Select/Special Topics in Classical Mechanics

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Module No.# 03

Lecture No.# 11

Polar Coordinates (i)

We will begin with unit 3 today; we are going to deal with physical quantities. What else we deal with in physics, other than physical quantities? These physical quantities are basically tensors; they are tensors of various ranks. They are tensors of rank 0, 1, 2, etcetera, accordingly these are some of these tensors, have got special names like a tensor of rank 0, is a scalar, tensor of rank 1 is a vector, tensor of rank 2 is often called a dyadic, but basically these are tensors of various ranks.

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We classify these physical quantities as scalars vectors in common language. So, we need to have a very accurate definition of what a scalar is, what a vector is and so on.

This is one of our concerns in today's class. Our learning goals or learn to, what these tensors are? And how these are recognized? These tensors are described by various components; depending on the rank of the tensor, they have an appropriate number of components.

Then, how they transform when these are viewed from different coordinate systems, which are rotated with respect to each other, is what determines their transformation properties. In particular, the whole issue boils down to observation of the physical quantity, because we observe physical quantities, then from these observations, we deduce the laws of natures, study them and so on.

So, these components are analyzed in various coordinate systems, so typically you are all familiar with the Cartesian coordinate systems. You have got like an x, y, z or right handed coordinate system, depending on which coordinate system, you are using either this one or this one or other the one. You can dance around and you can have a number of different coordinate systems, which are oriented differently with respect to each other. So, there is a rotation of the coordinate system that we are talking about.

The components of these tensors have different values in different orientations. How these are related to each other is what governs the important properties of physical quantities. So, we must examine how these components transform under rotation of a coordinate system.

The other learning goal for this unit, which will be - in the second lecture of this unit, which is to exploit symmetry, because if you deal with any observation of any physical quantity, means here you have got a biological. This is some sort of double helix or whatever, you immediately recognizes that it has got some sort of symmetry, there is a helical symmetry, it as if there is a strip which is going round the surface of a cylinder. The same is true if you look at the Qutub Minar, if you like right, there was a cylindrical symmetry and there is a tapering.

The distance from the axis of the surface keeps reducing as you go from bottom to the top, but no matter through what angle you have turned, there is certain symmetry. So, this is a cylindrical symmetry, this is the same in a nano tube. If you looked at one, which have cylindrical symmetries, here if you look at the solar system, then yes, there are the planes of orbits of the different planets.

They are not exactly in one plane, so they have a little tilt with respect to each other, but if you ignore it for of the time being, then you can think of this whole motion of all the planets in the solar system, to be taking place in one flat plane approximately. Then, you do not have to worry about the z axis. So, really have a two dimensional problem rather than a three dimensional problem.

You can - the cylindrical symmetry drops down to symmetry in a plane, rather than in three dimensions. Depending on the details of the orientation, depending on how things are lead out in the physical world around us, in either two dimensions or three dimensions or in some cases, you have to deal also with a multi-dimensional space and physics.

You want to describe the location of an object. Like, you want to describe the location, this is obviously Saturn and you can recognize it from the rings. You want to describe its location and then you need a reference point, which is observation point. With reference to this observation point, you need a coordinate system. With respect to this coordinate system, you will describe its positions. This is what develops the notion of a vector, so you may have a cylindrical symmetry. Here is a picture of the universe around the earth, which is in the middle, then you can have an equatorial plane, the celestial objects can be seen in a different geometry, you can make use of the three dimensional spherical symmetry.

These are various you kind of situations; it is quite obvious when we look at these figures that the Cartesian coordinate system will not be the most appropriate one to use. This you can use, it nothing wrong about it, but you may get much greater convenience if you use a different coordinate systems. We will also learn how to exploit symmetry of the situation and then develop in an appropriate coordinate system, so that our analysis is simplified; so these are some other learning goals for this unit.

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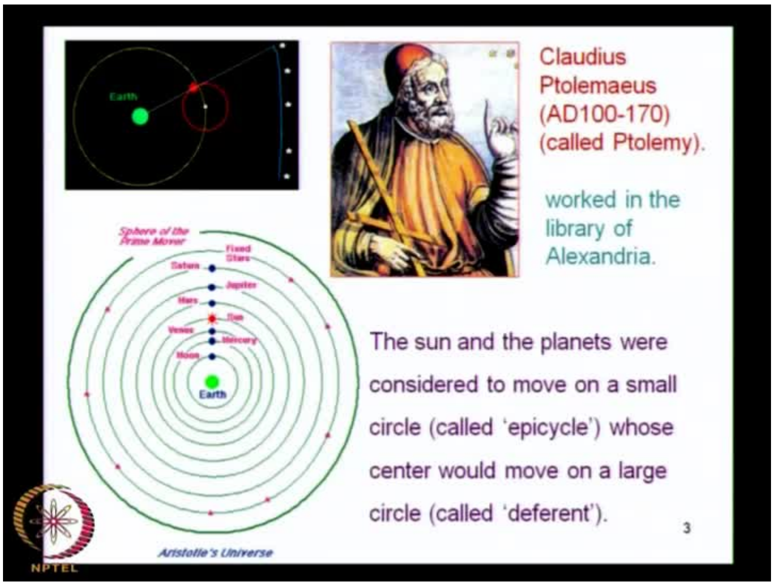

One of the first models was proposed by the Ptolemy - Claudius Ptolemaeus, he worked in the library in Alexandria. What he did was to presume that the earth is at the centre of the universe, this is what is called as the geocentric frame of reference, because you need an observation point. The natural observation point as one envy size, at that point of time, in the growth of science, this was nearly 2000 years ago and this was in the second century.

The geocentric frame of reference was the common one that people use, this was proposed by Ptolemy. Of course, the motion of the planets as seen from the earth would be obviously in the background of the stars. This could be analyzed in terms of the circles, as if you know all of these planets are revolving on certain circular trajectories about the earth. But then, this is was not a very convenient coordinate system, because it appeared that some of the planets, if they were going in one direction, then part of the year they would seem to go backward, this was called as electro kinetic motion in the background of the stars. This was explained by saying that on the circular trajectories, there are tiny circular trajectories, these objects move on those circles or circles, so these small circles were called as epicycles and the larger as the deferent.

The motion was somewhat of this kind, as you see on this picture. What it explained is how the planet in the background of the stars could be seen to be going in a backward kind of motion, for part of the year. So, this is a very complicated kind of coordinate system that was used.

