WAVE OPTICS

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Lecture - 64: Index Ellipsoid

Hello, student, welcome to the wave optics course. Today we have lecture number 64 and in this lecture we're going to discuss the index ellipsoid. So we have lecture number 64 and today's topic is index ellipsoid or simply refractive indices of uniaxial crystals. How to define the refractive index because the refractive index is no longer the same in the same direction. So refractive indices of uniaxial crystals are okay. So in order to define how the refractive index in the uniaxial crystal one can write I need to draw first this figure that for ordinary ray we have refractive index same in all the direction. Suppose this is y direction and this is x direction and the optic axis is along this direction as well, since the optic axis is along this direction we have an ellipse here. Let me draw this ellipse in this way for other polarization and this dotted line is basically the wave surface and this dotted line is the wave surface for the O ray and this is the wave surface for the E ray okay. So this is my origin O, this is B and this is a major and minor axis and if I draw a tangent here, let me finish this drawing then we will discuss. So this is the tangent and if I draw from here to here this point this is C, if I draw perpendicular this is point b and so this is t, so tb is tangent of this ellipse basically this ellipsoid three dimension and this is the thing we have this angle this angle, so this is theta. So let me define one by one then things will be, so n theta is the velocity of the light in vacuum in c the way we define velocity of light in vacuum which is C divided by the wave velocity of E-ray in the direction theta, so you can see that if I change these three time, if I just move the tangent then we can see the velocity is different, if I draw the perpendicular and this angle theta then if I change the theta in all direction the velocity is different. Along this direction however the velocity of the o ray and e ray is same and when it touches in this point and if I do the tangent so the angle here is theta equal to 0 for which it is happening. So that is the definition that means n which is the refractive index of e ray is not a fixed quantity but depends on theta that is something which is interesting here, so ob here which is equal to p that is the perpendicular distance drawn from the center on the tangent tb, d so we basically the perpendicular that is drawn as I mentioned. Now from this structure I need to draw this structure once again because I'm going to the next page because that is okay. So let me draw this. So we have this ellipse and then we had this circle this is X, this was Y and we had the tangent here and this is perpendicular to that, and this is the point over this, that was T, that was B, that was C and this was small b and small a characteristics of the ellipse. Now the equation of the tangent since we draw a tangent from outside point, I hope you know this equation that the equation of the tangent of an ellipse which is a straight line can be, so this is the equation of a tangent of an ellipse is y equal to mx minus root over of a square m square plus b square. So a b of the ellipse is given then you can draw a tangent and this will be the form where m is the angle. So here. I can write it as y minus mx plus root over of a square m square plus b square that is zero, that is the equation we have where m is this angle, this is phi, I mentioned that this is my theta and OB is a perpendicular point that we that we drawn.

So phi is the addition of theta plus 90 degrees. So M is equal to tan of phi and that is eventually equal to pi and this small angle is so that will be minus of the tan of this is tan of pi minus theta because phi is equal to theta because phi is equal to pi minus because phi plus theta has to be pi and and then it should be equal to minus of tan of theta. So okay let me check it once again. So phi plus delta, if this angle is delta, then phi plus delta is equal to pi and that is equal to theta plus pi by 2 plus delta. So phi plus phi is equal to theta plus pi by 2. So I made a mistake here. So it should be pi by 2 plus theta and if that is the case then this will be minus of tan theta because sine pi by 2 plus theta divided by cos pi by 2 plus theta, so it essentially should be that quantity and we have then p is a, this is the o point, let me check ob is essentially p and then what happened this p since the equation of the tangent is known the p will be simply the perpendicular distance from the point zero. So we can write it as zero multiplied by x plus zero multiplied by y plus whatever is there a square m square plus b square and this is root over of m square plus one okay but still let me check once again, so now p square here, so let me do that then I come back to this p square is equal to m square a square plus b square divided by m square plus 1. Now m which is tan phi and that we calculate that angle theta. So that is minus of tan theta, if that is the case then if I put this tan theta here then p square will be sine squared theta a square plus cos square theta b square divided by just putting tan and then divide into sine and cos it should be cos square theta then tan square theta plus one which is essentially p square is equal to a square sine squared theta plus b square cos square theta that I have, so this angle is okay, let me check once again. So theta plus delta is pi and then it seems to be okay at this stage. Let me go back. So, for e ray, the ray velocity or wave velocity are different in general, only along the optic axis they are the same. So, the ray velocity, let me draw this. So this is the tangent point C, this is the perpendicular that we are drawing B and this angle we say this is my theta and this is my x-axis, this is my y axis and this is phi and the ray velocity of light to move from the origin to the point C this tangent point, so the ray velocity is V is equal to OC.

