WAVE OPTICS
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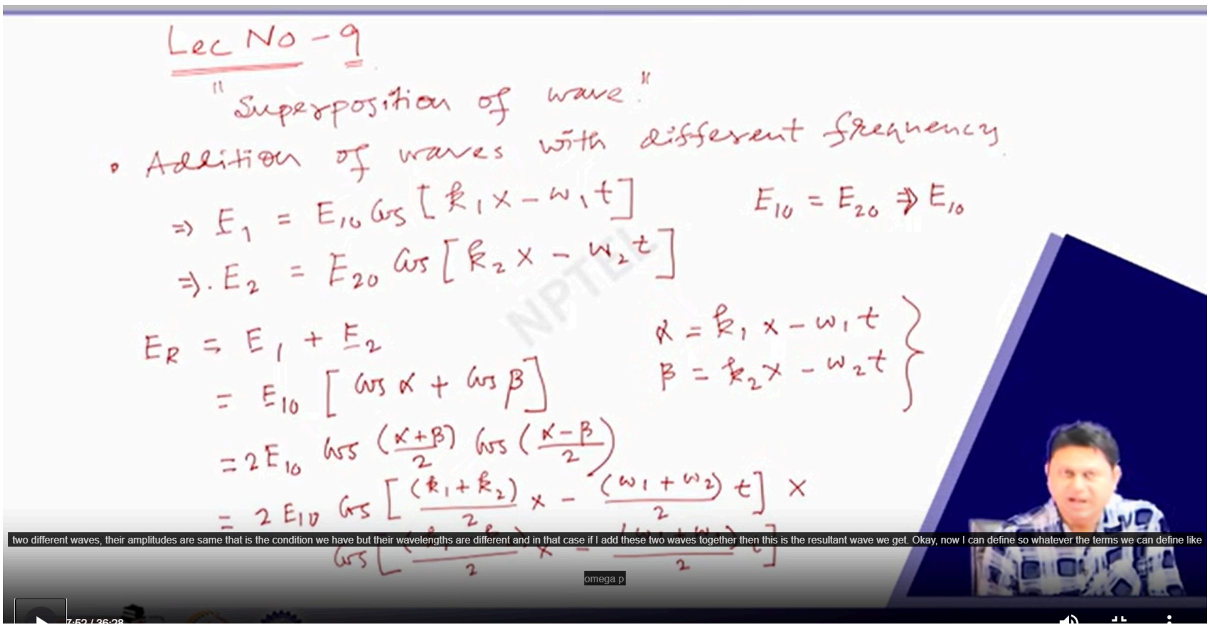
Lecture - 09: Group and Phase velocity

Hello, students to our course wave optics. Today we have lecture number nine and in this lecture we will discuss the concept of group and phase velocity. Okay, so let me start with the concept that we initiated in the last class. So today we have lecture number 9. So in the last class, we discussed the concept of superposition of different waves. So our basic concept was superposition of waves and in order to study that in the last few classes we superimpose two waves and their frequency the same. So now what we do today is we are going to superimpose two waves or we add two waves whose frequency is slightly different. So we will do it today. The addition of this is under the superposition of waves but with two different frequencies we will add two waves having two different frequencies. So, addition of waves with different frequencies. Suppose, we have two waves, say E 1 which is equal to E 1 0 cos of k 1 x minus omega 1 t. Previously, it was simply omega, but now we are going to discuss what happened if we add two waves. So, instead of having one wavelength, now we have two different wavelengths for two different waves. So, next is e 20 cos of k2 x minus omega to t. So their amplitude may be considered to be the same for simplicity and their wavelength is different. So I can take their amplitude to be the same. So I can write E10 is equal to E20 is simply say I write E10 because it is the same. So I write E20 as simply E10. So that is the condition we put that their amplitudes are the same. Now the resultant waves will be E1 plus E2 since their amplitudes are the same and we call it E 1 0 the amplitude of E 1 wave which is the same as E 2. So, I write E 1 0 and then simply write cos of alpha plus, cos of beta where my alpha is equal to k 1 x minus omega 1 t and beta is k 2 x minus omega 2 t. (Refer slide time: 07:52)



