Select/Special Topics in Classical Mechanics

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Module No. # 01

Lecture No. # 01

STiCM Course Overview

A very hearty welcome to this course on Select Topics in Classical Mechanics or Special Topics in Classical Mechanics, as you might want to call it. Classical mechanics is a very vast field, and in a course that would go over 40 lecture hours, there is no choice, but to provide only an introduction to a selection of topics. This is my choice and I have chosen these topics for a personal reason - my experience; my approach to this subject may have its own limitations.

Nevertheless, I believe these are exciting topics and they touch the very foundations of classical mechanics. If you develop a robust handle on these topics, then it will provide you with the foundations to learn classical mechanics from more advanced sources. So, this is what I would consider as an introductory course, suitable for students who are fresh out of the high school, in their under graduate curriculum.

If you master some of the topics that we are going to deal within this course, then it will prepare you to take more advanced courses in intermediate level or advanced level classical mechanics, electrodynamics, subsequently into quantum theory and more modern formulations of physical theories of the laws of nature.

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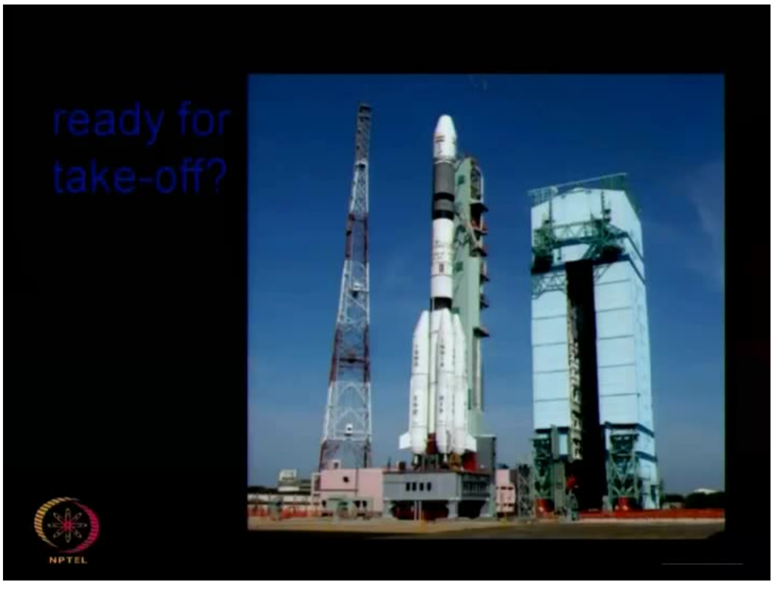

What we are concerned with when we observe around us? All of you live in modern era - the space era. You want to understand how a rocket is launched and what will be its trajectory? If you are having some satellite payloads and so on, to carry out some observations or monitor satellites for atmospheric studies or communication satellites or whatever.

The basic version is, you want to understand, how does this happen? What is it that makes the rocket choose a particular trajectory and how do you design this trajectory, and then, how do you work out the detail dynamics of the engines which will propel this rocket, so that it will follow the intended trajectory?

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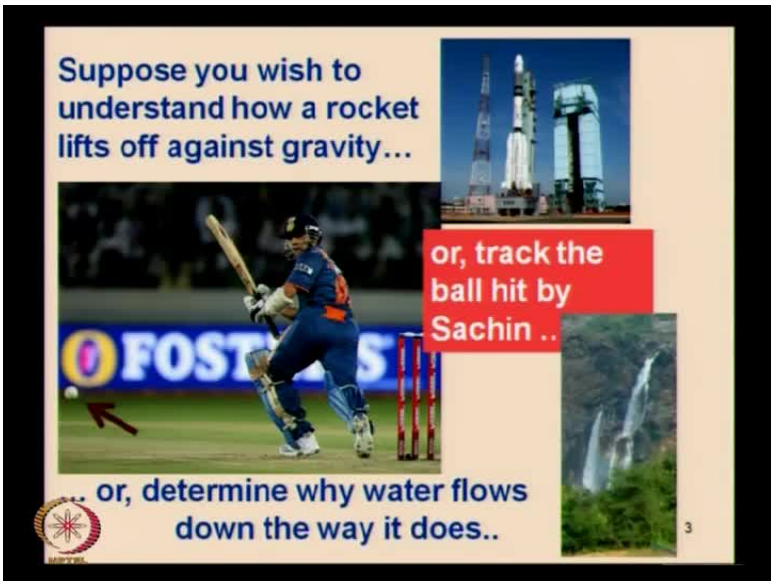

In designing the trajectories, you must understand what are the laws of physics, what are the laws of nature which govern this dynamics? If you are ready for takeoff, we can go ahead and ask other questions. You may want to track the ball hit in a cricket match, you are watching your favorite player and you want to see with this ball hit by Sachin is going to make it, is it going to cross a ropes?

You might have other questions, why does water flow the way it does? Why does it follow a particular path? It all seems so beautiful and so natural, but then, the water does follow a particular path and none other. So, what is it does makes water follow the path that it does, when it is flowing?

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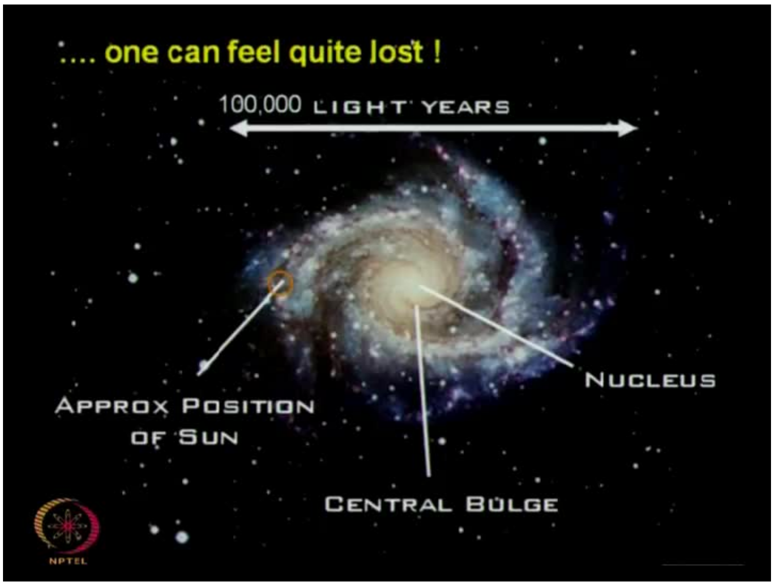

These are some other questions they seem unrelated rocket trajectories, the path taken by a cricket ball or how waterfalls but then there are some common answers to this. One can feel quite lost while addressing these questions, because it do not know where to begin, you are coming straight out of the high school, you have learn some basic laws of physics, you have some introduction. Now, you are prepared to take up these questions, address them in some further detail. One can feel quite lost in this big universe it is a huge, one may this is just one galaxy; the picture you see on the screen is just a single galaxy and there are so many of them.

This is the average size of a typical galaxy if you might consider, it takes like a 100000 light years - the distance is about a 100000 light years. The distance travelled by light at its huge speed, which at one time people thought is infinite speed, it is only more recently that we expect that the speed of light is finite and it has got important consequences in physics, but even this finite speed which is extremely high nothing ever can be accelerated to those speeds. At that speed light takes a hundred thousand years to go from one end to the other, it is such a huge universe and this is just one of - I do not even know how many galaxies that you need to talk about.

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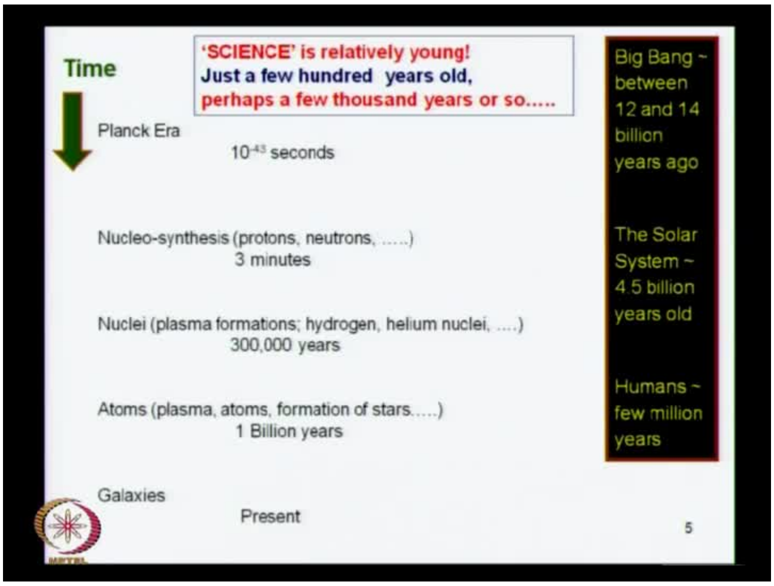

This is our own milky way as we call it. In our milky way, we are somewhere towards the exterior, this is the approximate position of our sun and our planetary system is somewhere over here. If you ask, when this galaxy was formed and so on, then if you go back to the origin of the universe, it is something which did not happen at the time of your great grandfather or anything of that kind, it was as back as 12 to 14 billion years ago, some of the best estimates are in the neighborhood of 13.7 billion years ago or there about.

In this solar system itself is about 4.5 billion years old and us humans - the Homosapiens - perhaps all are few million years old. So, there was no understanding amongst the humans at any time prior to this, because there were no humans. Our knowledge of the solar system of the universe is quite young consider to the life of the universe itself.

If you have time running from top to bottom, the first 10 to the minus 43 seconds is what is called as the Planck era and very little is known about this time. This is very short interval of time, you break a second divide it into 10 parts and then into a 100 parts and then into a 1000 parts and you keep doing it till you have the smallest unit as 10 to the minus 43 seconds. So, it is an extremely shocked time interval and nothing is known about what happened during this time in, this period is known as the Planck era.

Then, in the first 3 minutes the nuclear synthesis to place protons, neutrons were formed and then over the next 300000 years nuclei were formed and then hydrogen, helium, nucleus and so on, the formation of these entities took place. Then, the atoms about a billion years since the beginning and then, you come down to the presence.

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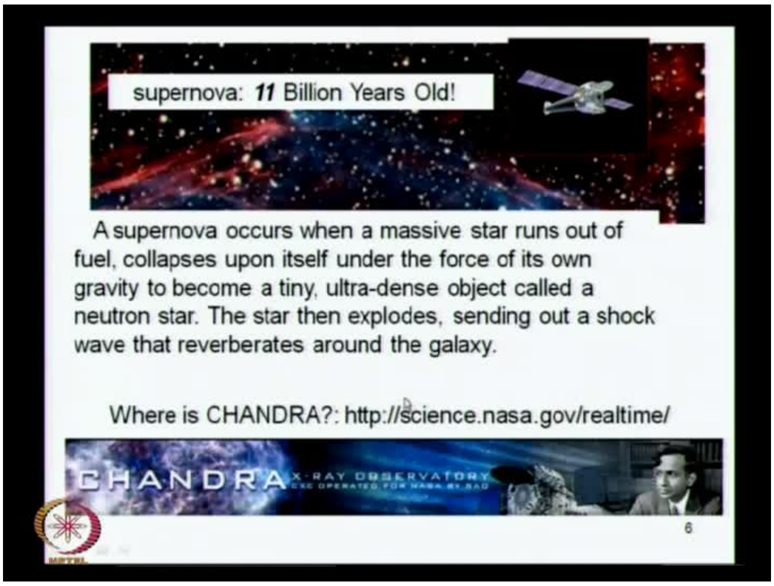

If you look at this whole scenario you discover that science is relatively young; it is just a few 100 years old or perhaps a few 1000 years old. These are not just peculations; you can actually estimate this by carrying out observations. Here is a picture of a supernova which took place 11 billion years ago. This is the picture taken by an observatory which is named after the distinguished Indian astronomer and astrophysicist S Chandrasekhar; this observatory is known after him, it is known as the Chandra observatory.

This is a picture taken by Chandra, you can view this at the NASA website many of these lovely pictures are available at the NASA website; you can actually track the position of the Chandra observatory also if you visit the NASA website.

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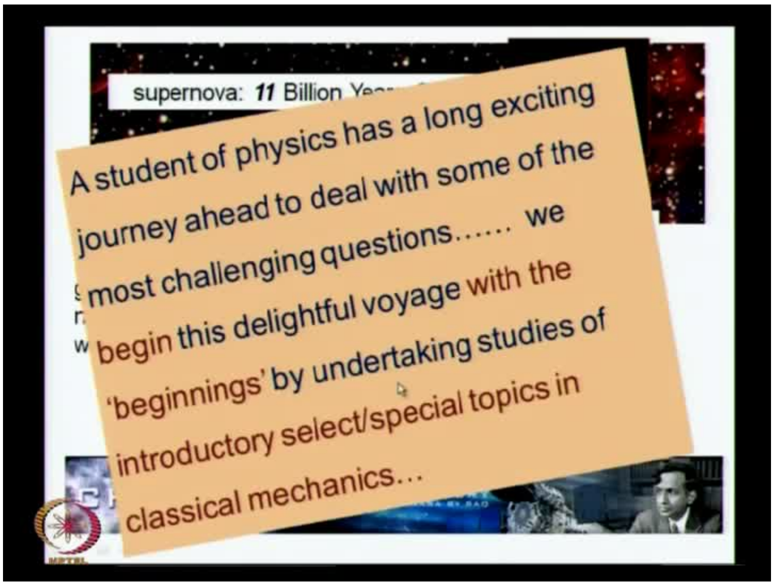

If you want to understand all of this and this is our primary query as a physicist, as a student of science we want to understand all of this. You have to learn things from the very beginning. What we are going to attempt in this course is to introduce you to some very introductory select topics, which will help you to prepare the background for more advanced courses or intermediate courses, before you get to the advanced courses. But then, soon enough you will get to a point that you can really deal with some of these questions and presumably some of you were actually provide the answers to these exciting questions.

