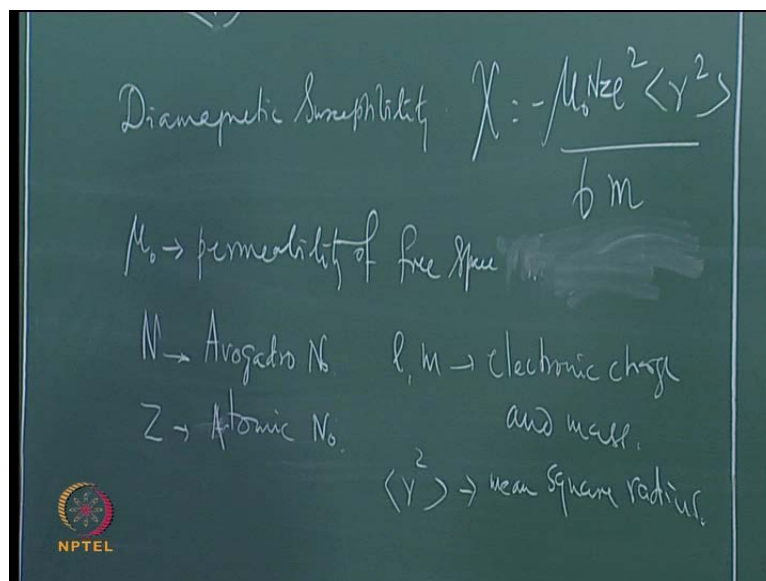


Condensed Matter Physics
Prof. G. Rangarajan
Department of Physics
Indian Institute of Technology, Madras

Lecture - 24
Magnetism and Magnetic Resonance-Worked Examples

Today, we will do some examples to illustrate the concepts on magnetism and magnetic resonance, which we already discussed.

(Refer Slide Time: 00:41)



Diamagnetic Susceptibility: $\chi = -\frac{\mu_0 N z e^2 \langle r^2 \rangle}{6 m}$

μ_0 → permeability of free space
 N → Avogadro No. e, m → electronic charge and mass.
 Z → Atomic No. $\langle r^2 \rangle$ → mean square radius.

NPTEL

Let us go to example in which we are as to calculate the diamagnetic susceptibility of a helium susceptibility means, in this case magnetic susceptibility given that the average atomic radius this 0.5 the angstroms is them as 5.3 nanometer. We know that the land remain expression for the diamagnetic susceptibility for any system atoms per molecules is given by χ equals minus μ naught e square r square into $n z$ by $6 m$.

(Refer Slide Time: 01:47)


Worked Example 54

Problem
Calculate the diamagnetic susceptibility of He atom given its average atomic radius is 0.53 Å.

Solution
Diamagnetic susceptibility per unit volume is given by

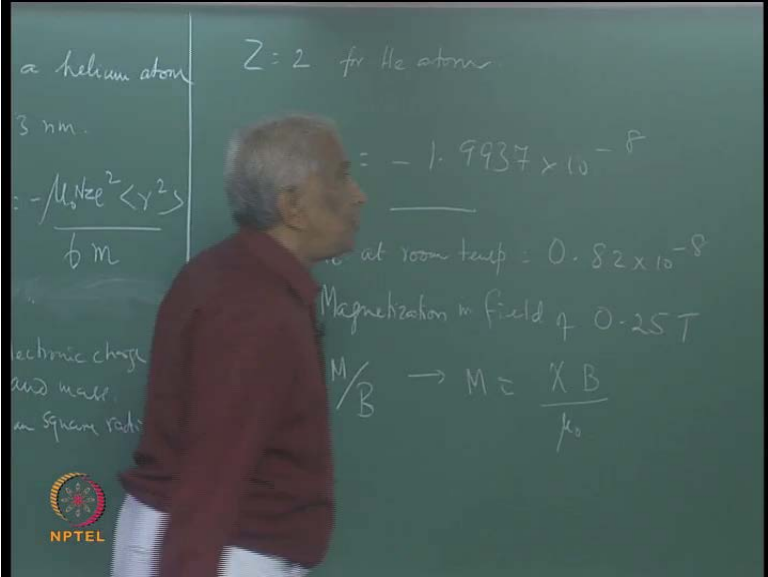
$$\chi = -\left(\mu_0 e^2 \overline{r^2} Z N\right) / 6m$$

where N is the number of atoms per unit volume. If for N we substitute the Avogadro number, we get the diamagnetic susceptibility per kg.mol or the molar susceptibility.




Where μ_0 is the permeability of free space, which is equal to $4\pi \times 10^{-7}$. We will substitute later and then n for a for a molar susceptibility, this is just the Avogadro number Z is the atomic number e and m are the electronic charge and mass and r^2 is the mean square radius.

(Refer Slide Time: 03:17)



a helium atom
3 nm.
 $= -\frac{\mu_0 N z e^2 \langle r^2 \rangle}{6m}$
electronic charge and mass in square radii

$Z = 2$ for He atom
 $= -1.9937 \times 10^{-8}$
at room temp = 0.82×10^{-8}
Magnetization in field of 0.25 T
 $M/B \rightarrow M = \frac{\chi B}{\mu_0}$



So, plugging in all these values we find Z is 2 for the helium atom. So, plugging in all these values we get the molar susceptibility diamagnetic susceptibility as minus 1.9937 into 10^{-8} .


(Refer Slide Time: 03:44)

Worked Example 55

Problem

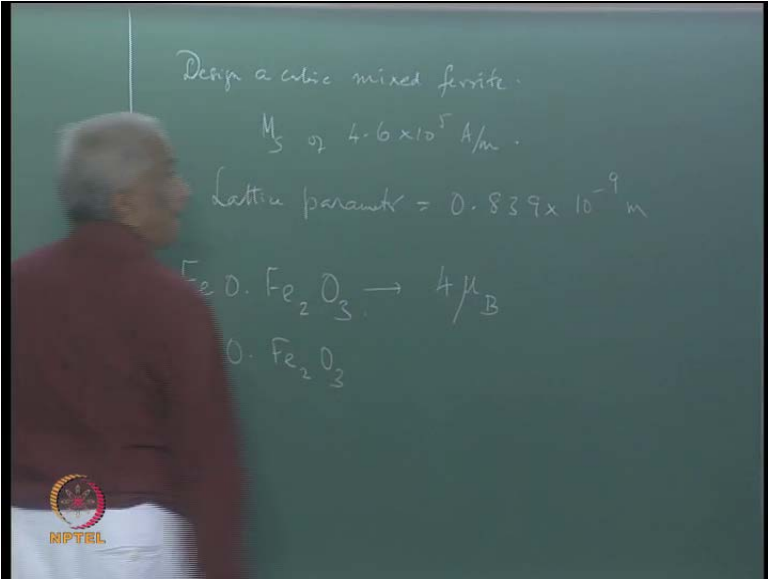
Magnetic susceptibility of aluminum at room temperature is 0.82×10^{-8} . Calculate its magnetization when a field of 0.25T is applied.