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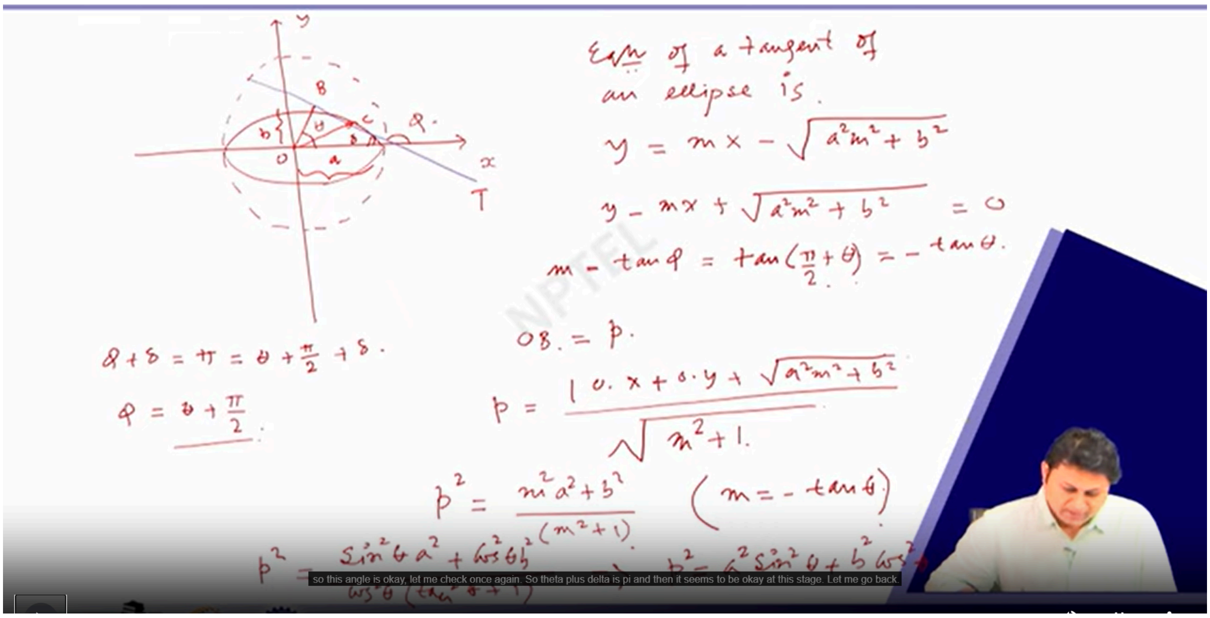

So this is OC divided by T and the wave velocity of the light to move from the origin to the point B which is perpendicular to the tangent and normal to the B which is normal to the e-ray wavefront. So now OB is P and it is V theta multiplied by the time t, A is because along this direction we have an optic axis. So ordinary velocity multiplied by t and the distance b will be extraordinary velocity multiplied by t along this direction. So we have p square is a square, I think we made a small mistake here. So I think this will be cot theta with plus sign okay and then here this will be cos because m was cot and everything will same, so this will be sine this will be cos, cos will replace by sine, this will be cot anyway, in the next class, I'll redo this because I made a mistake here. So that's why this carry forward. So it will be cos squared theta. It seems to be associated with this. So then p square is a square cos square theta plus B square sine squared theta that we have, once we have this then P, if I write it is V theta, T square is equal to A, I write V naught square T square cos squared theta plus ve square t square sine squared theta, so this t square will cancel out and we find an interesting equation and that equation is v theta square is equal to v naught square cos square theta plus v e square sine squared theta. Now v one can associate this v with the refractive index that we know. So v theta we know that n theta that was v theta divided by c n is the ordinary velocity divided by c, n extraordinary will be the extraordinary velocity divided by c. Replacing this, we can see if I divide these things over c. So, this is the inverse of that. Many mistakes here also did, I just made the same mistake earlier, let me see once again. Now this velocity in the vacancy is divided by the velocity of the e wave in the direction of theta. So that e sine is supposed to write it c divided by v theta. Now here v theta is there. So it should be then this equation we can simply get 1 divided by n theta square is equal to divided everything by c square 1 divided by n o square cos square theta plus 1 divided by any square sine squared theta, where that means this is the expression for refractive index ellipsoid or refractive index variation for extraordinary day so that means if I draw ellipse here, so depending on this angle theta so the value of the refractive index is going to change. Note that for this particular case when theta is zero that means along this direction is my optic axis so when theta is equal to 0 that means I am talking about this particular direction, n theta will be equal to n ordinary refractive index however,