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The curriculum for this course will be covered in 11 units. My first slide regarding this overview is a tribute to Chandrasekhar Venkatraman or what we will do is to provide an interaction in 11 units.

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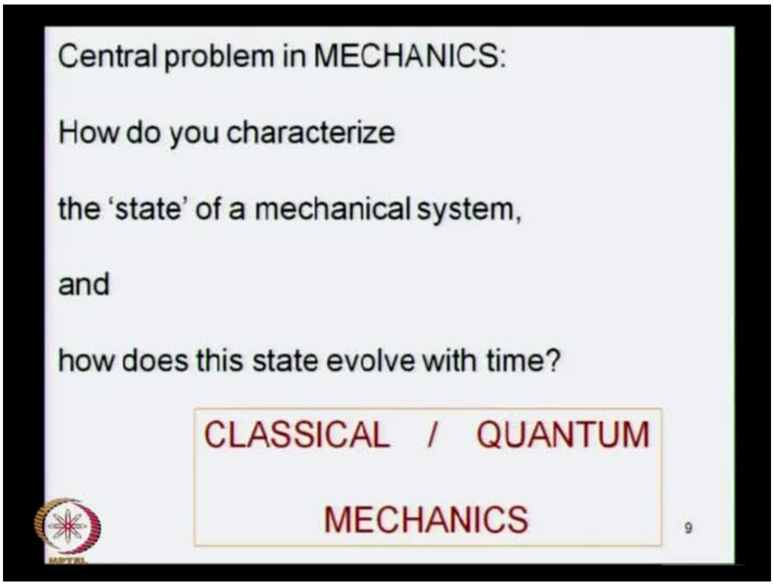

We consider the primary question in mechanics, how do you characterize this state of a mechanical system, this is the first question, how do you characterize it? Means, you look at this water bottle and you can say that it can take a certain amount of waters, so what is its volume, what is its mass, what is its shape, what is its color, there are large number of properties that you talk about any object.

Amongst these which are the mechanical properties, which characterize the mechanical state of the object? The shape of the object is a property of the object, but it is not something which describes the mechanical state of this bottle. The color of this bottle means, you have got our blue colored label on this; this is an attribute, it is a physical attribute, it is a physical property, but it does not provide information about the mechanical states. There is some peculiar property which comes to your mind when you talk about the mechanical state of an object.

Mechanical state of an object does not mean that you must specify all the properties of that object, but only those properties which are characteristic to the mechanical temporal evolution of the state of the system. So, you must first of all, figure out what is it that describes this state of a mechanical system, how do you characterize it in a precise unambiguous terms? Subsequently, how does this state change with time? How does the mechanical state evolve a time, what is it is temporal evolution? This is the fundamental question that is posed in mechanics.

We will get the answers in what is known as classical mechanics as oppose to quantum mechanics, which deals with this question in a different way, but in classical mechanics we will have to figure out what is it that characterizes the state of a mechanical system. The answers to these are different in classical and quantum theory, because how it is described in classical mechanics is a matter of contention, when you subjected to test, experiments and so on, then you are required to reformulate your description of the mechanical state of a system.

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You have to reformulate your answer of how the system will evolve with time, nevertheless to appreciate how it is done in quantum theory. It is absolutely important fundamental to understand very precisely, how the mechanical state of a system is described in classical mechanics and how does this system evolve with passage of time.

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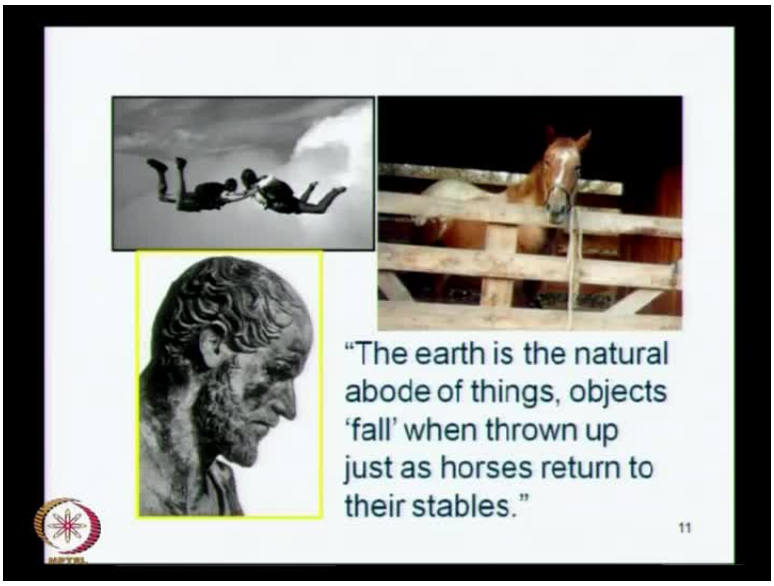

You ask these questions, why do objects fall? You take an object hold it let go, you see it falls, why do objects fall? Now, in the earliest times the Greek philosophers, mathematicians and scientists believed that, objects fall because the earth is a natural abode of things and objects fall when thrown up just a way horses would return to their stables cattle, horses, dogs, they just come back.

The general belief was that if you let go of an object earth being the natural abode of these objects they would come back home. This explanation was considered to be perfectly satisfactory, everybody was fully satisfied; the answer looked so logical and so reasonable, because it matches the experience of the time, no questions asked and people subscribe to this idea, they absolutely believed.

This was not like many thousands or millions years ago or something but as recently as just about 2000 years ago, means we have seen the life of the universe and even as 2000 years ago means, a humans themselves are a few million years old. For all these millions of years humans believed that yes, this is a perfectly complete answer.

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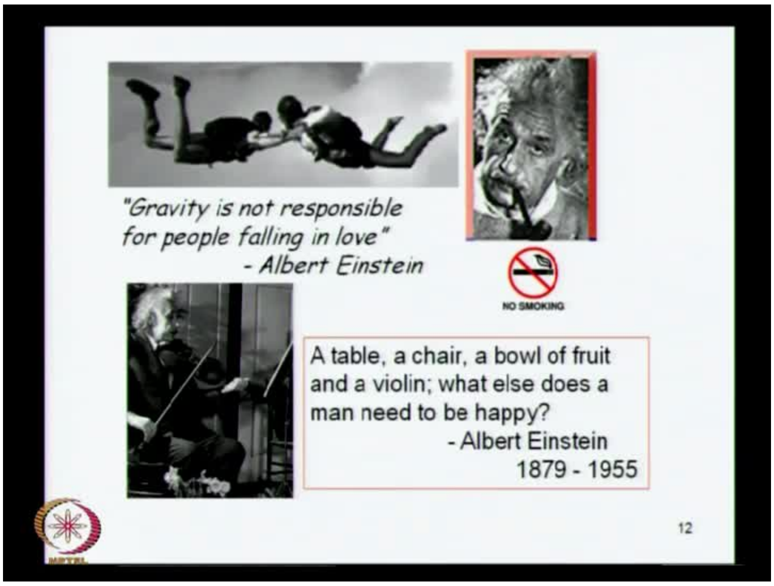

This answer as we know now is not really complete; we have to look for better answers. These answers as we now understand are the correct technical scientific answers, these answers are just a few 100 years old, not even a 1000 these are just a few 100 years old.

One of the classic contributors to modern theory is Albert Einstein and it is tempting to quote him that a table, a chair, a bowl of fruit and a violin, what else does a man need to be happy? If you want to follow Einstein and want to be happy, you do not really these two learn special theory of relativity or general theory of relativity or quantum mechanics, which also Einstein was not only a contribute to, but in fact perhaps the origins of quantum mechanics can be attributed to Albert Einstein's explanation of the photoelectric effect.

Then even classical mechanics, if you do not want to understand why objects fall. If the objective is just to be happy then, you do not have to learn any of this; you can just get a table, a chair, a bowl of fruit and a violin.

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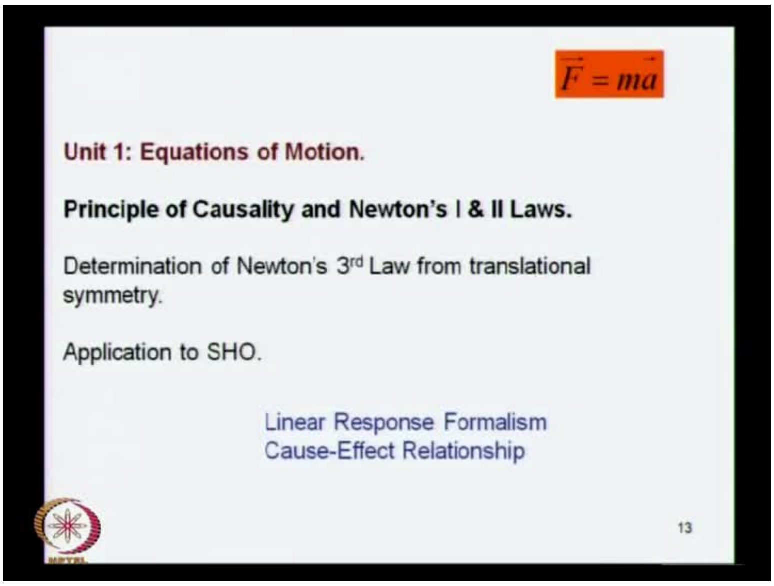

Let say you go beyond it, and you are really wanted to understand how physicists understand these issues? We will introduce you to the equation of motion in unit 1. Amongst the relevant units our first unit will set up the equation of motion for unit 1, we will discuss what equilibrium is this; object appears to be in a state of equilibrium in our frame of reference?

We will learn to define equilibrium in a very rigorous manner. We will then have to ask this question, what is it that is responsible for the equilibrium being disturbed? If I knock it off it falls; if I push it, it rolls, it rolls up to a certain distance and then it stops. When we want to ask all of these questions then, we have to ask what the cause which makes this happen is. What is a relationship between the cause and the effect?

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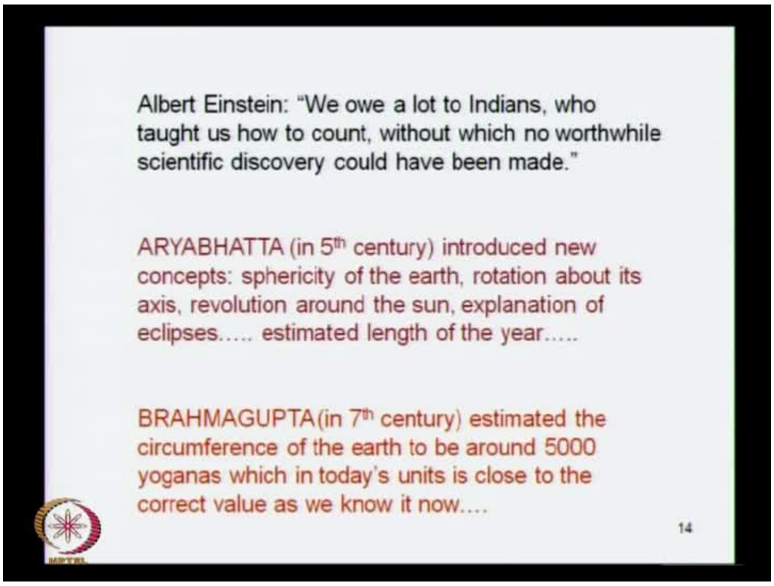

The cause-effect relationship is in fact in Newtonian mechanics a linear relationship - the effect is proportional to the cause. The effect manifest as the acceleration and the cause is a force. This scientific approach is like, I said just a few 100 years olds perhaps, a couple of 1000 at the most and again, I quote Einstein who points out that we over locked to Indians who taught us how to count, without which no worthwhile scientific discovery could have been made.

These are some of the most important contributors to the development of mathematics, physics, astronomy and the scientifics investigation carried out in India. Aryabhatta in the 5th century, he already understood the sphericity of the earth. He understood that the earth rotates about its own axis; he knew that the earth revolves around the sun, he could explain eclipses and it has been known from the time from the 5th century what the cause of the eclipses is? It remains a mystery to me as to why people still talk about the [FL] and the [FL] and then allow their lives to be governed by the [FL] and the so on. So, this kind of nonsense continues even in modern times. I hope that some of you will fight against this, which is part of a scientist's responsibility which is the eradication of superstition.

Then, Brahmaguptha in the 7th century, he estimated the circumference of the earth which he estimated to be like 5000 yoganas which in today's unit comes close to the correct circumference of the earth as we know it today.