Solution

$$\chi = \mu_0 M / B$$
$$M = \chi B / \mu_0 = 0.82 \times 10^{-8} \times 0.25 / 4\pi \times 10^{-7}$$
$$M = 1.63 \times 10^{-3} \text{ A/m}$$


Next we are has to calculate the susceptibility of aluminum at room temperature no we are being given susceptibility at room temperature as 0.82 into 10 to the power minus eight. So, we are as to calculate its magnetization in an applied magnetic field 0.25 teals again we know the relation between the susceptibility mu naught m by b. So, the magnetization is 5 times b by mu naught. So, substituting the value is turns out we have into 10 to the power minus 3.

(Refer Slide Time: 05:08)




Design a cubic mixed ferrite.

M_s of $4.6 \times 10^5 \text{ A/m}$.

Lattice paramter = $0.839 \times 10^{-9} \text{ m}$

$\text{FeO} \cdot \text{Fe}_2\text{O}_3 \rightarrow 4\mu_B$

O: Fe_2O_3




Next we are has to design a cubic mixed ferrite, which as a saturation magnetization of 4.6×10^5 A/m. Lattice parameter of the crystal = 0.839×10^{-9} m.

(Refer Slide Time: 05:40)

Worked Example 56

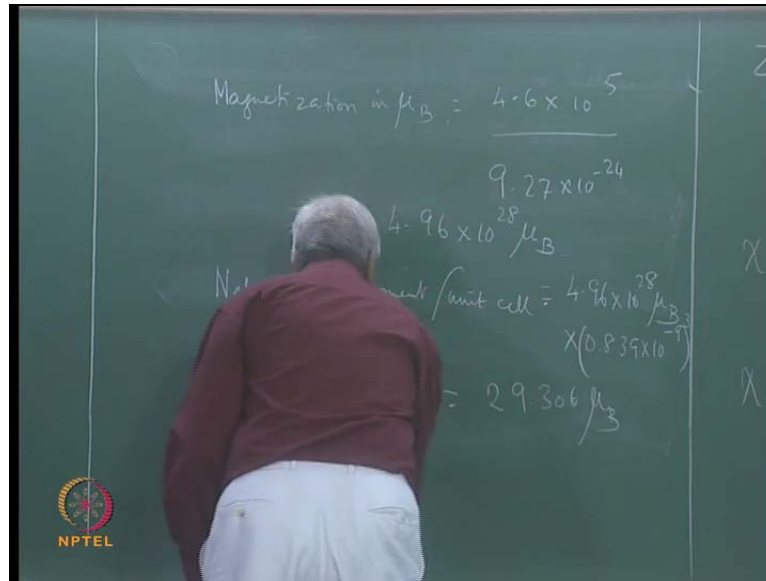
Problem
Design a cubic mixed ferrite magnetite material that has a saturation magnetization of 4.6×10^5 A/m. Lattice parameter of the crystal = 0.839×10^{-9} m.

Solution
One molecule of $\text{FeO} \cdot \text{Fe}_2\text{O}_3$ contributes $4\mu_B$ and one molecule of $\text{CoO} \cdot \text{Fe}_2\text{O}_3$ contributes $3\mu_B$ to the magnetization.



We are given the lattice parameters of the basic crystal is all eight three nine into ten to the power minus nine meters. So, basically we have the basic ferrite is magnetite in which the ion and contributes four bore magnetrons, and we mix it with cobalt right in which case the cobalt ion cobalt s ion as a magnetic moment of bore magnetron. So, we are to design a mix ferrite whose saturation magnetization is given from the value of saturation magnetization given.

(Refer Slide Time: 06:51)




We write the magnetization in bore magnetron. So, this is given value divided by the value of one bore magnetron.

(Refer Slide Time: 07:18)

The required magnetization in terms of Bohr magneton

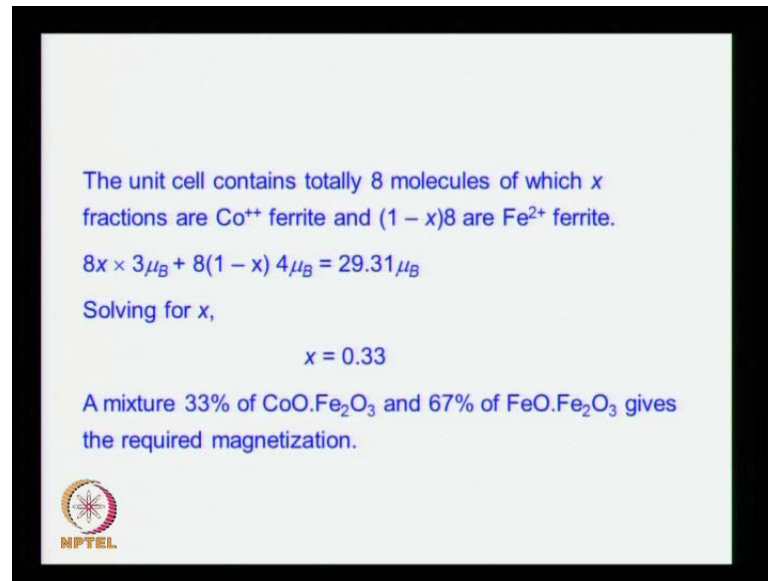
$$= \frac{4.6 \times 10^5}{9.27 \times 10^{-24}} = 4.96 \times 10^{28} \mu_B$$

The net magnetic moment per unit cell = $4.96 \times 10^{28} \mu_B \times$

$$(0.839 \times 10^{-9})^3$$
$$= 29.306 \mu_B$$


So, that would be now if we have a unit cell the net magnetic moment per unit cell is this is what we want times the volume of the unit cell the lattice parameter is given is a cubic lattice. So, it is just a cube or the lattice parameters. So, this turns out to be. So, we have to make a material which has a magnetic moment of 29.306 bore magnetrons.

(Refer Slide Time: 08:34)




The unit cell contains totally 8 molecules of which x fractions are Co^{++} ferrite and $(1 - x)8$ are Fe^{2+} ferrite.

$$8x \times 3\mu_B + 8(1 - x) 4\mu_B = 29.31\mu_B$$

Solving for x ,

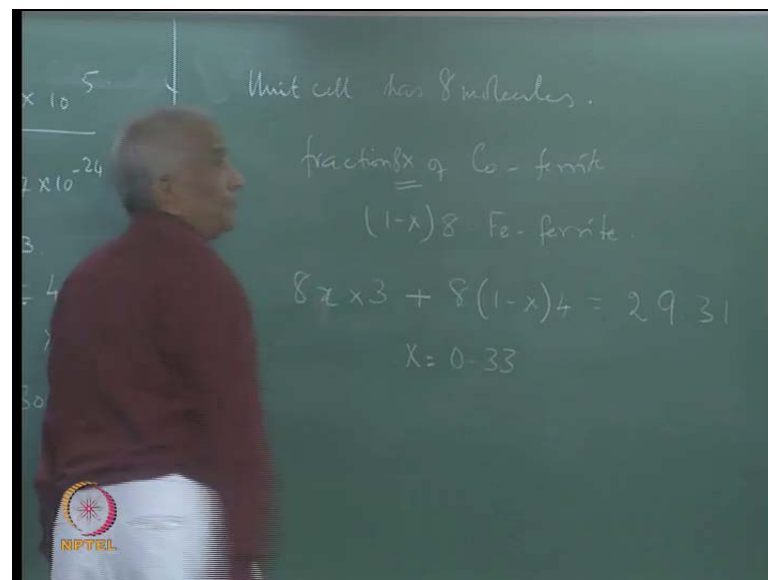
$$x = 0.33$$

A mixture 33% of $\text{CoO} \cdot \text{Fe}_2\text{O}_3$ and 67% of $\text{FeO} \cdot \text{Fe}_2\text{O}_3$ gives the required magnetization.



