1-1.

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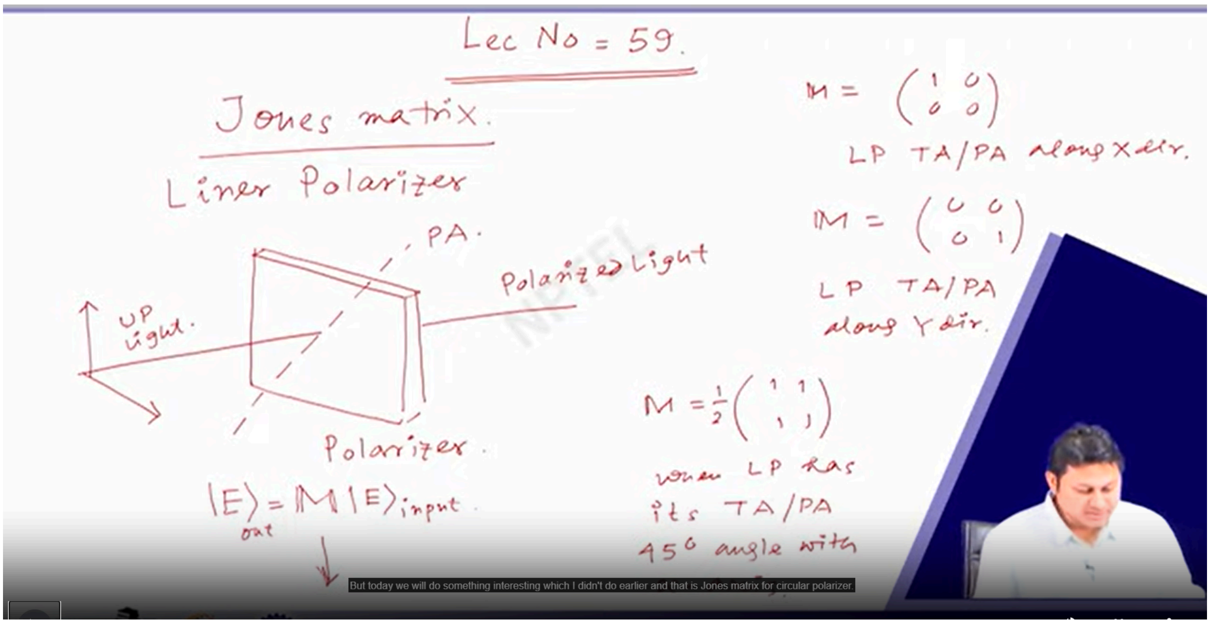

That was the structure with a certain factor half. And that is when a linear polarizer has a