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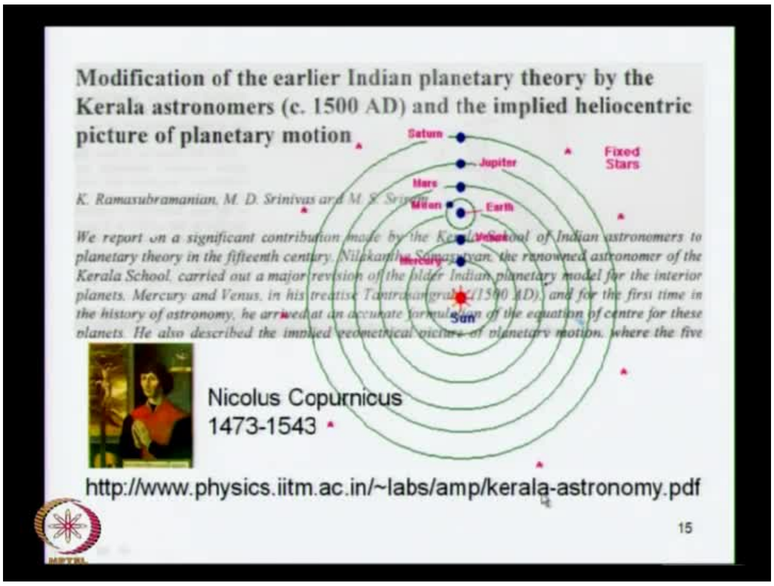

A lot of credit goes to Copernicus for coming up with the heliocentric frame of reference, because until then it was argued that it is the earth which is at the center of the solar system and everything else goes around it, because that is what you see. You see the sunrise in the east, sets in the west and the same thing happens to the rest of the sky.

Copernicus is given the credit of understanding this but before Copernicus, this was known to the Indian astronomers. These were not just speculations, these were rigorous estimates and in particular, the Kerala astronomers have done this study very precisely, their compositions have been investigated. I carry a copy of this paper at my website; a paper by my colleagues and friends Ramasubramanian, M D Srinivas and Sriram which is loaded at my website which is over here, but you can find this very easily on the internet. Then, in very precise terms which are mathematically rigorous, how the whole revolution around the sun is to be explained in mathematically correct forms.

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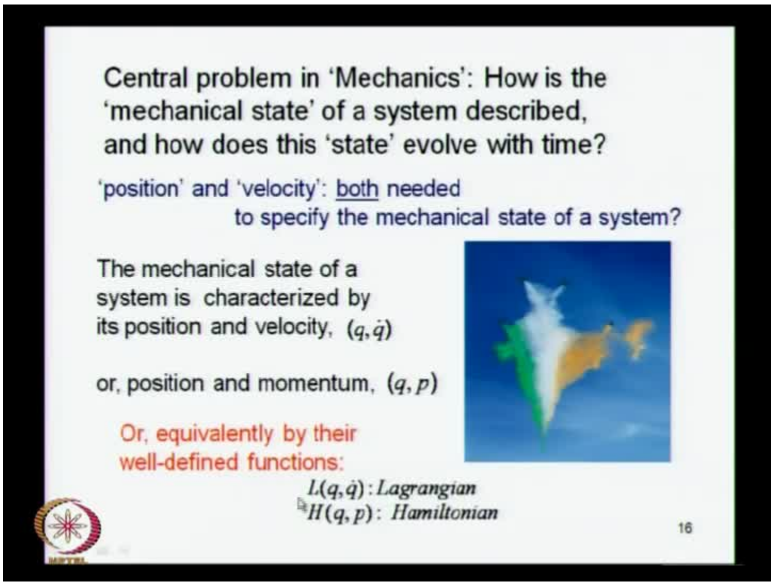

Let us take up this question that what that characterizes is the mechanical state of a system and we know it is not the color, it is not the shape, it is not the mass, it is not the volume and so what is it? The mechanical state of the system is characterized by two parameters: one is the position of that object, the other is a velocity of the object. And I need two parameters, no more than two and no less than two if there is only one degree of freedom of course. There are more degrees of freedom then you need more, for you need a similar pair for each degree of freedom, unless there is constraint that is a matter of detail. But, if there are independent n degrees of freedom, for each degree of freedom you need the position and the velocity. Now, I will like to dismiss one doubt, which some students sometimes have. Most of you will not have this doubt, but some of you may have, as to why is velocity an independent parameter? Because after all velocity is the rate of change of position.

If you know the position, you might think that all you need to do is to take it is derivative with respect to time and you will get the velocity. So, why is it an independent degree of freedom?

Now, it turns out that this question is not so uncommon, but when you get the answer everybody discovered that it was a silly question. The answer is not so silly perhaps, all you have to do is to look at this beautiful map of India generated by the lovely tricolor spread by the Indian air force.

If you locate Chennai over here on this map, so it is somewhere over here or the eastern coast and you consider a plane which starts out in Chennai moves toward the west at 300 kilometers per hour. Which starts from Chennai over here at 300 kilometers per hour, in 1 hour it will be somewhere over here, which will be over Mangalore, roughly speaking.

On the other hand, if it starts at the same velocity but, the starting point was not Chennai but, Bangalore itself then, in 1 hour it will be at the west coast somewhere over Mangalore. Velocity alone is not enough to predict where the aircraft will be after an hour, you also need the position.

If you knew the position alone, you still cannot predict where the aircraft will be after 1 hour, because if it starts out from Chennai at 300 kilometers per hour towards the west it will be over Bangalore whereas, if it started out at the speed of 600 kilometers per hour it would be over to Mangalore. So, these are two independent parameters, a unique both to predict the temporal evolution of an object. These are independent parameters and you would need both of them. You need the position and the velocity, I will represent the position by q and the velocity as dq by dt which is the time derivative of q and I will denote these time derivatives by placing a dot on q. So, q and q dot both are needed to describe the position and velocity of an object.

Equivalently you can describe the position and momentum. So, these are two alternative but equivalent descriptions of the mechanical state of a system but then, what you can do is describe it also by a function of the position and velocity; if you come up with a welldefined function of the position and velocity, then also you can characterize the mechanical state of a system.

This is done in one formulation of classical mechanics which is called is a Lagrangian formulation of classical mechanics, what it does? It begins with the position and velocity which is essentially the primary ingredient, but then what you define is a very rigorous well defined function of the position and velocity which is a Lagrangian but, the equivalent alternative available to us was the formulation of the system in terms of the position and momentum. So, you might as well defined the system by if well-defined function of the position and momentum, which is done in another equivalent formulation of classical mechanics; namely the Hamiltonian formulation.

The Hamiltonian is a function of position and momentum, the Lagrangian is a function of position and velocity. These are some of the alternative but equivalent formulations of classical mechanics.

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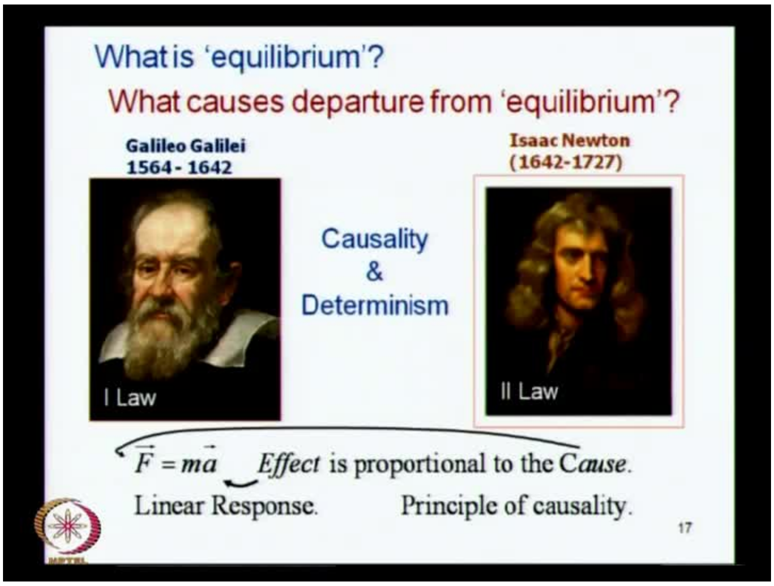

The first understanding of the mechanical equilibrium of an object was first rigorous understanding that we got was from Galileo Galilei. He explained what equilibrium is in very rigorous terms which we now recognize as the first law of mechanics. We often call it as Newton's first law, but it was actually formulated by Galileo before Newton. Newton was born the same year that Galileo died 1642, what Newton contributed further beyond Galileo is an explanation of what is it that causes departure from equilibrium.

Galileo explain what equilibrium is, Newton explained what is that causes departure from equilibrium. Galileo's discovery was that equilibrium is self-sustained as long as you do not disturb an object, an object continuous to be in its state of equilibrium as long as you do not disturb it.

This would happen depending on how you observe the object which connects you to the frame of reference and which the observer is. This law is known as the first law, it has to be the first law you cannot do anything else, unless you understand the first law, because that is what helps you understand, what an inertial frame of references is.

In this inertial frame of reference, once you understand what equilibrium is and then, you ask what is that causes the departure from equilibrium? Then, the explanation comes from Isaac Newton who tells us that a departure from equilibrium - first of all, he quantifies this departure from equilibrium. So, the departure from equilibrium is quantified as the acceleration of the object. Body in constant velocity is in equilibrium only if the velocity changes, so when there is dv by dt, there is an acceleration and this is an essential requirement of a departure from equilibrium.

This acceleration is a result of a certain cause, the causes over here, which we call as a Newtonian force in today's language. The result is acceleration and the acceleration is proportional to the cause. So, this is a linear response theory - a stimular response formulation - in which this stimulus generates a response in the system on which the system will let operate and this cause effect relationship which is sometimes called as a principle of causality or the determinism that the cause determines the effect.

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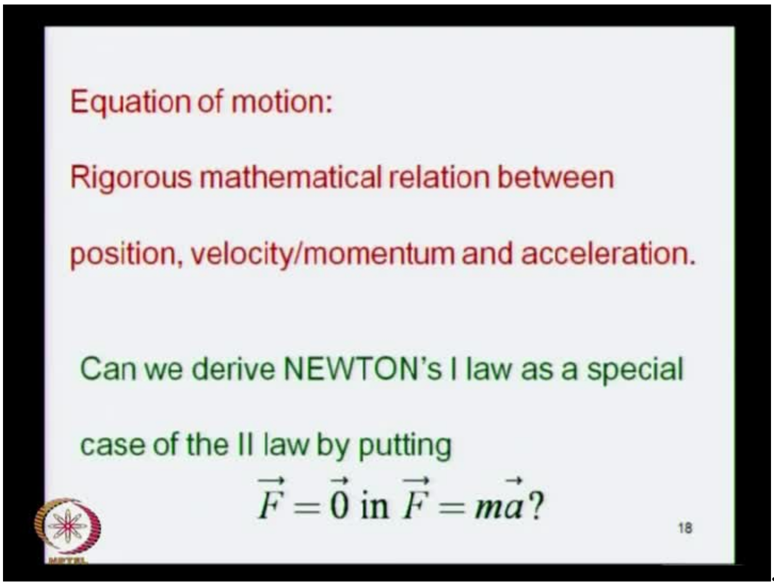

This is what we recognize as Newton's second law but, the first law of Newton is actually due to Galileo. Then, we will develop the rigorous relationship between position, velocity and acceleration or position, momentum and acceleration, and it is this mathematical relationship which is called as an equation of motion. So, we will meet the equation of motion in the first unit.

We will discuss if we can derive the Newton's first law as a special case of the second, because if you put F equal to 0 in this equation, which is the statement of second law it would appear that if F is equal to 0 acceleration is 0, which means that the velocity is constant, which sound so much like a statement of the first law. So, it would be very tempting to suspect that Newton's first law is just a special case of the second and then you might worry, why do have to learn it as a separate law at all. So, that question we will discuss in some detail in unit 1 (Refer Slide Time: 32:30).

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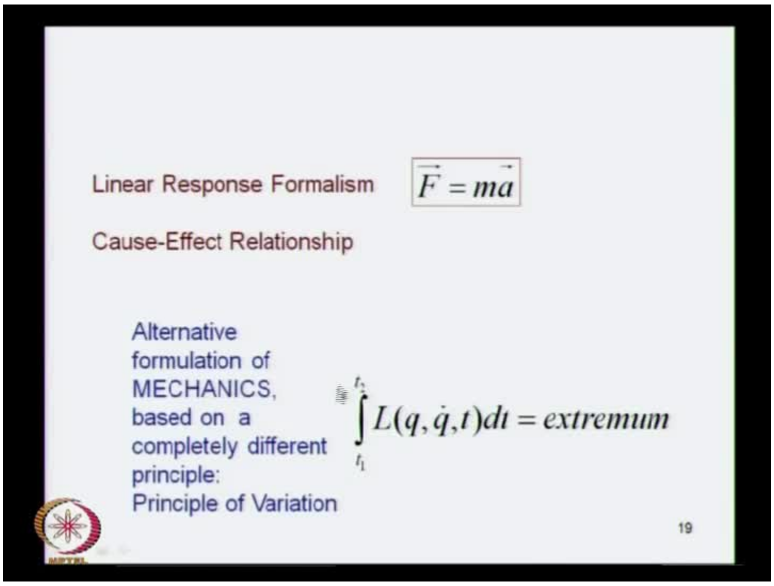

We will then also meet an alternative formulation of mechanics, alternative to what? Alternative to a Newtonian formulation, so this is the heart of Newtonian formulation F equal to m a that a departure from equilibrium is caused by a force and this linear cause-effect relationship is a heart of Newtonian mechanics.