And the unit cell of the ferrite has eight molecules, therefore we have to determine the fraction x of cobalt ferrite.


(Refer Slide Time: 08:56)



Unit cell has 8 molecules.

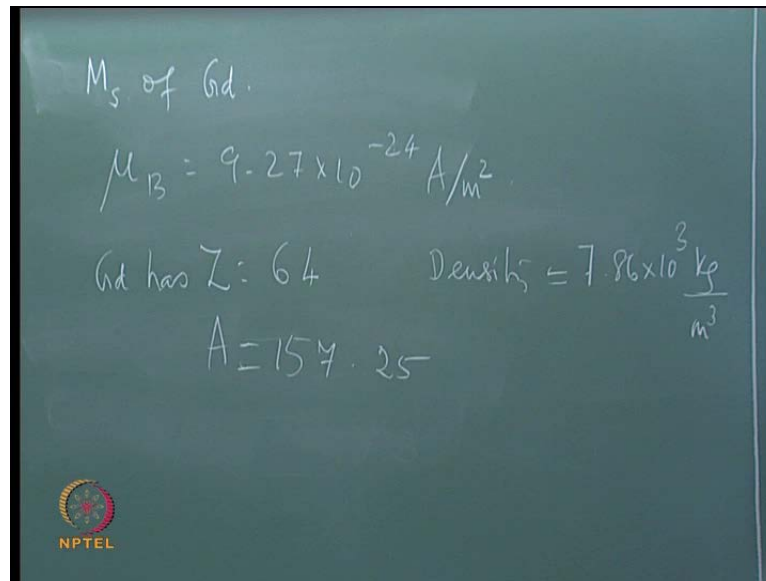
fraction x of Co-ferrite

$(1-x)8$ Fe-ferrite.

$$8x \times 3 + 8(1-x)4 = 29.31$$
$$x = 0.33$$


So, that the remaining one minus x times 8 are ferrite. So, we have this is $8x$ we have $8x$ times three bore magnetron plus 8 into 1 minus x times 4 bore magnetrons that should be 29.31. So, x turns out to be 0.33. So, that mean that we have to mix 33 percent of cobalt ferrite with 37 percent of ion ferrite the magnetize.

(Refer Slide Time: 10:22)



So, that the answer we next move on the problem in which we have to calculate the saturation magnetization of gadolinium for which we are given the data that there are one Bohr magneton.

(Refer Slide Time: 10:40)

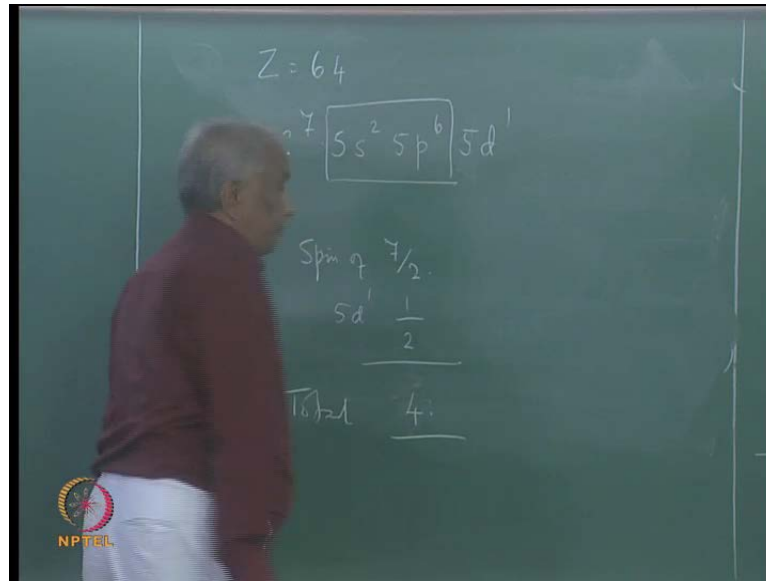
Worked Example 57

Problem
Calculate the saturation magnetization of Gadolinium given that Bohr magneton = $9.27 \times 10^{-24} \text{ A/m}$, atomic number of Gd = 64, Atomic weight = 157.25, density = $7.86 \times 10^3 \text{ kg/m}^3$.

Solution
Atomic No: 64
Electronic configuration: inner core $4f^7 5s^2 5p^6 5d^1$
spin
 $4f^7 \rightarrow 7/2$
 $5d^1 \rightarrow 1/2$

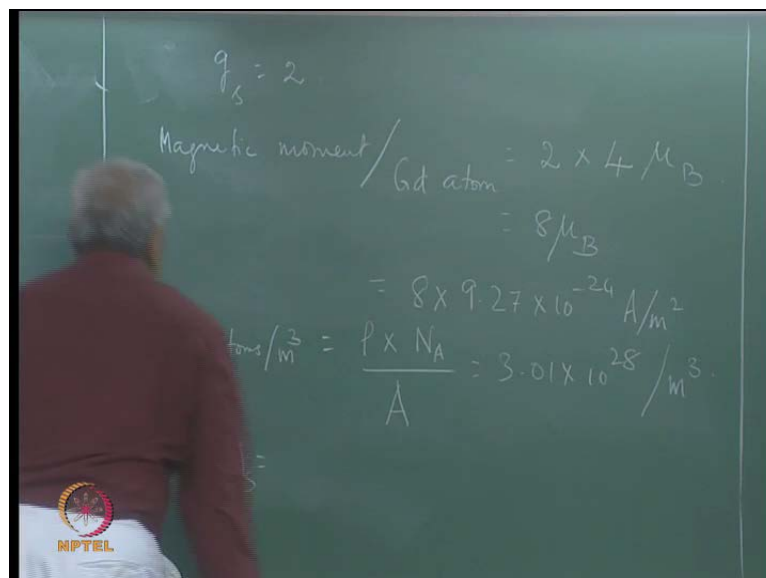
Gadolinium has the atomic number is 64 and then atomic weight, which is hundred and 57.25 it has a density of 7.86 into 10 to power 3 kilogram per meter cube in order to calculate the saturation magnetization.