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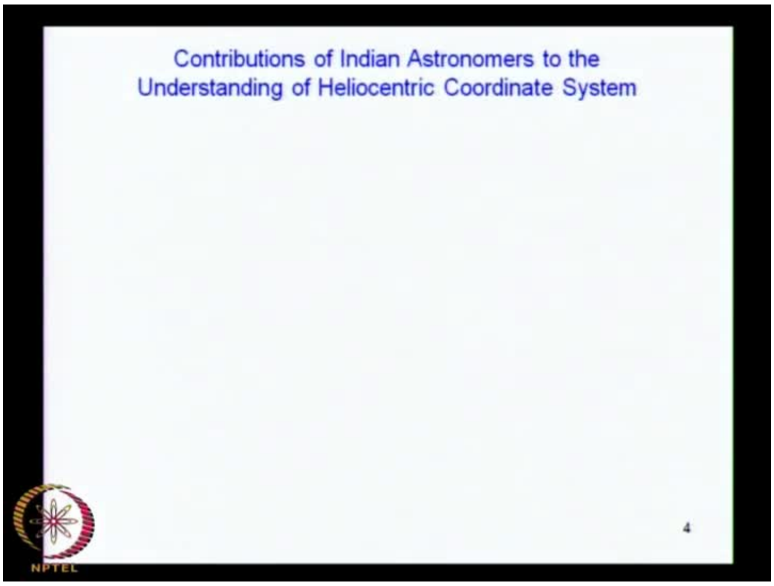

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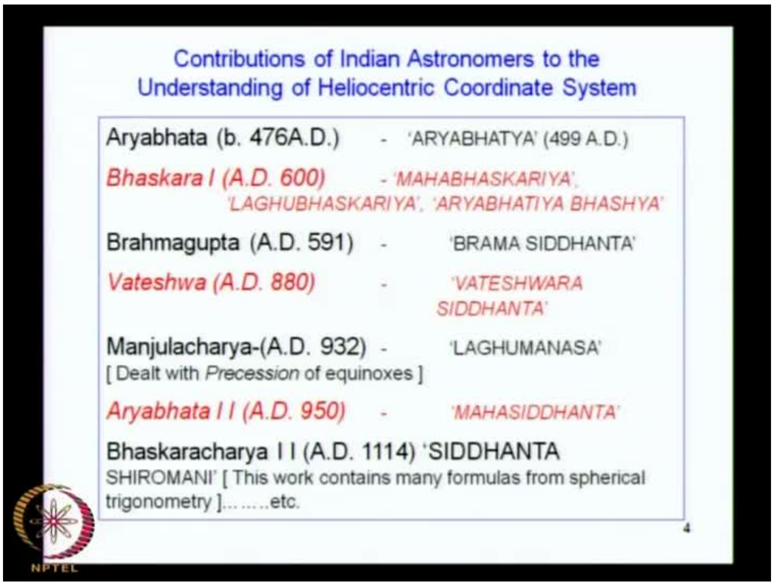

I should point out at this junction that Indian astronomers - especially the astronomers from - whereas gods own country from Kerala, they contributed a lot to the analysis of planetary motion. There is a huge contribution by Indian astronomers, in fact I should mention the work of Aryabhata in the fifth century, Bhaskara who wrote this

Mahabhaskariya, then Brahmagupta, Vateshwara, Manjualacharya, Aryabhata, Bhaskaracharya's Siddhanta and all of these were great contributions.

In fact, many of these astronomers were quite aware of the fact that the tries of earth as the center, as having a frame of reference, which is geocentric, which is earth centric, is not the most convenient one, that a heliocentric frame of reference is more convenient. They were quite aware of the fact that what causes the sun to rise in the east and then set in the west, is because of the earth's all rotation about its axis, this is something that we understand very well now.

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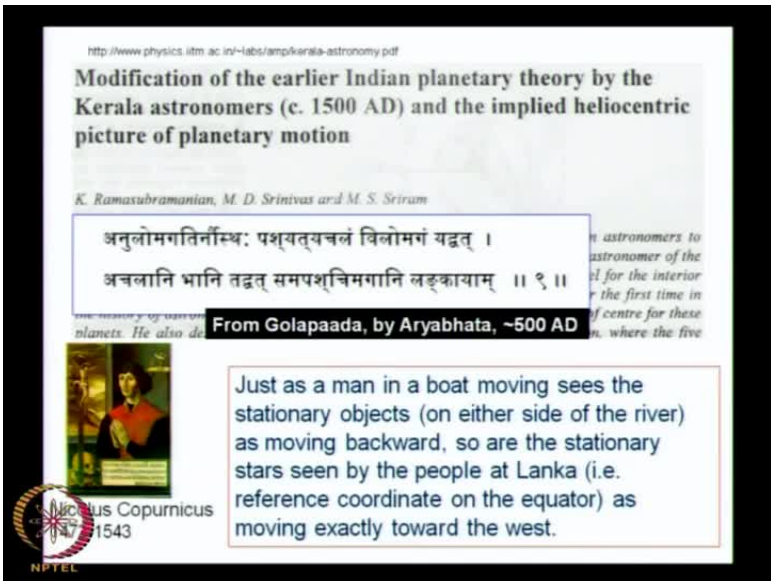

We credit Copurnicus for this work, for recognizing the heliocentric frame of references are more convenient frame of reference, but much before Copurnicus or many of the Indian astronomers were quite aware of this. This has been analyzed in great details by some astronomers – astronomer, physicists here in Chennai, at the University of Madras. This is a paper which I have uploaded at my website, so this is the link. This is the contribution of the Kerala astronomers and the implication of the heliocentric frame of reference.

If you see these words, this is a words from Golapaada, which is composed by Aryabhata in the around 500AD. What it really means is that just as a man in a boat moving, sees stationary objects which are on the either sides of the river at the banks, he sees that these objects are moving backward related to him. So do the stationary stars seen by the people at lanka, lanka is a reference coordinate on the equator. That is part of the reason the srilanka is called as srilanka, because it is all most on the equatorial plane.

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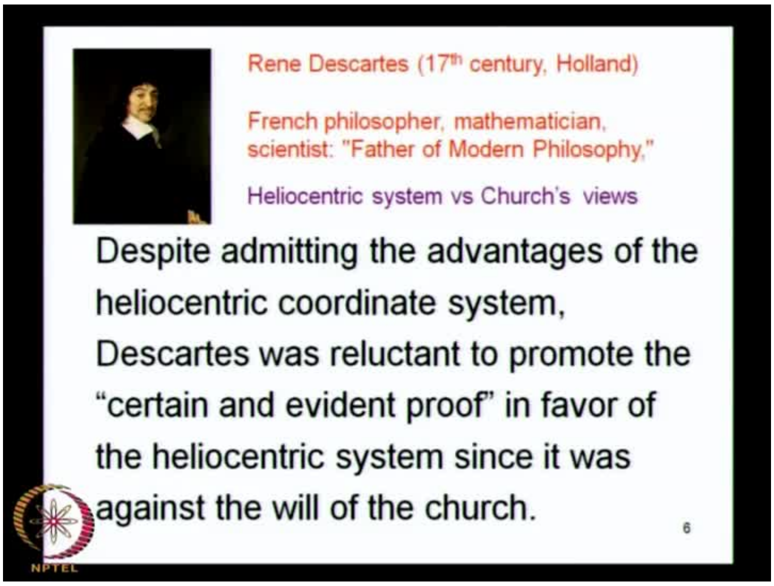

This is moving exactly toward the west, some of these things were well known to these Kerala astronomers. Having pointed this out, the history of the development of the heliocentric frame of references is very interesting one and a very challenging one, because even after Corpurnicus proposed the heliocentric frame of reference, there was very strong opposition to it.