This is the value of alpha and beta and now we simply try to figure out what this value is. So simple addition of cos alpha and cos beta gives us 2 of E10 then cos function of alpha plus beta divided by 2 and then this is cos a plus b. So cos a plus, b by 2 and cos a minus, b divided by 2 that is the next one. And now alpha plus, beta and alpha minus, beta I can calculate because it is already defined here and if I do we are going to get this 2 of E 1 0 it will be cos of alpha plus, beta and simply k1 plus, k2 divided by 2x minus omega 1 plus, omega 2 t divided by 2 that is the first term multiplied by cos of k1 minus, k2 whole divided by 2x minus, omega 1 plus omega 2 whole divided by 2 t. So that is the total term we have when I simply add two waves having this form, two different waves, their amplitudes are the same that is the condition we have but their wavelengths are different and in that case if I add these two waves together then this is the resultant wave we get. Okay, now I can define whatever the terms we can define like omega p that is the sum average omega that we had. Similarly I can define Kp, which is the average of the propagation constant of these two waves divided by 2, also we can write omega g that is the difference between these two. So I have omega 1 minus, omega 2 divided by 2 and kg. I define k 1 minus k 2 divided by 2. So we just defined a few more variables omega p kp, omega g kg but they are related to the initial frequencies that the wave had that is omega 1 omega 2 and the initial k vector they have, the propagation constant value they have which is k 1 and k 2. So if I write all together in this new notation then my ER becomes 2, it becomes 2 e 1 0 that is amplitude and cos of kp x minus omega p t multiplied by cos of k g x minus omega g t. So we can see that two propagating waves with different frequency when we add together then the resultant wave is also propagating but it has the amplitude. If I write this portion as an amplitude then the amplitude is also moving. What is the meaning of amplitude and what is the meaning of propagation we're going to understand here. So, before that we can see what the frequency is, here we can see that this function cos k p x minus omega p t, this is a propagating wave and this is also propagating. So, initially we have, so let me picturise what we get. So, we have two waves with equal amplitude, but different frequency, so this is my say E1 and another wave with relatively higher frequency is E2.

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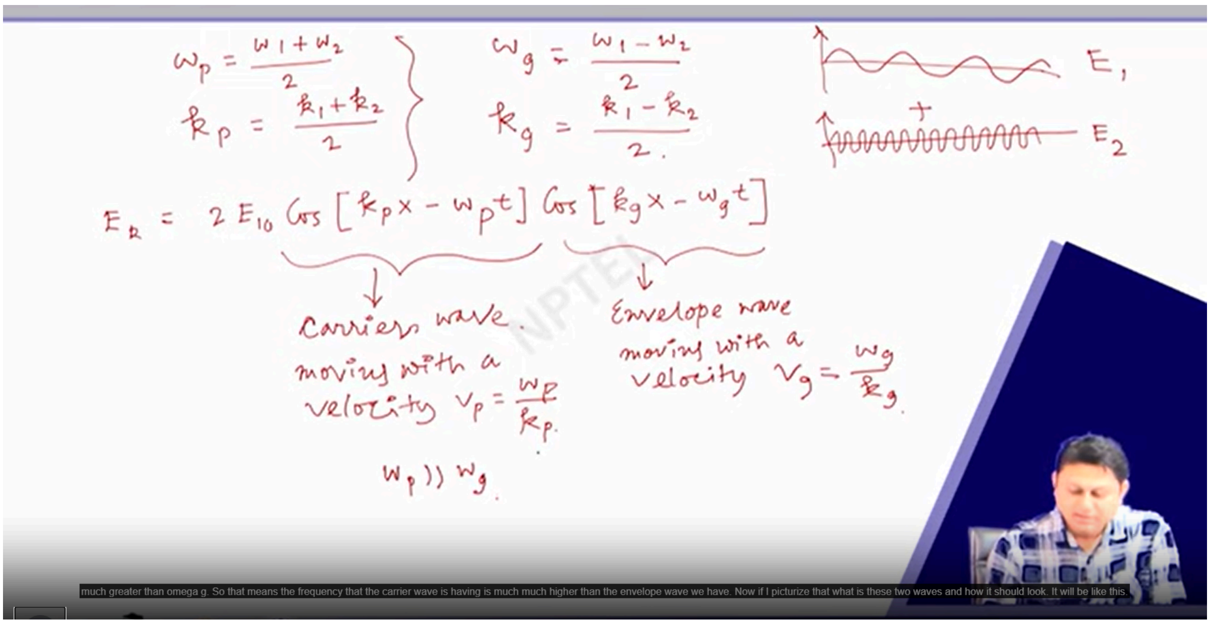

