Condensed Matter Physics
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Lecture - 32
BCS theory (Continued): Josephson Tunneling: Quantum Interference

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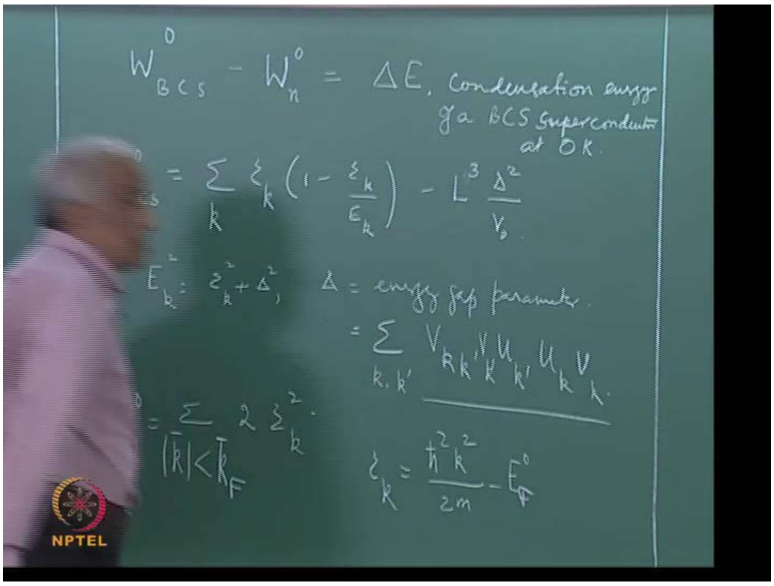

Last lecture we learned how to calculate the condensation energy as the difference between the brown brown state energy of the BCS theory of a superconductor, and the energy of the normal state that difference gives you the condensation energy of BCS superconductor at 0 k. We learned that W BCS 0, which is the ground state energy of a BCS superconductor and absolute 0 is summation overall k psi k into 1 minus psi k By k minus l cube l plus square by v 0, where E k is given by psi k square plus delta square and delta is the energy gap parameter defined by, and the normal state energy W n 0 is given by, because only state up to the fermi energy or fill in the normal state and psi k square psi k is h cross square a square by 2 m minus fermi energy at absolute 0.

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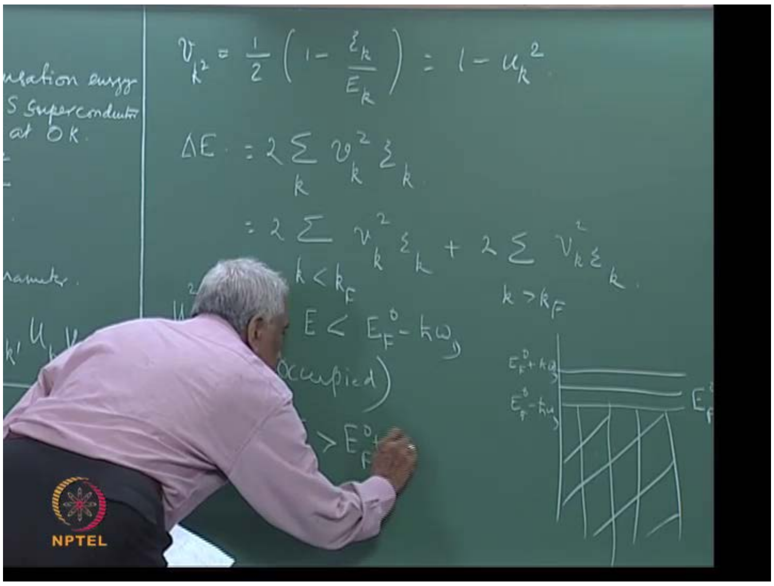

So, here we also use the parameters v k square they occupancy factors forgiven as half of and this is 1 minus using this we can calculate delta e, and in order to do that this split the summation into 2 parts here we can write this and the normal state is just has this energy. So, this for u k square is 0 energies less than E f 0 minus h cross omega d this is because for such energy in the gap, if this is E f 0, this is E f 0 minus h cross omega d, this is E f 0 plus h cross omega d. Now for state below this all states are occupied. So, there are all occupied states in therefore, u k square is 0 v k square is 1 and v k square is 0 for E greater than E f 0 plus h cross omega d, that is these state are all empty space. So, v k square 0 is for them using this we can simplify delta e.