Descarte, after whom the Cartersian frame of reference is known. The Descarte quite supported the heliocentric frame of reference, he understood it and he grasped it. But, the heliocentric frame of reference was against the views of the church; he therefore had great difficulty in supporting it. Descarte was reluctant to promote the certain and evident proof in favor of the heliocentric frame, because the church would not accept it.

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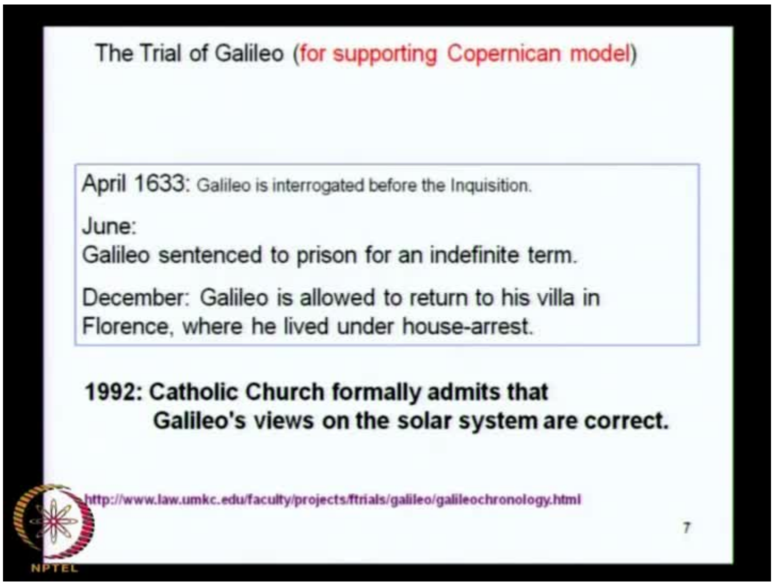

So, there was recognition of scientific value, scientific attitudes, it took good bit of effort to be consolidated. In fact Galileo, he also supported Copurnican model, but then he was tried, because this was again against the view of the church, in 1633 Galileo was interrogated. In June 1633, he was sentence to prison for an indefinite term. In December, he was allowed to return to his villa in Florence, but he was kept under house-arrest.

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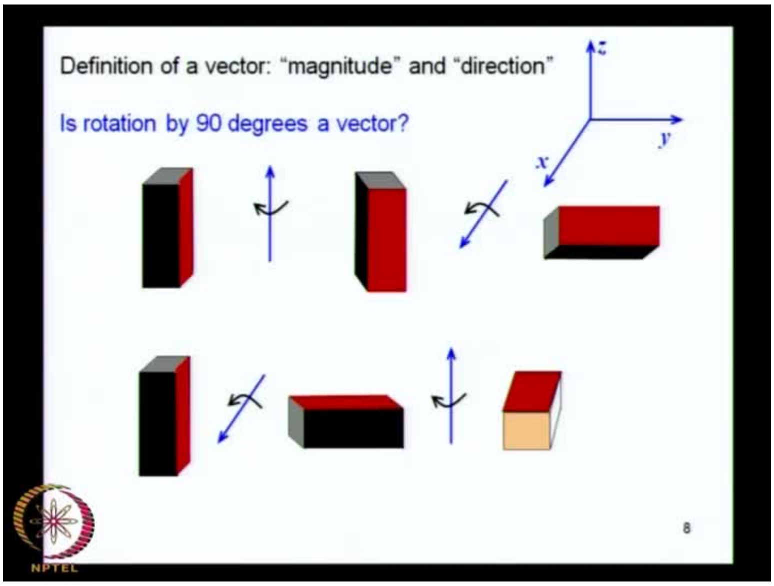

So, it was only recently, just about 20 years ago, the Catholic Church formally admitted that Galileo’s views on the solar system were correct. It has taken the long time to accept scientific point of view. All these are centered around the basic requirement that you have to look at physical objects, whatever you look at. To look at these physical objects, you first of all you need an observation point. It could be the centre of the sun, it could be the centre of the earth or it could be not a geocentric or a heliocentric frame of reference, but just an egocentric frame of reference, you have say that the centre of the universe is at the tip of my nose and I will observe everything just from there, wherever, whichever way I turn my neck.

So nothing wrong about it, one can carry out coordinate transformations exactly, you will get exact answers. You can analyze anything and everything, nevertheless you need a point of reference and then you need a coordinate frame of reference, so this is what you refer to physical quantities. So, with reference to that point, you look at it and then, you ask yourself, do you need just one parameter to describe it or you need more than one parameter. Do you need to look at just the distance of an object or also the direction in which that object is located, is it toward east or west right.

Very often in the high school, a vector is defined as a quantity, which has got magnitude and direction. Now, this is fine, but it is not sufficient, because if you look at a rotation through 90 degree - take some object, I think, I got an object, can I pick it from my bag, I think I have an object, I hope.

So, if you look at some object like this, then I turn this object through 90 degrees, so this is a clockwise rotation as seen from my side, from your side it is an anti-clock wise right, but then the direction of this as clockwise or anticlockwise obviously depends on who is talking about it. Because, you will call it as anticlockwise and I will call this as clockwise.

Nevertheless there is a direction; we can talk about a direction. Now, a rotation through 90 degrees, the 90 is a specific angle which is larger than 80 degrees and less than 100, so that is a magnitude I am talking about, which is well defined in this case. Obviously, I have a quantity, which has got magnitude as well as direction.

So, I meet the criteria, which the high school definition provides. That here I am talking about a quantity, which has got a magnitude and the direction and then I could ask, is this vector? Because, I defined my vector is a quantity, which has got magnitude and direction

Now, actually tells out this is not a vector. The reason is the following that if you take this quantity, I can carry out a 90 degree rotation about three mutually perpendicular axis. I can have an axis, a vertical axis, which goes through this, so here is an axis and I can turn it like this. I can have an axis through this, right, in the horizontal plane and I can rotate it through 90 degrees like this.

What I will do is I consider an axis going through the top, through the bottom, rotate it through 90 degrees. Now, you have got this black face, which you can locate and I rotate it through a 90 degrees, it comes here. Now, I take this axis, which is from you toward me, so this axis, about this axis, I rotate it again through 90 degrees, so I rotate it like this. Now, I have got this black face at the bottom, now this is the final row orientation that I get at the end of these two 90 degrees rotations.