What we did is we add these two by using the superposition principle that is we simply linearly add these two and I get two waves with the multiplication of this cos function suggesting that this resulting wave is also propagating because it is having the form kx minus omega t form. Well, this wave is called the carrier wave having a frequency of omega p and omega p as I mentioned here it is omega 1 plus, omega 2 divided by. So, carrier wave is wave having a average frequency whatever the frequency we have for e 1 and e 2 the frequency of the carrier wave is average of that and it is moving with a velocity like if I write vp that velocity is omega p divided by k p, this is the velocity at which this carrier wave is moving. Also we have another part of this wave and this part which is having relatively lower frequency. If we look at omega g carefully, omega g is the difference between the frequency of the two waves. So, this is called the envelope wave. This is the envelope wave and moving with a velocity I write v g is equal to omega g by kg, this is the velocity at which it is moving clearly. It is given that the velocity of these two are not the same. It can be different, it can be the same in a few cases but in general this is not the same and omega P is much greater than omega g. So that means the frequency that the carrier wave is having is much much higher than the envelope wave we have. Now if I picture what these two waves are and how it should look. It will be like this. So, let me draw in the same figure. I'm going to draw the two waves. So in one case, let me fix the amplitude and this is a wave having high frequency, this is the one wave and this is my E1. Another wave I also draw whose frequency is less and it is something like this. On top of that I draw and this is my E2. So, if I add this E1 and E2, the resultant wave will have a form like this. We have an envelope like this. And inside this envelope, we have the carrier waves. The solid line that I am drawing here will be the structure of the resultant wave that one can expect. So this is along x direction and this is moving, this is the resultant one. So resultant one means ER, which is equal to e1 plus e2 and ER is a moving wave, it is having a modulated amplitude inside this envelope. We have the distribution as well which we called the carrier wave having a frequency omega p, which is omega 1 plus, omega 2 divided by 2

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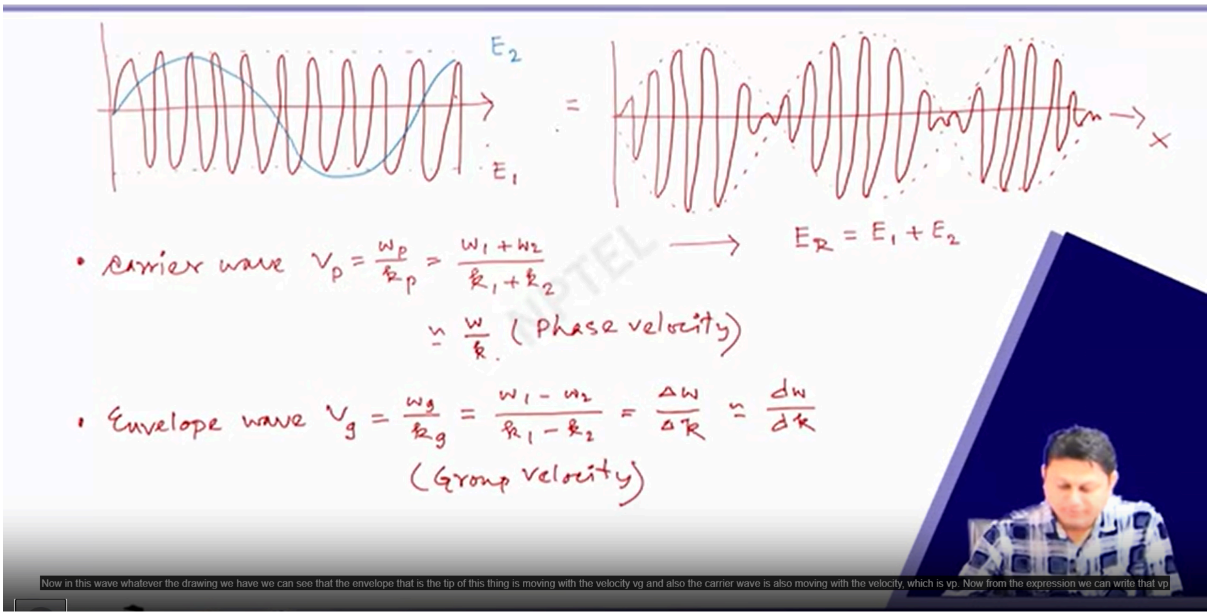