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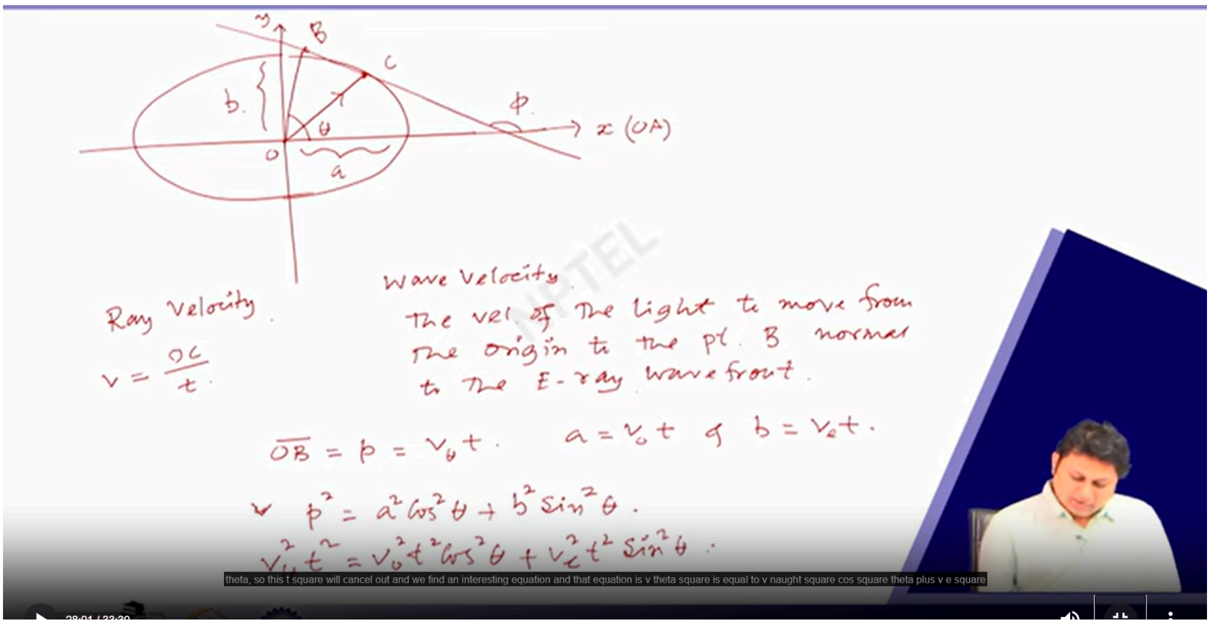

if I go perpendicular along this direction, for example when theta is pi by 2 then n theta will be equal to n extraordinary. So this is the equation for extraordinary rays which is not the same for uniaxial rays, first it is not the same in all directions, it is different in different directions and what should be the refractive index that can be defined by this equation. During the calculation however if you look carefully I made a small mistake here. So you need to check this I wrote initially it is a tan theta, it is a cot theta and rest of the thing if you do then you will find that sine theta will be simply replaced by cos theta and cos theta is sine theta that I made a mistake I'm not doing once again and then what we get is p is a square cos squared theta plus b square, sine squared theta due to my mistake I initially got a square sine squared theta plus b square cos square theta which will be opposite sine theta will be cos theta and cos theta will be sine theta because of this mistake. Now I rectify this mistake and now I get the correct expression which is this one. So I don't have much time to discuss more about that. So I suggest that you should redo this calculation by yourself to get more idea how these things are working. In the next class that I do, we will put some examples of a few crystals which show this uniaxial property and exploiting this property of the crystal you can polarize light and also analyze the light. So with that note, I would like to conclude here. Thank you very much for your attention. See you in the next class for more.

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