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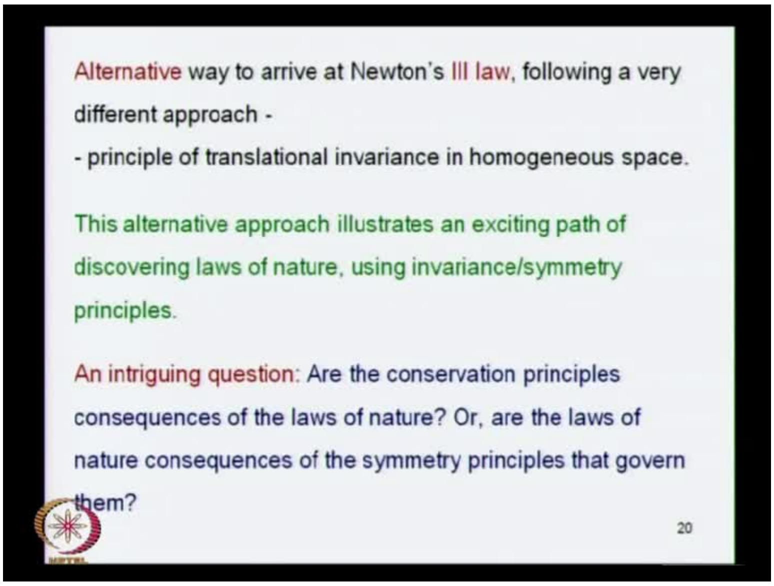

To this formulation there is an alternative which makes no reference to the idea of force. It makes no reference to the cause-effect relationship, but it explains a mechanical evolution of a system in terms of a different entity called as the action which is this integral of the Lagrangian and this formulation tells us that the system evolves in such a manner as would make this the action integral an extremum. So, this is the principle of extremum action; this is the principle of variation and we will discuss this formulation also in the first unit.

We will also discuss the Newton's third law, but we will interpret it in a different way not merely as an action and reaction being equal and opposite. We will connect this to the principle of translational invariance in homogeneous space. We will see that, Newton's third law is in fact a statement of conservation of momentum which is intimately connected to the principle of translational invariance in homogeneous space.

Now, this will introduce us to an a very exciting path of studying physics, which has implications also in modern science, because it will illustrate to us an exciting alternative path of discovering the laws of nature using invariance and symmetry properties. There is a fundamental question because we also learn conservation principles as consequences of laws of nature which can be studied as consequences of laws of nature. You can formulate the laws of nature, you can formulate F equal to m a as the equation of motion as a law of mechanics.

From this law you can deduce the conservation of energy, you can deduce the conservation of angular momentum, you can deduce various conservation laws, but you can also do the opposite. That either laws of nature consequences of the symmetry principles that govern them, is also a question that can be raised or it turns out that the answer is not only yes, but a better answer and a better way of doing physics. So, I will discuss some of these possibilities in the first unit.

What is done in contemporary physics is that symmetry and invariance is place ahead of the laws of nature. This approach began with Albert Einstein once again, through his analysis of laws of electrodynamics and then, very rigorously formulated in a very famous theorem known after Emmily Noether; it is known as the Noether's theorem and very lucidly explained by Eugene Wigner using group theoretical methods.

So, this whole approach has got very fascinating applications in modern physics, which is further reason I have chosen to develop this branch of classical mechanics following this particular route. You will see this connection between symmetry and conservation laws as - I would not say as a central theme, but certainly one of the main backbones of this course.

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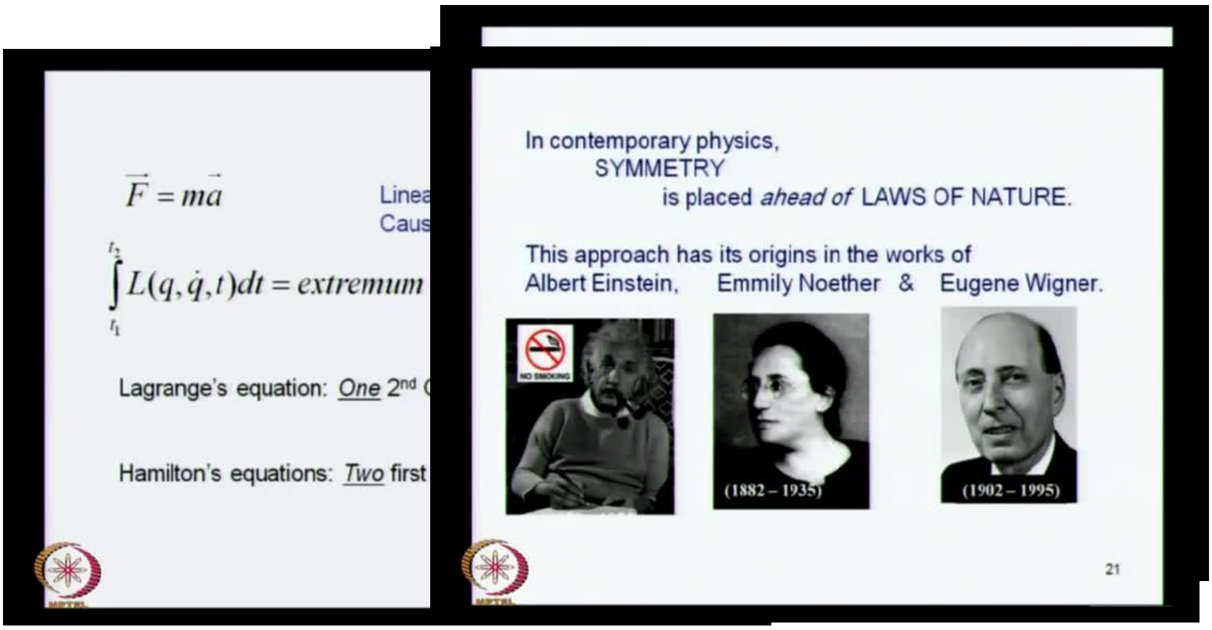

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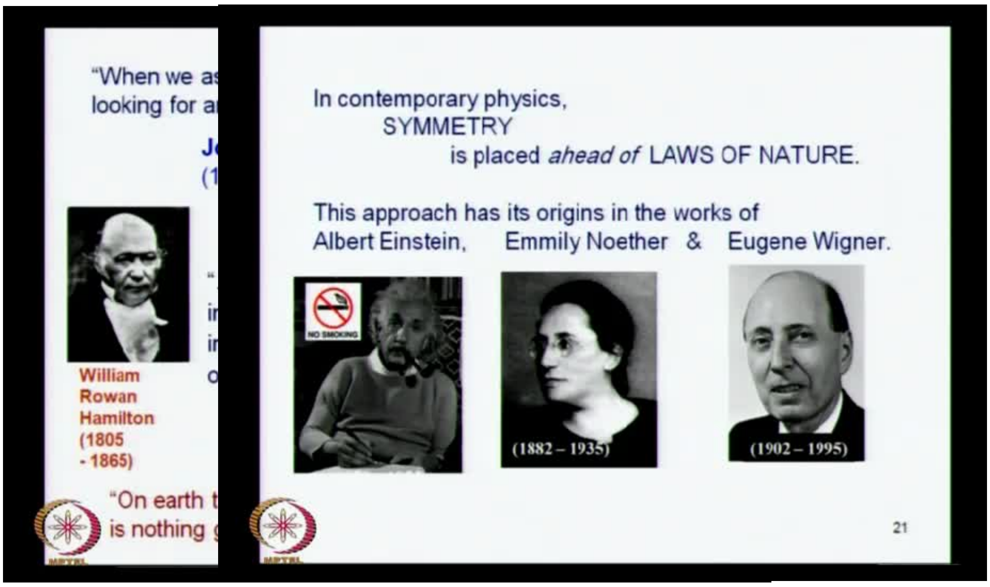

We will develop these formulations; a Newtonian formulation, we will meet an introduction to the Lagrangian's formulation, Hamilton's formulation of classical mechanics, these formulations are more recent. Both Lagrange and Hamilton were brilliant mathematicians and physicist. Here is a very well-known quote of William Hamilton who was the elder son of the Hamilton after whom the Hamilton's principle is known who was William Rowan Hamilton.

Hamilton was a very studious person, he was a workaholic, he would just sit in his office and his son tells us that we use to bring in a snack and leave it in a study, but a brief nod of recognition of the intrusion of the chop or cutlet was often the only result, he was completely engrossed in his studies. He was very peculiar mind he came up with brilliant ideas and what he said is that on earth there is nothing great but man and in man there is nothing great but mind. That is what was perpetually active in Hamilton.

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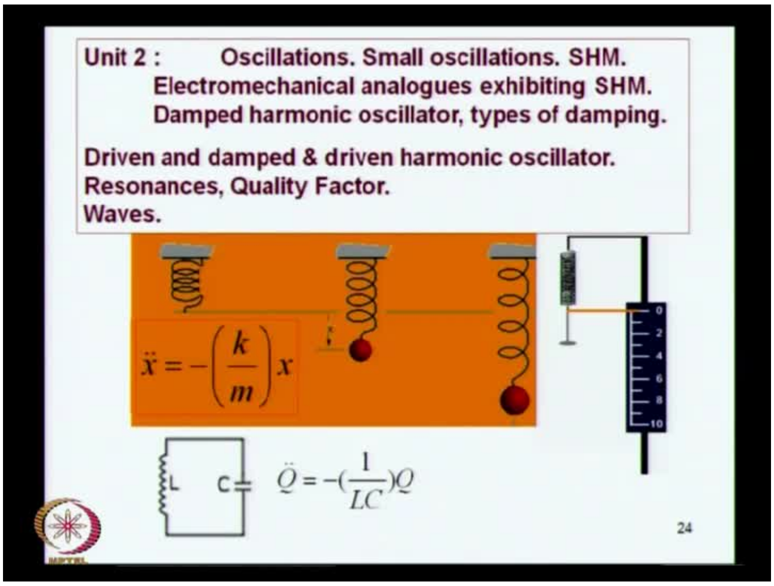

In unit 2, we will begin to apply these ideas to problems on mechanics. Having introduce you to the Lagrangian and Hamiltonian formulation, I will not take that branch of mechanics any further, it is a very worst area, , it needs a full course in fact it needs perhaps two courses, which is not the intent of this introductory course on select topics to introduce you to ideas of classical mechanics.

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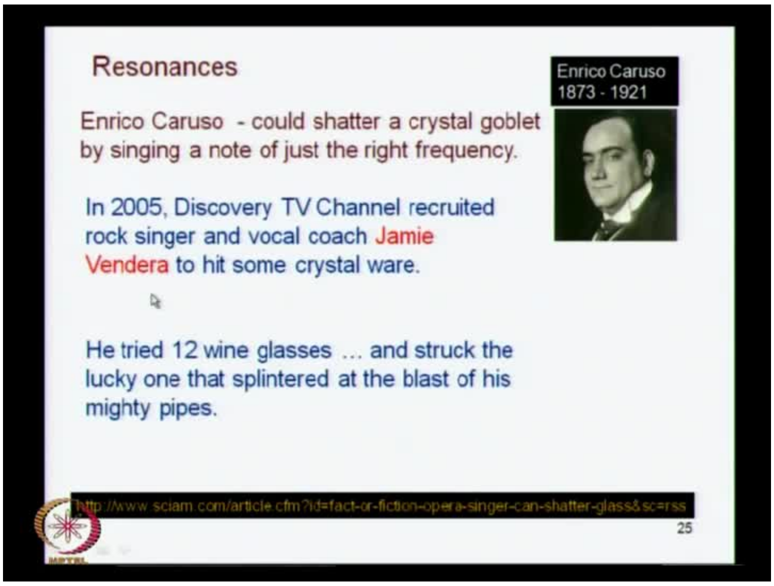

So, I will not be developing classical Lagrangian or Hamiltonian formulation but, I will just take up some simple applications like, the motion of a simple harmonic oscillator, whether it is a mechanical oscillator like a mass spring system or a simple pendulum in small oscillations or in electrical oscillator like, an LC circuit. So, we will take up some simple applications of this kind.

It leads to very exciting physical phenomena such as resonances; this branch of mathematics and physics develops into some very beautiful applications. You all know resonances and one of the manifests in our observations all the time and Enrico Caruso was credited with the ability to shatter crystal goblet by singing a note just of the right frequency.

These are more modern examples of distinguish musicians like Baiju and Tansen; Tansen was in Akbar's court and Baiju was also a very distinguish in the complex musician perhaps who even excelled our Tansen in many ways. These musicians were credited with abilities of this kind. This happens because of the phenomenon of resonance, because of the musical note that you produce has a frequency and phase relationship with what is called is a natural frequency and phase of an object. Then, what results is a resonance and this can lead to huge effects. This is not just stories from history books, but the discovery channel just five years ago in 2005 actually invited Jamie Vendera to produce a vocal note, which would shatter a wine glass. Jamie Vendera actually did it in his twelfth attempts and he performs; he shows and many of these videos are available on the internet on the YouTube and so on.

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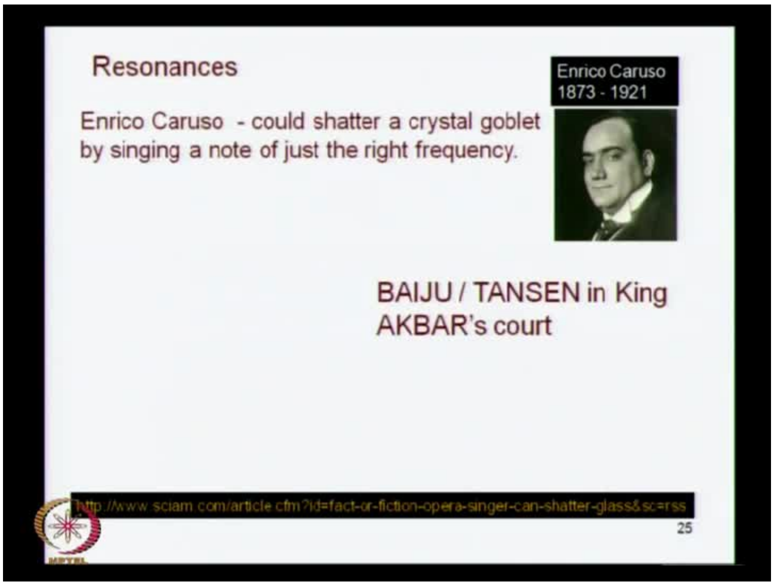

So, if you just Google Jamie Vendera and resonance or any keywords of this kind you will be easily let to that and you can actually see, this happen in front of your eyes and this was recorded by the discovery channel in 2005, so it is a very visible effect. These are the Baiju's and Tansen's of the modern hero in some sense.