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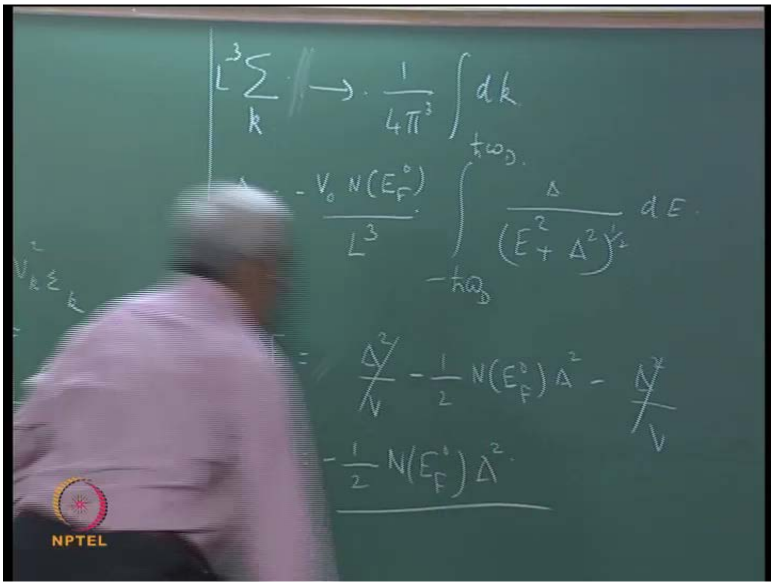

And we go from l to the power minus 3 summation over k to 1 by four pi cube integral d k in other words, we convert this summation into a integral use since states are all most quasi is a quasi continuous states with a density of state function using all this. And we are also used the energy gap expression delta can be shown to be minus v naught N of E f 0 by l cube integral. So, this can be readily shown from this definition here using the expression from k and k. So, using all this we can show that delta E the condensation energy at absolute 0 turns to be. So, that these 2 cancel leaving out a particularly simple expression that is very makeable result, because physically it means this this means that the condensation energy of all the cooper pairs in the superconducting state of a BCS superconductor at absolute 0. In other words all the electrons all the conductional from prop.

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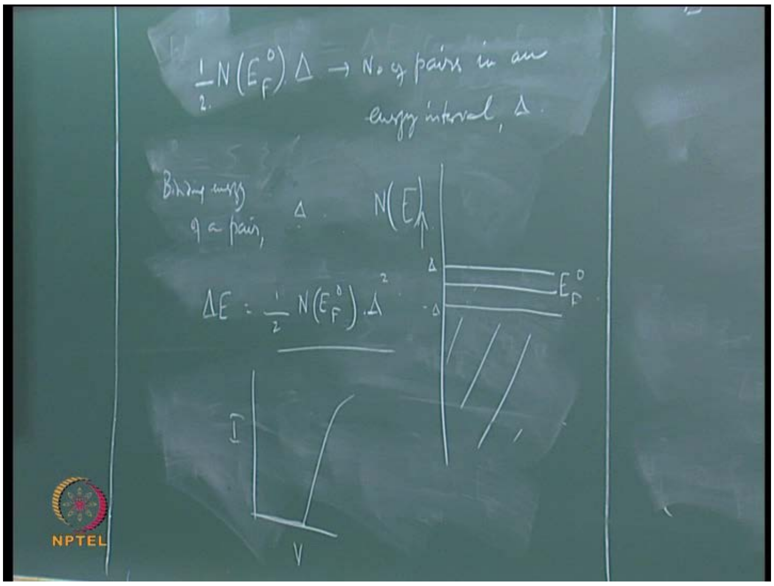

And how many electrons are there the number of electrons is given by the density of states times the victim energy which is the order of energy gap, and this is the number of electrons you have to take half this in order to find the number of pairs. And this number of pairs in an energy interval delta. Therefore, if each of them as an binding energy delta binding energy of a pair is delta than delta E the condensational energy is just the number of pairs times delta times delta which is delta square which is result that we have here. So, we have a simple physical interpretation for the energy gap parameter delta which is as just the binding energy of a cooper pair at 0 k.

So, this as a very important physical interpretation in the sense that this says this is the 1 of the important result of the BCS theory that gap opens at the fermi energy and its magnitude is the order of delta. So, this is the physical interpretation of this. So, there is an excitation spectrum there the normal state all the states are filled, and all this states are empty now at the superconducting transition at absolute 0 for this is as to superconductor there is an gap which opens symmetrically about the fermi energy and therefore, this is minus delta and this is plus delta.