(Refer Slide Time: 11:19)




We have to start from the atomic number. So, the electronic configuration use for gadolinium is 4 F 7 neglecting the inner shells the outer electronic configuration five s 2 5 p 6 5 d 1. So, the 5 s 2 5 p 6 is also a closed shell. So, the 4 F 7 we know as a spin of 7 by 2 because there are seven electrons in the 4 f shell, and then there is a 5 d 1 electron which as the spin of half. So, the total spin is 8 by 2 which is 4 gadolinium has half fill shells. So, it has a s ground state.

(Refer Slide Time: 12:36)



So, the angular momentum is only due to spin and this spin only g value is s 2. So, the magnetic moment per gadolinium atom is the g value 2 times the spin in bore magnetrons.

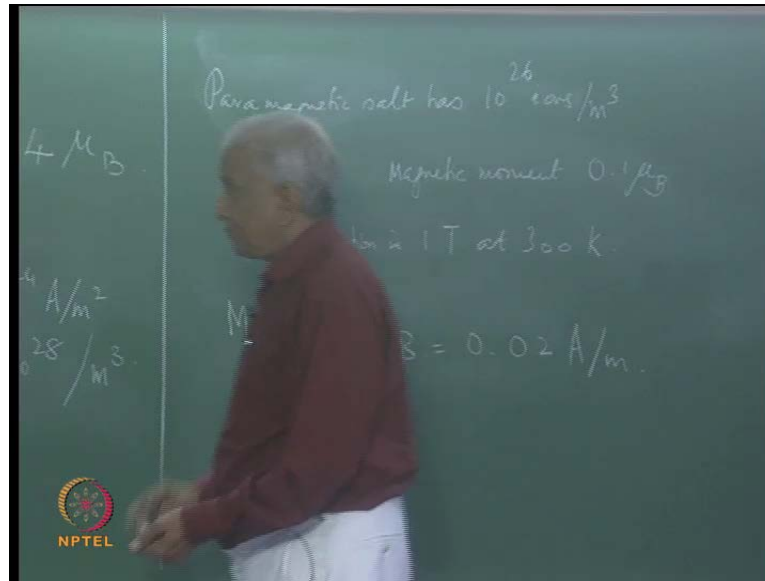
(Refer Slide Time: 12:57)



$$\begin{aligned} \text{Total spin: } & \frac{8}{2} = 4 \\ \text{Magnetic moment per atom} &= 2 \times 4\mu_B \\ &= 8 \times 9.27 \times 10^{-24} \text{ Am}^2 \\ \text{Number of atoms per unit volume} \\ N = \frac{\rho \times N_{Av}}{A} &= \frac{7.86 \times 10^3 \times 6.023 \times 10^{26}}{157.25} = 3.01 \times 10^{28} / \text{m}^3 \\ \text{Saturation magnetization} &= 8 \times 9.27 \times 10^{-24} \times 3.01 \times 10^{28} \\ &= 2.2 \times 10^6 \text{ A/m} \end{aligned}$$

So, this is eight bore magnetron and in since bore magnetron is given that is for one gadolinium atom and will have to calculate the number of atoms per meter cube which is just density times the avogadro number divided by the atomic weight. So, this works out to be substituting the numbers per meter cube. So, the saturation magnetization is therefore, just number of atoms per meter cube times the magnetic moment of atom and that works out two.

(Refer Slide Time: 14:17)




Next we have told that a paramagnetic salt contains 10^{26} ions per meter cube as the concentration of paramagnetic ions each ion has a magnetic moment of point one Bohr magneton.

(Refer Slide Time: 15:00)

Worked Example 58

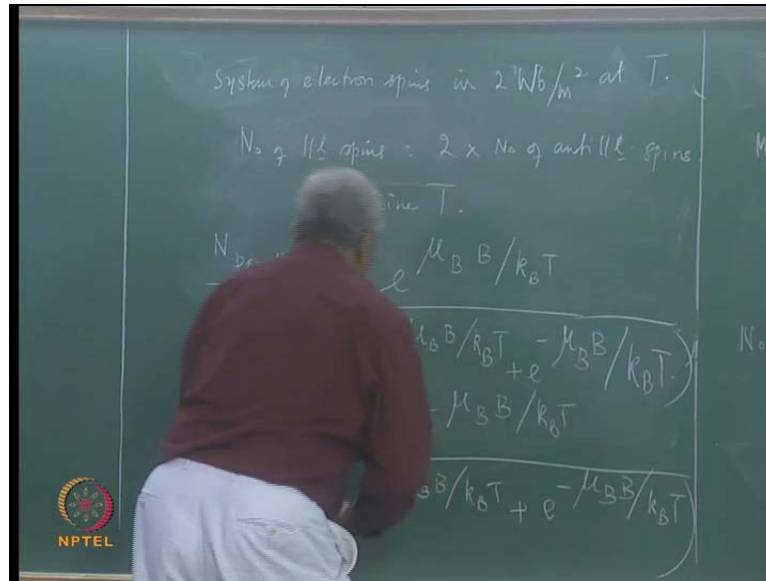
Problem
A paramagnetic salt contains 10^{26} ions/m³ with magnetic moment $0.1 \mu_B$. Calculate the magnetization in a field of 1 T at 300 K.

Solution

$$M = \frac{N\mu^2}{k_B T} B$$
$$M = \frac{10^{26} \times (0.1 \times 9.27 \times 10^{-24})^2 \times 1}{1.38 \times 10^{-23} \times 300} = 0.020 \text{ A/m}$$


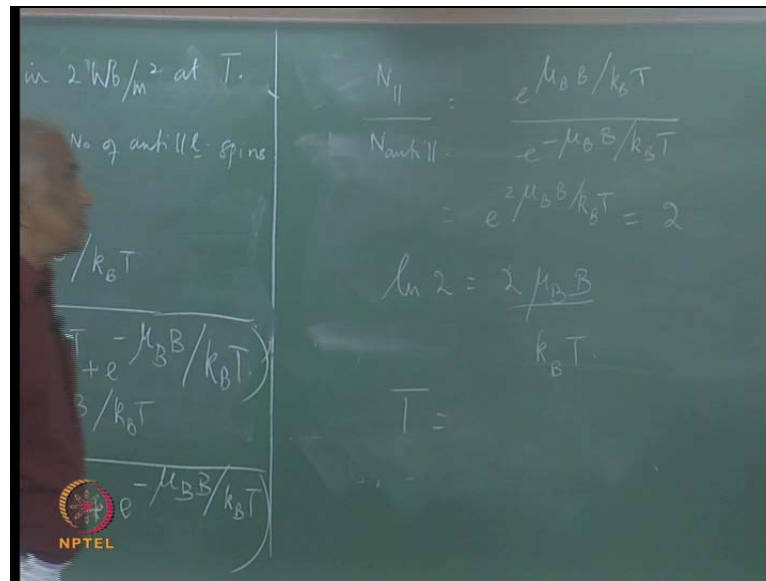
So, we are as to calculate magnetization in a field of one tesla at 300 Kelvin well we know that we are magnetic moment, if it is μ then the magnetization is $n \mu^2$ by three $k_B T$ times B applied magnetic field. So, substituting these values we readily get the answer as...