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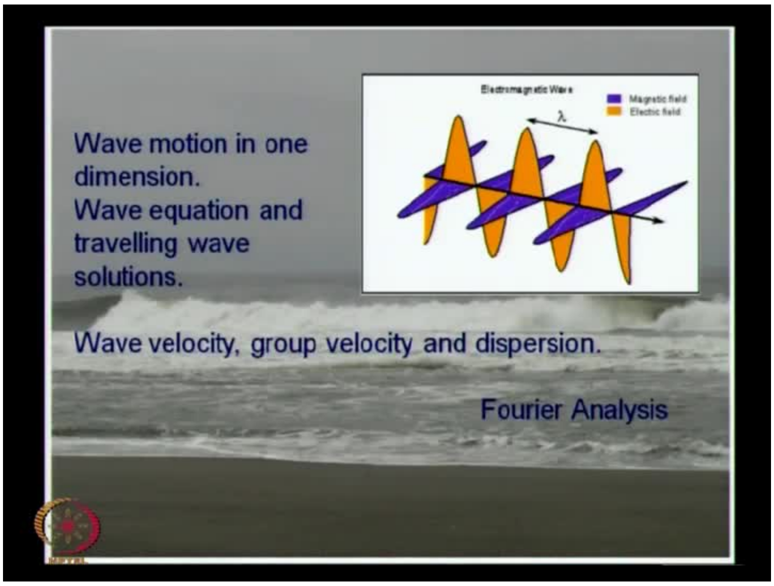

This whole subject deals with - it is the central subject which goes into the study of wave motion. We will study wave velocity, we will understand what is phase velocity, what is group velocity, what is dispersion, what is refraction. So, many of these things in mechanics and optics so on and also electromagnetic theory, so many advanced applications are also connected with these topics.

There is probably no power

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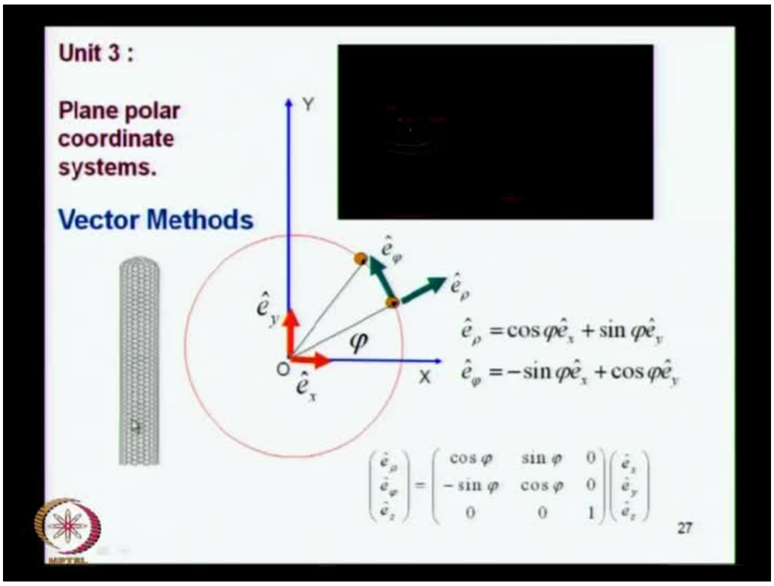

We will see the applications in the study of wave motion in general, we will also introduce ourselves to methods of Fourier analysis. Then, will be going over to unit 3 in which we will study various coordinate systems, because when we want to study these objects like, nanotubes or some other objects of various kinds. Then, you exploit symmetries and using natural symmetry which is best suited to study the mechanics and the dynamics of an object in motion. You employ different coordinate systems, you may use the plane polar coordinate system, you may use the cylindrical polar and so on.

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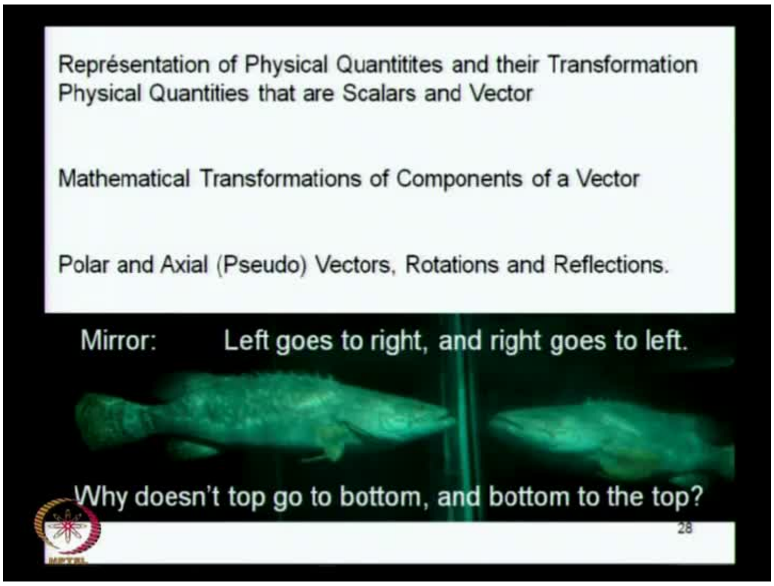

For this you must understand, what a vector is? We will define a vector very precisely; we will raise such questions like, what is a polar vector? What is a pseudo-vector? What is the rotation, what is a reflection? When we deal with these questions in precise terms we get the correct definitions of what a scalar is, what a vector is. We can ask this question that when you look at a reflection, why is it? That the left goes to the right and right goes to the left, but not the top to the bottom and the bottom to the top.

Now, it is important to understand the answer to this question, it looks like a very simple question but the answer is very tallying. One learns a lot by discovering these laws of rotations and reflections. They introduce us to the correct definitions of a vector and a scalar and so on.

We will introduce ourselves to these rigorous definitions it is not sufficient to just say that is scalar is defined by magnitude alone and a vector by magnitude and direction. We will find that these high school definitions are somewhat in adequate and one must have rigorous definitions of these quantities. So, we will introduce ourselves to this in our units.

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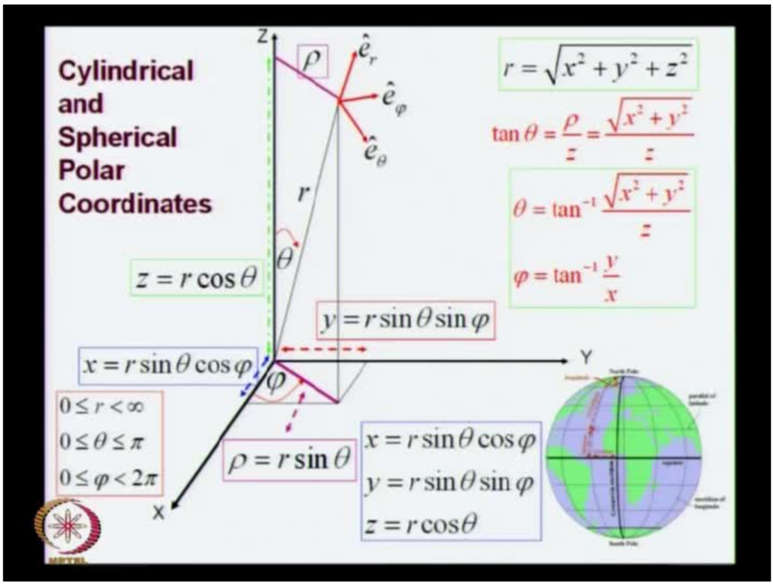

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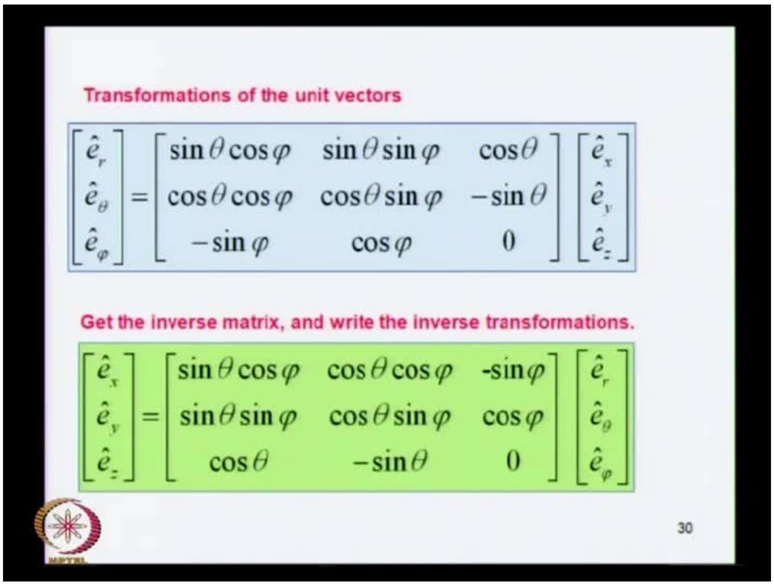

In unit 3, we will consider other coordinate systems like the spherical polar coordinate system. We will understand how to transform from one to the other in using rigorous mathematics. These transformation laws can be very nicely placed in compact form using matrix methods.

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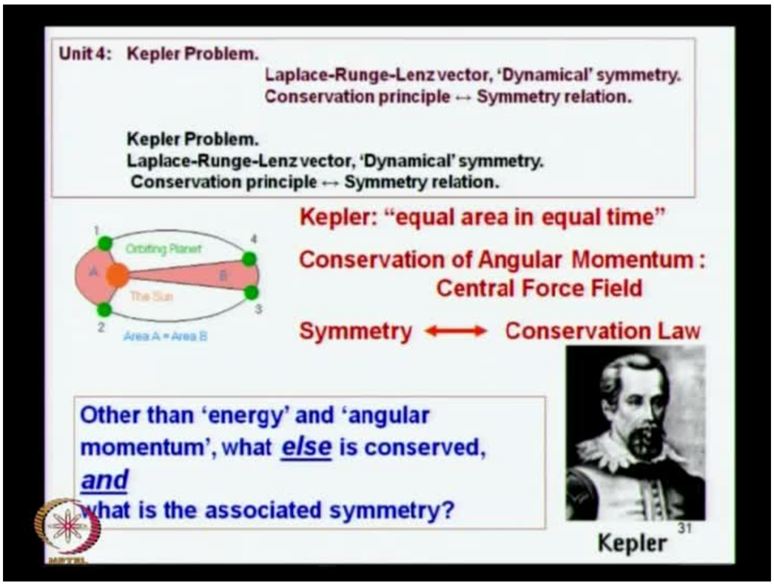

We will then go over to unit 4 which we will study the Kepler problem. Kepler problem which I am show you have studied in the high school physics, you know that angular momentum is conserved and that is connected with the fact that a planet describes equal areas in equal times.

The question that we will raise is that yes, this is due to the conservation of angular momentum; there is also conservation of energy. We will ask is there any other physical parameter which is conserved in the Kepler problem, other than energy and angular momentum.

What we definitely know is that - yes, other than energy and angular momentum there is a conservation principle which is involved in the Kepler problem. We must then look for an associated symmetry because I mentioned in our over view of unit 1 then, there is a direct intimate connection between symmetry and conservation law.

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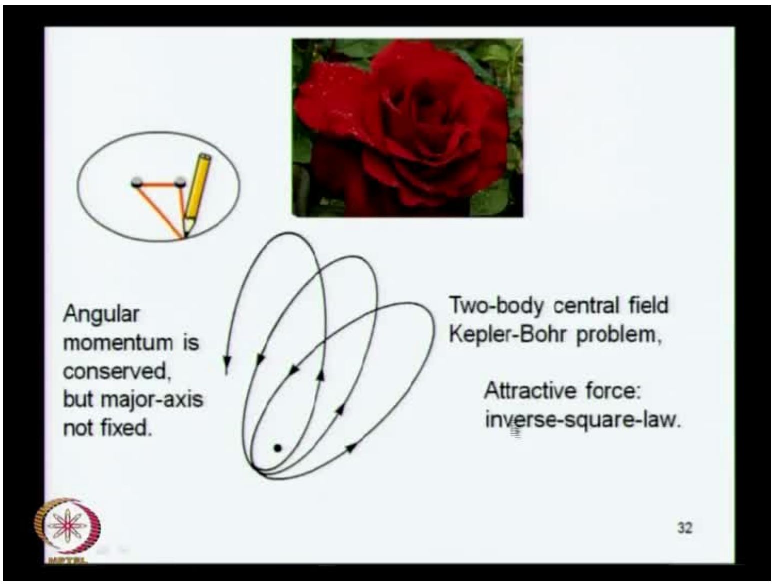

If there is an additional quality which is conserved in the Kepler problem, we will also need to explode the associated symmetry. What is conserved in the Kepler problem in addition to energy and angular momentum is the fact that the ellipse itself does not precise and conservation of angular momentum alone does not explain this.

Conservation of angular momentum only tells you that the ellipse must remain confined to 1 plane, it cannot come out of the orbit, but in that plane, why it does not precise, why it is major axis would not turn? So that if there is a precision or motion of the ellipse that would take place this is called is a rose at motion, because if you were to see it from a distance this will look like the petals of a rose, so it is called as a rose at motion. So, why is it that an ellipse does not execute the rose at motion? Tells us that in addition to energy in angular momentum there is an additional physical property which is conserved, which is a fact that the ellipse does not precise and then, if there is an additional conservation principle, what is the associated symmetry is a question that we must ask.