So, this is the gap and no states are allowed here, and this explains there is an y, there is an excitation energy of the order of 2 delta is needed in order to excite electrons from this occupied states into the gap explaining the exponential temperature dependent of the specific heat of a superconductor at low-temperature. This also explains, why in experiments. We find that there is no current in the i v characteristics and till we reach the energy the order of the energy gap than there is a current. So, this is the i v characteristics of a superconducting tunnel junction. So, and this also explains why the microwave absorption as well as the ultra-phonic absorption shows the barriers, and excitation energy of the order of the energy gap these are all important results of the BCS theory now let us go back to this parameter the definition of the gap parameter and try to get an expression of this.

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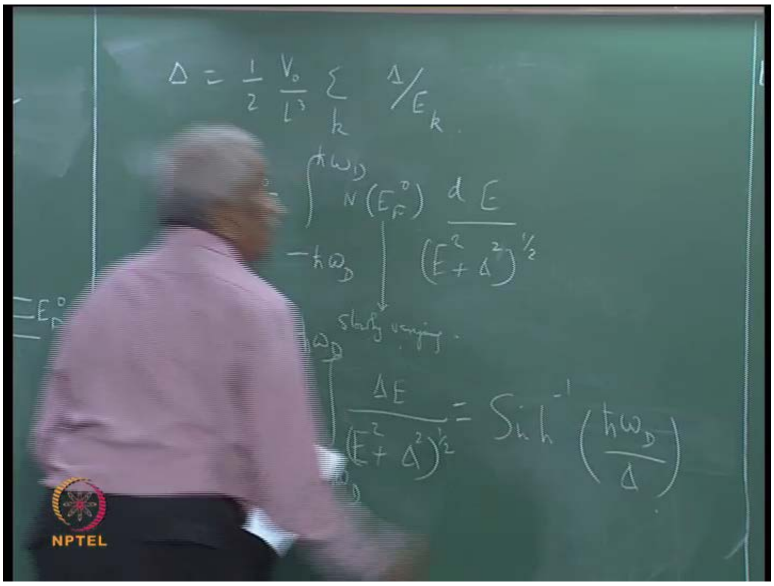

So, we see the delta is from what we are given half v naught by l cube sigma k delta by k. And this gives the condition 1 equal to v naught by 2 integral minus h cross omega d to plus h cross omega d n of E of 0 d E by E square plus delta square 2, the power half therefore, we get 1 by if he density of states you slowly varying with respect energy. Then we can factor it outer the integral, and write 1 one by n of E of 0 E equals integral minus h cross omega d to plus h cross omega d the psi E square plus delta square to the power half which integral is known to be sign h inverse h cross omega d by delta.

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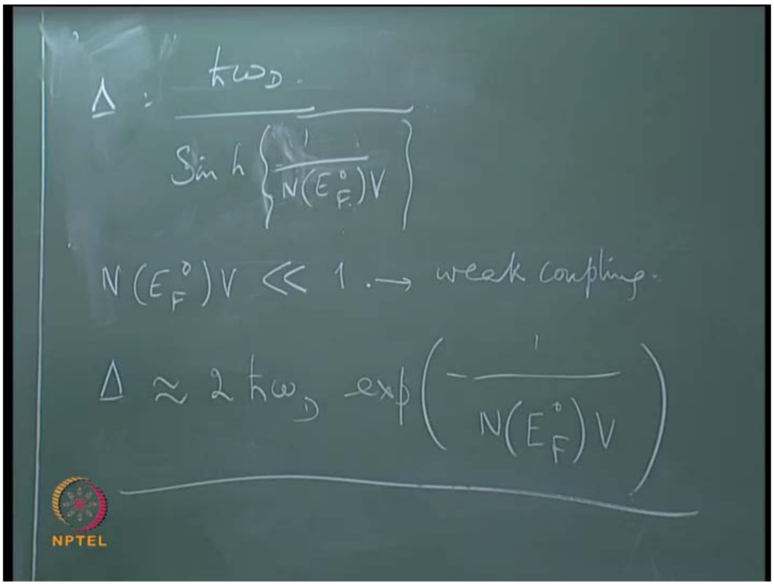

So, turning it over we can get an expression for delta as h cross omega d by 2 sign h 1 by n of E of v. And since N of E f 0 v, if it is very small compare to 1 which is the condition known as the re-coupling the electron phonon coupling is weak. And the attractive interaction bringing about the superconducting transition is rather weak in that limit delta can be approximately written as 2 h cross omega d exponential minus 1 by N of E f 0 v. So, that is that gives the expression for binding energy in terms of the electron density of space and the electron phonon coupling the pairs exponential factor is not very important except to show the it involves of phonons mechanism, and hence involves the d by frequency here. So, this is the parameter which strongly determines the value magnitude of the delta. And therefore, we know that a material with a large electron density of state the fermi energy is likely to have a larger delta.