transfer axis or pass axis making 45 degree angle with say x axis, this we discussed. But today we will do something interesting which I didn't do earlier and that is the Jones matrix for circular polarizers. So what is the meaning of the Jones matrix for circular polarizer? So we have a polarizer here and this polarizer what it does is if I launch a polarized light with any arbitrary polarization then in this site we will always get a circularly polarized light. So now the question is what is M here? How to calculate the M for this case? So let us start with right circularly polarized light. So the right circular polarizer makes any arbitrary polarization to the right circular polarization after through this system. If the linear or any kind of polarized light is passing through then the output we always get is the right circular polarization. So what we have in order to find out, So our aim is to find out m. So what should be the first condition? If I launch a right circularly polarized light then allow it to pass through this system then in the output, I always get a right circularly polarized light that is the first condition. And the second condition is that since it is producing right circularly polarized light, if I launch a left circularly polarized light, which is perpendicular to the right circularly polarized light, these two are orthogonal to each other then here we are going to get 0. So, nothing will come. Exactly like linear polarization, if I have a linear polarization, the way we calculate the linear polarization with vertical pass axis and horizontal pass axis, what should be the form we calculated? Exactly the similar way we are going to calculate by considering these two conditions. Now, we consider, after having this, we consider my M, matrix is a-b-c-d simply so we need to find out a-b-c-d using these two conditions 1 and 2. So let us do that quickly. So from condition 1, what I get is a-b-c-d and this matrix that I am writing here 1 minus I is representing the right circularly polarized light. So it will produce a right circular polarized light because I am trying to find out the value of m for the right circular polarizer. So that is the equation one can have and then I can get an algebraic equation like this: a minus ib is equal to 1 and C minus id is equal to minus i. So a set of equations I write 3, also from condition 2 I can get a similar kind of thing.
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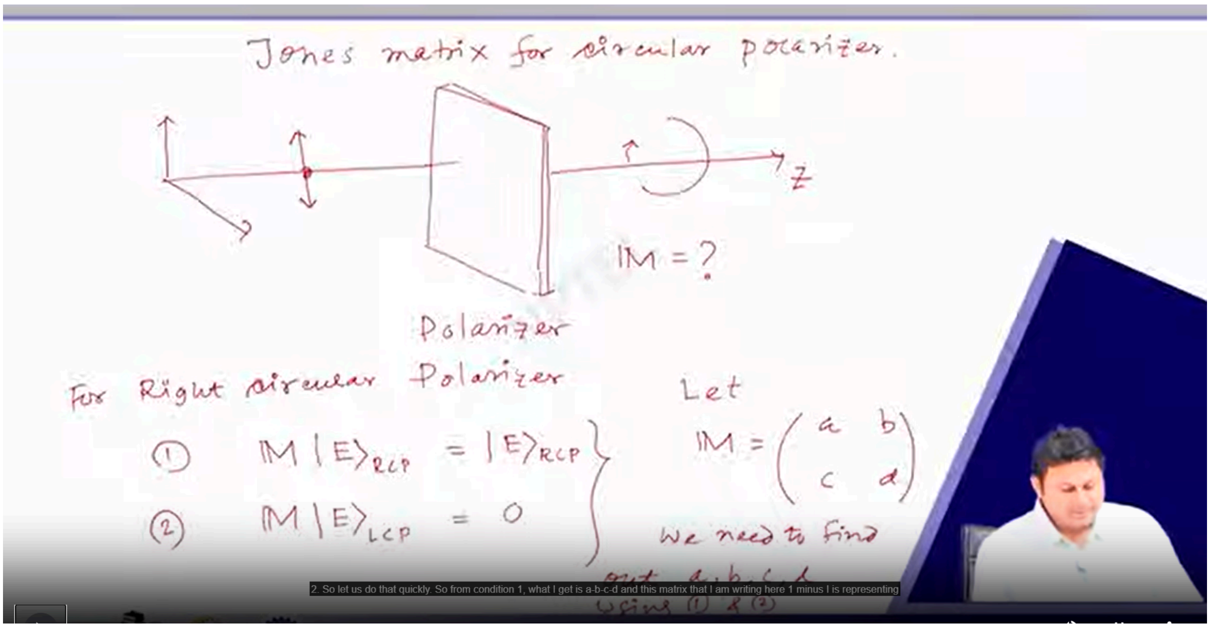

But here, instead of having right circular polarized light, I put a left circular polarized light, which is orthogonal to the right circular polarized light. So, that means I can have here 0-0