We will get the answer to this it turns out that a necessary condition why the ellipse does not precise is the fact that the dynamics between the sun earth systems is governed by the gravitational law which is an inverse square law. The force must be an inverse square force as a necessary condition for the ellipse not to precise. So, this is a necessary condition it comes from F equal to m a relationship that the dynamical equation we describe this motion F equal to m a must have a force, which is an inverse square force.

The dynamical law itself is a necessary feature of this particular property of this particular conservation principle and the associated symmetry therefore, called as the dynamical symmetry of the Kepler problem.

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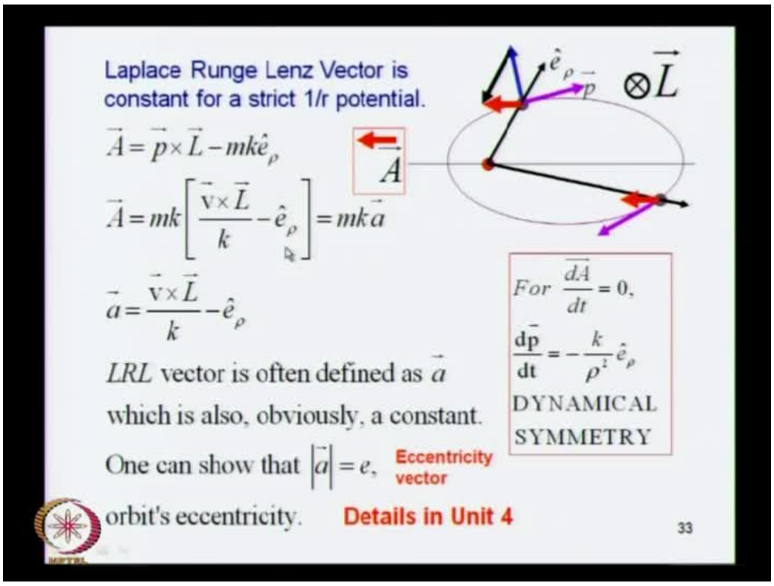

What is conserved is expressed in terms of a vector quantity which is called as a Laplace Runge Lenz vector. So, this is just a preview of what you are going to meet in unit 4. This is called as a eccentricity vector and this is conserved eccentricity or the Laplace

Runge Lenz vector and this is conserved it is connected to the dynamical symmetry of the Kepler problem and we will study the details in unit 4 (Refer Slide Time: 48:54).

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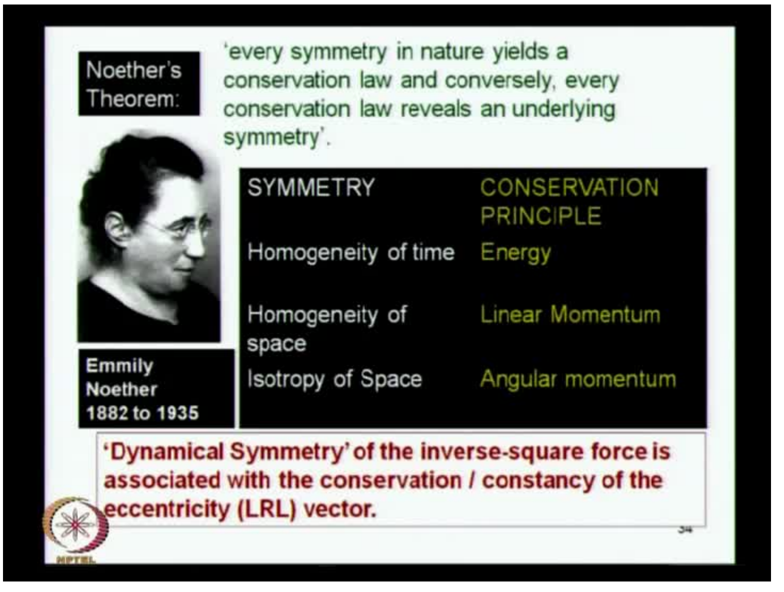

This will develop deeper appreciation of the connection between symmetry and conservation principle. You know that if there is homogeneity of times, so there is a symmetry associated with that and there is a corresponding physical property, which is conserved namely, the energy of the system then there is homogeneity of the space associated with the translation invariance.

There is a conservation principle which is the conservation of linear momentum likewise; isotropy of space is connected with angular momentum. This is a more general principle known as the Noether's theorem that every symmetry in nature yields a conservation law and conversely, every conservation law reveals an underlying symmetry. This is a very powerful principle which is of fundamental importance through the exploration of the laws of physics and laws of nature that we continue to remain after. This is known as the Noether's theorem and we will illustrate it from Newtonian principles and this will be discussion on the dynamical symmetry of the Kepler problem.

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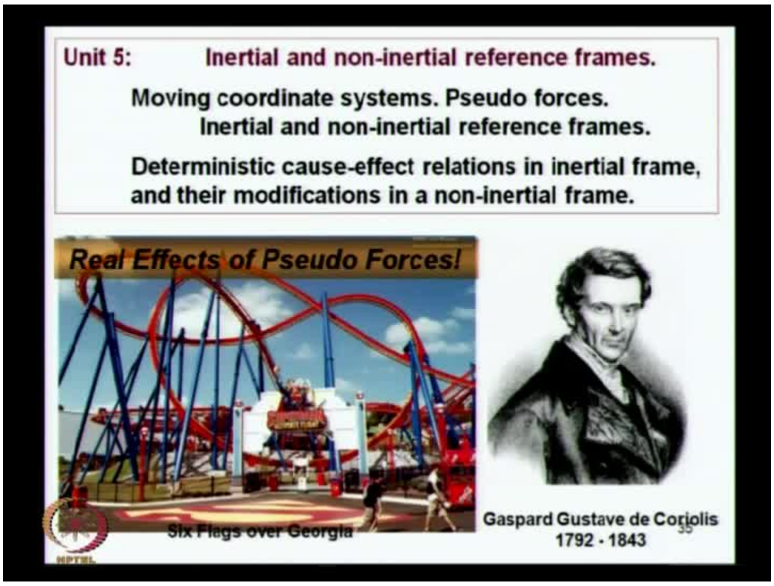

In unit 5, we will discuss motion in non-inertial frames of references means, we consider motion around us but, then when you look at your own experience you could be in a car and then the car turns or a train which is accelerating suddenly or which decelerates you could be in an elevator on the ground floor and then, when you go up certainly there is an acceleration, there is also a deceleration when it comes to halt. So, very often you have to carry out observations in accelerated frames of references.

The earth itself is an accelerated frame of reference because you know that it is rotating about its axis, you know that it is also revolving about the sun. The whole solar system is moving within our own galaxy towards the constellation Hercules. There are all kinds of accelerations which are involved and we must interpret the laws of nature correctly depending on which frame of reference we belong to or the observer belongs to.

To understand this if you want to retain the form of the equation of motion then, we are left to the requirement that you must invent forces which do not exist. So, these are mathematical constructs of our theoretical formulation. These are not physical forces, these are mathematical cryostats but they lead to real effects which are observed on accelerated frame of references, I like to call it as real effects of pseudo forces.

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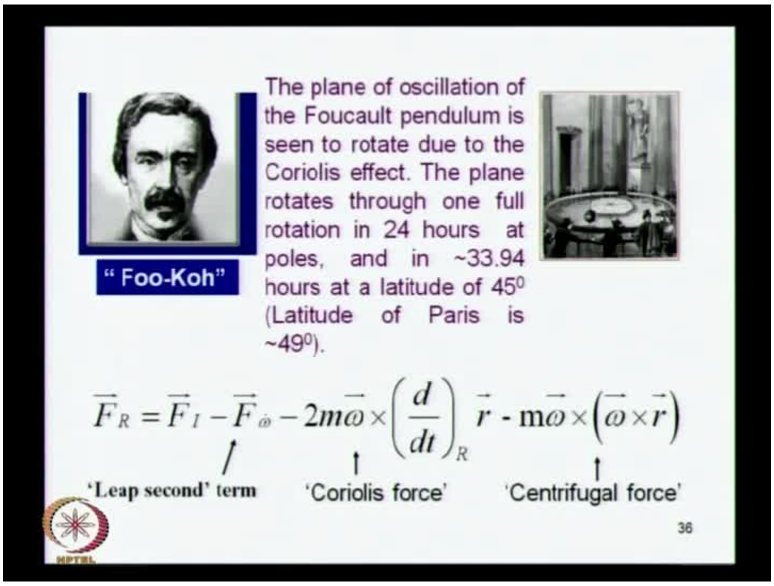

If you setup the equation of motion in earth's rotating which is a rotating frame of reference then you need to invent the pseudo forces, there is leap second collection, there is a coriolis term and there is a centrifugal term in all of these we will study in some detail in this unit 5.

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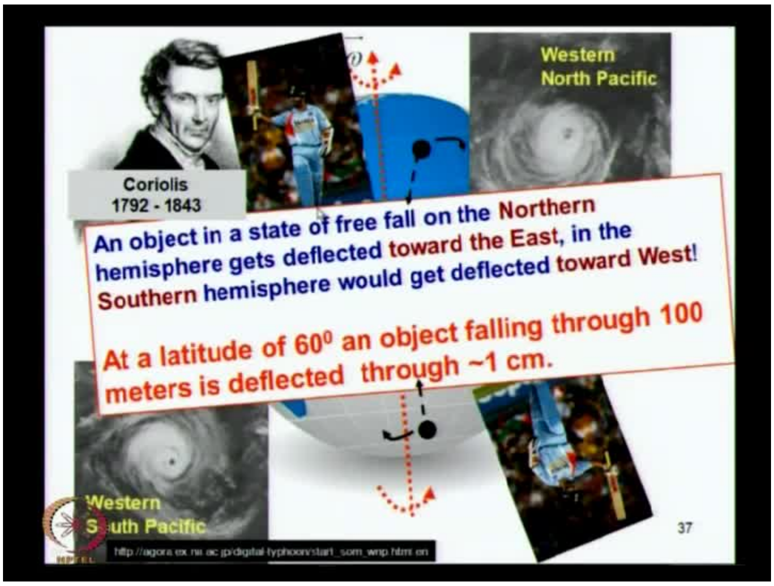

It also explains the facts as to what are the weather forms. If you see storms in the south pacific or the north pacific they rotate in opposite directions; one is clockwise, the other is the anticlockwise. Objects which are in a state of free fall, if Tendulkar is batting in

Europe in England and hits a ball for the 6 then when it falls, would it deflect toward the right or the left toward the east or west? But the answer would be different if he were playing this game in Australia or New Zealand which is south of the equator. We will see some of these things at a latitude of 60 degrees for example, an object which is falling freely just through 100 meters is actually deflected through almost 1 centimeter.

One really as to study this in great detail, this is of importance to all of you because many of you will be involved in designing rockets trajectories or interprocedural-continental ballistic missiles. You must take all of these effects correctly rigorously into account, so that you can design these trajectories correctly.

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Whether it is a rocket or a satellite, here is the picture of the chandrayaan launched by ISR0, so in designing all of this you need to have a very rigorous understanding of motion observed in rotating frames of references.

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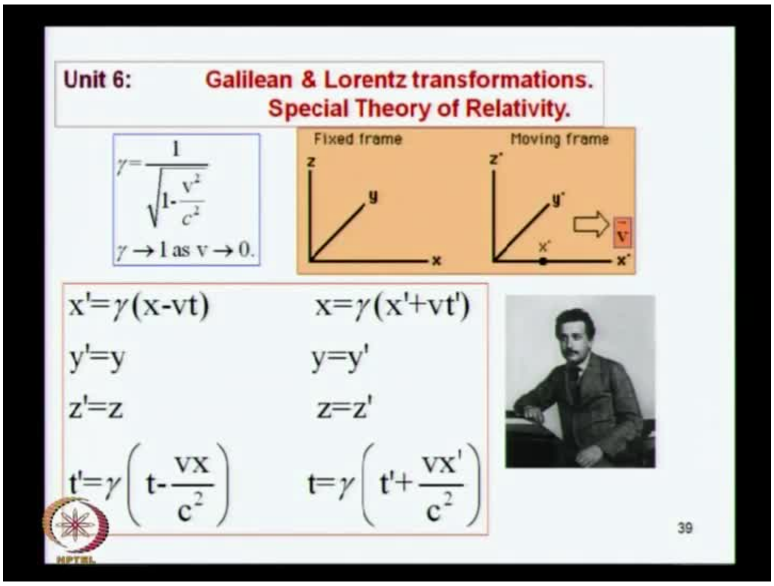

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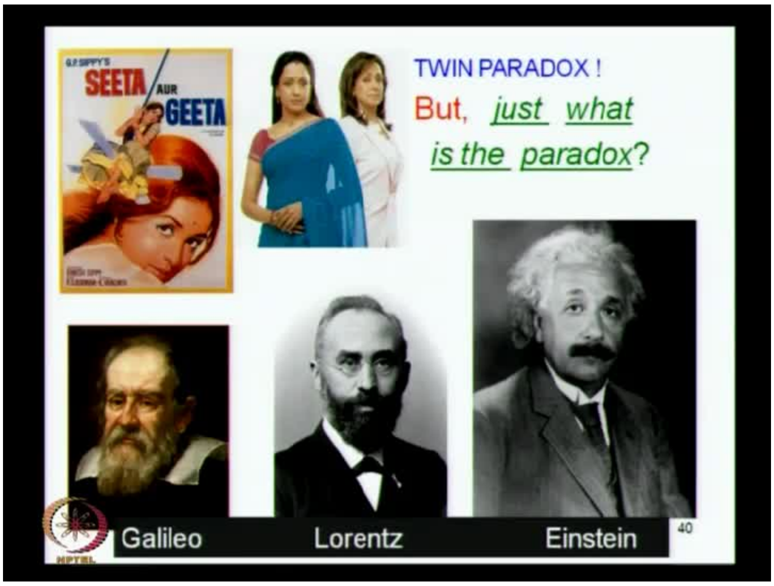

In unit 6, we will come to terms with the fact that the speed of light is huge extremely large, very large but finite. This came as a fairly modern discovery and till very recently it was believed that the speed of light is infinite but then it leads to very important consequences. This came with the study of Albert Einstein and he develops the special theory of relativity. We will study Lorentz transformations, how they depart from Galilean transformations? We will also discuss, what is known as the twin paradox? We will also discuss, what exactly is the twin paradox? Because it is often formulated as that if you have twins then one could age at a different rate than the other if one is stationary and the other travels relative to the first.