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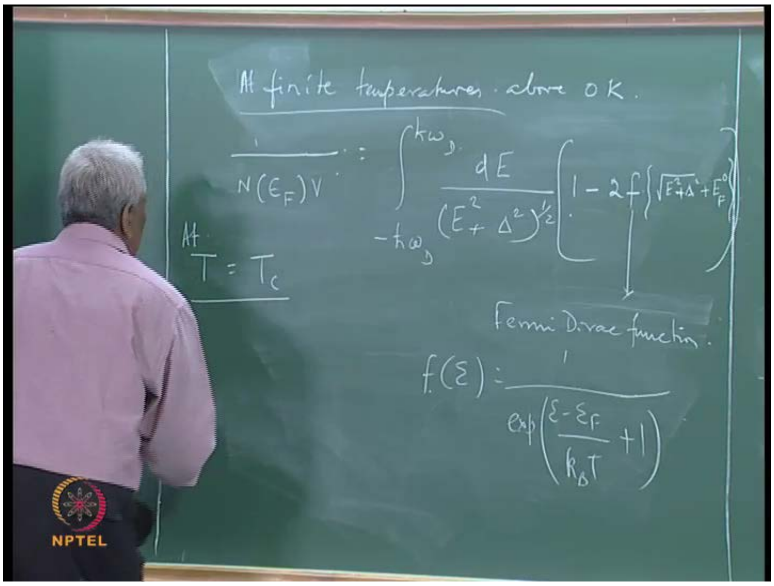

And similarly these also connected the strength of the electron phonon coupling trans, we are is what happens at finite temperature, when on goes to finite temperature above 0 kelvin. We have 1 by n of E of v very similar to this becomes again minus h cross omega d to plus h cross omega d of this integral d e, but there will be a temperature dependent factor. Because of the statistical occupation and that is given by 1 minus 2 s of root of E square plus delta square plus E f 0, this is the f is the fermi dirac function del known plus from the conduction electron theory.

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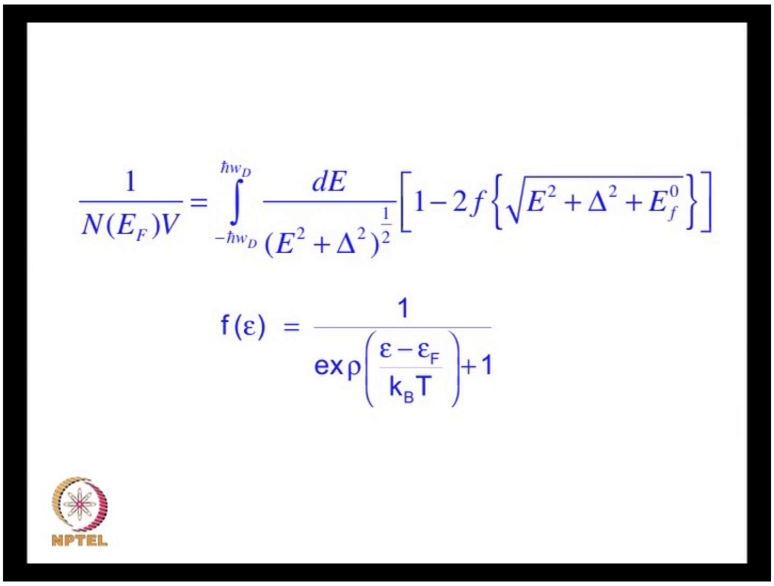

And f of E is 1 by exponential E minus E f by k B t plus 1 filed, we have we here fermi dirac function describes the probability of occupation of an energy state of energy E by an electron. And the factor to in front of f of this function is because we have a pack. So, you get your probability is double, and the probability that the state is empty is 1 minus 2 s and therefore, substituting this fermi dirac function. And then calculate in this we can find the temperature T c at temperature equal to t equal to T c the energy gap vanishes because he energy gap is, now the order parameter of the superconducting transition and at the superconducting transition the energy gap opens happened becomes nonzero about two. This superconducting face transition the gap becomes 0 vanishes. So, the energy gap is the order parameter these superconducting transition. So, the superconducting face transition temperature T c is defined as that temperature at which energy gap vanishes. So, from this expression, we can find the condition for delta to vanish in order to do that we have to evaluate this integral.

