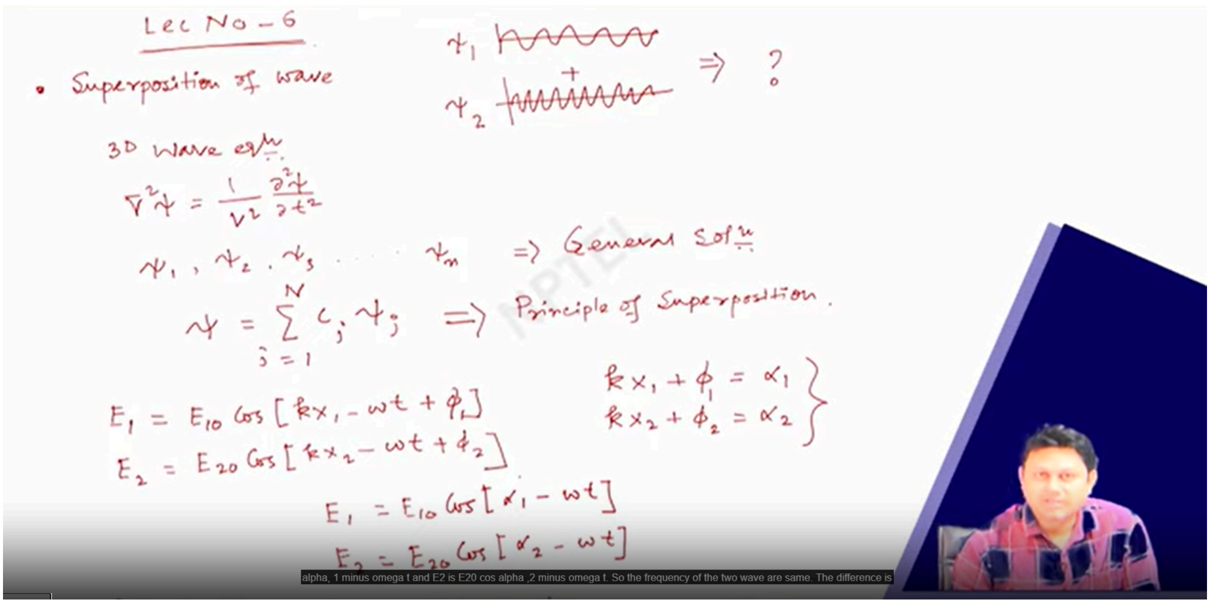
WAVE OPTICS
Prof. Samudra Roy
Department of Physics
Indian Institute of Technology Kharagpur
Lecture - 06 : Superposition of waves

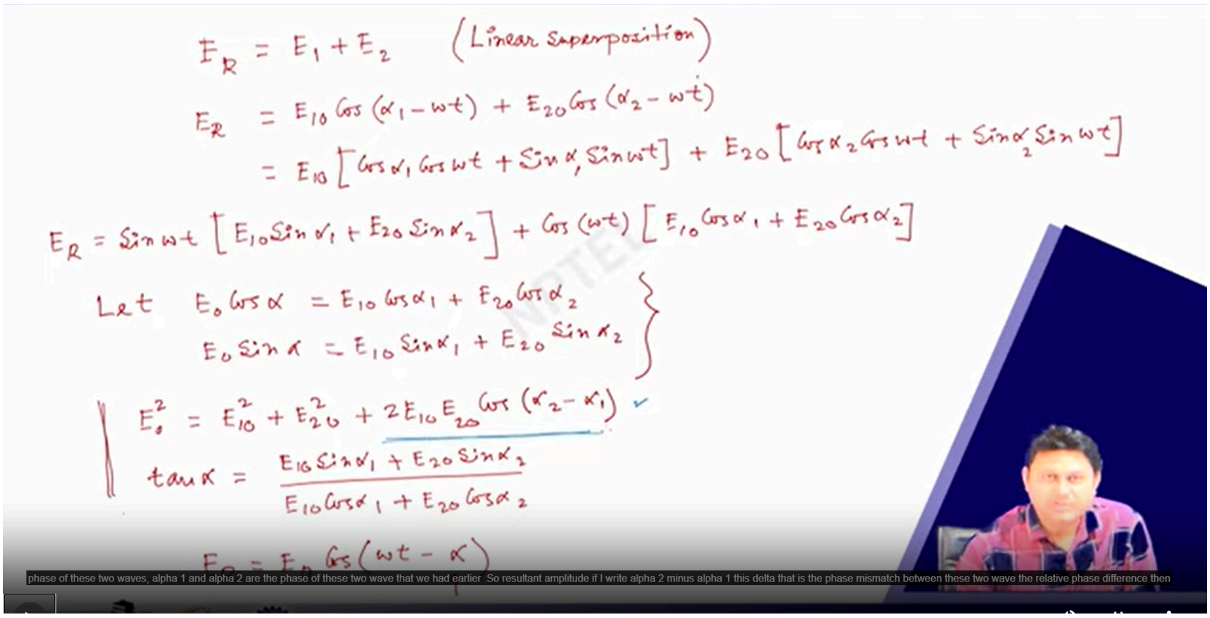
Hello, students to the 6th lecture of our course Wave Optics. Today we will discuss the superposition of waves and the concept of superposition. So, we have lecture number 6 students. So our topic today is very interesting and that is the superposition of waves. As the name suggests, what I am trying to say is simply that if we have two waves, they can be different, they can be the same, their frequency can be different, and their amplitude can be different. If I superimpose these two waves, say, psi 1 and psi 2, then what will, what will we get here as a sum? So, let me write down the 3D wave equation once again. And we have del squared psi is equal to 1 divided by v square del 2 psi del t square, v is the velocity. I am just writing for a general wave, not for electromagnetic waves. Now, if psi 1, psi 2, psi 3, and psi n are the solutions of this wave, then general solution I can write as these are the individual solutions of this wave, then the general solution we can write simply psi is equal to a linear combination of this wave, j can go from 1 to n so that is this is essentially is the principle of superposition. This is essentially the principle of superposition. So now we try to understand that, if two waves because this kind of situation we always have when we deal with optics, especially when the optics of two lights interfere with each other to light,

(Refer slide time: 6:55)



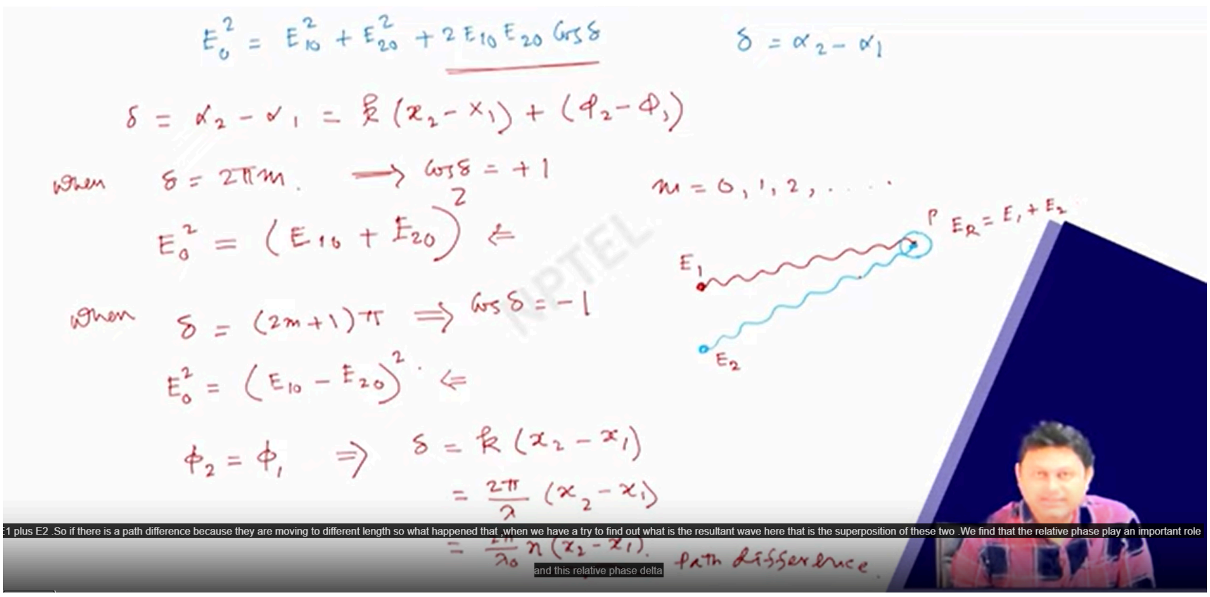
if I combine two lights for example, with the same frequency then essentially we superimpose these two waves. That means superposition principle we apply on them and then what should be the situation and how we deal with this situation. Let us do it in today's class. So suppose, we have one wave E1 and that is to say E10 amplitude is E10. And then it is a propagating wave. Let us consider the X direction. So I write this as KX 1 minus omega T plus a phase phi. So these are the same frequency. Another wave I can write is E2. I write E2 and this is E 20, cos I write kx2. I am