What exactly is the twin paradox is something that we will formulate the twin paradox the way it was formulated by Einstein. We will also discuss how it is result in the special theory of relativity.

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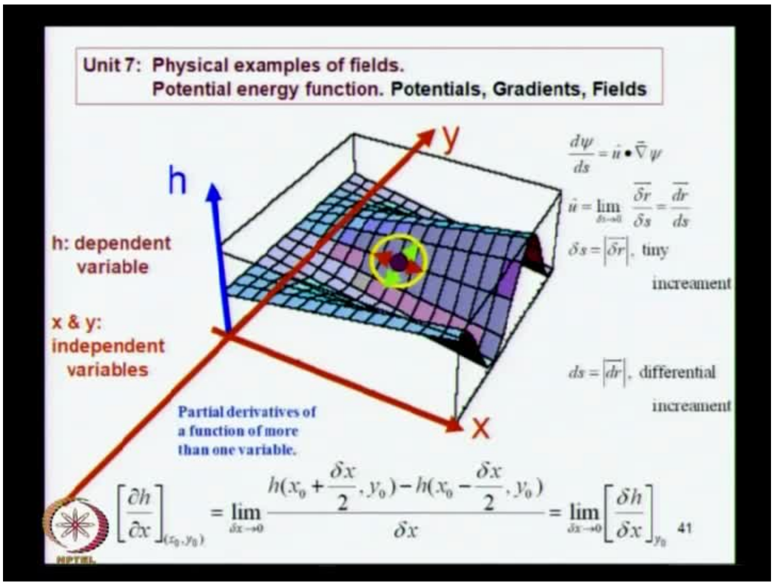

In unit 7, we will study the connections between potentials, gradients and fields. It will introduce us to the idea of a directional derivative and the gradient. The directional derivative is a quantity which is a scalar; it is a ratio of two scalars. Here, the numerator is a scalar, the denominator is also a scalar, the ratio will also be a scalar, but it will be a scalar which has got a directional attribute. You will learn that this is part of the reason why you cannot really define a scalar just by a quantity which is defined by magnitude alone, because here you will meet a scalar, which nevertheless has a directional attribute.

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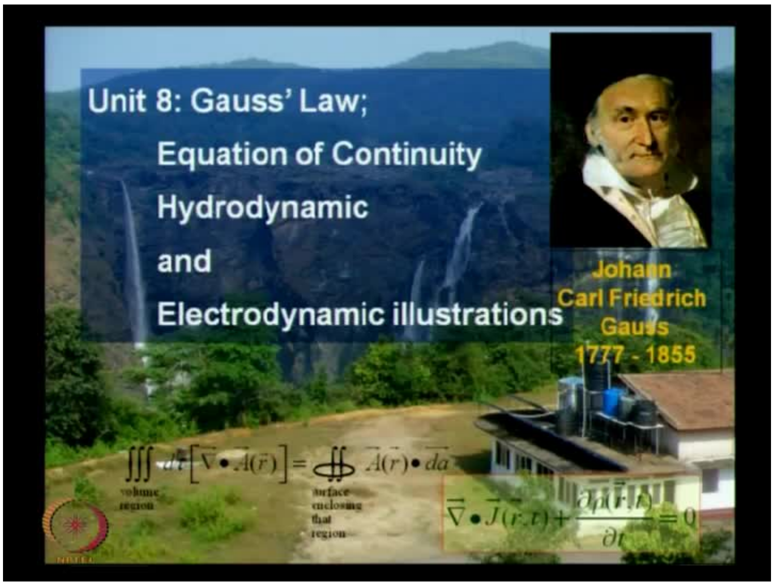

We will discuss the connections between potentials, gradients and fields then, we will apply these ideas to fluid dynamics to hydrodynamics; it has applications in electrodynamics as well or we will formulate the Gauss's divergence theorem.

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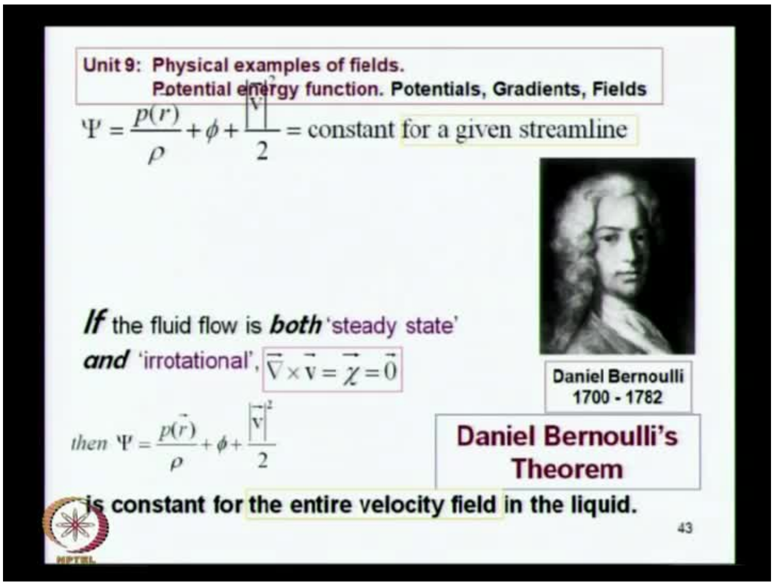

We will then go over to applications in some other branches of fluid dynamics such as a Bernoulli's theorem. This is known after Daniel Bernoulli use one of the distinguished Bernoulli's and it is a conservation principle that a is certain quantity is conserved along a given streamline, but in the under certain circumstances it is conserved for the entire velocity field of the liquid.

There necessary conditions for these two are different because for the first case, you certainly need a steady state, but for the second you need the state of this velocity field to be not only a steady state but, also in irrotational. For this you have to define what is called as the curl of the vector and we will define and introduce these terms very precisely in our formulation.

Mathematics and physics really develops hand in hand and we will introduce very rigorous mathematics when needed. It is a prerequisite but we will see that it is an integral part of the formulation of physics. We will not really see the difference between physics and mathematics; it is like a seamless homogeneous formulation.

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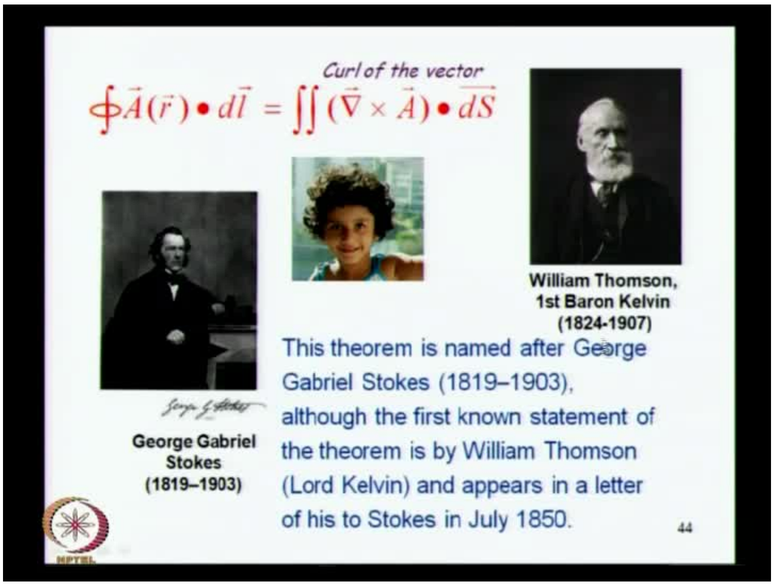

We will meet rigorous mathematical formulations such as stokes theorem which is named after George stokes, but it was originally formulated by William Thomson who wrote about it in a letter to stokes and stokes popularize this. We will study this and study its applications in electrodynamics as well as a hydrodynamics and fluid dynamics.

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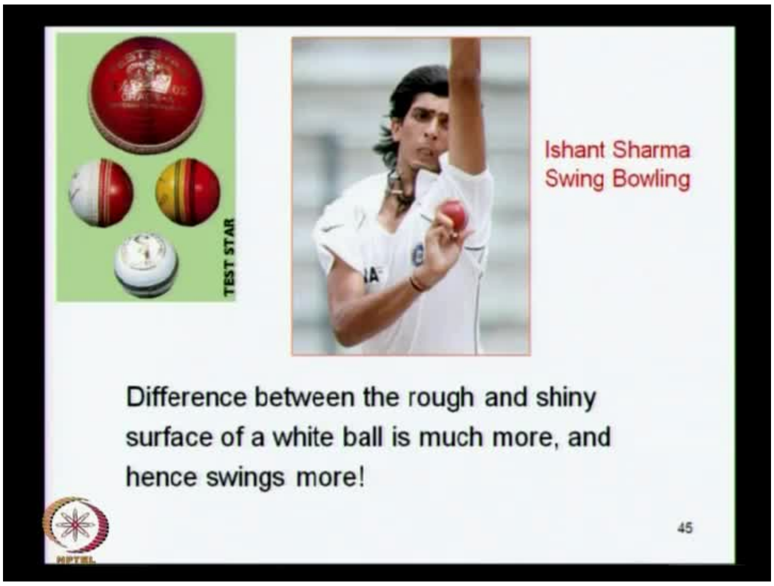

We will also see how these laws influence the velocity fields, because when Ishant Sharma or any fast bowler is bowling through a medium of air then, there is a relative velocity across the surface of the ball. This is important contributions in the dynamics of the ball. These come from the Bernoulli's principle from an appreciation of both the divergence and the curl of the velocity field. You must understand both the divergences and curl of the velocity field to appreciate these dynamics.

This is part of the reason why you choose a white ball for a 20-20 match and not a red ball? Because, it is the seam bowlers who play a bigger role and not only in cricket, but in all forms of games like tennis, squash and even soccer ball, so these are important considerations.

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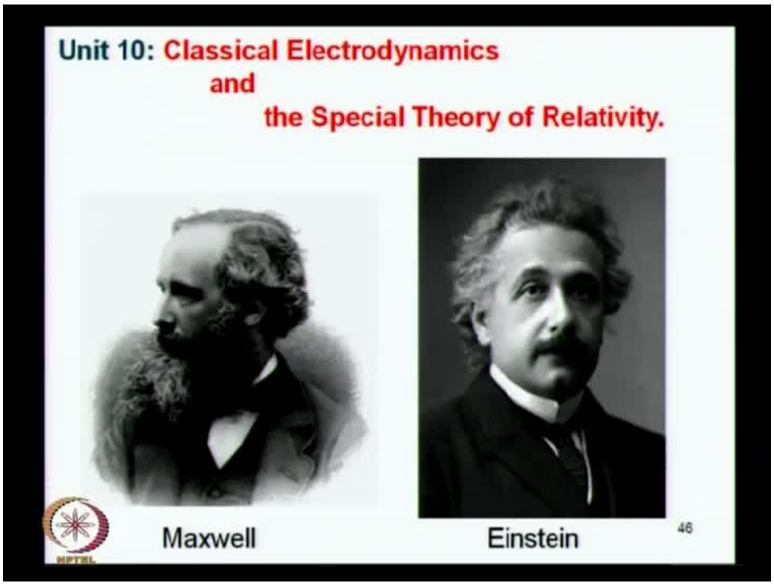

We will then meet the applications of the idea of divergence at the curl in electrodynamics. Here, we will recognize the fact that, to understand the electromagnetic field, we will exploit the fact that both the divergence and the curl are required. The Maxwell's equations are precisely the divergence and curl of the electromagnetic field.

Maxwell's equations - the four equations - for two vector fields, electric and magnetic field. You must specify there are divergences of electric field as well as the curl of the electric field, and you must also specify the divergence of the magnetic field as well as the curl of the magnetic field, when you do that you get the 4 Maxwell equations.

This is a consequence of an important theorem known as a Helmholtz theorem. There is vector field defined by both the divergence and the curl, you need something more than that you need the boundary conditions as well, but that is a matter of detail. I will introduce you to the laws of classical electrodynamics and the Maxwell's equations.

We will see that they really result on account of the fact that there is an interpretation of the Faraday Lenz law which is very exciting. This was the reason which led Einstein to develop the special theory of relativity. There is a very intimate connection between classical electrodynamics and the special theory of relativity, so I will introduce you to some of these connections.