(Refer Slide Time: 15:37)



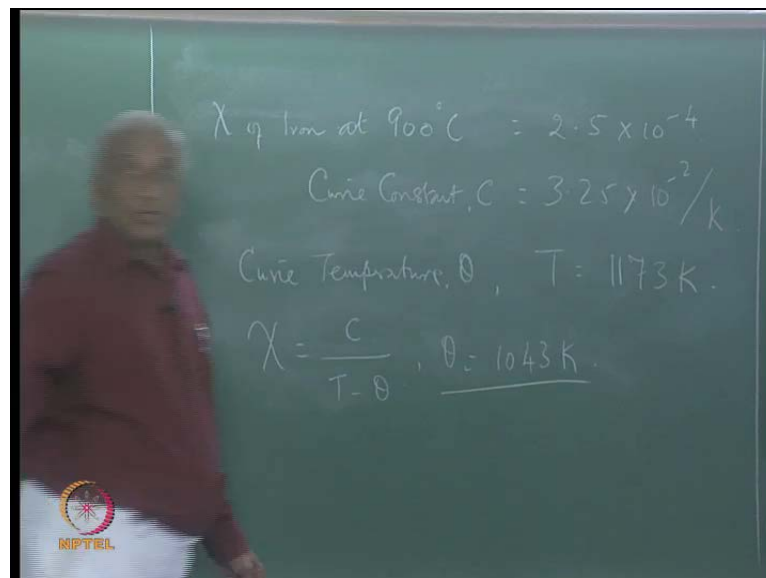
Next, we are told that a system of electrons spins is placed in a magnetic field of 2 watt per meter square idea temperature t . And we are told that the number of spins which are oriented parallel to the field is twice the number of spins, which have spins ant parallel to the field that the data given from this information, whereas to determine the temperature at t . So, let us write down the expression for the number parallel spins the ratio of the number of parallel spins to the total number of spins and that is given by because this is the ground state. So, it will be the total number is proportional to inflame maybe number of anti parallel tense the ratio with respect to the total number is just given by exponential minus $\mu_b b$ by $k_b t$ by the same therefore, the ratio of the number of parallel spins those of anti parallel spins which is given by the ratio of these 2.

(Refer Slide Time: 18:07)



And that will be dividing one by the other therefore, this is just and we are told that this ratio is just two therefore, taking logarithms, Therefore, substituting for everything we can easily calculate the temperature it turns out to be 3.87 k.

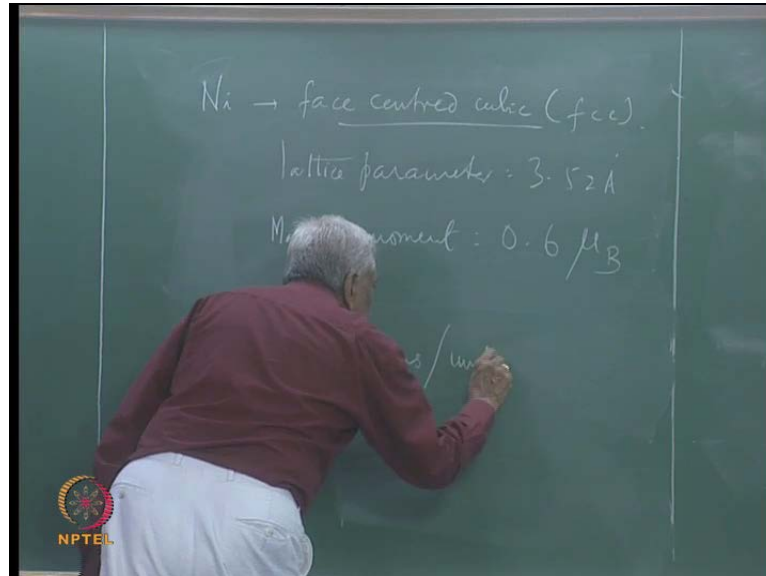
(Refer Slide Time: 19:12)



Next is a problem on the magnetic susceptibility of iron at 900 Celsius is 2.5 into 10 to the power minus 4. Here also given the curie constant c as 3.25 to 10 to the power minus 2 per k we are as to find the curie temperature for which the standard symbol is theta in the expression for the susceptibility. And we are told that the temperature is 900 Celsius

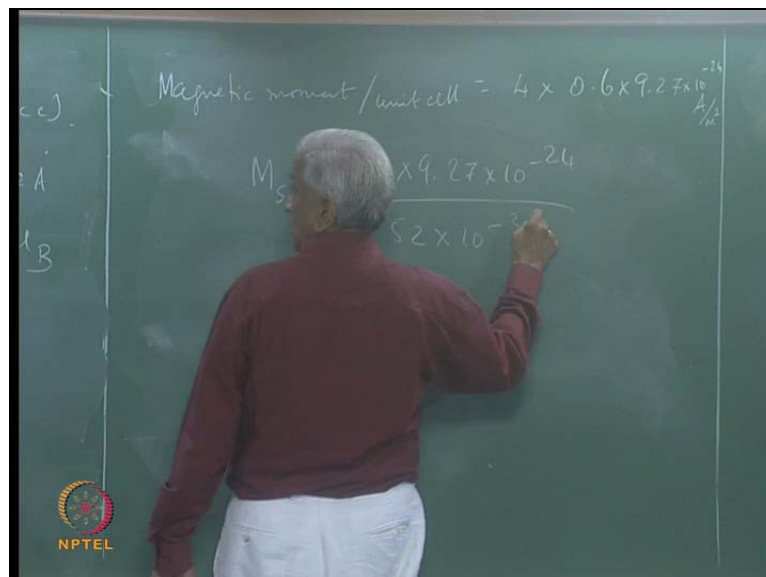
which is 1173 Kelvin. So, substituting here for χ_c and t we get θ as 1043 K. So, the Curie temperature of iron is 1043 Kelvin.

