Select/Special Topics in Classical Mechanics

Prof. P. C. Deshmukh

Department of Physics

Indian Institute of Technology, Madras

Module No. # 02

Lecture No. # 10

Oscillators, Resonances, Waves (iv)

Greetings, the subject is vast and in today's class we will be concluding the second unit on Oscillators, Resonances and Waves. One can discuss very many very fascinating aspects of resonance quality factor and so on and so forth.

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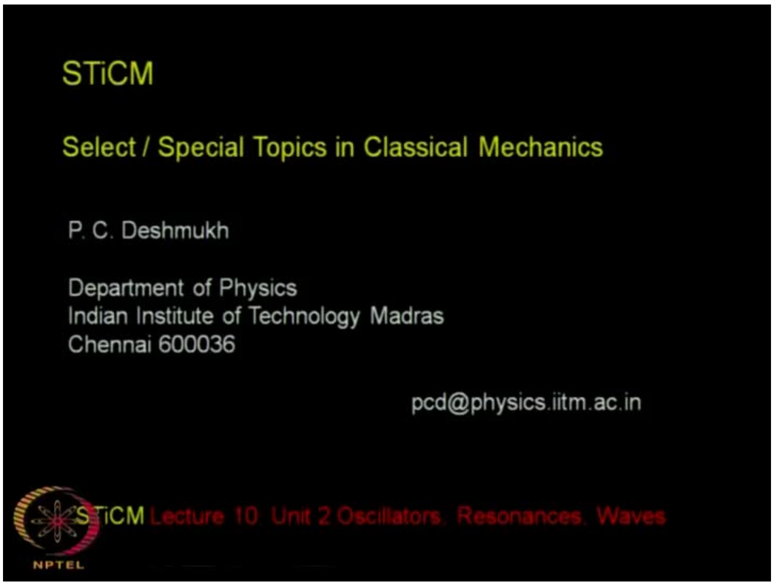

Our plan in this course is not to give a very exhaustive review of these topics, but to introduce and graduate students to basic principles of oscillatory motion and damping, and other things that we have been talking about to get some sort of an introduction. We will provide some further acquaintance with important consequences of oscillatory motion leading to the behavior of resonances and waves.

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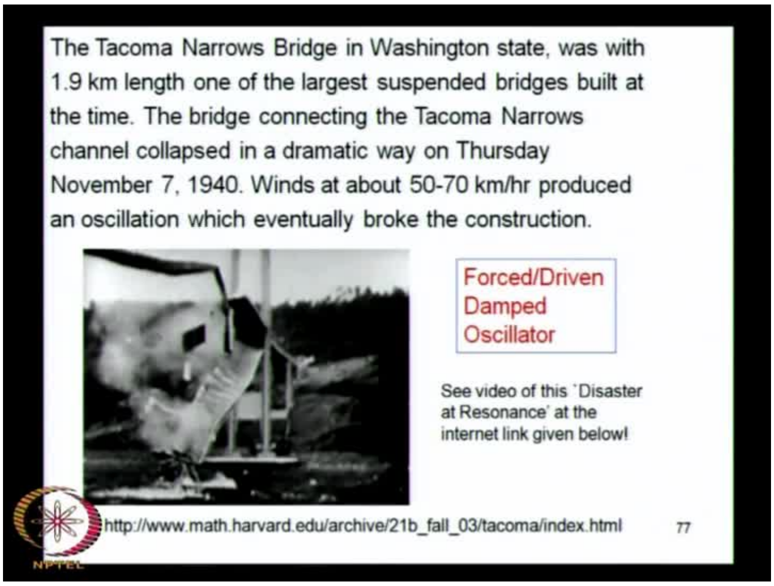

So, here is a picture and you can actually see a movie. There is a website at the bottom here and you do not have to write down this link but, if you just Google this Tacoma Bridge, you will get this link easily on the internet. You can actually see a video; this is Tacoma Bridge in Washington State. This was a bridge about nearly 2 kilometers long, one of the largest suspended bridges of that time. This was in 1940 but, this is a very famous story. What happened is that this bridge collapsed in a very dramatic way on the 7th of November 1940.

Actually it turns out that it was for (( )) reason that this happened because of the strong winds which were blowing. These winds generated certain oscillations in the bridge; the bridge had its own internal natural frequency of oscillation. The wind speed at 50 to 70 kilometers per hour they produced a driving force; a periodic driving force because of the manner in which the winds were blowing and it led to a resonant type of phenomenon which eventually causes the amplitude of oscillation to become so large that the bridge could not sustain itself and it actually snapped.

So, this is the movie that you can see and the basic phenomenon. It is not nice to think about tragic events in terms of mere physics and differential equations but, essentially that is precisely what it really voice down to. The important thing over here is to understand that the differential equation that we set up in this is not just abstract mathematics; it is a real, it describes real physical phenomenon and then it is real events which can be analyzed and understood in terms of the differential equations and the damping coefficients and so on.

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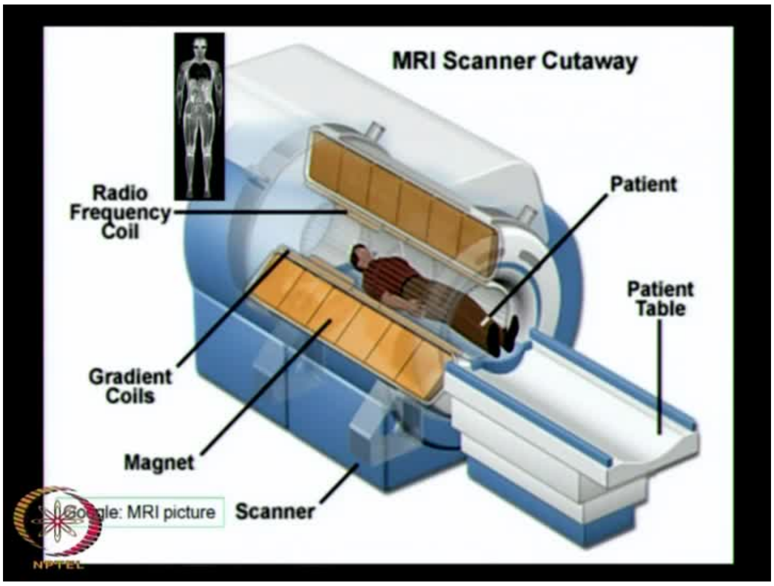

This has got consequences not just in classical mechanics but, also in quantum theory and here is a situation in which I do not want to find anyone of you because I hope that you will not need it.

This is the picture of MRI scanner (Refer Slide Time: 03:47) and this is a very powerful tool, diagnostic tool in modern medical technology. This is based on the principle of nuclear magnetic resonance, so this is again a resonant phenomenon. This is the quantum phenomenon in which, you have got an intrinsic property which a nucleus has called as the spin and I will of course, not discuss that over here but, it is this spin which when subjected to a certain periodic electromagnetic oscillation that leads to a resonant phenomenon.

Then when you explode the properties of the magnetic resonance, you can use it to map individual biological trends in the body at a cellular level and that because of very powerful diagnostic tool in medical technology. So, the phenomenon of resonance will come in classical mechanics, quantum mechanics or I do not know 100s 1000s many examples and it is for this reason that this is an extremely important subject to learn.

