

Condensed Matter Physics
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Lecture - 10

The Free Electron Theory of Metals – Electrical Conductivity – Worked Examples

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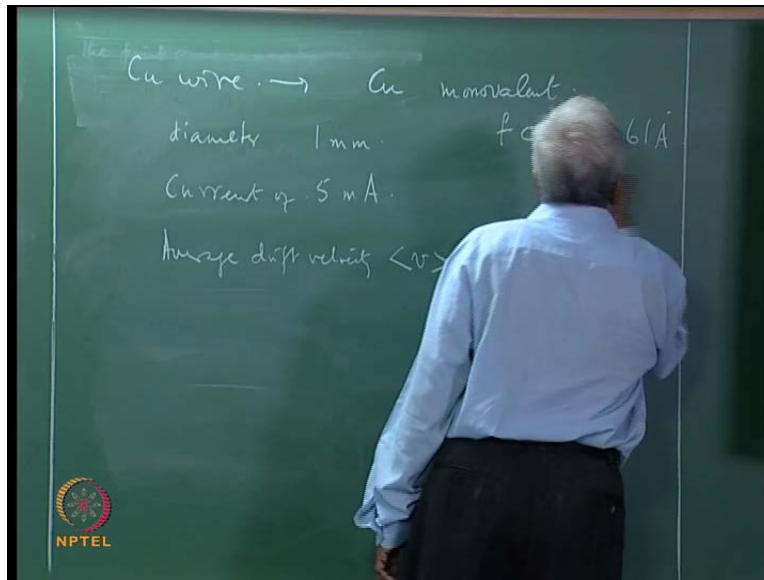
Problem 27

A copper wire of 1mm diameter carries a current of 5 mA. Calculate the average drift velocity of the free electrons. Copper is monovalent and crystallizes in FCC structure with a lattice parameter of 3.61 \AA .

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Next we go on to the problem regarding the conduction electrons in copper - in a copper wire. We are told that it has a diameter of one

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1 millimeter, and it carries the current of 5 milliamps. Now we are asked to calculate the average drift velocity. We are given the copper is monovalent, elemental copper is monovalent; and crystallizes in a face centered cubic structure with a lattice parameter of 3.61 Angstrom. These are the data given.

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Solution:

$$J = ne\langle v \rangle$$
$$\langle v \rangle = J / ne$$

Copper is FCC and so the number of atoms per unit cell is = 4

The volume of the unit cell = $(3.61 \times 10^{-10})^3 \text{ m}^3$

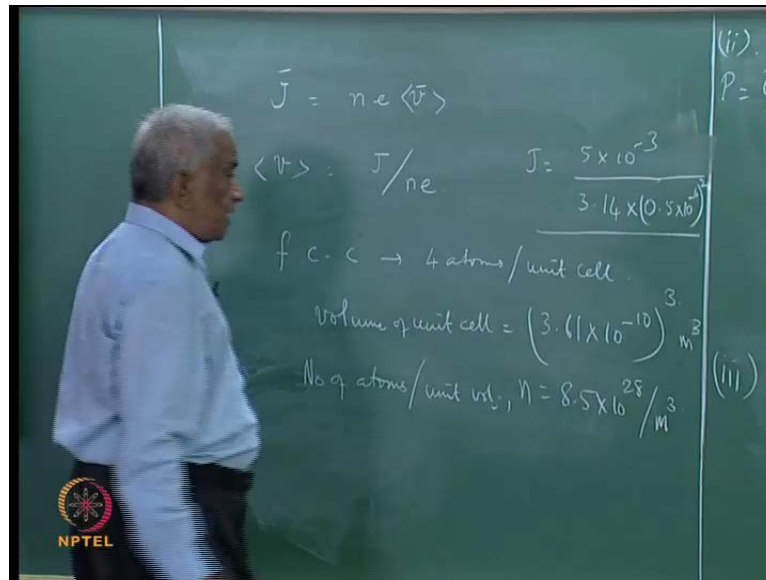
Therefore, the number of atoms per unit volume

$$= \frac{4}{(3.61 \times 10^{-10})^3}$$
$$= 8.5 \times 10^{28} \text{ atoms / m}^3$$

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So we know that the current density J is $n e v$.