(Refer Slide Time: 20:50)



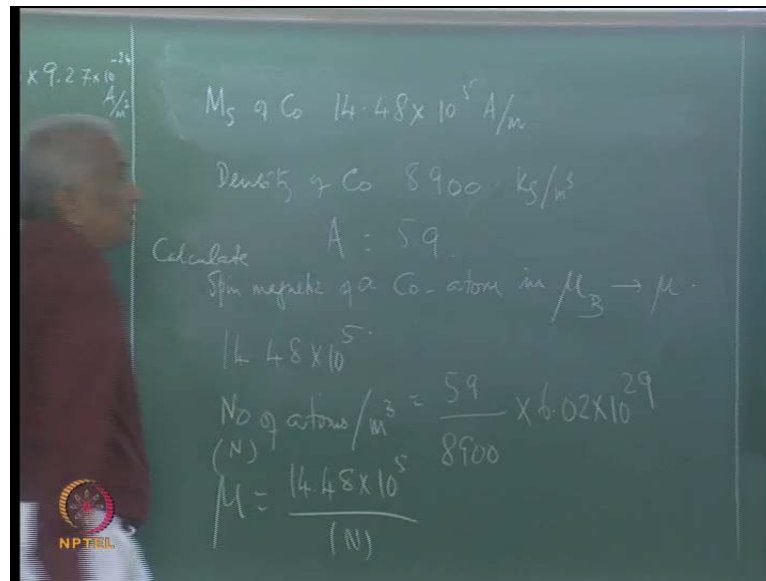
Next is a problem on nickel which we are told has an fcc structure phase centred cubic and its lattice parameter is given to be 3.52 angstroms the magnetic moment of nickel atom is 0.6 Bohr magneton. So, we are as to find the saturation magnetization for this is start for the fact that it is a phase centered cubic structure. So, it has 4 atoms in the unit cell.

(Refer Slide Time: 21:52)



We are told that the magnetic moment per unit cell since there are four atoms curie 4 times the magnetic moment of one nickel atom which is 6 bore magnetrons. So, the saturation magnetization is just 4 an 2.6 is 2.4 into 9 point to 7 into 10 to power minus 24 divided by the volume of the unit cell which is 3.52 into 10 to the power minus 30. So, this will give me 5.17245 anti per meter.

(Refer Slide Time: 22:59)



The next equation tells us the saturation magnetization of cobalt is 14.48 into 10 to power 5 here also given the density of cobalt as 8900 kilograms per meter cube and the atomic weight is 59. So, we are as to calculate the spin magnetic moment for cobalt atom in bore magnetron to do this is start again from the saturation magnetization.

(Refer Slide Time: 23:59)

Solution

Saturation magnetic moment = $14.48 \times 10^5 \text{ A/m}$.


Let spin magnetic moment of Co = μ Bohr magneton

Density = 8900 kg/m^3 ; atomic weight = 59.

No of atoms per unit volume = $(59 / 8900) \times 6.02 \times 10^{29} / \text{m}^3$

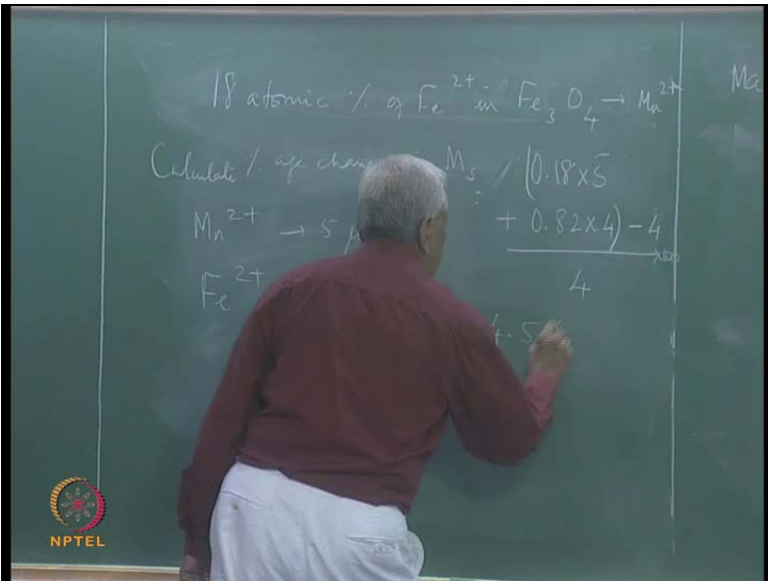
$$\mu = \frac{14.48 \times 10^5}{\{(8900 / 59) \times 6.02 \times 10^{29} \times 9.27 \times 10^{-24}\}}$$

$= 1.71 \text{ Bohr magneton}$



Let's assume that this spin moment is μ in Bohr magneton the density is 8900. So, the number of atoms cobalt atoms per meter cube is given by atomic weight divided by the density times the Avogadro number and therefore, μ is from 14.48 into 10 to the power 5 divided by suppose bcc n substituting for n well into Bohr magnetons and that will be 1.71.

(Refer Slide Time: 25:25)



18 atomic % of Fe^{2+} in $\text{Fe}_3\text{O}_4 \rightarrow \text{Mn}^{2+}$

Calculate % age change


$\text{Mn}^{2+} \rightarrow 5 \mu_B$

$\text{Fe}^{2+} \rightarrow 4 \mu_B$

$$M_s / (0.18 \times 5 + 0.82 \times 4) - 4$$

4

4.5



Next is a problem on ferrite in the we are told eighteen atomic percent of ion 2 plus in magnetite is replaced by manganese 2 plus whereas, to find in the percentage change in


m s saturation magnetization now we know that the m n 2 plus ion as 5 bore magnetron whereas, ion 2 plus as only 4. So, the percentage change will be 0.18 which is the percentage of iron replaced by manganese times 5 plus the remaining 0.82 time 4.

(Refer Slide Time: 26:41)

Worked Example 63

Problem
 18 atomic percent of Fe^{2+} in Fe_3O_4 is replaced by Mn^{2+} .
 What is the percentage change in the saturation magnetization?

Solution
 Mn^{2+} has a magnetic moment of 5 Bohr magneton whereas Fe^{2+} has only 4 Bohr magneton.
 Therefore % age change in saturation magnetization is
 $(0.18 \times 5 + 0.82 \times 4 - 4) / 4 = 0.045 = 4.5\%$



This is the new magnetization minus 4 divided by the original magnetization into 100 of course, it turns out to be 4.5 percent.

(Refer Slide Time: 27:09)


Proton magnetic resonance in H_2O in 1 T.

rf pulse drive NMR.

↓

rf magnetic field ^{has} amplitude of 1 G.

Width, $t_w \rightarrow$ Tilt the proton spins onto the rf field direction.



We now born to a problem on magnetic resonance proton magnetic resonance in a sample of water is placed a magnetic field of one Tesla. We have in r f pulse used to


observe NMR this magnetic field associated with this r f magnetic pulse as an amplitude of one gauss.

(Refer Slide Time: 28:06)

Worked Example 64

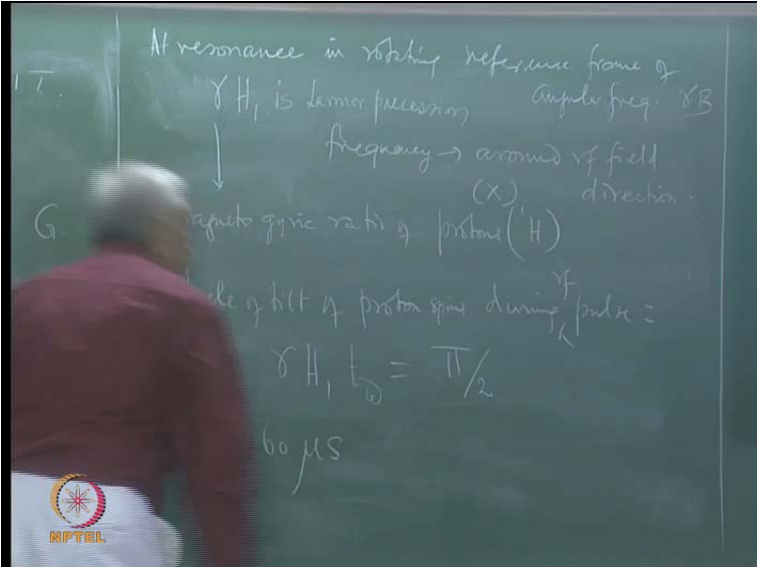
Problem
 A sample of water is placed in a magnetic field of 1 T. An rf pulse is used to observe the proton magnetic resonance in water. The rf magnetic field has an amplitude of 1 gauss. What should be the width of the pulse were to tilt the proton spins onto the rf field direction?