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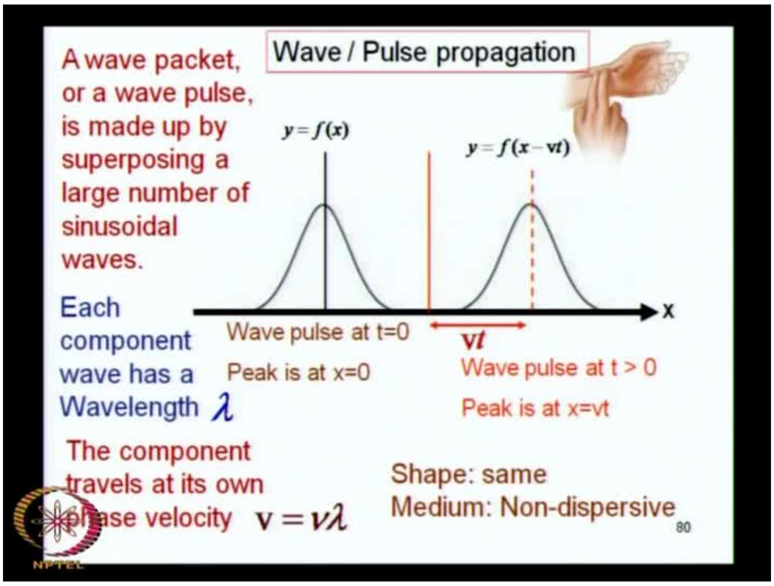

Furthermore, the subject of oscillations is a fundamental importance in understanding how information is transmitted from one part to another, how propagation of energy takes place from one part to another. Digital signal processing, for example and we will get some kind of acquaintance in this last class to some of these ideas; what is the wave phenomenon and how is the oscillatory behavior at the bottom of all of this analysis? So, we will first look at a pulse and you know what a pulse is; you just feel at a wrist, you can feel your own pulse and you see periodic pulsations which are generated by the bio rhythms at a certain regular frequency with which the heart is pumping blood through your body.

So, what is crossing a particular point is a pulse and this pulse is sometimes also called as a wave packet. It is actually made up of a large number of sinusoidal waves and these sinusoidal waves they come from the solution to the oscillatory problem that we have discussed earlier. So let us see, what a pulse is really like; so here you see in this picture a pulse at time t equal to 0 and it has got a peak which is centered at x equal to 0.

You have got an x axis and the x equal to 0 over here under this vertical black line and this is where the peak of the pulses and then this pulse which is a superposition of a large number of sinusoidal waves. These different sinusoidal waves each has its own wavelength which is lambda, each component wave travels at its own speed which is called as a phase velocity. This phase velocity is equal to the product of the natural frequency nu times the wavelength.

At a later time, at t equal to greater than 0 the pulse will be at a different location. It will move away from the origin through a distance vt depending on the time at which you are seeing it. It will be described by since it is moving with time, the argument will be phase shifted through the factor vt.

Now, the shape of the pulse in the example that I have considered, has not changed. This is going to happen when each component you know, all the components travel together. This happens in a medium which is said to be non-dispersive. So, each component you know many different component which must be superposed to generate a wave packet or a pulse when they travel together and they can travel together only in a medium which is said to be non-dispersive.

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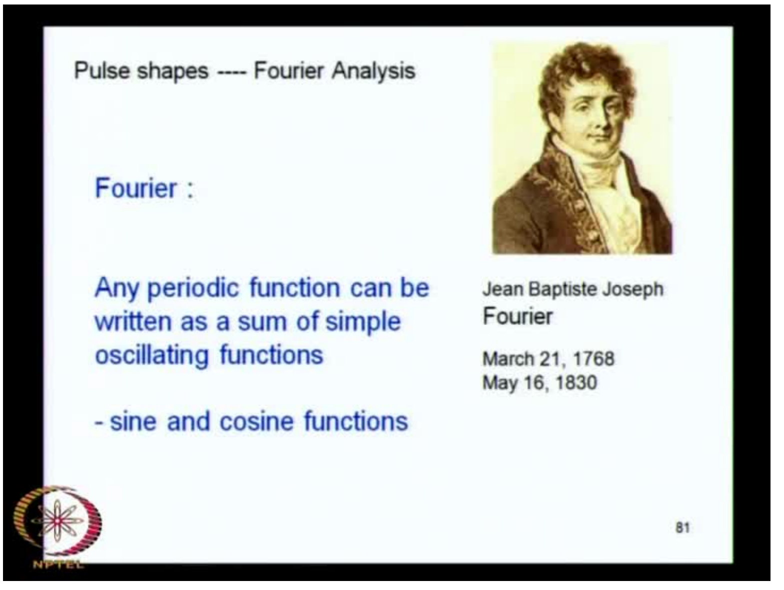

So this also gives us the definition of what is a dispersive medium and what is a non-dispersive medium. The shape of the pulse must remain intact as time progresses. Now, lot of this analysis is due to Jean Fourier and he lived between 1768 and 1830. He was at the Ecole polytechnic at Paris and one of the very brilliant mathematician and physicist who contributed so much to the analysis of wave motion.

We find this being used in just about every branch of physics in engineering with including quantum mechanics and optics electro dynamics, you name it and we make use of Fourier methods. So, will get some idea about what this analysis is about. The Fourier theorem can be stated in some more simple terms and I am not going into the rigorous mathematics of this because we really do not have the time for it.

But essentially in some sense, what it does is tells us that any periodic function, any phenomenon which occurs with a certain periodicity. It can be anything, which is a repetitive phenomenon which repeats itself at a certain frequency. It can be written as a sum of simple oscillating functions namely the sine and the cosine functions; now this is an amazing feature.

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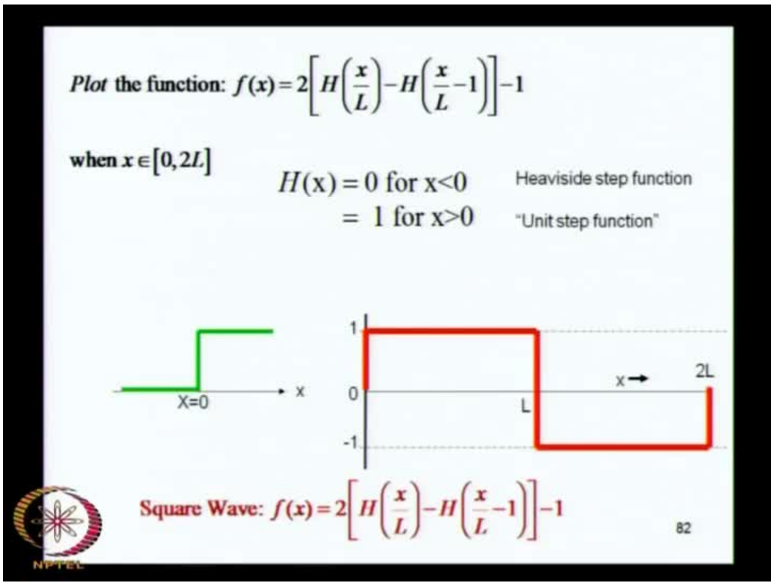

This is absolutely incredible; that you look at any periodic function and you can always write it as a superposition of the sine and cosine functions which are very simple functions to use. Everybody knows what a sine and cosine function is and in terms of the sin and cosine functions, you can write anything and everything that has got any kind of periodicity.

This comes from the Fourier methodology and we will see how these methods work. Let us plot a function which I have written on this equation for f x this function is defined in terms of another function H, which is called as a Heaviside function (Refer Slide Time: 11:20). So, H of x is a Heaviside function, it is also sometimes called as the step function and the definition of the Heaviside function is very simple one.

Let us first define the Heaviside function. Once, we know what the Heaviside function is then instead of the argument x, we must put the argument x over L and then we get the first term; subtract from it the Heaviside function for a different argument which is not x over L but, x over L minus one. Then you subtract from this result after multiplication by 2 a factor of 1 and you get the function f of x; so that is how you will get the function of x.

So, everything is defined in terms of the Heaviside function and the definition of the Heaviside function is very simple. It is equal to 0, if x is less than 0 and if it is greater than 0, it is equal to 1. So, that is part of the reason it is called as a step function because at x equal to 0 its value suddenly in one step shoots up from 0 to 1. We are interested in plotting the function f of x in the range 0 to twice L, where L is some length parameter.

Let us first have a look at the Heaviside function. So for x less than 0, it is 0, so it is indicated by this green line and for x greater than 0 it is equal to 1; so that is what generates this step, so this is the Heaviside step function (Refer Slide Time: 13:03). Now that we know what H of x is, we have to find how to plot H of x over L in a range of x which goes from 0 to 2L. That is the fairly straightforward thing to do and I will let you work it out in details in your notebooks. I will plot the result over here but, now you know how to do that.