Now, what I do is, I can start with the same first configuration. The first orientation as it was, but now I carry out these two rotations in the reverse order, so I first carry out a rotation about this axis, from you toward me, so here it goes. Now, I carry out the rotation through the axis going from the top through the bottom, so I turn it like this and this is now the final orientation.

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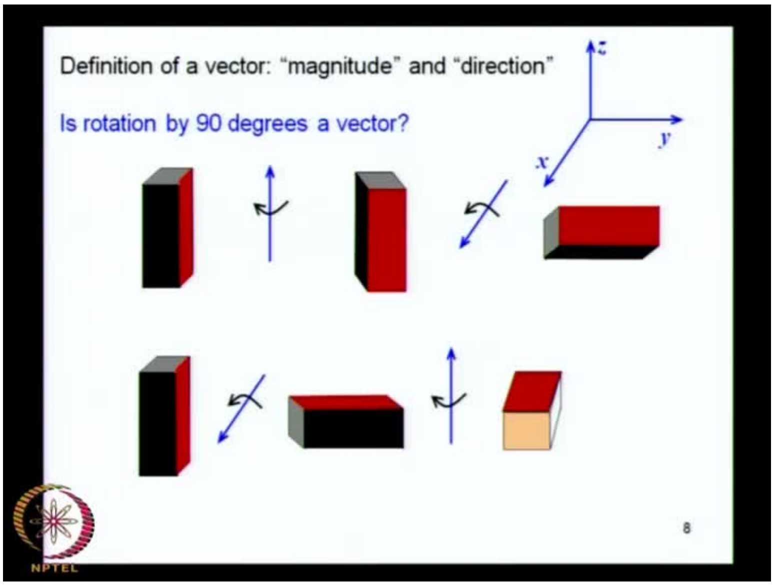

Now, this final orientation leaves this black face on the side, whereas earlier we had it at the bottom right. So that is indicated in these figures over here that if you rotate it about this axis first, this axis next, you get a certain orientation. If you do the same operation in the reverse order, you get a different orientation, which means that if you have to look at this as a vector, then a plus b would not give you b plus a.

So, the definition of a vector, as a quantity which has got just a magnitude and a direction is not adequate. You have to qualified further, this definition is clearly is not adequate at all to describe what a vector is, because if you follow this definition, you get inconsistency.

Actually, you need much more than saying that it should simply meet the law of addition of vector. So that is an additional requirement, which some book suggest that you should define it not only as a quantity which is defined by a magnitude and direction, but it also should fulfill the triangle law of addition. But, even that is not quite sufficient, so we will define a vector in a precise manner now.

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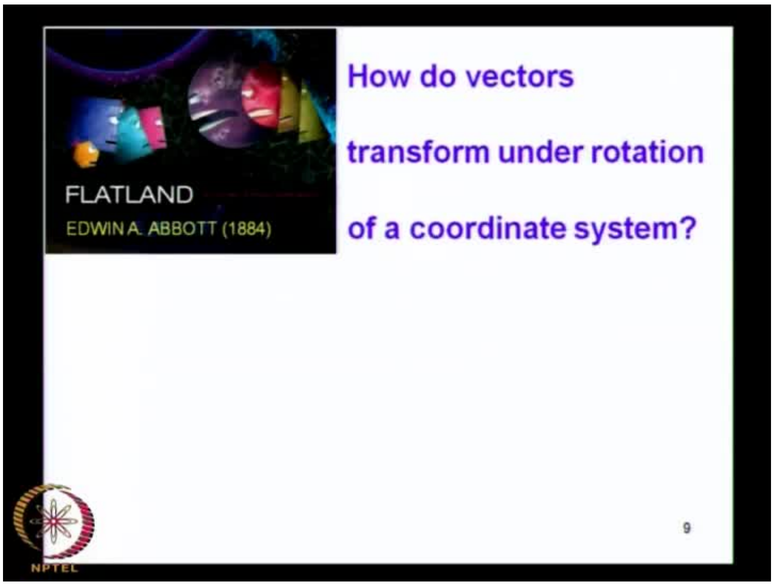

So, the question that we are going to ask is, how do vectors transform under the rotation of a coordinate system. Because, you can have a coordinate system like this and this is what I started talking about that you can describe physical quantities from certain observation points that is the origin of your coordinate system. In this coordinate, if you require a certain set of components of that quantity, which certain number of parameters, which describe that physical quantity, if you look at those components from a different coordinate system, which is oriented, which is turned, which is rotated with respect to the previous one, how do the components transform when you go from one coordinate system to another? Now, this is the main question.

I am remained of a very famous book, which was written by Edwin Abbott, called a Flatland. This is very famous thing, movies have come out of it and you will enjoy seeing it very much. What you do is you begin analysis by looking at flat objects in 2 dimensions, develop your vocabulary, develop your analytical tools and subsequently you can extend it to higher dimensions. What this movie does is, it has a flat square, who meets creatures from high dimensions like, so what he does is he meets a sphere, which is got a third dimension. Then, there is very fascinating conversation between them, because the square lives just in the two dimensions, in this sphere, in the three dimensions.

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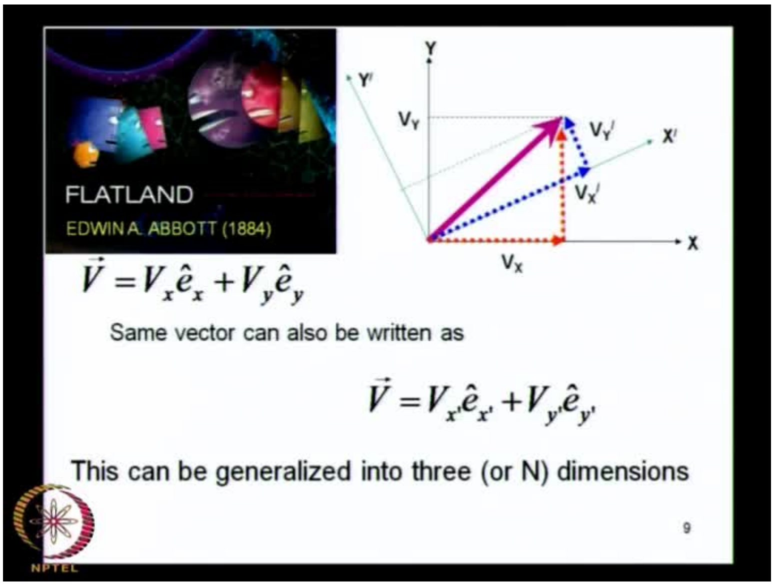

Then, through these conservations you see the relationships between different spaces, which have different dimensions. So, this is an enquiry into higher dimensions, we can extend this kind of technique to learn about high dimensions 3, 4, whatever. What we need is a coordinate system; we begin with a flat world, in which there are two dimensions. We worked with a Cartesian coordinate system; we have got a nice vector, nice because, it has got a color which I like. You need two parameters to describe it, the projection along the X axis and the projection along the Y axis.