because it is not going to, so, that light will not be allowed by this polarizer. So I will get another set of equation which is equal to minus ib and c is equal to minus id. So here we have four unknown a-b-c-d and we have four equation set of equation in 3 and a set of equation in as 4 and we can use this. So once we have a equal to this and if I replace this thing to equation 3. So from the set of equations 3 and 4 we can readily get if I replace this. So, I can get minus of 2 ib is equal to 1 and that gives me simply b equal to 1 by minus of 2 i or if I multiply i in both sides, so 1 by 2 multiplied by i with a plus sign that. Similarly, if I put 3, then we get minus of 2 id that is equal to how much? So, from here, I can get d equal to i minus minus will cancel out, i i will cancel out, d equal to half. So I get b equal to i by 2, d equal to half and that value again if I put in equation 4 then that gives me simply equal to when we have a equal to minus of ib that we get and b is equal to i by 2, so it is minus of i multiplied by i by 2, this is half. So my a is half and c which is minus of id, so I have minus of i and d what d value we got d is half, so it is simply half. So a b c d we calculate, my m matrix if I take half common should be half of a is 1 b is what b my b was i if I take half common, so here we have i, c is minus i and d was half, so 1. So, this is the matrix we have and this matrix corresponds to the right circular polarizer. In a similar way, for left circular polarizer the Jones matrix M will be, I am not doing that, I strongly suggest, exactly the same way one can calculate this, the student should calculate it, the condition exactly the way we put it should be put that m that will operate over left circularly polarized light will produce a left circularly polarized light that is condition one and condition two that m if it is operate over a right circularly polarized light we will get 0 because it is orthogonal to left circularly polarized right. So, this thing I wanted to show in today's class because that was missing and we can also check that actually that for any arbitrary polarization whether this is happening or not. So, let us do that. So, for right circularly polarized light is this, so m for right circularly polarized light we know this is half 1 i minus i 1 that is the matrix, that is defining the right circular polarizer and any arbitrary polarized light this is the most general polarized light I am writing. Now which is elliptically polarized light?
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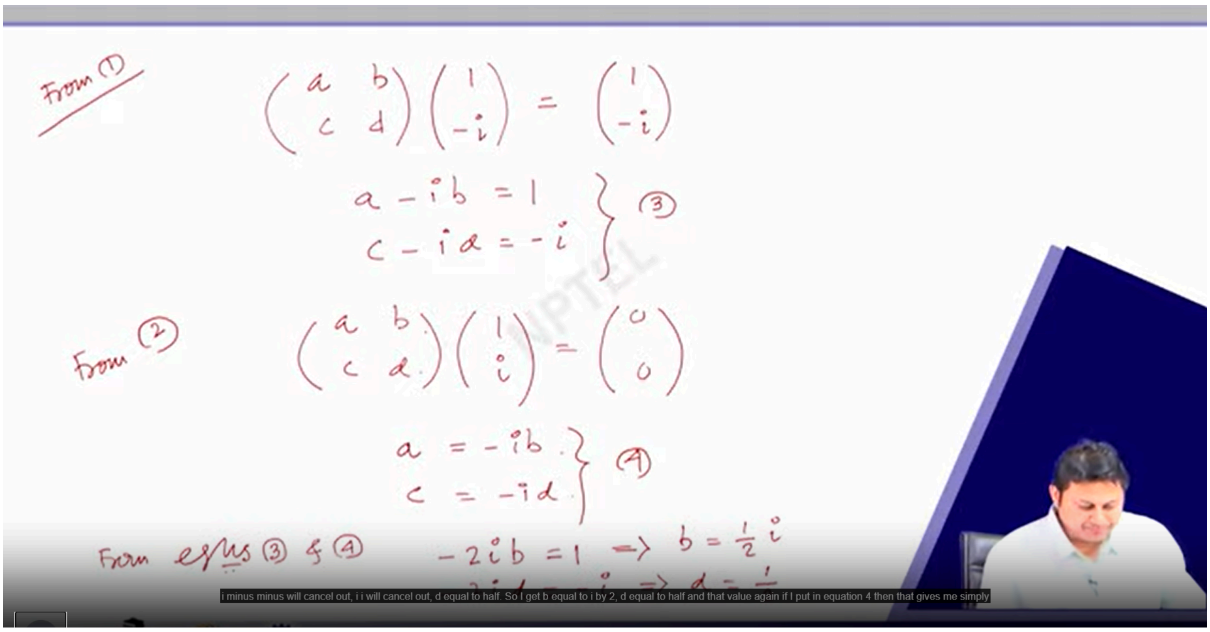
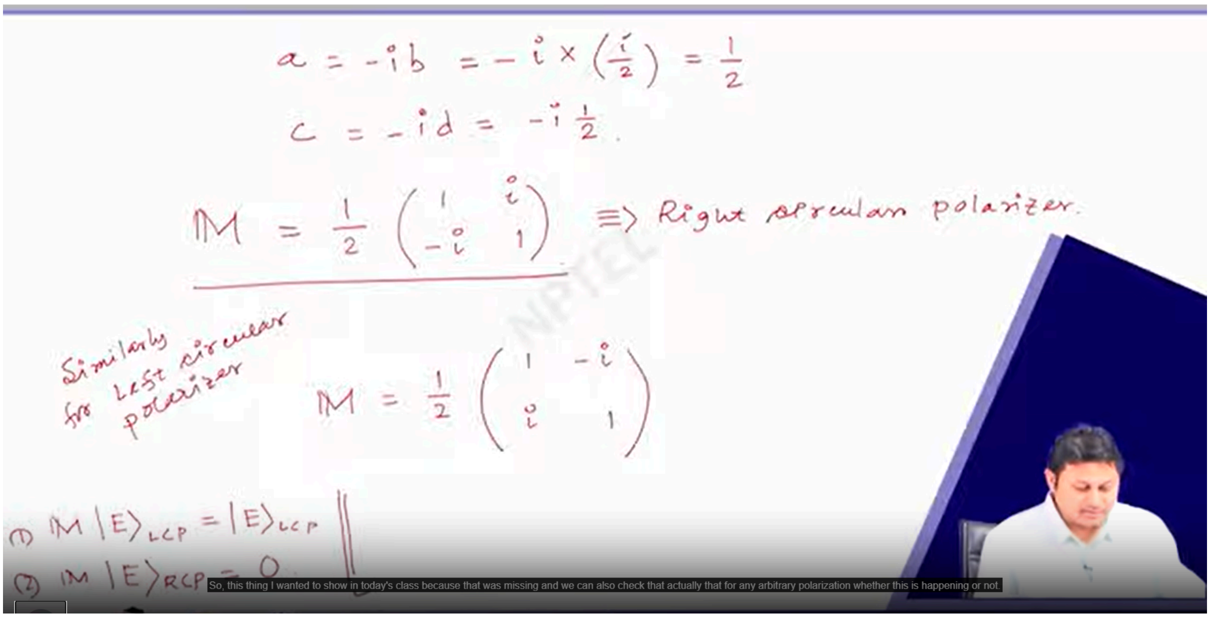