just trying to find the superposition of these two propagating waves at the same frequency, two different points x1 and x2. This is phi1 and this is phi2. So, this I can write. So, this quantity is kx1 plus phi. I write this quantity as alpha1, and kx2 so, sorry this is phi1 plus phi2, the phase of the other wave I write alpha2 if I do that then I have two waves like E1 is equal to E10 cos alpha, 1 minus omega t and E2 is E20 cos alpha, 2 minus omega t. So the frequency of the two waves is the same. The difference is this, so two different points I am trying to find out what is the superposition. So of this two-way using the linear superposition principle if I have the superposition then ER is the superposition of these two waves is E1 plus E2 which is called the linear superposition And E1 and E2 will be represented by the form we have. So, essentially ER will be E10 cos of alpha 1 minus omega t plus E20 cos of alpha 2 minus omega t. Well, I can expand this cos and if I do, I have E10 then cos A cos A minus B so it is cos A cos B plus sine A sine B. So you have another expression which is cos A cos B plus sine A sine B and then what we do, just let me write ER here. We just take these sine and cosine components so sine omega t, if I take common then I'm going to get a set here like E10 then sine alpha 1 plus E20.

(Refer slide time: 15:35)



Here we have sine 1 sine is here sitting here we have 1 here we have 2 and then we have sine alpha 1 and then E20 I should have sine alpha 2 plus similarly, if I take cos omega t, this term common, I am going to get E10 and then cos alpha 1 plus E20 cos alpha 2. Now, let us consider that this entire term, I write E0 cos alpha, which is E10 cos alpha1 plus E20 cos alpha2. E0 sin alpha we write this term which is E10 sin alpha, 1 plus E20 sin alpha, 2 straight forward algebraic calculations. I am doing nothing special. And then if I want to find out what is the magnitude we have for E0 and alpha in terms of this, that we can also find. And you can see that, by the way, this term, this is the time-independent part that I am taking aside. And I can find E0 square. That is what I do in order to, so I make a square and add. And if I do that, then I will go to get E0 because cos