These two are the Cartesian components of this vector; you know that this component is given by the scalar product of this vector with the unit vector, in the corresponding direction. So, this is the description of the vector, V x is a scalar product of V with e x, it is the shadow of V on the x axis. V x and V y are the two parameters we describe this vector.

Now, what you can do is, you can choose another coordinate system, which is rotated with respect to the previous one. There is no reason to say that X and Y is the only coordinate system, if you have this, you can also choose another coordinate system. As long as you have a linearly independent basis, you can always describe the vector.

What you need is a linearly independent basis, in a different basis, which is along the x prime axis and the y prime axis, the very same vector has got different component. The component in the first basis were V x and V y, the components in the second basis are V x prime and V y prime.

Another question that we can ask is what is the connection between the components of this vector, in the new frame to the components in the old frame? Is there a certain relationship? That is question that we need to ask. Subsequently, using the technique of the square meeting, the sphere in this flat land movie, you can extend this to higher dimensions, so it is a very straight forward process.

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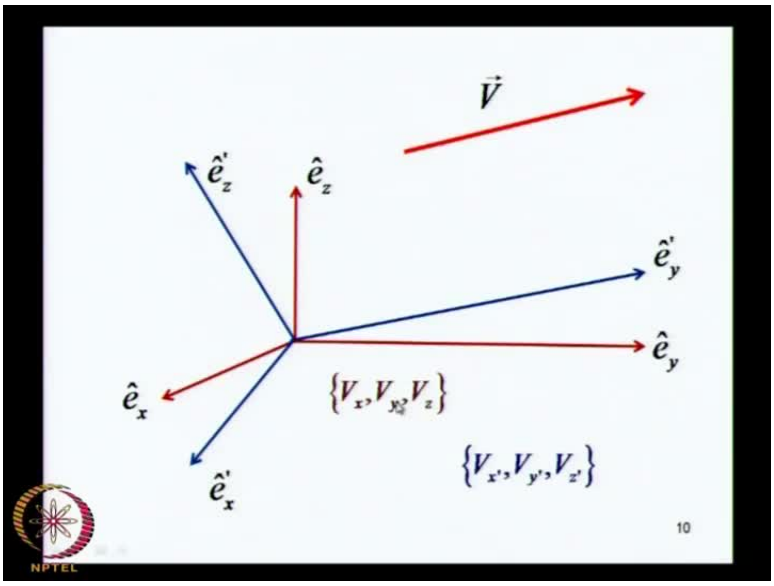

So, here you have a vector, in one frame of reference you look at it, in a three dimensional space e x, e y, e z and that is a Euclidean space that we have conscious of. Although, the real space in which we leave may have more dimensions. In this space, the components of the vector along the three axis, let us say are V x, V y and V z. Then you have another frame, which is turned with respect to the previous one, which is a primed of reference, which is the blue of reference in this picture. In this frame of reference, the components are V x prime, V y prime and V z prime. The same question we now ask as to what is the connection between V x prime, V y prime, V z prime and V x, V y, V z.

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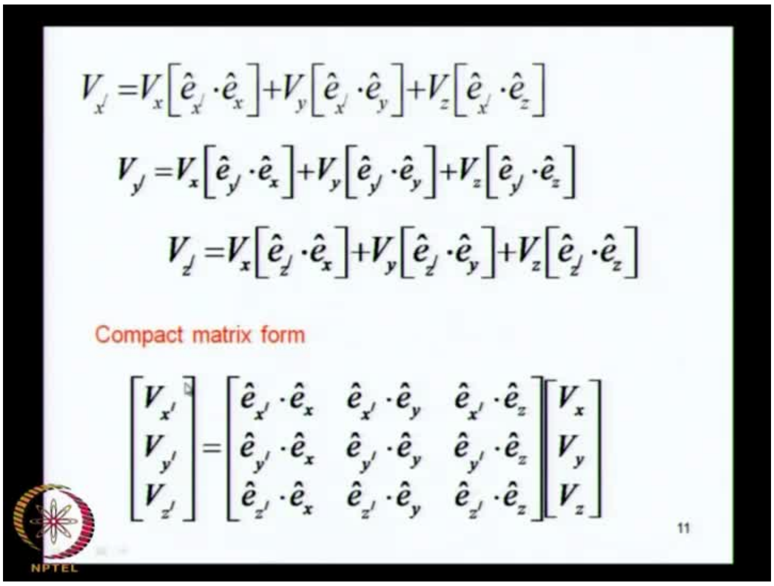

So, can we write V x prime in terms of V x, V y and V z? So, obviously yes, this is the relation that V x prime will not be equal to V x, but it will be scaled by a cosine factor here. This dot product is the cosine factor, then you have a V y scaled by another cosine factor and then V z scaled by another cosine factor; so there is a cosine law, this is called as the cosine law for obvious reasons. Because, the scaling factors are cosines of the angles between the unit vectors, so this is called as a cosine law.

So, V x prime has got this relationship with V x, V y and V z. Likewise, you can write a similar relation for V y prime, which is also a similar superposition. Likewise, you write V z prime as well. You can combine these three relations and write them in a very compact form in using matrix algebra, because instead of writing these equations in the form, in which we see them on the screen, if we write them as a matrix equation, we get a very compact form.

So, this V x prime is this cosine multiplied by V x plus this cosine multiplied by V y plus this cosine multiplied by V z, likewise for V y prime and V z prime. So, just using matrix multiplication rule, you can write this relationship in a very compact form. This is the cosine law of transformation of components of the vector as seen in one frame of reference, to another frame of reference, to the components of the same vector in a different frame of reference, which is rotated with respect to the first frame, so this is called as the cosine law.

Obviously, this cosine law tells us something about the nature of the quantity. If you were to talk about the mass of water in this bottle, for example, this is the physical quantity that you are talking about; you choose a coordinate frame of reference. You have an origin and you have got an x y and z axis. With respect to this, you say that the mass of water in this is let say 700 grams or whatever, then if you were to rotate this coordinate system, then ask what is the mass of water, it is still 700 grams, it does not change.

So, whether I see it from this frame of reference or this frame of reference or this frame of reference or no matter how would dance and sink, it is always invariant, right. This is what gives me the definition of a scalar that the scalar is a quantity, which remains invariant, no matter how you orient the frame of reference, but that is not going to happen for the components of a vector. Because, V x, V y, V z, which have got a certain set of values in one frame of reference, they change to V x prime, V y prime, V z prime in another frame of reference and the connection is through the cosine law.