and also there is a modulation in amplitude and it is periodically modulated. We call this envelope function through which it is modulated and this envelope function on envelope waves has a frequency which is omega 1 minus, omega 2 divided by 2. So this is also moving in one direction. Now the point is what should be the velocity of this envelope that is moving. And what should be the velocity of this carrier that is moving. Already it is mentioned that the carrier wave will travel. So with the velocity, let me write here, S for carrier wave the velocity if I define v p, this is w omega, p divided by k p and this is essentially omega 1 plus, omega 2 divided by k1 plus, k2. Now this value is nearly equal to omega by k, where omega, I mean this is where omega is the average one and k is some way if I just write the average of omega and omega 2 is omega p. So I write omega k and k 2 the average of that thing is k. So these values will be simply omega divided by k. Well on the other hand the envelope wave have a velocity vg, which is omega g divided by kg and that is essentially omega 1 minus, omega 2 divided by k 1 minus, k 2 which is delta omega divided by delta k, where delta omega is a difference between these two frequency and delta k is a difference between the wave number they have. So this is essentially d omega if they are small then I can write it as d omega dk. So, this velocity has a special name it is called the Group velocity and the carrier wave which is propagating with a velocity omega divided by k is called the phase velocity. So, this is called the phase velocity. So, these two velocities we define in one case it is omega by k and another case it is d omega by dk. Now in this wave whatever the drawing we have we can see that the envelope that is the tip of this thing is moving with the velocity vg and also the carrier wave is also moving with the velocity, which is vp. Now from the expression we can write that vp is simply omega by k and vg is d omega by dk. So vg I can write in terms of vp. So vg which is d omega by dk I can write ddk multiplied by Vp into K. So, this essentially gives us Vp plus K into del Vp dVp dK. So this is the relation between the group velocity and the phase velocity. So group velocity is equal to phase velocity plus k multiplied by the derivative of the phase velocity with respect to k. So if we had a situation when the change of phase velocity with respect to k vanishes then one can simply have then we can have vg is equal to vp that means the phase and group velocity are same.

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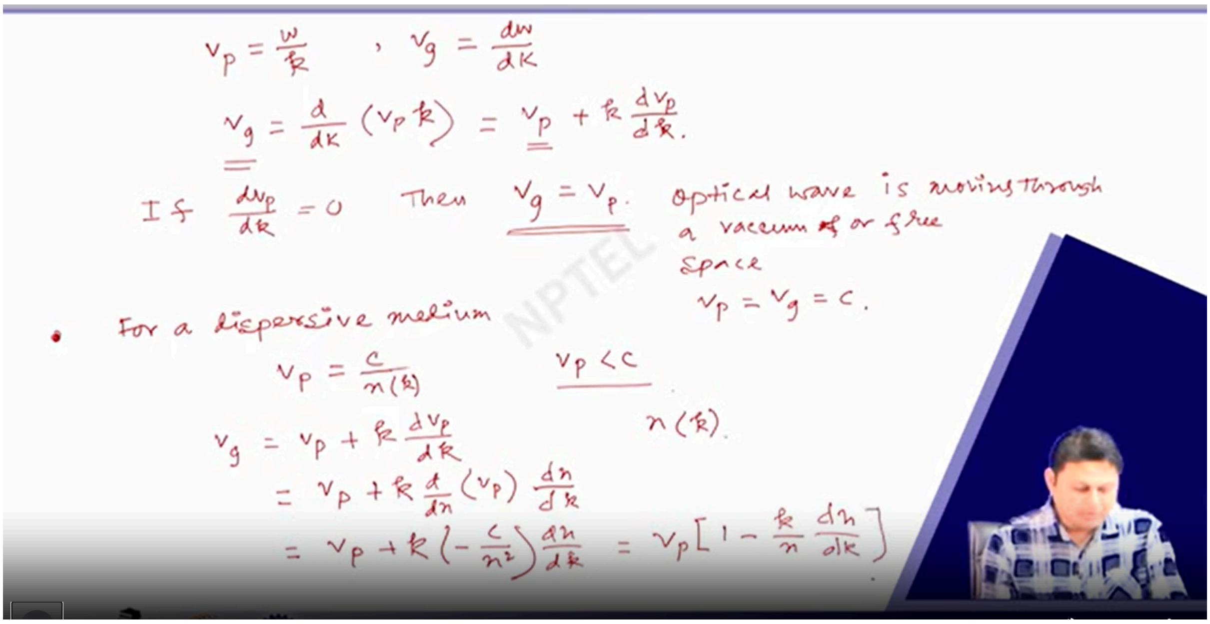