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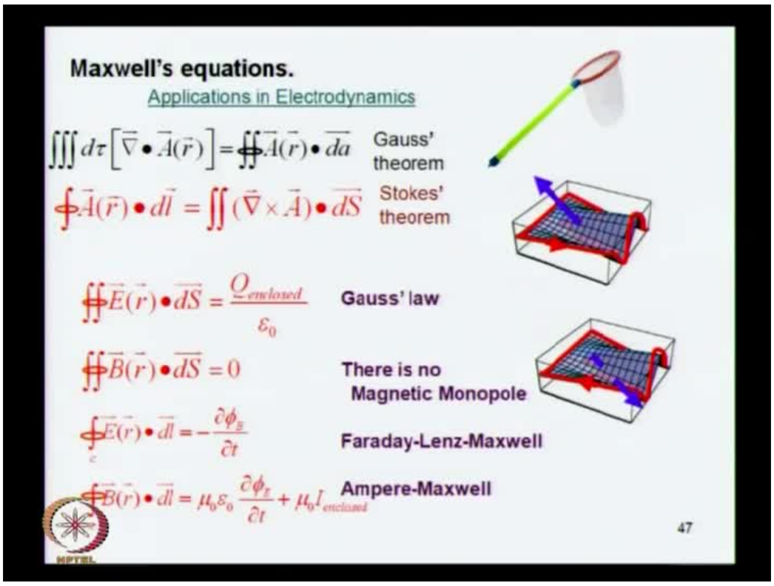

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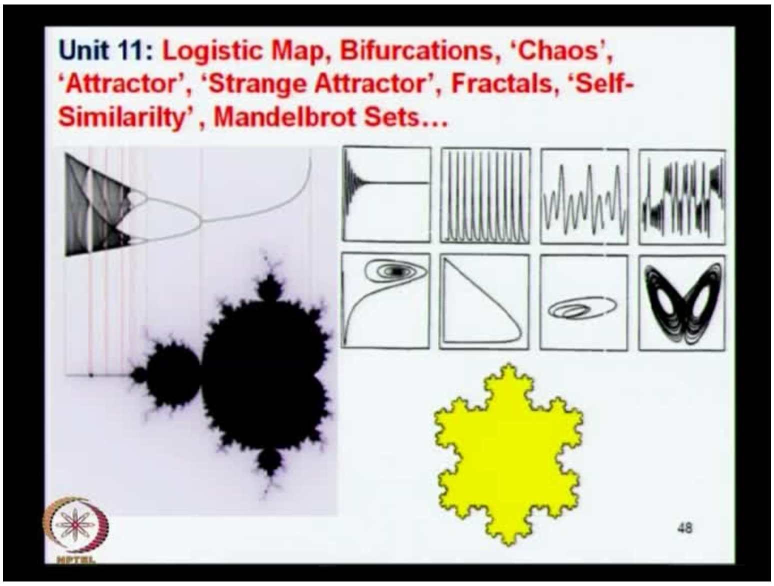

We will meet the Maxwell's equations in this unit and then in our last unit in unit 11. We will study an important branch of classical mechanics which is of great importance in modern physics, which is the branch of chaos as it is called as the as the non-linear dynamics. We will introduce ourselves to what is a logistic map? What is meant by bifurcations? What is chaos? What is an attractor? What is a strange attractor? What are factors? What is self-similarity? What are these Mandelbrot sets?

Some of these things I will give you an elementary introduction, that really bring us to the conclusion of the 40 lectures which I have planned for this course, only so much can be done in 40 lecture hours.

We will have to come to terms with the fact that we will have like to go beyond this in our understanding of Lagrangian and Hamiltonian formulations. We would have like to study electrodynamics beyond the Maxwell's equations. We would have like to study the special theory of relativity certainly, and we will study the Lorentz contraction, the time dilation which is fundamental to the understanding of the twin paradox.

We will also make some comments on, what general theory of relativities? But, we will not discuss the general theory of relativity. We will certainly feel that we will have like to study non-linear dynamics and Mandelbrot sets and these topics beyond what we introduce you to, but then only so much can be done in 40 lecture hours, in our 40 lecture hour course. This will give you the basic introduction to these topics and then you will take courses at a slightly more advanced level and then, you will be formulating physics in your own way.

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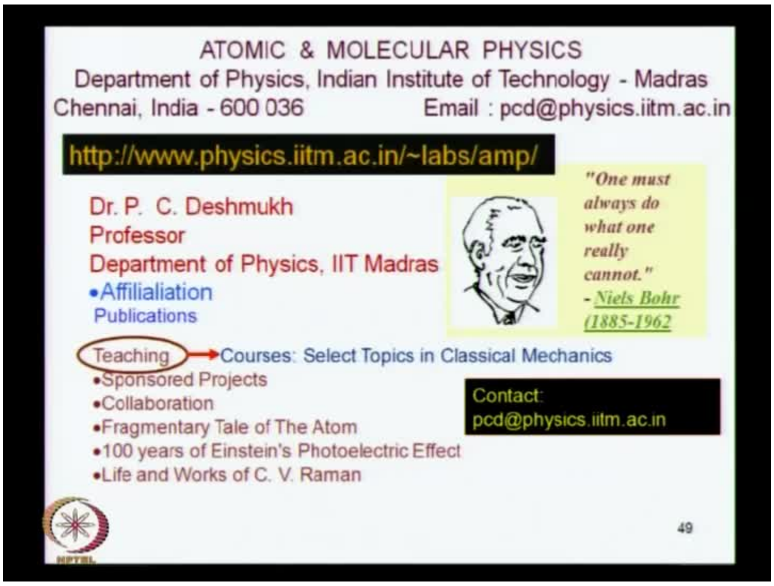

So, this is part of your training and I hope that you are enjoying it, I am looking forward to it myself; I will invite you to my website. This is the link to my website and you will find some material related to the course works occasionally listed at this website, you will find it easily.

You might want to enjoy some general articles of interest like, the life and works of C V Raman and 100 years of Einstein's photoelectric effect. So, there are some top articles of general interest which you might want to read about, but most importantly you are most welcome to contact me by E-mail anytime if you have any questions. You can always talk me during the class for those of you were present in the classroom and those of you are distinct and viewing these lectures electronically over the electronic medium. You may send your questions to me by E-mail at this address P C D at physics dot IITM dot ac dot in.

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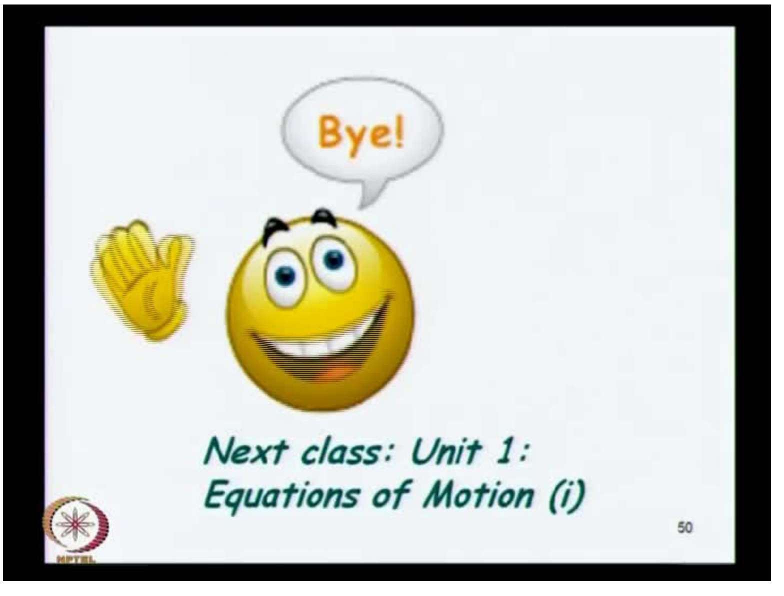

I hope you will enjoy these studies of 40 class hours it is good bye for now. Our next class will be on unit 1 and it will be the first one on the equations of motion. So, I look forward to the whole course and invite you to this course, thank you all very much.

If there is any question I will be happy to take?

Sir, what is the physical meaning of homogeneity of time?

Homogeneity of time, the question is, what is the physical significance of homogeneity of time? It is a good question. Homogeneity already has a meaning in your minds, it means that, there is some property which remains uniform, it does not change and it does not change from something to something.

Let us talk about homogeneity of space first. Homogeneity of space suppose, you consider a region of space over here between these two hands and I move my hands from here to here and I ask, how the properties of space changed when I move it from here to here? (Refer Slide Time: 01:18:00).

Now, if you ask this question the answer really depends on what detail an answer are you looking at, because if you look for answers with such details that - in this space there is a certain amount of oxygen and nitrogen and what is the velocity of each molecule of oxygen at a given instant of time and how is that in this region? Then, the answers are different, but if you ignore these differences.

Good way of doing it is to consider this space and vacuum then have the properties of vacuum in this space and in this space are they different. Again, the answer depends on to what detail you are looking at, because if you hold some mass over here in this region of space and drop it, it will fall towards the center of the earth along this whereas, if it is over here, it will fall along this, if the center is in the middle.

These are some details if you ignore these details then; the properties are exactly the same. This is what we will mean by homogeneity of a space or if you travel inside a crystal. If you consider a travel inside a perfect crystal and you go in one direction, any direction, it does not matter, does not even have to be along the axis of the crystal, but if there is one single perfect crystal and you travel in any direction in that crystal then the properties of the crystal will remain invariant. The invariance will be with reference to the number of steps, it will be in integer number of some basic unit steps which is the unit cell parameter and this is what we mean by homogeneity of space. Now, with this background let us deal with the question that you rose, what is meant by homogeneity of time?

Time, how do you define time? Time is an experience and more precisely as was defined by Einstein and none other, time is what you measure as a part of this experience.

You have a certain perception of time which you measure and you measure by your wrist watch and you measure time from 6 am in the morning, when you wake up or at least the latest time by which you should wake up, that you wake up at noon is a different matter, but from 6 am to the 6 am next morning, there is a certain time interval this is what you measure in your clock.

Now, if the properties of time from the start to the end, from t 1 to t 2; t 1 is when you start ticking the clock and t 2 is when you stop it and then you look at the interval t 2 minus t 1 this is what you measure.

Now, if the properties of time have change from yesterday to today or from today to tomorrow, because time itself is some physical property. It is something that you measure, but over subsequent time intervals that you are going to measure by your watch, by your clock whatever be the nature of your clock; it could be a wrist watch, it could be any periodic phenomenon and it could be just the count of the number of pulses assuming that these are absolutely regular.

If you measure it using some periodic event using some clock then the properties of time itself - if they remain invariant from yesterday to today to tomorrow what is meant by homogeneity of time. If this were not to happen energy would not be conserved.

Now, this is a very fascinating relationship between energy and time. This is a very fascinating relation yes; I will take your question in just a moment. It is a very fascinating relationship, because it is mathematically rigorous, it will turn out that energy is a form of a momentum, which is canonically conjugate to time. The relationship between energy and time is the same as between momentum and coordinate.

Conservation of the momentum is connected with the coordinate having some sort of an in various property which is rigorously expressed as the coordinate being cyclic. So, when you write this in Lagrangian formulation, there is a very famous theorem which says that if the Lagrangian is cyclic in a coordinate then the corresponding momentum is conserved.

When time does not explicitly appear in a Lagrangian then energy is conserved. When it does not take appear explicitly in the Lagrangian is when properties of the system are invariant with respect to a time that happens on a homogeneous time scale. Conservation of energy is connected to this homogeneity with respect to time. This is very similar to the conservation of linear momentum in translational invariant space or conservation of angular momentum in a rotationally invariant space. This is in a sense and illustration of the Noether's theorem.

These are conjugate parameters, there is more to it, the more you exploit this question the deeper you get into physics, because if conservation of momentum is connected with a coordinate being cyclic, you must then ask are these measurable and how accurately are they measurable? It turns out that you cannot measure them simultaneously, accurately with infinite accuracy.

There is a certain limitation which brings you to the limitation of classical mechanics, because there is a fundamental intrinsic uncertainty between position and momentum. There will also be this intrinsic uncertainty between energy and time. So, there is this delta q delta p relationship for position and momentum likewise, there is a relationship delta e delta t in quantum mechanics, which expresses the principle of uncertainty. It is somewhat different from the uncertainty of relationship for position and momentum, but that is a matter of detail that is because there is no operate of a time in quantum mechanics whereas, for q and p of course, you formulate corresponding operators.

You have a question?

Select/Special Topics in Classical Mechanics

Prof. Dr. Pranawa C Deshmukh

Department of Physics

Indian Institute of Technology, Madras

Module No. # 01

Lecture No. # 01

Principle of Causality

For a because for time to homogeneity the interval of time should not change suppose, t 1 and t 2 are time interval times and the time intervals is t 2 minus t 1 that should remain constant that should remain invariant, but sir, in case of why it is true for any this frame or inertia frame with them but, when frame is moving in its velocity compare to the velocity of c then the interval will definitely change then the homogeneity of time is does not appear in that case. ((Student talks))

Yes, what happens is that you measure energy in terms of certain dynamical variables in terms of position and momentum. So, energy for an object for example, it is some of kinetic energy and potential energy. The kinetic energy is p square by 2 m the potential energy some function of the position.

Now, position and momentum also do not remain invariant in frames of references which are moving relative to each other. So, they will also undergo a transformation and if you use the correct transformation laws, because what is position in 1 frame of reference if you look at it in another frame of reference, which is moving a relativistic speeds. Then the transformation of this position coordinate from this frame of reference to the other position in the second frame of reference is not governed by Galilean relativity but by Lorentz law transformations.

Position then gets scrambled by position and time likewise, the new time in the new frame of reference is also a superposition of position and time. The equations of transformation really mix space and time, what is invariant is neither space nor time, but what is called as the event interval in Minkowski space in the space time continuum. So, you have to interpret all these parameters differently when you do relativistic mechanics. You will get some introduction to this in our unit 6 I believe, any other question?

Thank you all very much and goodbye for now.