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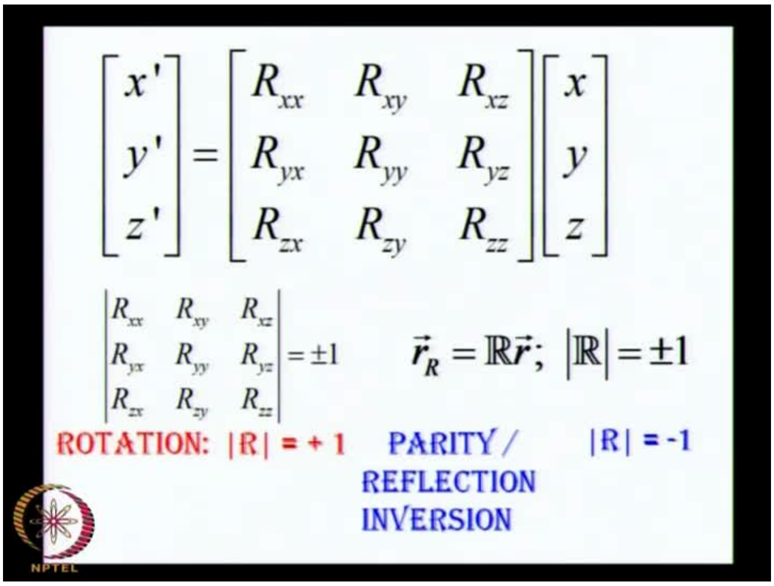

So, typically, when you are carrying out these rotations, you can write the relationships between over vector, whose components are x, y and z to the components x prime, y prime, z prime seen in a rotating frame of reference, through the kind of matrix relation that I have written.

These matrices are the orthogonal matrices, there are transpose and inverse are the same, which is what tells us what an orthogonal matrices. So, these are orthogonal matrices, if you look at the determinant of this matrix, typically this determinant is either plus 1 or minus 1, both the possibilities are open.

If you have a rotation, only a rotation, then the determinant of this matrices always equal to plus 1, but in addition to this rotation, if there is a reflection or in inversion, then the determinant of the matrix is minus 1, so that is what you have that is a parity operation. It is important to understand the parity operation in this context, because obviously it gets connected when you are dealing with transformations.

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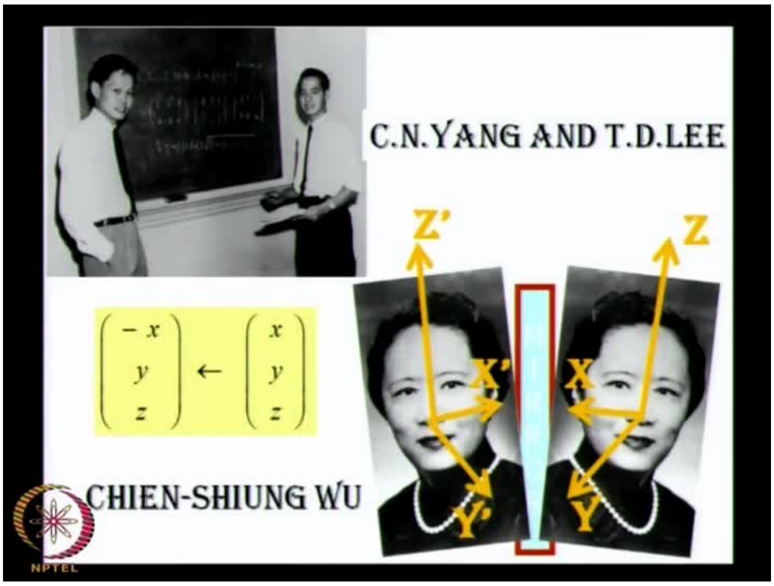

Now, parity is a great importance in physics. One believes that most of the physical interactions are invariant, under parity - under if you carry out an experiment in one world and then you carry out the same experiment in the mirror world. In the image world, one would except that the physic does not change. Then, parity violation was sort of suspected by yang and lee, observed in the experiments dealing with nuclear decay, in the beta decay, in particular by Madam Wu's experiments.

Essentially, it comes because of the left right business, as you go through the mirror world. What is happening in this is that if you see, this is a picture of Professor Madam Wu, this is how this picture will look in a mirror. What you see is that the Y and the Z coordinates remain invariant, but the X coordinate would change and that is a one which will go change of sign.

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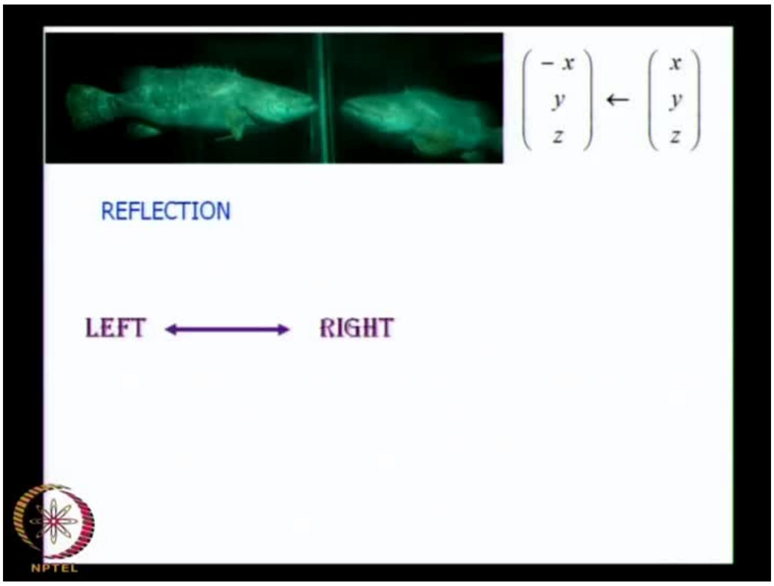

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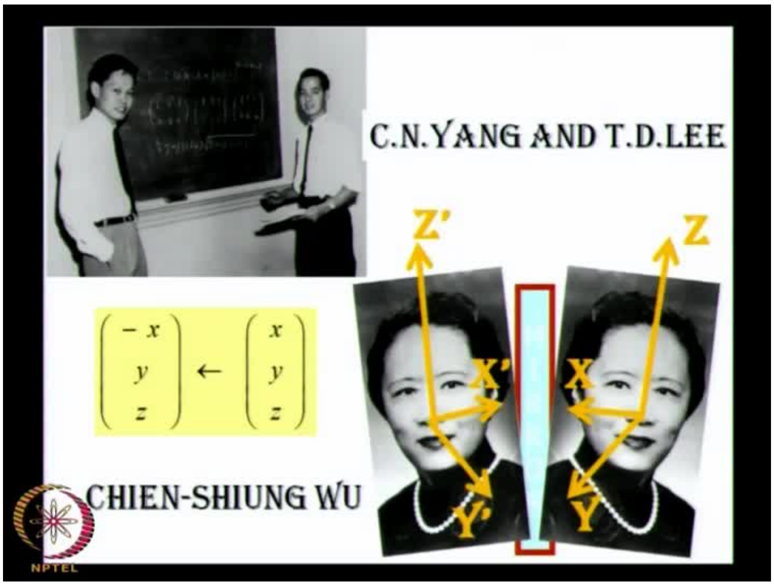

So, x goes to minus x, x goes to minus x, y goes to y, it does not change, right and z also does not go to change. If you go back and look at this picture Madam Wu, you can see that what is her left ear would look like a right ear and vice versa. So, there is a left right you know transposition in the mirror world.