Solution:
 At resonance γH_1 is the frequency of Larmor precession of protons around the rf field direction.
 Here γ is the magnetogyric ratio of protons. If t_w is the rf pulse width then $\gamma H_1 t_w = \pi/2$.
 Hence $t_w = 60 \mu\text{s}$



We as to find the bit the pulse r f pulse needed for this pulse to tilt the the proton spins onto the r f field direction. We know that at resonance the Larmor Precessional frequency in a static magnetic field the frequency with which a rotating frame xi used and in this rotating frame. Since it as the angular frequency of the Larmor precessional frequency in the static field, it will stay constant.

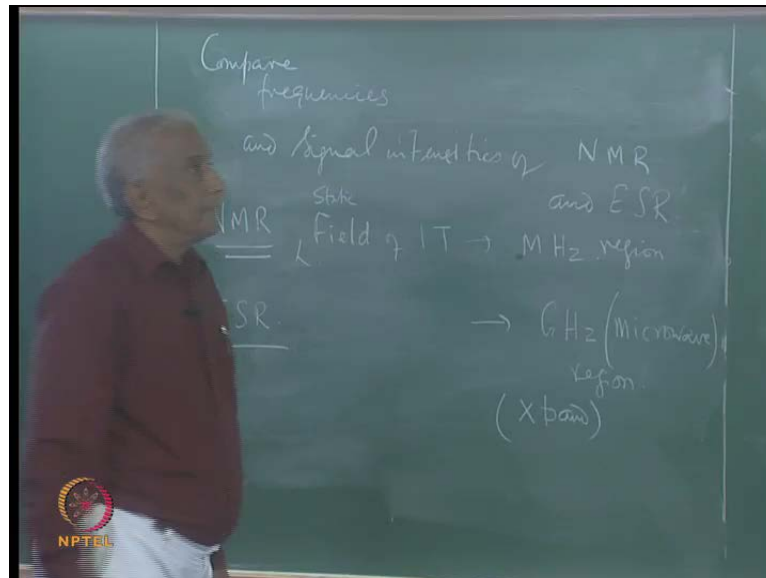
(Refer Slide Time: 29:21)



So, the only field which is their which causes Larmor precession in this rotating frame at resonance the only frequency of precession is the factor γh . So, this is the frequency with which the spins are processing in the rotating frame around the r f field direction the r f field is applied in a plane perpendicular to the static magnetic field. So, let s take this as the x direction.

So, γ is the magnetogyric ratio of protons which is known we are also told that this pulses use tilt the protons spins which are originally aligned along the z direction onto the x direction, which is that are the r f field. Therefore, the angle of tilt of protons spins during pulse is γH the frequency angular frequency a precession times the pulse width. And this we are told since it goes from the z direction to the x direction this is that the r f field. So, this angle is given to the π by 2. So, substituting the value of γ and the field amplitude we get the pulse width as sixty microseconds.

(Refer Slide Time: 32:12)




Next, we as to compared to the frequencies and signal intensities of NMR, and ESR the assumption is that they are bore used in the same applied magnetic field static magnetic field.

(Refer Slide Time: 32:41)

Solution:

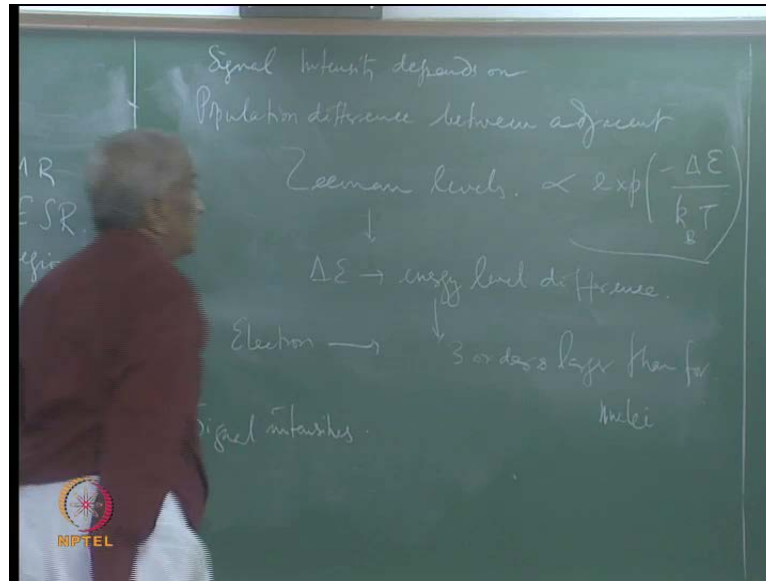
For NMR the frequency in a typical static magnetic field of strength 1T is in the rf region while that of ESR lies in the GHz (microwave) region of the electromagnetic spectrum.

The signal intensities depend on population differences between the ground and excited states which in turn are determined by the Boltzmann factor, $\exp\left(\frac{-\Delta E}{k_b T}\right)$ Since the energy level separations for an electron are three orders of magnitude larger the signal intensities are also correspondingly higher.



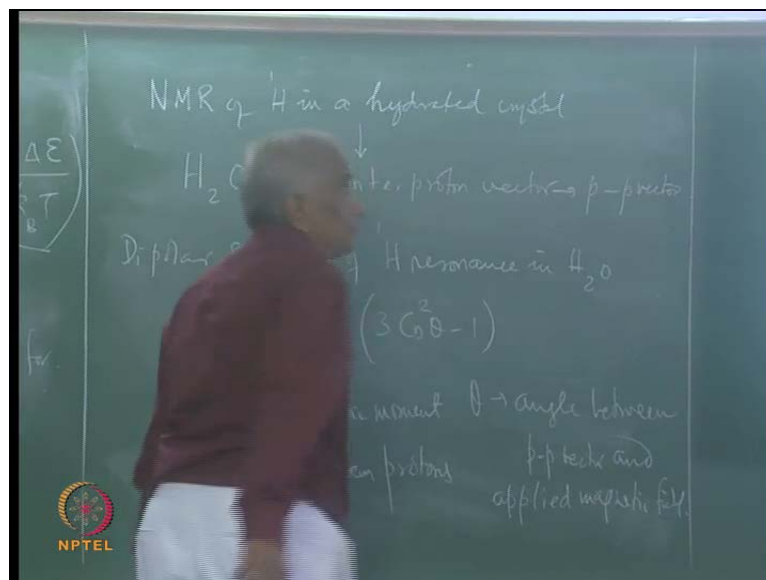
So, in the case a NMR we know from the magneto gyric ratio of nuclei that for a field static field of one Tesla the frequency is somewhere in the megahertz region 42.5 megahertz for example, for protons. Whereas, in the case is the electron spin resonance the same since which magneto gyric ratio involves the mass in the denominator. And since it is electron which is thousand time or 1800 times lighter than nuclei order a magnitude three order, therefore the corresponding gamma value is larger. Therefore, the same frequency resonance frequency which involves the gamma is going to be if this is in megahertz this is going to be in the gigahertz or microwave region typically in the x band for as spin only sample. So, that is regarding the frequencies in order to combat the signal intensities the signal intensities are related to the population.