square alpha plus sin squared alpha should be 1. And on the right-hand side, I do the same thing. It is easy to show that it is nothing but E10 square plus E20 square plus 2 of E10 E20, and cos of it should be the sine cos combination and it should be cos alpha 2 minus alpha 1, and if I want to find out what is alpha then that you also can find that tan alpha is equal to E10 sine alpha 1 plus E20. Just make it divide, this divided by this if I do sine alpha 2 all divided by E10 cos alpha, 1 plus E20 cos alpha 2. I can also figure out what is my E0 and tan alpha so that I can find this value E0 and tan alpha using the known values E10, E20 and etc if I put this back here the point is I can have the resultant wave as E naught, cos of omega t minus alpha. So the resultant wave the initial two waves should have the form of cos and the final wave one can also have the form of cos. The important thing is the frequency will remain the same if I add this to the frequency will remain the same but there might be something here that is sitting and that is a phase term. So there will be something related to the phase. So the resultant amplitude, whatever the resultant amplitude we have, is this for the superposition of these two waves. So let us now check one by one, so this is the resultant amplitude. One can have this resultant amplitude now having E10, E20 that is the individual amplitude of these two waves, and also a term that is sitting here which is related to the individual phase of these two waves, alpha 1 and alpha 2 are the phases of these two waves that we had earlier. So resultant amplitude if I write alpha 2 minus alpha 1 this delta that is the phase mismatch between these two waves the relative phase difference then the resultant amplitude I can write down as E0 square is equal to E10 square plus E20 square plus 2E10, E20 and cos of the delta. Where the delta is alpha 2 minus alpha 1. Now depending on the value of the delta one can have this value differently. That is the interesting part. So delta again which is alpha 2 minus alpha 1. So let me write down explicitly in terms of known value. So delta what we have here is alpha 2 minus alpha 1. So this is essentially k x 2 minus x 1 plus phi 2 minus phi 1. So