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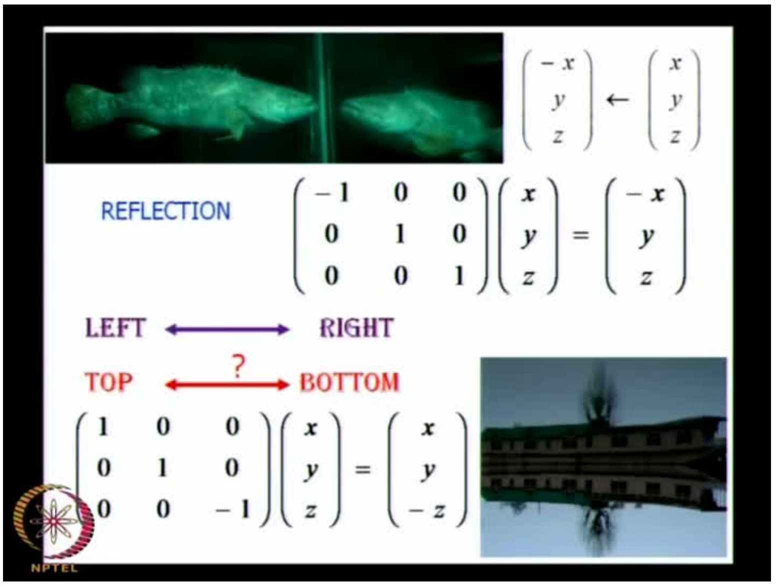

But the top remains at the top and the bottom remains at the bottom. I do not know, if you have a wondered as to when you look into the mirror, why is it that the left goes to the right and right goes to the left, but the top does not go to the bottom and the bottom does not go to the top; now, this does not happen at least for most of us.

So, the reason for this transformation is that under reflection, the x goes to minus x, the y goes to y, z goes to z and this transformation is indicated by this matrix, whose determinant as you can see just from the sight of the determinant, is obviously minus 1 rather than plus 1. So, the parity, the reflection is a different kind of symmetry than rotation.

But then, for some of us, the top does go to the bottom and the bottom does go to the top, because if you are sleeping already and if you look at the mirror, then you will in fact see that the top goes to the bottom and bottom goes to the top. Because, the corresponding orientation of the mirror will be different, this operation will be indicated by x remaining at x. So, this is x, you know so the x and y do not change in this mirror, which is a beautiful lake and it is in the horizontal plane obviously.

The x remains at x, the y remains at y, but it is the z which goes to the minus z, so yes and no, big deal about it. Essentially, the determinant of these matrices, whether this one or this one is equal to minus 1, so that is the main difference between reflection and rotation.

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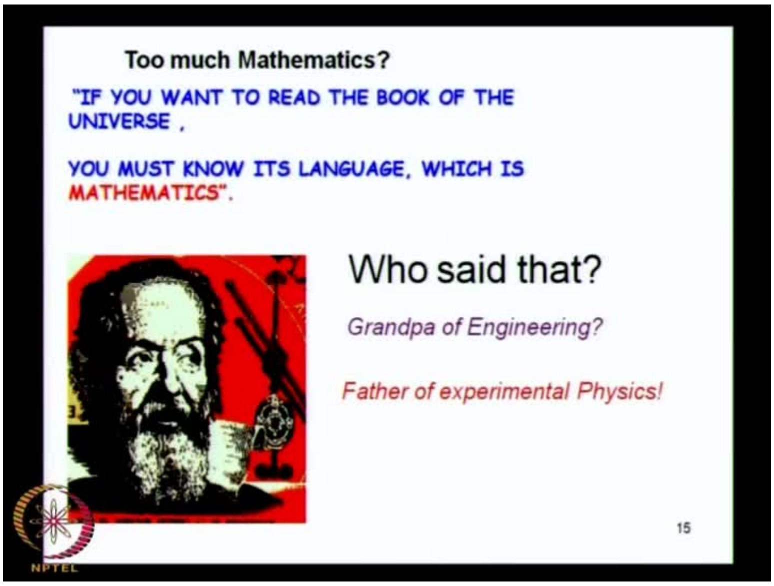

So, we have to deal with matrices, we have to deal with vectors, we have to deal with vector components, we have to deal with tensors, because of vector is a tensor of rank 1, or scalar is a tensor of rank 0. What determines these tensors to be what they are, are their properties in terms of how their components transform under rotations.

So, one has to deal with a good bit of mathematics, if it bothers you as to why, we use so much of mathematics, then I think it is a good point at which I should quote. That if you want to read the book of the universe, the book of nature, then you must know its language, which is mathematics, you might as well learn.

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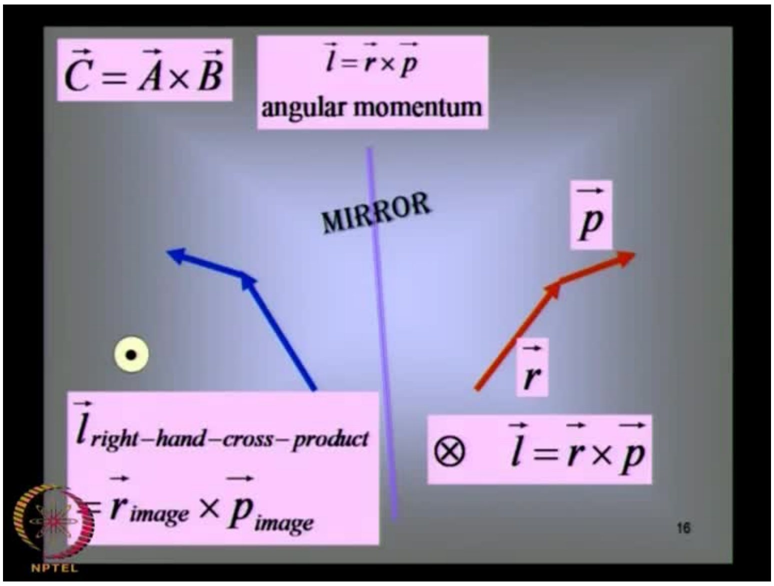

I presume that you know who said this; this is the grandpa of engineering. If you have not recognized him as yet, I should tell you it is a father of experimental physics that of course is Galileoso. This is a great court I hope that it will motivate you from using and learning and whatever mathematics one is to learn to do physics.