So i a b plus i c, so that is an elliptically polarized light, so what I am doing that. Okay let me draw this scenario. This is my right circular polarizer optical element that polarises the light in the right circular direction and what I am doing here is I am launching a light which is elliptically polarized. So the polarization here is an elliptical general form. So I am just drawing a elliptical shape here and now this polarized light is allowed to pass through this system and what I get in the output is a circularly polarized light, not only circularly it should be a right circularly polarized light, this is Z direction, so that I am going to get. So, let us know whether I am really going to get it or not. So, what I do is I put this m value. So, half of 1-i-minus i-i and this I put which is a and then b plus i c, so, that means I am operating m over this e, so that thing is this one. So what do I have here? So here I am getting like half of a then minus c plus IB that I get and another I get minus of IA plus, B plus, IC that is the value I get in the right hand side but this I can do in this way I can write it if I take common A minus c plus i b, if I take it common outside then inside the matrix what I get is interesting and that is 1 and minus i. Note that if I multiply a minus c plus i b with minus i then I am going to get this value which is here and this is if I want to find out the state of polarization, this is nothing but a right circular polarized light. So, that tells us that the right circular polarizer can be defined by a matrix with this particular form. Obviously, half will be there, half 1i minus i 1, so this is the way the right circularly polarized light can be generated by using a system which can polarize the light in the right circular direction. Well now I quickly summarize the Jones vectors that are generated and summarize the Jones vector for all the cases quickly like we did for the polarizer and phase retarder and phase rotator for the optical systems we do. Now here we will list out the Jones matrix for Jones vector rather for all the all kinds of polarization. So the summary of Jones vectors are not matrices but vectors for different polarization for different states of polarization and different states of polarization. Now, check it. So, linear polarization, the general form, if it is x direction, phi direction, and this is the way it is vibrating, the general form is cos alpha, sin alpha. Well, this is alpha and special case when it is vibrating along x direction e naught will be 1-0, when it is vibrating along y direction it should be 0-1 and when it is vibrating with 45 degree angle if this angle is 45 degree, (Refer slide time: 17:42)



then I can write it as with a normalization factor 1-1. That is for linear polarized linear polarized light. Now what happens if I put a circularly polarized light? For circular polarization we have two states one is left circularly polarized light and can be defined by this and another is right circularly polarized light defined by these two then also we can have elliptically polarized light. Let me do that on a different page. Then we have elliptically polarized light. In elliptically polarized light, we have ellipses whose major and minor axes are coinciding with the x and y axis. So this is x, this is y, so e 0 x is a and e 0 y is b and a here is less than b, one case also we can have with this orientation where x and y where a is greater than b but the general form is e naught tilde 1 divided by for normalization factor I put this and a i b and that is for for either left circularly either this system depending on the value of a and b or this system but both the cases it is left circularly polarized light, so the symbol will be this. So this is left elliptically polarized light. Similarly if I have this notation, this structure depending on the value of A and B I can have two structures either this or this, but here it will be written elliptically polarized light. That is when the major axis and the minor axis coincide with the x and y axis. A more general condition can be figured out and that is when suppose this is my x-axis, this is y-axis, and the ellipse is defined in this way. And it is supposed to move in this direction. So, E will be 1 divided by root over a square plus c square and it should be a b plus i c and that is again it is moving in this. So, again, it is a left elliptically polarized light. But the thing is here, the x and y axis are not coinciding with the x and y axis. Sorry, the major and minor axis of the ellipse does not coincide with the x and y axis. In a similar way, we can have another orientation with opposite direction, this is right elliptically polarization x and y and e will be very much same only thing is that we have a minus sign here minus i c and the angle also this angle whatever the angle it is making whatever the angle it is making we can figure out here for example e 0 x or x 0 whatever is a and e 0 y is this and the delta will be tan inverse of c by b this delta is a phase of these two components and we can generalize all these things and we can also figure out the angle actually.