the delta which is the relative phase between these two waves that are superimposing is related to this quantity so if I have x 1, x 2 or phi 1, phi 2 this value is multiplied by k in such a way that delta is 2 pi into m then what happened that I'm going to get E0 square a value like , E10 plus E20 whole square of that okay. So that is the value one can have when the value of the relative phase is simply 2 pi by m if m is 0 then also we have a cos delta value in a simple case. Let us put m equal to 0. So, if the delta is 0, that means the relative phase between the two waves is 0, then we have this quantity, which is the interfering term. This is a cross-term, which is basically the interference of these two different waves. This term goes maximum. And when we have a delta equal to 0, this term is maximum. And then the delta is equal to 0, cos delta becomes 1. And we have E1 square plus E2 square plus 2, E10, E2. So that means it is essentially E10 plus E20, the whole square of that. On the other hand, this is when the delta equals 2 pi, when 2 pi n. When delta is equal to, say, 2 m plus 1 and pi then what happened, then this term cos delta will be in this case what happened, let me write down cos delta will have plus one value in this case. What happens to the cos delta you will have a minus 1 value, m here is integer 0, 1, 2, and so on. So, what happened in that case? E0 square that is the resultant amplitude will simply become E10 minus E20 square of that. So that means based on the value of the relative phase we can have two waves when they interfere or they are superimposed, then two waves sometimes can get a higher value which is this one and can also sometimes get a lower value given by this. So these two values differ because of the value of the delta based on the value of the delta we can have this. So inside the delta, we have two phases, and this is k x2 minus x1 term. So if we say, forget about this phase term, let us take these two phases the same. So two waves that are coming out from a point have the same phase. So if phi 1 is equal to phi 2 then delta is simply defined by k multiplied by x2 minus x1. So k multiplied by x2 minus x1 is essentially 2 pi divided by lambda x2 minus x1 or I can write.