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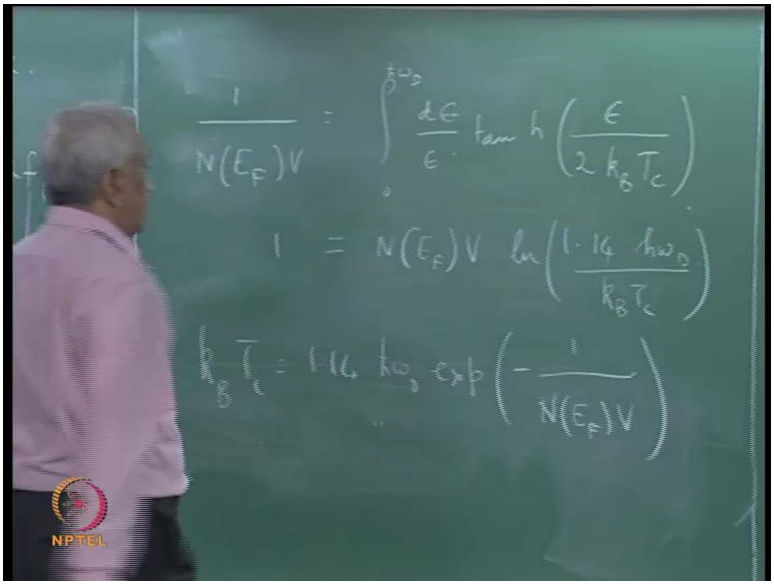

So, we get 1 by n of E of v equal to that is from this and this definition therefore, the evaluating this numerically we get 1 equal to n of E of v times log 1.14 h cross omega d by k B t c. That means, k B T c equal to 1.14 h cross omega d exponential minus 1 by N of E f of v is 1.14 is result of obtained numerical integration of this integral.

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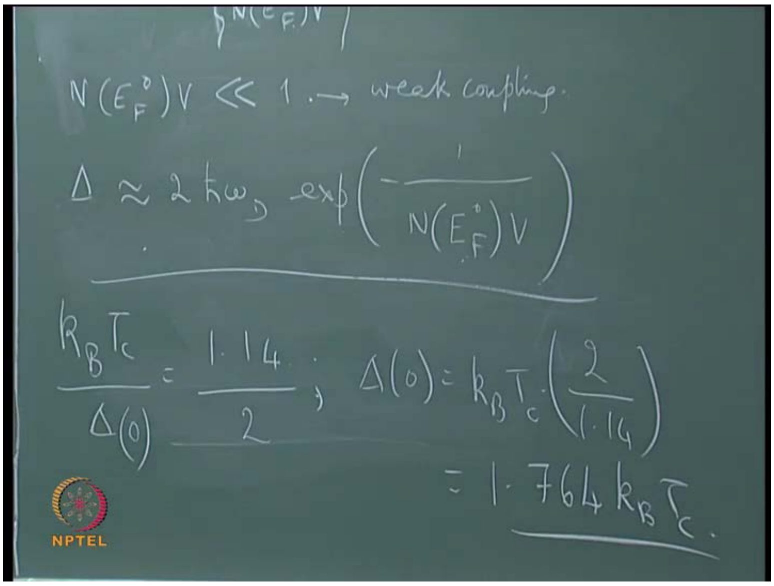

Therefore comparing these 2, we can write the ratio of k B T c for delta as 1 point 1 four by 2, therefore this is delta 0. In other words delta 0 is k B T c times 2 by 1 point 1 four which is 1.764 k B T c. So, this is another important prediction of the BCS theory of finite temperature for the superconducting transition temperature, the superconducting transition temperature is given in terms of the energy gap at absolute 0 by this factor 1 point seven six four k B. So, that is a very important prediction by this BCS theory which can be experimentally verify for superconductor superconducting transition temperature have been measured, and whose energy gap can also be measured by the techniques that we described above already therefore, this gives a very nice way test to the validity of the p c s theory. So, based on this the superconductor are in classified as weekly coupled superconductors strongly coupled superconductors in which to BCS theory this is not obey and intermediate coupled superconductors.