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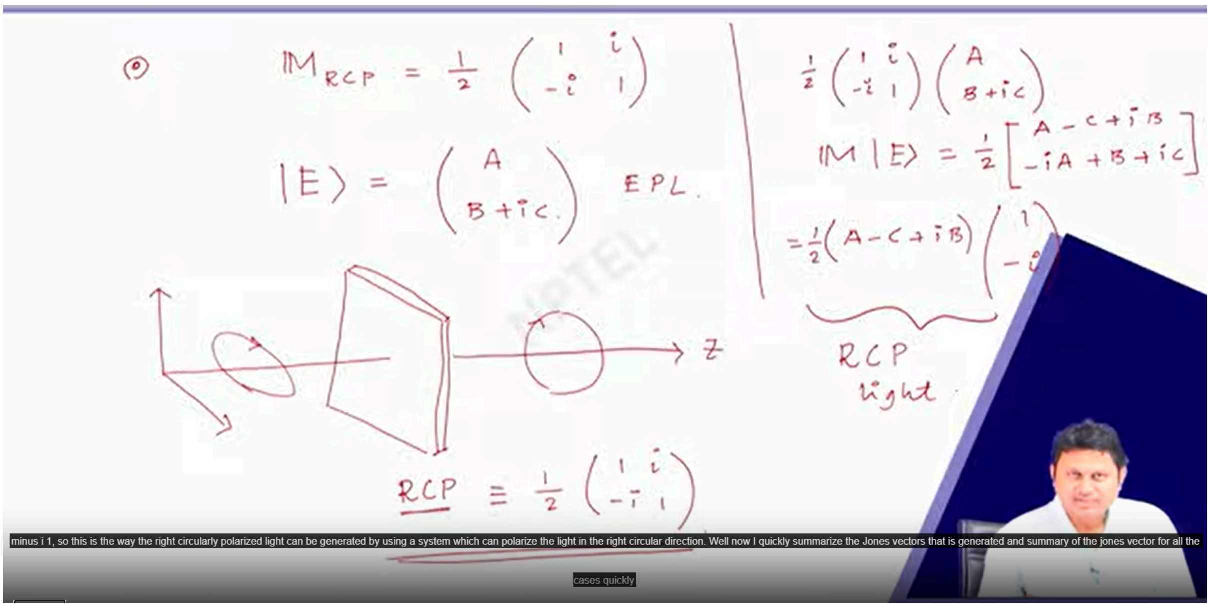

So, if this angle is alpha, then tan of 2 alpha, as per our expression, this is 2 of, let me write down first, E0x E0y whole divided by e 0 x square minus e 0 y square and then we have cos of delta so cos of delta you can find out with c and b e 0 x is a e 0 y is b squared plus c square root over of that and this is the quantity. So using that one can find out what is the value of the alpha and with exploiting this expression which is written. So, if a general matrix is given in this way a plus a and b plus i c or b minus i c. So, one can understand readily that this is an equation of this is representing a polarized light and this polarized light is elliptically polarized whether it is a left elliptically polarized or right elliptically polarized you can find out from this sign and what are the values of these angles alpha and what is the phase lag between these two components and what are the amplitude of this one can figure out everything from this matrix notation only. So with that note I would like to conclude today's class because I don't have much time. So in the next class, we will try to understand how polarization can be done. So what is the way to polarize the light? That thing we are going to understand. So beforehand, we study the Jones matrix method, which is very very important to systematically understand the polarization of the light. Now we are in a position to understand how light polarization can be mathematically represented. So, now, we will discuss how polarization can be done, and what is the way through which we can polarize light. So, with that note, I would like to conclude here. Thank you very much for your attention and see you in the next class.