So, you have an x axis, this is the 0 of the x axis, here is a point at x equal to L and here is a point at x equal to 2L (Refer Slide Time: 13:36) and it is in this range that we have to find what this function f of x is. If you just substitute the value of the argument to x over L and then construct this difference multiplied by 2, subtract 1 out of it you will get the function f of x. Now this is what the function f of x turns out to be, you see that it is a square wave. So, it is coming from a sequence of these step functions the manner in which you have defined the function f of x and this is actually a square wave.

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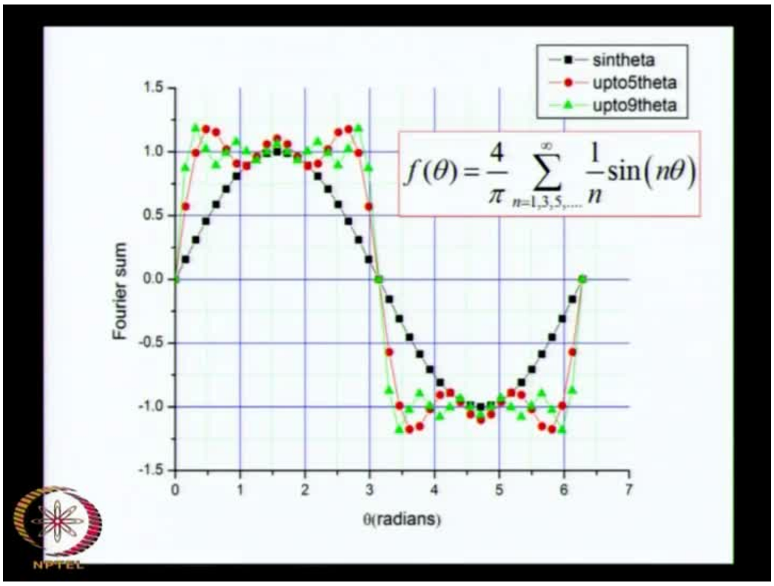

Now suppose, this square wave wants to repeat itself, then you have a phenomenon which is periodic. You would expect that this can be written as a sum of sine or cosine waves, now this does not quite look like a sine function. Everybody has a picture of a sine function in your mind, it has some similarity to a sine function but, the difference is more visible than the similarities. Because this has got sharp edges right, a sine function does not have any cuts, there are no angularities in the sine function. It is very smooth function which varies between minus 1 and 1.

So, here also we have chosen a function which varies between minus 1 and 1; it is a repetitive function. It does not have to be minus 1 and 1; it can be from minus n to plus n you can always scale it by an appropriate factor and normalize it to unit magnitude, so that is not an issue.

Now, it turns out that this can actually be written as a sum of sine waves and I strongly encourage you to do this exercise on your own using some graphical plotting routine on your personal computers. I have used graphics software which comes in the software called region to plot this graph and this function, this is the function of theta which I have generated here which is written as a superposition of the sine function, but it is a very peculiar superposition. The argument is n theta where, n can take only odd numbers all the way from 1 through infinity but, it will take only odd numbers 1, 3, 5, 7, 9 all the even numbers are omitted.

Then each sine term is divided by the corresponding n, so it is 1 over n sine n theta summed over n going from 1 through infinity. This is just some normalizing factors, so do not worry too much about it; if you do it, you can get depending on how many terms you put in this summation, this summation is go up to infinity and if you take it too seriously, you will spend your life time and beyond doing this summation. Not just your life time but, also beyond that; so I do not want you to spend the rest of your life time to doing it.

So, just take the first term, n equal to 1; n equal to 1 is sine theta divided by 1 which is again sine theta and then you get this black curve, this one (Refer Slide Time: 17:25). This is sine theta, here it goes and this is the usual sine theta that you see; then, you add to that sine 3 theta divided by 3; then you add to that sine 5 theta divided by 5 and if you go up to 5 theta and just stop there. You are supposed to go up to infinity but, you decided that you will not do it. So, you stop just at two terms, forget the infinity.

At 5 theta if you stop, you get the red curve (Refer Slide Time: 18:08), this is the red curve which goes like this, it vigus over here, vigus further and then it dips to negative values, vigus over here and gets back; if you go further up to 9 theta, if you add to the previous sum the 7 theta by 7 and then also sine 9 theta by 9, you get the green curve and now you see that it is looking more and more like the square wave that you have met in the previous curve.

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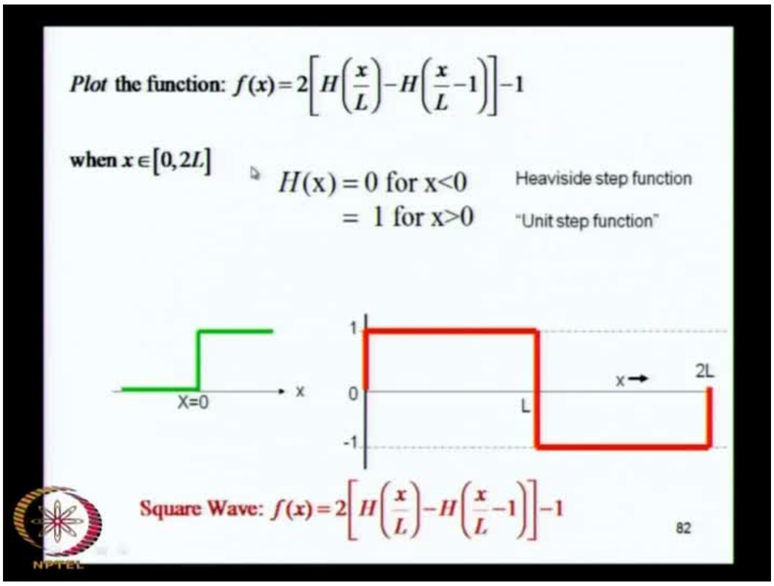

This is the square wave and you see that this summation is looking more and more look it is looking more and more like the square wave of this picture. I strongly urge you to do this, because all you have to do is to add just a few terms and you can do this plotting on a small graphics calculator if you like or you can plot these graphs on a piece of graph paper or you can do it on some computer with a graphics plotting routine.

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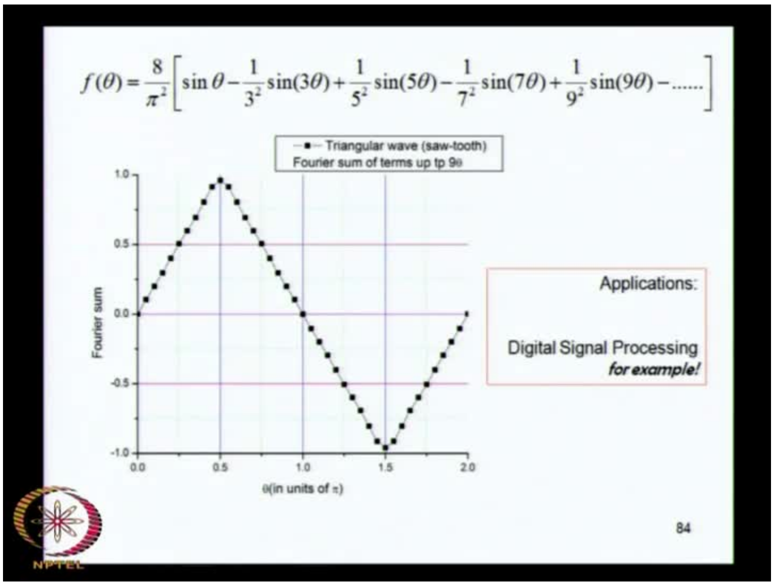

Now, here is another periodic wave which is a triangular wave. This is a triangular wave it is also called as a saw tooth wave, for obvious reason it looks like a saw tooth (Refer Slide Time: 19:30). This is a summation in which you have again the odd multiples of the angle theta, so you have sine theta in the first term, sine 3 theta in the second term, sine 5 theta is next term and so on but, then the denominator is not the corresponding n integer but, it is the square.