So, this is in brief the microscopic theory of the superconducting face transition, which accounts for the superconducting face transition as been brought about by the formation of cooper pairs by the conduction electron in the pharmacy of a normal state superconductor. So, that gives in brief basics of BCS theory.

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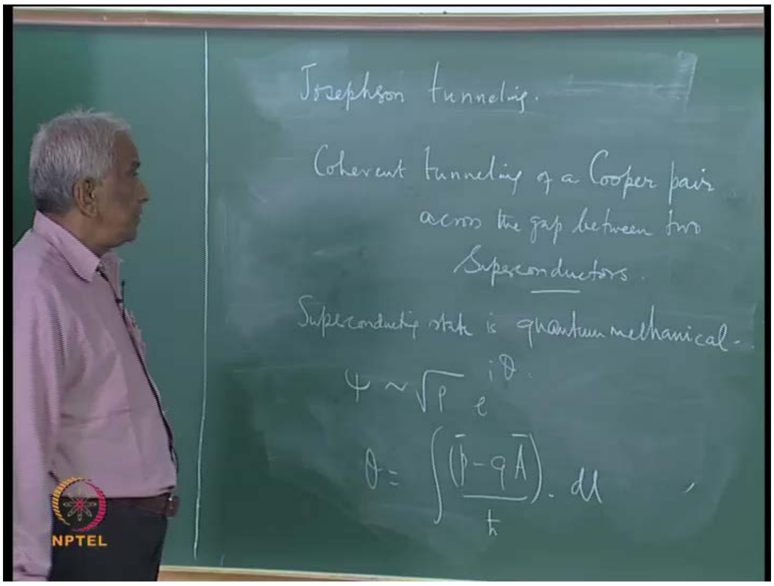

And we now perform to and another important class of phenomena connected to the superconductors namely the Josephson are the Josephson tunnels, which was a phenomenon which was discovered by Josephson.

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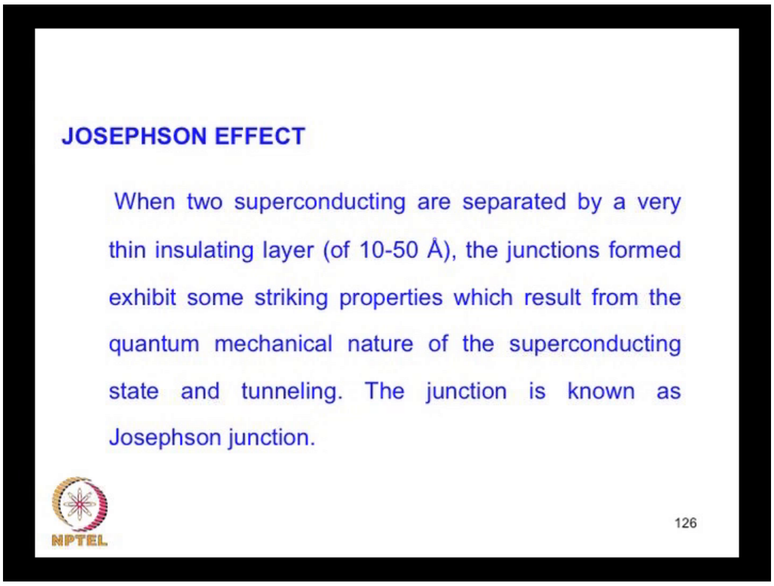

And this is describes the coherent tunneling of the cooper pair across the gap between to superconductors crucial to understand, in this is the idea that as superconducting state is quantum in nature. And the given superconductor it is a microscopic quantum mechanical states with a wave function which is given by where rho is the charge density and theta is the phase factor. So, the theta is the quantum mechanical phase factors which is given by as we already saw as the line integral of this factor this is very important, because you are describing the tunneling of cooper pairs in a coherent manner from 1 region described by 1 wave function across a barrier into another region described by a different not necessarily in the same quantum mechanical wave function. So, this is similar to the tunneling of carrier across a gap someone quantum mechanical system to another which is well known in quantum mechanics. So, there is a results are easy to described. And the important of this effect is that this a provides hat and other means of observing of cooper pairs and the microscopic effects produced by them.