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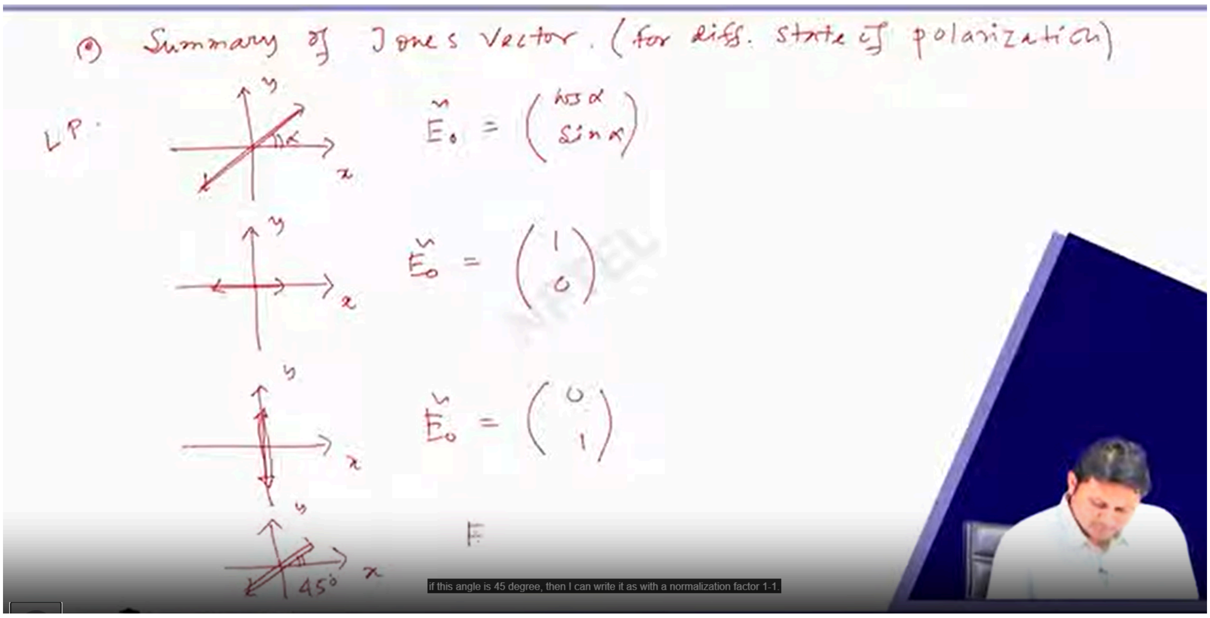

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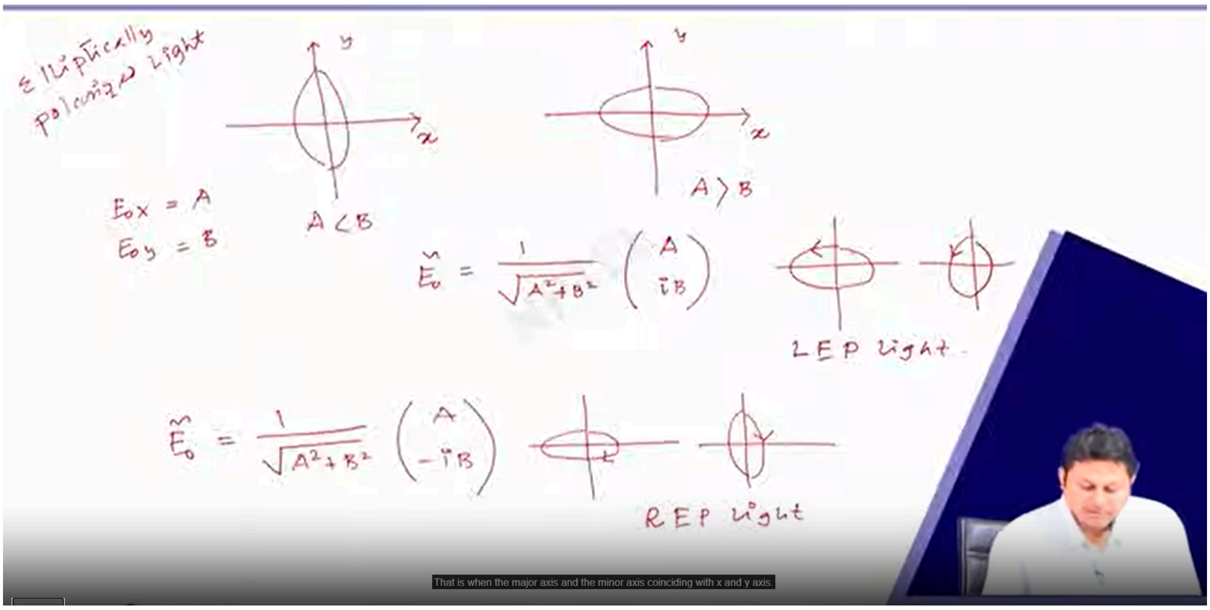

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