So sine 3 theta divided by 3 square, sine 5 theta divided by 5 square, sine 7 theta divided by 7 square and notice that the alternative signs are different. This is the minus sign; you have got a plus sign and then a minus sign over here and a plus sign over here. So, if you construct a superposition and for different periodic functions the details will be different but, I have illustrated this for two functions to two periodic functions: one is the square wave, the other is triangular wave. You can see that you can generate any periodic phenomenon in terms of a superposition of just sine and cosine functions.

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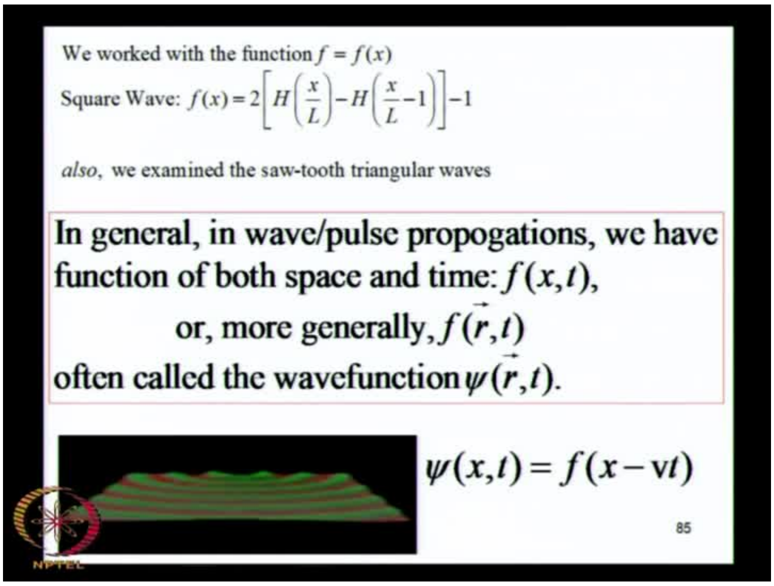

So, no matter what the nature of the function is and if it is true for the square function and for the triangular function for which we could see the Fourier decomposition so easily. It is in fact true for any periodic phenomenon, no matter what the shape of the pulse may be as long as it is a repetitive phenomenon. In general means what we did was to plot only a function of x but, a physical phenomenon may be a function of both space and time.

A pulse will be written as a function f of x as well as time or more generally in three dimensions as a function of the position vector which will have three components x, y and z or no matter what coordinate system you are using. So, it will be a function of space coordinates and the time coordinate and this is typically called as a pulse or a wave function or whatever.

So, it is and you can expect it to be decomposed into sine and cosine functions, if it is got a periodic element. Now, the periodicity can be either with respect to x - with respect to the space coordinate or it can be with respect to the time coordinate or with respect to both. In general, it is periodic with respect to both, which is how the wave packets you know traverse in space and time. So, you have got in general a function of space and time and you have these pulses or wave packets which propagate with time and you can easily do a Fourier analysis of these functions. Here the basic solutions are coming from the sinusoidal and the cosine function. These were the solutions of our simple harmonic oscillator, the solution was e to the i omega 0 t that was a basic solution.

What is that? That is the cosine term and a sine term. So, the solutions of the simple harmonic oscillator are fundamental to this analysis, fundamental to the phenomenon of oscillators, damped oscillators, damped and devin oscillators, resonances, wave motion as well.

Transmission of energy, electrodynamics, quantum mechanical wave functions everything comes under the application as a part of an application of the basic analysis that we have learned from solving the differential equations for the simple harmonic oscillator.

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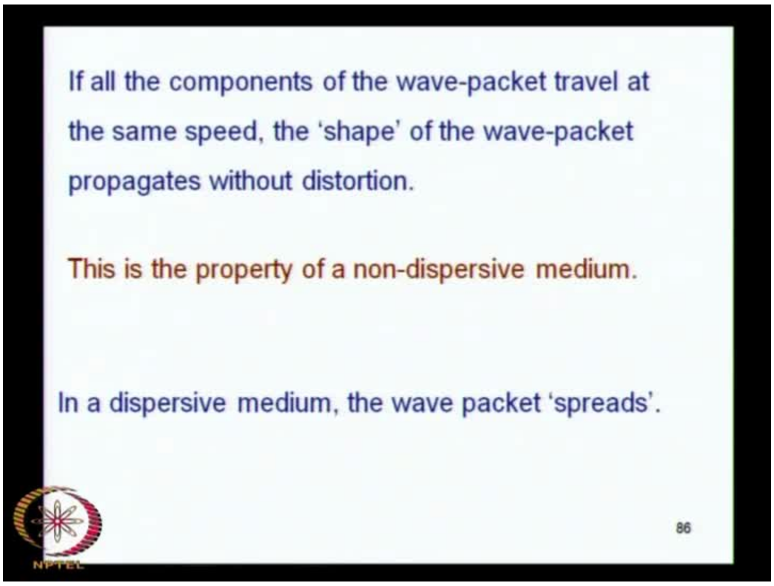

Now, as I mentioned a typical wave packet will have a superposition of different sinusoidal waves; each sinusoidal wave will have its own wave length, each sinusoidal wave will be traversing at its own speed and as long thus the speed of all these different components is the same in a medium and that will depend on the properties of the medium obviously, right?

So if this speed remains the same, then the shape of the wave packet will not change; so it will traverse without any distortion and this is the characteristic feature of a medium which is non-dispersive whereas, in a dispersive medium the shape will change. You talk about the wave packets spreading because it sort of it does not retain its shape and it spills out of the original shape that it is started out with.

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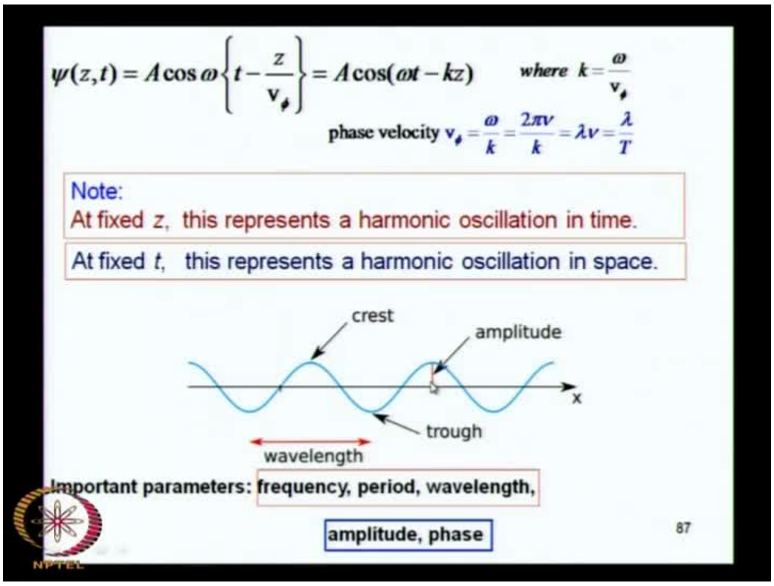

Let us look at this wave function. This is the function of z and t it is a function of both space as well as time, so the space parameter z or z and the time parameter is t (Refer Slide Time: 24:42). So, the phase velocity of this particular wave is given by this ratio omega over k or nu lambda which is a product of the wavelength and the frequency; the frequency is nothing but, the inverse periodic time.

Notice that at a fixed z at a fixed position, this is the harmonic function in time. So as time changes, at a fix point the value of this function will change harmonically like a sinusoidal function or a cosine function. Likewise at a fixed time, this will represent a harmonic oscillation space. So, if you plot it as a function of the space coordinate, at a fixed time if you take it snap shot in your camera.