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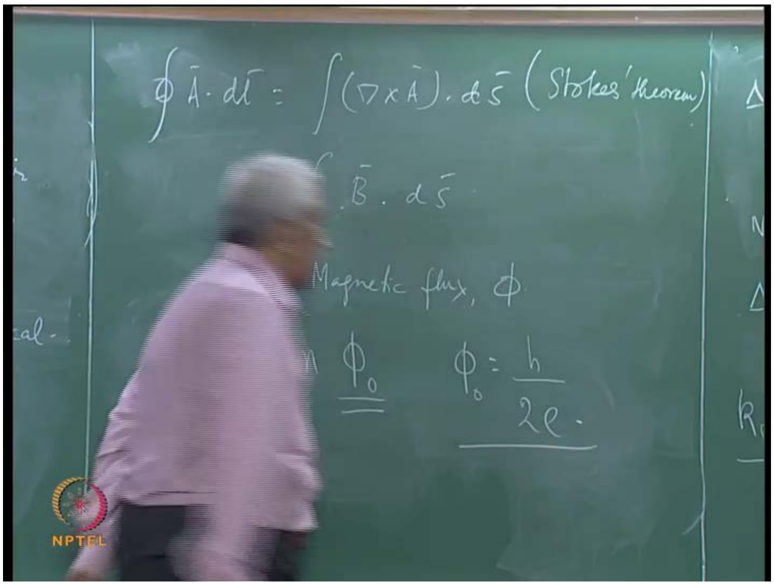

And superconductor in particular enables as, because of the line integral a dot d l over a close loop is related to surface integral del cross a dot d s by stuck theorem. And del cross a is nothing but the magnetic induction and the surface integral of the magnetic induction is what we understand by the magnetic flux trading this gone to this loop. And this as we solved already let naturally to the quantization of the magnetic flux in terms of the fundamental unit of the flux quantum, which is h by 2 E where E is the electronic charge.

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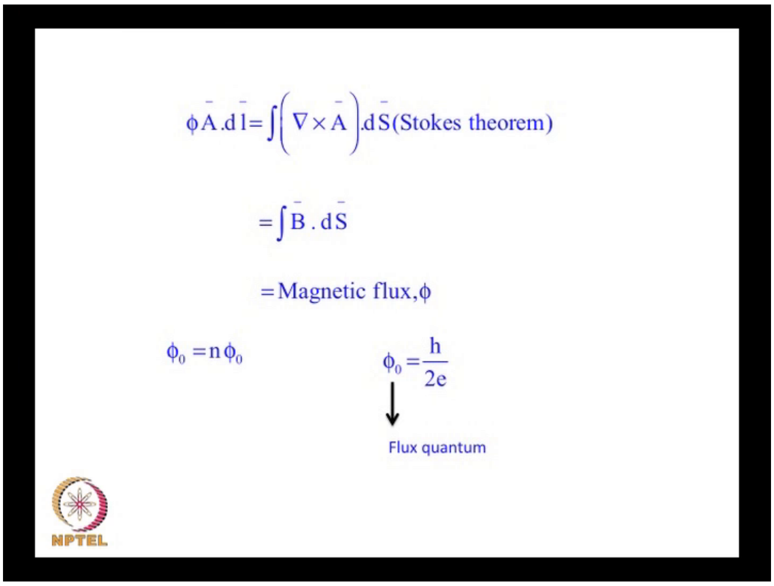

So, this is the flux quantum Josephson and tunneling enables are to observe and measure this flux quantum I forming a tunnel junction made up 2 superconductors, and observing the tunneling current produced by the passage of this cooper pairs across a barriers separating this junction. So, this is know this forms the basis of what is known as superconducting quantum interference device.

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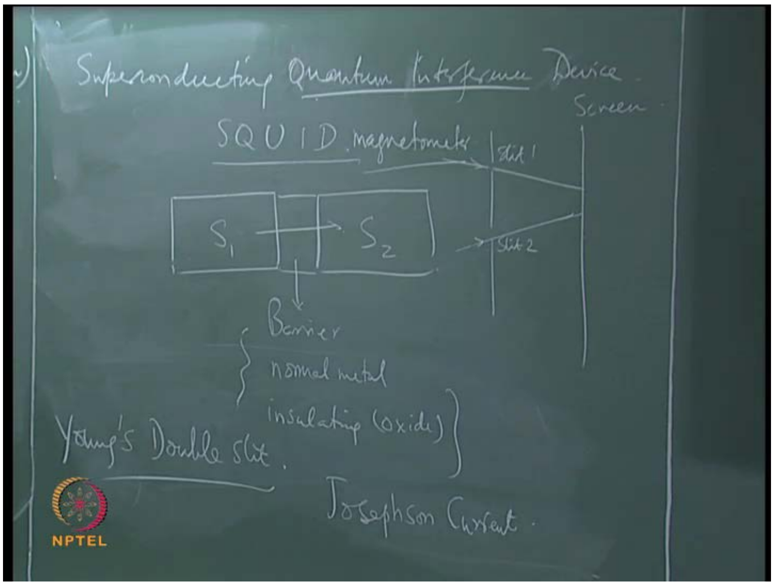