(Refer Slide Time: 34:30)



So, population difference depends on population difference between adjacent, and same level. And this population difference is proportional to exponential minus delta e by k b t then delta e is the energy level difference again the energy level separation for the electron. This energy level differences are three orders larger than for nuclei therefore, the exponential factors are much larger. So, the signal intensities are much stronger in the case of the electron resonance by the same factor, where this is three orders higher next question concerns NMR of protons in a hydrated.

(Refer Slide Time: 36:26)



So, there are water molecules of hydrations. So, the H₂O molecule has two protons, and we are as to find explains how one attains information on the inter proton vector in the water molecule is usually call the p-p vector proton-proton vector.

(Refer Slide Time: 37:00)

Solution

The dipolar splitting of the proton resonance absorption peak has the form:


$$\Delta H = \mu/r^3(3\cos^2\theta - 1)$$

where μ is the magnetic moment of the proton, r is the distance between the two protons in a water molecule and θ is the angle between the interproton vector and the direction of the applied magnetic field.

The separation between the proton resonance doublet in a single crystal has a maximum value at $\theta = 0$ and a minimum at $\theta = \pi/2$.

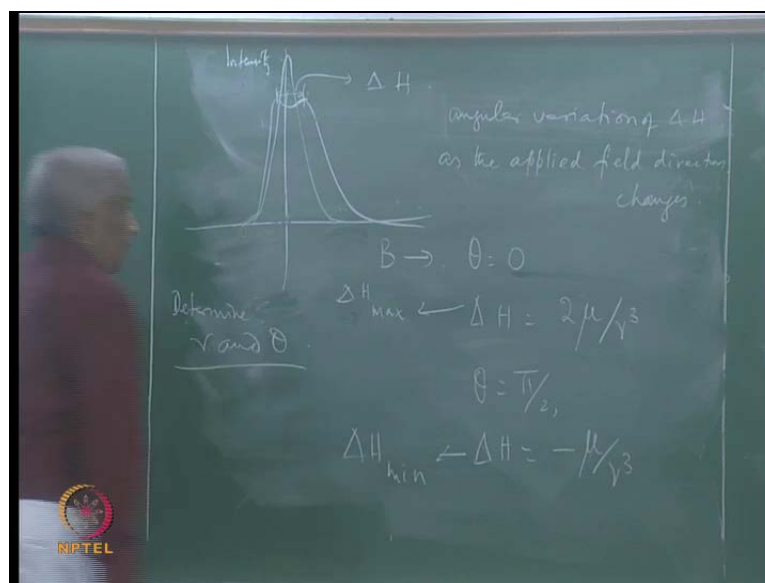
$$\Delta H_{\max} = 2\mu/r^3 \text{ and } \Delta H_{\min} = -\mu/r^3.$$

From a knowledge of r and θ one can determine the length and orientation of the interproton vector in the water molecule.



In order to answer this we have to start from dipolar splitting of the proton resonance in water, this is given by we discussed this already μ by r^3 into three cause square theta minus 1, where μ is the proton magnetic moment which is known r is the distance between protons. And theta is the angle between p-p vector and the applied magnetic field. So, this means that this dipolar splitting of the photon resonance gives a doublet structure like this.

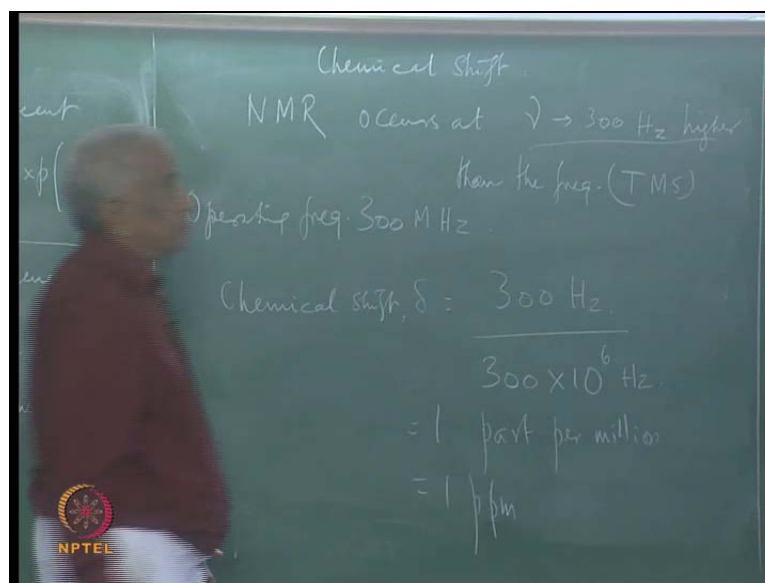
(Refer Slide Time: 38:39)



So, instead of where there is a single proton resonance speed we will have doublet structure like this somewhat like this. So, this is the separation which we call delta H. So, as one rotates the applied magnetic field the angle theta changes in therefore there will be an angular variation of this dipolar splitting as the applied magnetic field direction changes in particular in theta is 0 three cross square in theta is three.

And therefore, delta h is just 2 mu by r cube whereas, theta equal to pi by two delta H as the minimum value which is minus mu by r cube. So, this is delta h minimum and this is delta H maximum. So, this meeting where maximum and minimum values, so one can determine these two and from a knowledge of these one can find r and theta from the angular variation. So, we know the separation or the distance between two protons and the orientation the inter proton vector in the water molecule. So, this is the information that one can get from the splitting dipolar splitting next question concerns the so-called chemical shifts of the nuclear resonance.

(Refer Slide Time: 41:21)



So, we are told that the NMR occurs at a frequency 300 hertz higher than the frequency corresponding to standard sample which is tetra methane TMS the operating frequency of n m r. So, 300 mega hertz spectrum meter therefore we are asked to calculate the chemical shifts standard symbol is delta this define as the ratio of the shifts which is 300 hertz 2 operating frequency, which is 300 megahertz. So, this is one part per million for short it is written as 1 ppm.