At a particular instant of time, it will look like this picture on this screen and this is the obvious definition of a wave length which is the distance between two points of corresponding phases and then the peak here is a crest and then the bottom here is the trough, the maximum displacement is what you call as the amplitude. So, these are important parameters for this analysis which is the frequency, the time period, the wavelength, the amplitude and the phase.

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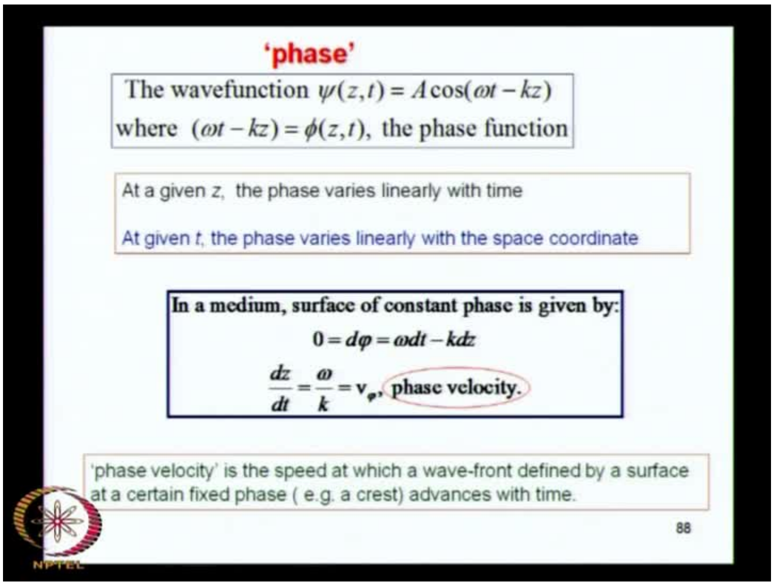

Now, let us spend some time discussing this phase. The phase function is omega t minus k, this is the phase function (Refer Slide Time: 26:36). At a given z, at a fixed value of z the phase varies linearly with time; omega t is a linear function with time, if you plot this angle as a function of time it will be a straight line because it is scaled by a factor omega which is a constant and then the power of t is t to the power 1.

So, this is a linear function of time it goes as a first power of time and for a fixed times it is a linear function of space because the function of space so far this angle is concerned, the phase angle is concerned is this kz. So, for a fixed time this is the linear function of z.

In the medium, if you look at how the surface of constant phase would propagate then this omega t minus kz, this phase will have to be constant for all points on that surface of constant phase. So this argument would be 0, so d phi is equal to 0 and d phi is the differential of omega t minus kz which is omega dt minus kdz. So, essentially what you have is omega dt minus kdz equal to 0. So from this relation you immediately get that dz by dt is equal to omega over k. This is what gives you the phase velocity.

The phase velocity is given by through what distance the surface will have to move in a certain amount of time and that ratio delta z by delta t, in the limit delta t going to 0 this ratio will be exactly equal to omega over k and this is the definition of a phase velocity for a particular wave. So, this is the speed at which a wave front which is defined by a surface at certain fixed phase. That could be any fixed point, it does not have to be the crest and it does not have to be the trough, any particular phase that you track but, you see that all how that particular point propagates in space and time and this will give you the corresponding phase velocity.

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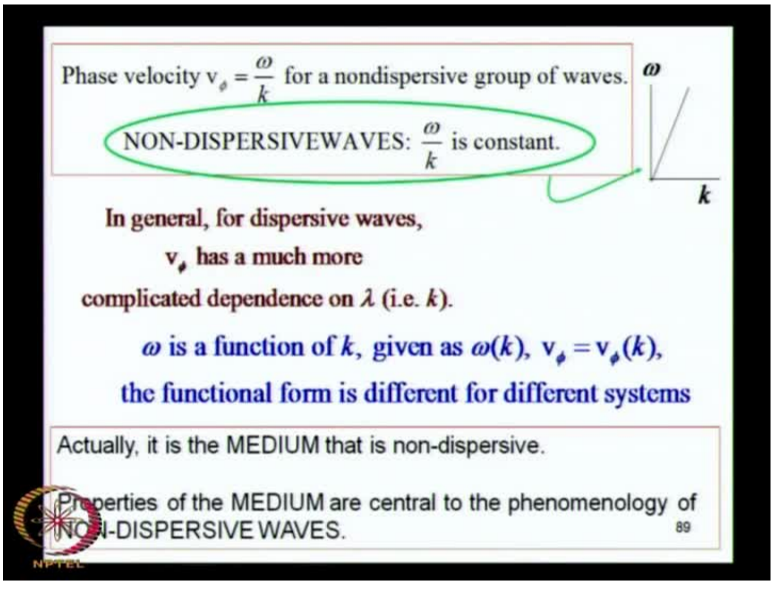

So, omega over k would be a constant and this is the property of non-dispersive medium these are called non dispersive waves. It makes, it sounds as if this is the property of wave but, it is more a property of the medium in which the wave is propagating because whether this omega over k will be a constant or not depends on the medium rather than on the waves. So, it is the property of the medium, so this is like transferred a method in which you have a description of the medium but, you use it to describe the wave.

So in a dispersive medium the behavior is not so simple because the omega versus k relationship in a dispersive medium is not just a linear straight line relationship. It is a little more complicated, it depends on the wavelength that you are talking about.

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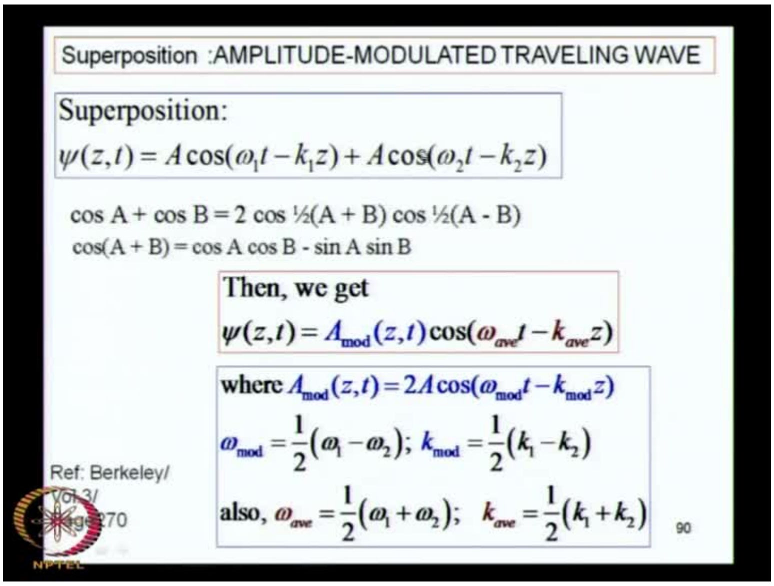

So, this omega which should be plotted actually as a function of k and the reason it happens is that depending on the properties of the medium, you may have different dispersive relationships. I will show you some of these examples. When you deal with refraction; for example, this is the kind of thing which leads to refraction in a medium because the medium disperses the different wave lengths, different frequencies so that you get when light travels through a prism for example, its spreads out in different components.

So before we get to that, let us deal with superposition of waves because typically a wave packet will consist of several components which are super post on each other. So here, you have an example of a superposition of two waves one for which the frequency is omega 1 and the wave number which is 2 pi by lambda is k 1 and for the other the corresponding parameters are omega 2 and k 2.

Let us assume for the sake of simplicity that both have the same amplitude A. So, we construct a superposition of such two waves and all you do to work out the analysis is simple trigonometric relations like the addition and subtraction of cosine and sine angles. So you can work out the algebra quite in a simple manner and if you play with these terms using ordinary trigonometry relations which are the usual trigonometric identities, you can construct the superposition of this by writing. Thus cosine of these two terms and then combining the corresponding terms and you find that this net function psi the superposition can be written as a single sinusoidal wave. Now the cosine wave is also called as a sinusoidal wave, the cosine function after all looks just like the sinusoidal wave it is only phase shifted by pi by 2.