So, this is known as squid a squid is a superconducting quantum interference device, which basically consist of 2 superconductors separated by a barrier this barrier can be a normal metal can be normal metal can be insulators touch as an a oxide and so on. So, it can be a weak link between 2 superconductors also shows such as a cooper pairs tunnel across this gap superconductors s 1 to superconductors s 2 in a coherent manner, so as to preserve the quantum coherence. So, we are able to measure the presence of they applied magnetic field. We are in a position to know manipulate the quantum phase difference the quantum mechanical phase difference between these 2 superconductors the situation here is almost similar to the young's double, its experiment in optics we used in which you have 2 slides. And the light is transmitted across the leave them and then the intensity of light is measured on a screen. So, you have 2 slides and delight goes like this and then converges on to a screen here, and this is the screen slit 1 slit 2 1 a and the light is coherent light is passed through this, and the intensity distribution their allowed to interfere to constructive on the screen. And the intensity distribution here is measured and that is a function the intensity distribution is a function of face difference between these 2 waves which go through the 2 slides.

So, this is the essence of the Young's double slit experiment we have a complete analog effect. So, instead of 2 slides we have 2 superconductor and the pairs passed through this and then you measured the current across this tunneling junction. So, you major Josephson, and current which is the tunneling current across these 2 superconductor which is now just like the intensity on the screen this tunneling current is a function of phase difference between these 2 quantum mechanical system formed by the 2 superconductor, and this quantum mechanical phase difference is a function of the applied magnetic flux.

Therefore, by controlling the wearing the magnetic field applied one can control the phase factor, and therefore control the josses and current by absorbing the intensity distribution maximum minimum josses. And current one can link into the applied magnetic flux and therefore, 1 is in a position on the actually measured the quantum mechanical phase difference which is modulated by this which is produced by this variation in the magnetic flux. So, that is the E and y you are 1 is producing 1 is actually carrying out a 1 quantum interference experiment just like in the young double split you have a optical interference. So, 1 performs quantum interference which is a direct verification of the quantum mechanical nature in the superconductor another important reason, why this squid is of interest is because the squid is enables us we will check write the relevant equations. In the next lecture the squid enables us to measure magnetic flux in units of the flux quantum which is an extremely small quantity this is something order of the ten to the power minus 34 by 10 to the power minus nineteen. So, this is about ten to the power minus fifteen the flux quantum which is measure is that the order of ten to the power minus fifteen waver permit a squire. And therefore, this is an extremely small flux. So, 1 has the possibility to measure extremely small variations in the applied magnetic flux using such a squid. So, this is known as a squid magneto meter changes in the magnetic flux in units of the flux quantum in a very sensitive faction due to extremely small changes in the applied magnetic flux.

So, this gives a very sensitive method for measuring magnetic flux and this is a very important fact because the entire gamut of measuring instrument such as a volt meter ammeter and. So, an in the electrical instruments measuring instruments they all or based on the measurement of the magnetic flux due to current magnetic effect of a current and uses this magnetic flux change, and it is insensitive magnet anon vitamins an equally sensitive ammeter are a voltmeter and so on. So, the entire process measuring extremely small current are extremely small voltage is rendered to very accurate using the squid magnetometers in addition to measuring magnetic flux.

So, these are the summary basic applications for squid magnetometer that is all these are summary of the article applications of superconductors. So, the superconducting quantum interference device in one struck demonstrate the pan can quantum mechanical nature of superconductors on the theoretical side enables to a measure the flux quantization flux quantum, and also provide the bases for extremely sensitive method of measuring magnetic flux and hence current and voltage. So, these a application of a superconductor which is of very for reaching in it is a scope. So, we will considerer some of these in the next lecture.