(Refer slide time: 23:50)



So this if is in free space, so if I write in a medium it should be 2 pi divided by lambda naught, lambda naught is a wavelength. Lambda naught is a wavelength of the light or the electromagnetic wave in free space. If it is in a medium then I need to multiply the term n which is the refractive index and then x2 minus x1. So this term is actually multiplied by x2 minus x1, this term is essentially called optical path, or optical path difference in this case. Because we have x2 minus x1. So this is in general called path difference. So if two waves are moving in such a way there is a part difference between these two that one can calculate by this. So two waves are moving and they are reaching some point and if there is a part difference between these two, that part difference can lead to a phase difference between this and when so let me try to visualise these things because that is what we're going to use. So suppose we have two sources like this one is red and another is blue, so I will have a wave that is emitting from this and it is moving like this and reaching to a point say p. Here is another wave that is also moving and going to that same point p. Here now at this point at p point these two waves are superimposed to each other so if I write in our previous notation the field that we are having here this is for E1 and this is E2 and this is my ER, which is E1 plus E2. So if there is a path difference because they are moving to different lengths what happens when we try to find out what is the resultant wave here that is the superposition of these two. We find that the relative phase plays an important role and this relative phase delta is coming through the path difference between these two. When we consider the wavelengths of these two are the same, the lambda or wavelength or the frequency is the same for E1 and E2. So interference is solely coming through the relative phase between these two phi 2 phi 1 or the path difference that creates a relative phase with this, we already considered that there is no initial phase difference between these two. Only the phase difference comes through the path difference and we get the interference pattern there and this interference pattern essentially gives you either a wave That is more amplitude with the initial combination of the two waves or sometimes it is less. So if I try to show this figure, I mean try to understand this figure simply so, this is say, one wave so this is my E1 and I can have another wave with a different amplitude. I don't know maybe we take the same amplitude now we take different amplitude because in E1 we take this amplitude was E10 and for E2 let us take another amplitude but the same frequency may be less amplitude and this is my P20 and if I add these two if these two are in phase over the time I'm propagating if these two are in phase then I'll go to get a resultant wave something like this and in this case if it is the resultant is E0, so this is my ER which is E1 plus E2. This is my E2 that is vibrating and my resultant amplitude here is E10 plus E20 square of that when the delta is 0, In another case we can also consider if I show that pictorially so that in this schematic figure I have the same wave and this is my E1. This magnitude is E10 and I have other waves of this relational amount of magnitude but same frequency and this value is E20 and this is my E2 wave. And if I add these two, I am going to get a resultant wave with less amplitude than the previous one. This is my ER, which is E1 plus E2. This value is my E0 and here E0 square is equal to E10 minus E20 square in this case my delta is the phase. Okay, here I am making a mistake in this figure because of the phase difference between these two, I mean they are not in the same phase and this phase. Suppose, the value is such that this phase is pi the phase difference between these two is pi if that is the case then what happened, that we were going to get a resultant wave with less amplitude? Note that if E1 is equal to E2, in this case, the resultant amplitude vanishes. That means when we have two waves and if their phase relationship is such that they interfere in a destructive manner, there is a possibility that even if you add two waves, they will cancel out each other and we are going to get zero intensity. On the other hand, if we add two lights there is a possibility that we are going to get more higher intensity light at least the distribution will be high in one and we will get a wave with a high value of amplitude, and peak amplitude if they are in phase. So superposition of these two waves we discussed today and we understand that when two waves are superimposed with each other then what happened, is that they can also lead to a higher amount of magnitude. The resultant wave that is coming out through the mixture of these two waves the superposition of these two waves can have a higher magnitude or they can have a lower magnitude as well and this entirely depends on the relative phase between these two. So interference, The concept of interference will come through this picture that we discussed today. What if I add these two waves, then what happens? If they are in phase, then they will interact in a constructive manner and we are going to get constructive interference. And if they are out of phase, then their superposition leads to something called destructive interference and we are not going to get any light from that even though we are signing. We are signing the screen with two lights so that thing we are going to discuss in detail in our future classes but the concept of the superposition, the concept of the propagating wave, and how one can deal with this superposition mathematically that we described today. So with that note maybe I would like to conclude in the next class what we do, that we will discuss how these superpositions of wave can be treated in a complex, complex notation, and when we do in a complex notation, we find that things will be a little bit simpler we can understand that instead of adding two waves if I add more and more wave say three four five waves or an infinite number of the wave how to do deal with this addition of wave in terms of the superposition principle. So with that note let me conclude today's class, thank you very much for your attention, and see you in the next class.

(Refer slide time: 32:50)