So, it is still called as a sinusoidal wave but, this is a cosine function. So you write this wave function, this psi z of t which is coming from a superposition of two cosine functions as a single cosine function. That is the net result of the superposition with the difference that the frequencies and the wave numbers are however different. The frequency is neither omega 1 nor omega 2 and the wave number k is neither k 1 nor k 2.

It comes as a result of the superposition and these frequencies; so here, you have an average frequency and here you have an average wave number. So, the average frequency is just the arithmetic average of these two frequencies, average wave numbers is just the arithmetic average of these two wave numbers. These are the average frequency and the average wave numbers that you get in the argument of the cosine function (Refer Slide Time: 33:40).

Now when you construct the arithmetic average you take the sum of the two and divide it by 2 then, amplitude itself comes not from the sum but, from the difference of the corresponding terms. This amplitude is called as modulated amplitude. It is neither A, it is nor this factor but, it comes and it is modulated, so this amplitude is A mod and this requires another function which is coming from the modulated frequencies and the modulated wave number.

These modulated frequencies and modulated wave numbers are coming from the differences between the two frequencies omega. So omega mod is omega 1 minus omega 2 divide by 2 and the k mod is k 1 minus k 2 by 2.

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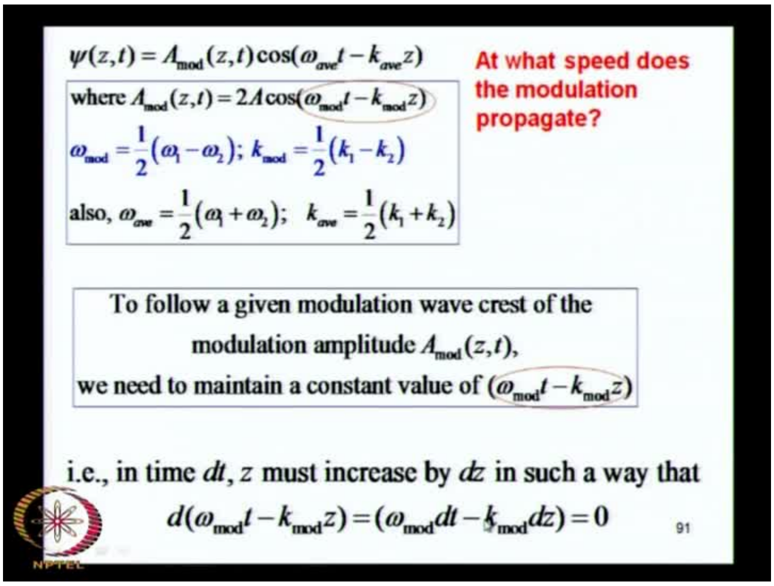

So this is how now this comes from the plane trigonometry, there is no magic over here. We can now ask the question, you now have a modulated function which is a sinusoidal wave a cosine function and at what speed does the modulated modulation propagate? Because, this will be a function of space and time and you can ask at what speed is this modulation travelling? How will you find that out?

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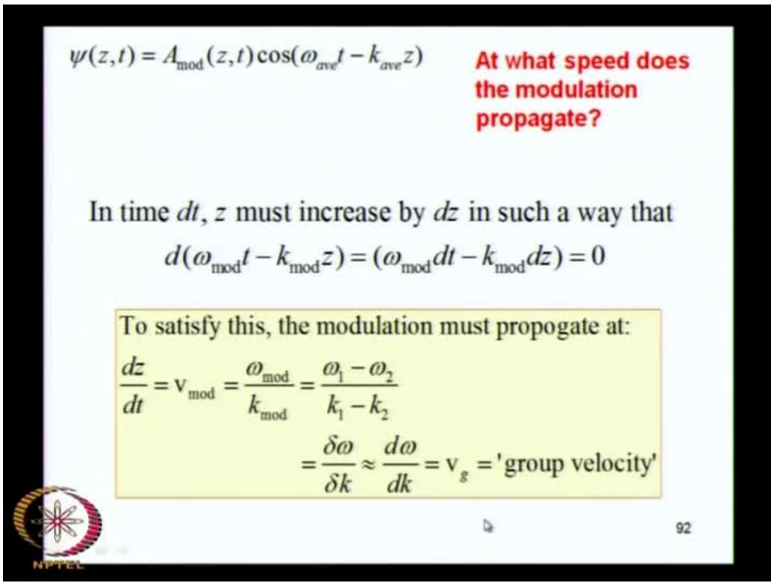

What you will have to do is to find, we look at this argument v mod t minus k mod t z. This is the argument and this must remain constant. What is the condition for that is what we will find and to do that you set the differential of this argument to 0. So this is differential of this argument z equal to 0 and you get a condition for the ratio omega mod over k mod, so that will give you the corresponding dz by dt.

So dz by dt which is given by the ratio of omega mod to k mod which is nothing but, the ratio of the difference in that two frequencies as we have seen in the denominator we have the difference in the corresponding wave numbers. It is delta omega by delta k and this modulation then travels at a different speed, this is what you call as a group velocity. So there is a difference between the group velocity and the phase velocity. So, the phase velocity is what we described earlier: this is the phase, this is the velocity at which the individual sinusoidal waves propagate and this is the entire modulation, how it propagates at what speed so that is given by the group velocity.

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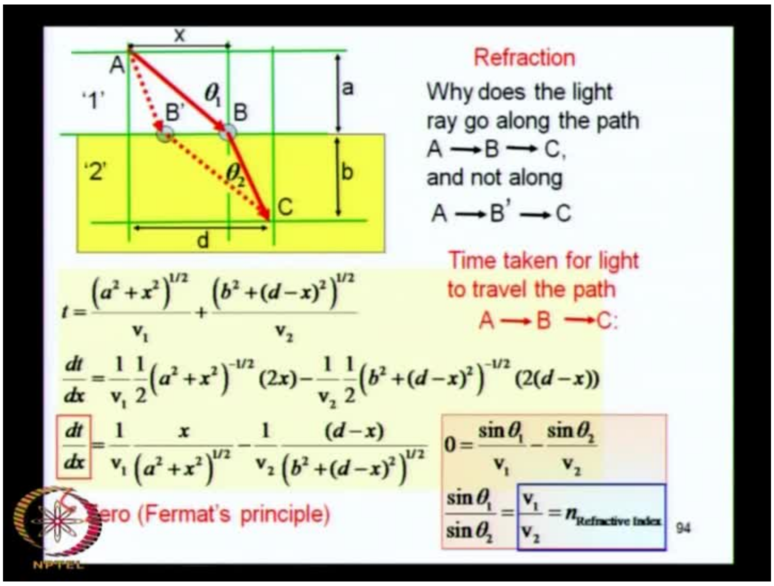

In non-dispersive media, the wave packet spreads, so let us see an example over here. Here, you have a case of refraction and you have got light which travels along this ray and then when it is crosses the surface of a medium it bends, this is the phenomenon of refraction.

We ask the question, why does light have to go from A to B to C? Why can it not go from A to B prime to C? Why should it take this the particular path, why should it not take some other path? Why should it not come here and then bend at this angle?

There is some reason why it takes a particular path and this comes from the variation calculus. This is the subject we dealt with the unit 1, in which we talked about the evolution of the mechanical systems being described not by the principle of causality. But by the principle of variation, namely the Hamilton's principle of variation, we argued that a system evolves in times such that action is an extremum and in this context it translates to what is called as the Fermat's principle. The time taken by light to go from A to B to C rather than A to B prime to C for any other path through some other point B prime anywhere on the surface, compare to any other path, this will be the least time.

So, this is the principle of least action and you can see it very easily because all you have to do is to write the expression for the time taken for light to go from A to B and from B to C and the sum total of this is the total time taken by light to reach C through the point B. What is this? This time is divided by the distance by the speed of light and the speed of light is not same in the two medium.