So, normally this case happens when the light or the optical wave is moving through a vacuum or a free space vacuum or free space, then vp is equal to vg and they are equal to c.

So then what happened was that vp is equal to vg and they are equal to the value of the speed of the light c but the problem is completely changed. The problem is different when we have a dispersive medium. So what is the meaning of dispersive medium? That means we have a medium with refractive index more than one. So for a dispersive medium what we have Vp is no longer C, it is basically C divided by nk because refractive index is a function of the propagation constant or wavelength. So that means in dispersive medium vp is less than c. So the refractive index as I mentioned is a function of wavelength or k then vg from this expression we had. So vg was how much it is vp plus k into d vp divided by dk. So vp I know what the functional form is. So that I am going to put. Okay let me do that first. So I have vp plus, k and then I need to make a derivative. So n is a function of k. So what do I do? I will write that d of n v p and d of n d k, so if I execute this quantity it should be vp and then plus k dvp dn. From this expression I simply have minus of c n square and then we have dn dk. So from here I can also find that c by n, I can write it is vp and this expression simply gives us vp, if I take vp common then 1 minus k by n dn dk. But normally what do we do? We don't put dn dk because n is a function of lambda. So normally we try to write down everything in terms of lambda. So this expression we slightly change as per our convenience. So let me write it down. So we have vg is equal to vp 1 minus, k by n, we have dn dk instead of writing what we do in this way. So k, I write as 2 pi by lambda. So dk d lambda will be minus of 2 pi by lambda and if I replace this derivative vg will be vp 1 minus, 2 k, I replace 2 pi divided by lambda then it is 1 divided by n and this derivative I write dn d lambda because n is essentially function of lambda. We know the explicit form of n as a function of lambda. We will show in the next class maybe then I write dk d lambda which is d and d lambda rather, d lambda d k, here I write. So let me do that d lambda dk, so d lambda d k I replace it from here. So we have v g equal to v p 1 minus 2 pi divided by lambda into 1 by dn d lambda and this quantity is minus of lambda divided by 2 pi. Here should be a lambda square so it should be a lambda square by 2 pi and essentially we get vp into 1 minus this, 2 pi will cancel out, 1 lambda will cancel out, this minus minus will be plus. So plus lambda by n then dn d lambda, so we have dn d lambda, so this is the expression we have which shows how the quantity v g and v p are related to each other. That is, the group velocity and phase velocity are related to each other and we find a very important term that determines whether the group velocity is equal to phase velocity. Under that condition what happens the dn d lambda has to be 0 that means n should not be a function of lambda and it only happens when we have the value of n equal to 1 that is in free space or in air. So where the refractive index does not change with lambda in that case we have vg equal to vp that we know but what happened when we have a component like this into the equation, that means when we deal with the velocity of the light in a medium then that medium should have a characteristic form in terms of his refractive index and if you know the refractive index as a function of lambda then from this expression we can calculate the group velocity of the light. So when the light with different frequency is passing through that medium the phase and group velocity will differ and also the group velocity is a function of lambda. So different wavelength will travel at different speed and that is the phenomena we called as the dispersion which is very important when we discuss the aspects of light as a wave and if it is passing through a dispersive medium then the velocity of the light for different frequency component will be different and that's why what happened we will going to get a very important phenomena called dispersion. So today we will not have much time to discuss the phenomena of this dispersion. In the next class we are going to discuss in detail the phenomena dispersion and then we will discuss more about the material aspects of the dispersion, how the refractive index is related to lambda, and then go forward with other topics. With that note I would like to conclude here. Thank you very much for your attention and hopefully we are going to see in the next class, where we are going to discuss about the material dispersion in detail. Thank you very much and see you in the next class.

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