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Therefore, the magnitude average drift velocity is just the magnitude of the current density by n e . Now J is of course we are told that it has 5 milliamps current in a diameter of one millimeter, so it is 3.5 into 0.5 into 10 to the power minus 4 square, that is the diameter, that is the radius, so that gives the value of the current density. We also know from the fact that it has the unit cell, which is face centered cubic, therefore there are four atoms per unit cell.

In the unit cell volume can be calculated from the given lattice parameter, which is 3.61 into 10 to the power of minus ten cube that is the volume in meter cube. Therefore, we can calculate the numbers, since there are four atoms in the unit cell, so what is the number of the atoms per unit cell number of atoms per unit volume that is what we call as n and that turns out to be 8.5 into 10 to the power 28 per meter cube. And since copper is monovalent, this is the number of, this is n equal to this, the number of electrons is just equal to this.

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Since copper is monovalent, the number of free electrons per unit volume = the number of atoms per unit volume = $8.5 \times 10^{28} \text{ m}^{-3}$

The current density

$$J = I/A = \frac{5 \times 10^{-3}}{\pi(0.5 \times 10^{-3})^2} = 6366 \text{ Amp / m}^2$$
$$\langle v \rangle = J/ne = \frac{6366}{8.5 \times 10^{28} \times 1.6 \times 10^{-19}} = 4.68 \times 10^{-7} \text{ m / sec}$$


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Therefore we have the current density, we have this know the electron charge, so we can readily calculate the drift velocity which turns out to be the average drift velocity by substitution in this as the value of 4.68.

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$\langle v \rangle = 4.67 \times 10^{-7} \text{ m/s}$

$n = 8.5 \times 10^{28} / \text{m}^3$

$\sigma = 6 \times 10^7 \text{ ohm}^{-1} \text{ m}^{-1}$

Relaxation Time, τ .

Drude expression $\sigma = \frac{ne^2\tau}{m}$


$$\tau = \frac{m\sigma}{ne^2} = \frac{2.5 \times 10^{-14}}{5} \text{ s}$$


We will take the data for a common metal a general metal.

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Problem 28

Taking the required data for a common metal show that the relaxation time is of the order of 10^{-14} sec.



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So and we are ask to prove calculate the relaxation time time tau from Drude expression for the electrical conductivity, where tau is the relaxation time, so tau is inverting this and sigma n e square.

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Solution:

$$\sigma = ne^2\tau / m$$
$$\tau = m\sigma / ne^2$$

For copper,

$$n = 8.5 \times 10^{28} \text{ m}^{-3}$$
$$\sigma = 6 \times 10^7 \text{ ohm}^{-1}\text{m}^{-1}$$

Therefore

$$\tau = \frac{9.1 \times 10^{-31} \times 6 \times 10^7}{8.5 \times 10^{28} \times (1.6 \times 10^{-19})^2} = 2.5 \times 10^{-14} \text{ sec}$$


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So we take the data of copper which we already found just now as 8.5 into 10 to the power 28 meter cube. And for copper the number of electrons per unit volume and the conductivity is, so

using these values, we can readily calculate the relaxation time as 2.5×10^{-14} sec. So this is the average time in which the electronic system the electron relax back in an electric field.


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Problem 29

5. Choose the correct alternative(s) in the following:

i. In the absence of scattering by phonons, the electrical resistivity of a metal


- A. is zero
- B. is infinite
- C. is independent of temperature
- D. depends on its purity



In the next question, we are given a set of correct alternatives answers then we are ask to choose the correct alternative, there may be more than one correct alternative.

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No scattering by phonons,
P. of a metal.
C and D.



So when there is no scattering by phonons, we are told this statement is the electrical resistivity ρ of a metal is it 0, is it infinite or is it independent of temperature and does it depends on its purity. Obviously C and D are the correct alternatives.