So, it is v 1 in the medium 1 which is the upper medium and it is v 2 in the lower medium. So this time is the distance over here, which is given by the sum of the squares of this distance and this by the Pythagoras theorem. So this distance is the square root of a square plus x square divided by the time by the speed in that medium which is v 1 and then over here the speed is different, which is v 2. This is the hypotenuse for this right angled triangle and you can see that the corresponding distance is from this green line to this green line is d and this to here is x. Then, the distance between these two green lines the second and the third green line is d minus x (Refer Slide Time: 40:00).

So, you have to take the square of this distance and this distance is b, so b square plus d minus x whole square and then you take the square root to get the length of the hypotenuse itself divided by the speed of the light in medium 2 and this gives you the total time traverse that is required for light ray to go from A to B to C.

You can ask, what will be the change in this time if the ray were to go not through the point B but, through some other point like B prime. If it were to go through some other point B prime what would change it is this distance x because when it goes through the point B; this is the value of x which I am pointing out by this pointer whereas, if it goes through the point B prime this will be the corresponding distance. So, this distance is a measure of which point of the surface the light ray will go through, so if you take the derivative of the time taken by t with respect to this measure which is x dt by dx all you do is to take the differential of the right hand side with respect to x.

So that is simple algebra, what you do is take dt by dx, find this out, solve it, no need to write down these terms in your notebooks. All you have to do is to take the derivative of t with respect to x and then demand that this derivative is 0, because by Fermat's principle, it is going to take the least time, so at that least time this derivative must vanish.

So put this equal to 0 which means that these two terms, this minus this and this difference must vanish. You will get a relation for the ratio of v 1 over v 2 and that is exactly what you get because this ratio will turn out to be given by the trigonometric properties of the right angle triangle that you have constructed. You get a very simple relationship that the sine theta 1 upon sine theta 2 is equal to the ratio of the two speeds and this ratio of the two speeds is what the refractive index of the medium is.

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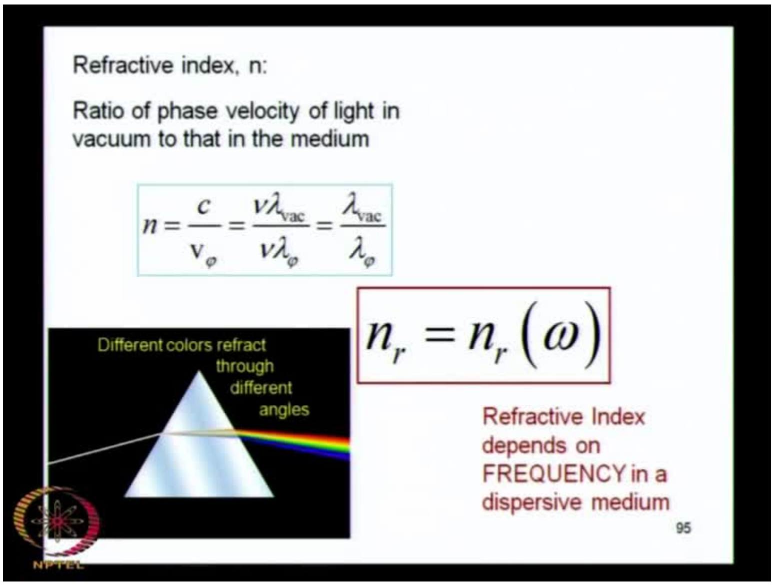

Now that we know what the refractive index of the medium is, we understand dispersion very easily by recognizing that this refractive index actually changes with frequency is not the same for all the components in white light. Because this refractive index changes for different frequencies, then you have the different waves going at different speeds in different media. Since, they go at different speeds then, they are going to spread out.

So that is exactly what happens and this is what generates for us what a dispersive medium is and you find that at the bottom of all of this is the simple algebra of simple harmonic oscillator. It is for this reason this is one of the 100s and 1000s of examples in physics and engineering why we learn this. It is not our intention here to study the dynamics of wave packets or the Fourier analysis or optics, let alone quantum optics or any details but, just to give a flavor of the applications of the range of phenomena for which a very clear and comprehensive rigorous understanding of the oscillator and the damped oscillations and the resonances and so on is important.

Now, this is the phenomenon of dispersion; you find that this refractive index changes with the frequency, so different wavelength will disperse when passing through a medium. This is the famous experiment by Newton and then you have the red at the top and blue at the bottom this is how the wave is spread out (Refer Slide Time: 44:44) and this is what is called as normal dispersion.

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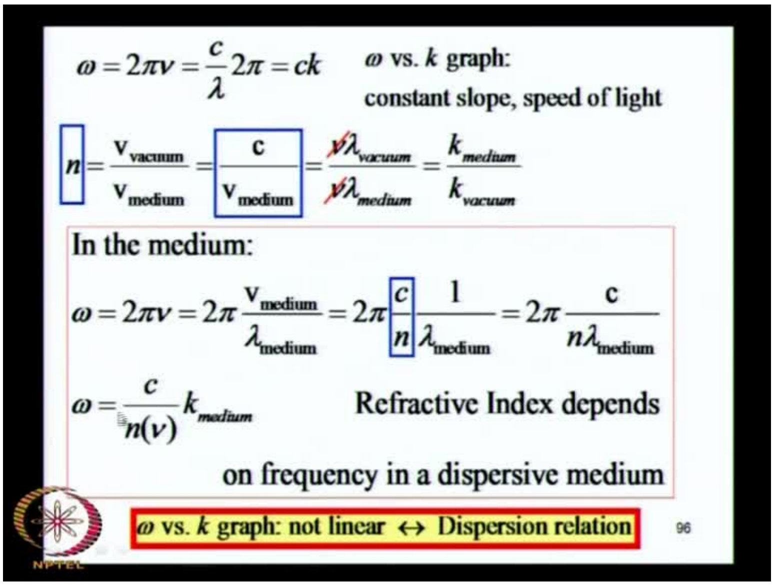

There are peculiar properties of different media and you find that the refractive index is given by the ratio of two speeds, so it is a speed of light and vacuum, it is the speed of light and the medium; the speed of light and vacuum is usually written as the letter C, which is a universal constant.

The speed of a wave as we have seen earlier, when we discuss the phase velocity is nothing but, the product of the frequency and the wave length. So, the product of the frequency and the wavelength is this nu lambda but, the lambda will have to be different for the same frequency. Because these two spaces are different, the refractive index is not equal to 1, only when the refractive index is equal to 1, these two spaces will be equal and the numerator and the denominator will be equal; otherwise the two spaces are different and the two wavelengths will be different. The corresponding wave numbers are different because the wave number is just the reciprocal wavelength 2 pi over lambda which is what defines the wave number.

So this is the ratio of the wave number for the medium divide by the wave number of the vacuum. So in a medium, you will have omega which is given by the v medium divided by lambda times 2 pi of course, because you are writing for the circular frequency.

Now, you know that this v medium over lambda medium if you just swap this term from this relation over here, this will come from dividing the speed of light by the refractive index. Because the refractive index change depends on frequency the medium becomes dispersive. So it is the medium which is dispersive but, you often say that the waves are dispersive.

So now you can see if you plot a graph between omega and k omega versus k, you plot k on the x axis and omega on the y axis then, omega versus k is related by this particular factor proportionality. Now if this proportionality were same for all the frequencies you will of course, get a linear relation but, if this proportionality changes with frequency you cannot get a linear relation. Now this is the characteristic feature of a dispersive medium; the omega versus k graph will not be linear because for different frequencies you will have a different proportionality between omega and k.

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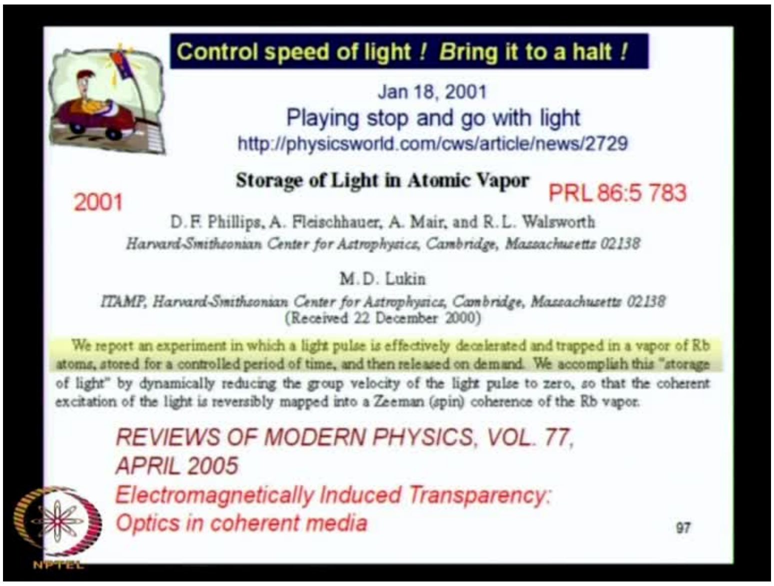

Now, this is exploited in modern physics and I am not going to be able to discuss this at all but, physicists play with this and they are able to control the speed of light actually bring it to a halt depending on how you control the properties of light propagation; this leads to very fascinating applications like electromagnetic induced transparency and so on. These are all specialized topics in quantum optics and this is only to excite some of those who will be interested in some applications.

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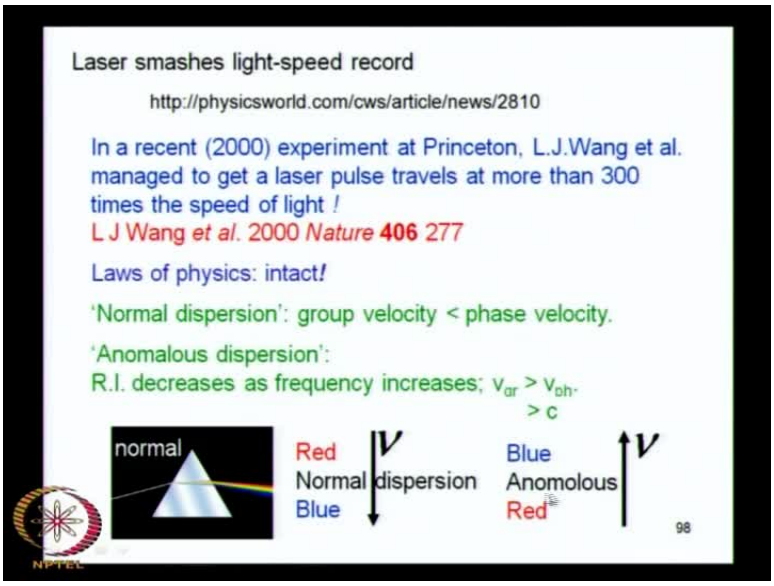

But the properties are very fascinating, you can also find situations in which the here is an example of a paper which was published in nature about a decade ago. This was an experiment carried out at Princeton by Wang and his teammates in which they manage to get a laser pulse travels more than a 300 times a speed of light. Now, it does not mean that you are violating any fundamental rules in physics.

How all this reconciles with the fundamental rules in physics is a matter of detail, it is very fascinating; you have to deal with very complex dispersive phenomena; there is this normal dispersion, there is a anomalous dispersion, in which the refractive index actually decreases as the frequency increases not increases but, it decreases; that is the case of anomalous dispersion. So what you will find in an anomalous dispersion is that if light were to go through a medium which has got anomalous dispersive properties, the ordering of the red and blue will be reversed.

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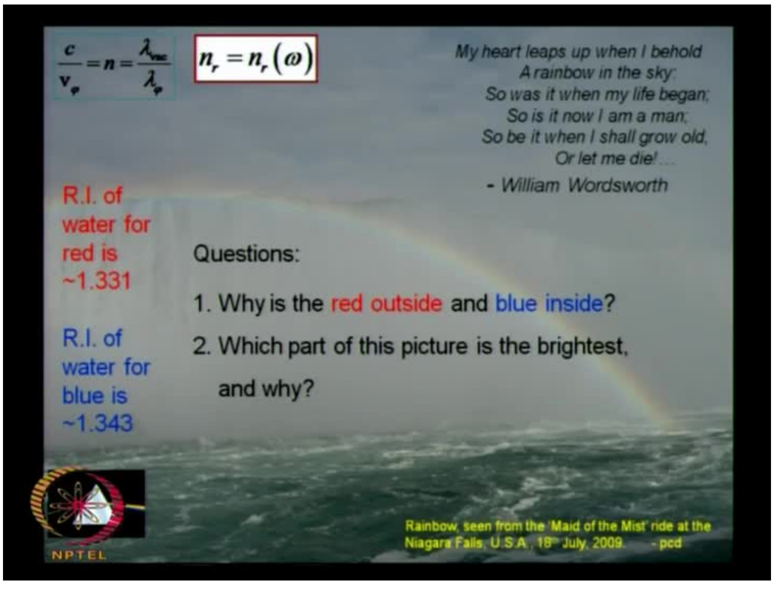

There are even more fascinating materials which are like meta materials in which you even have negative refractive index and so on, so that is something for you to read about. I will leave you, with one picture before we conclude this unit. This picture taken from the maid of the mist boat ride near the Niagara Falls and this is the picture of a lovely rainbow. This is taken because there is a mist and the fall of the Niagara Falls and here I want to leave you with one or two questions for you to ponder over.

Questions which probably come to your mind are ready on seeing this picture. Why does the rainbow have red outside and blue inside? The other question is which part of the picture is the brightest and why? So this is not going to be a part of our discussion today but, these are the questions I would like to leave you with, if there are any questions.

Rainbows is always such a lovely sight; in fact sometimes you see a double rainbow and then you have to ask yourself what is the ordering of red and blue in the second rainbow. But I let you worry about these questions and the answers come from the simple phenomenology that we have talked about together with some simple techniques that you use in and doing optics; it comes from this essential consideration that the refractive index is frequency dependent.

I will give you as a hint that this refraction of course, is in water droplets. So water has got a refractive index which is different for the red and different for the blue; for the red the refractive index is about 1.331 and for the blue it is 1.343. So they are not exact, they are slightly different and the answer lies in this, the details are for you to work out.

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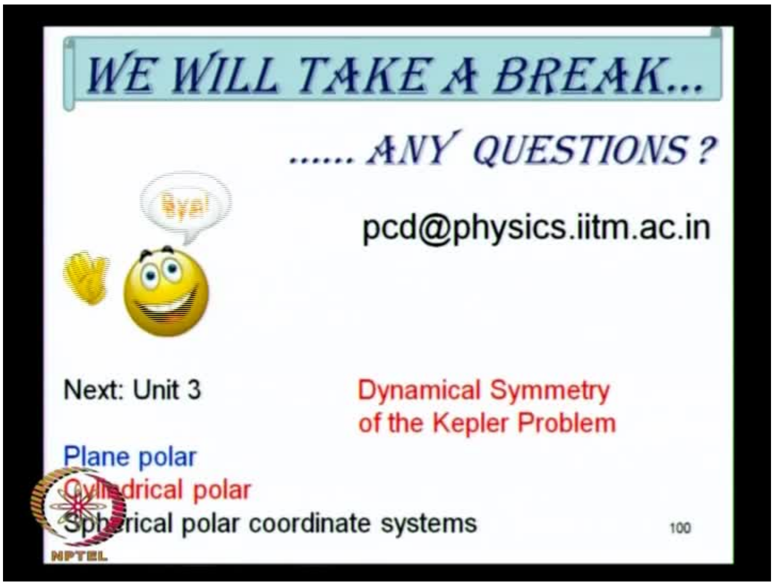

I guess, I will stop here and then next time we meet we will go over to unit 3 in which we will learn about various coordinate systems, we will begin with the plane polar coordinates and then the cylindrical polar and the spherical polar coordinate systems. We will learn about the dynamical symmetry of the Kepler problem and various other things as the course will progress. So, thank you all and we will conclude unit 2 and begin with unit 3 the next time.