Now, let us look at some further properties of reflection. Now, here you have a mirror, you have got an arrow here, another arrow here. You look at the image of the first arrow, which is this, I am not suggesting that the colors change under reflection, but this is just to show the picture, just to tell you that this is the image world and this is the real world, you could do it the other way round either.

But, anyhow, so you have got a vector here, another vector here, these are the corresponding images. Now, if this was to be a position vector, this was to be a momentum vector and that is what we are going to talk about in physics. Because, we talk about position, we talk about the momentum; we talk about the equation of motion, acceleration and forces, etcetera, right.

So, let us start talk about these physical quantities, if one of these physical quantities is a position, the other is a momentum and then the cross product of the two would give us the angular momentum. If you look at the angular momentum in the real world, then now do not go into the mirror yet, do not go into the mirror yet like Alisha, she did when she went through the looking glass, but look at this in the real world, which is a cross product of position and momentum. This is angular momentum, which is a vector, which is perpendicular to the plane right, which contains r and p. This would be represented by an arrow, which is going into the screen that you see in front of you.

This is the angular momentum vector, which goes into the screen. Now, do the same with the images. Take the image of the position vector and take the image of the momentum vector, if you construct their cross product and how to construct the cross product? You have the sin theta factor and then you must go from the image of r to the image of momentum through the smallest angle and ask what would be the forward direction of a right hand screw that is the rule, so you apply it.

Where do you find the angular momentum in the image world coming? Right, it would be represented by an arrow, which is coming out of the plane of the screen, rather than into the plane of the screen. What it is going to tell you that the image of the arrow does not coincide with the image, with the arrow itself, which is the angle of momentum vector.

Which means, under parity, the angular momentum behaves in a different way, right. Angular momentum of course is a vector, but now it is a vector of a different kind. There are two kinds of vectors, one of the vectors like the position, momentum, acceleration, force and etcetera. Angular momentum is the different kind of vector, its transformation properties are not given by the same cosine law, which gave you the transformations for the vector that we earlier talked about. So, this is what is called as a pseudo vector, it sometime also called as an axial vector.

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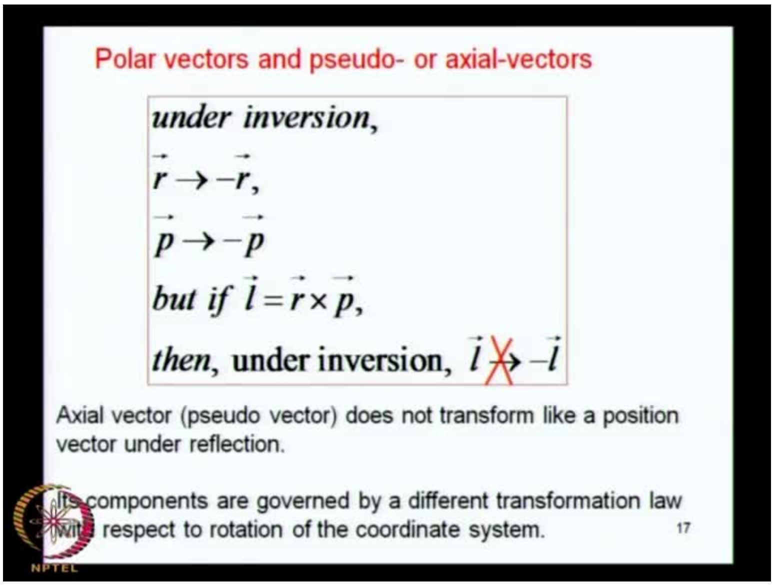

So, there are vectors of two kinds, one the usual ones, which are now to be specifically given a particular name, so we called them as polar vectors. Then, there is another class of vector, which are the axial vectors or the pseudo vectors. These two kinds of vectors, under inversion, the position vector goes to minus r, momentum goes to minus p, but if angular momentum is r cross p, then under inversion, l does not go to minus l. Because, when you take the cross product of minus r with minus p, you still get r cross p, because minus 1 into minus 1 is plus 1 even in physics.

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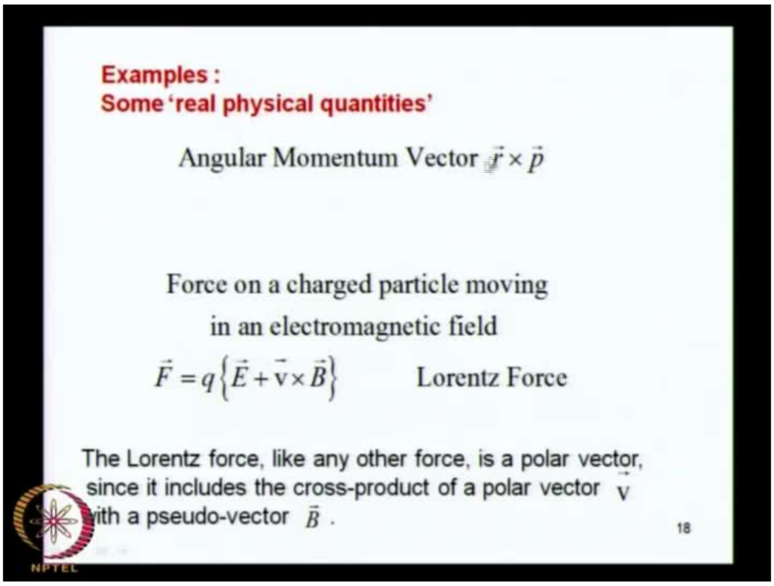

So, this is the different kind of vector, this does not transform like a position vector. So, there are two kinds of vectors, in general you talk about two kinds of tensors, tensors and pseudo tensors, if you like. That way you will also have scalars and pseudo scalars, because the scalar is a tensor of rank 1, you can easily construct certain physical quantities, so angular momentum vector is a pseudo vector. Then, if you look at the force on a charged particle in an electromagnetic field, then force of course is a polar vector. So, you never equate a polar vector within axial vector in a mathematical equation.

So, on the right hand side what you see, must also be a polar vector, but then on the right hand side, you have got that charge, which is one of the best scalars that one knows - so this is the scalar. Then, you have the electric intensity, which is a polar vector, but what about here? You have got a cross product of two vectors, is it an axial vector? Then, you will end up equating an axial vector with the polar vector, right.

Now that does not happen, because the magnetic field is a pseudo vector. So, this is not a cross product of two polar vectors, it is a cross product of one polar vector, which is V, with one axial vector, which is a magnetic field B. So, one has to keep track of these thing when you deal with physical quantities. So, electromagnetic fields, the electric field are a polar vector, the magnetic field however is axial vector, so they have different transformations properties under rotations.

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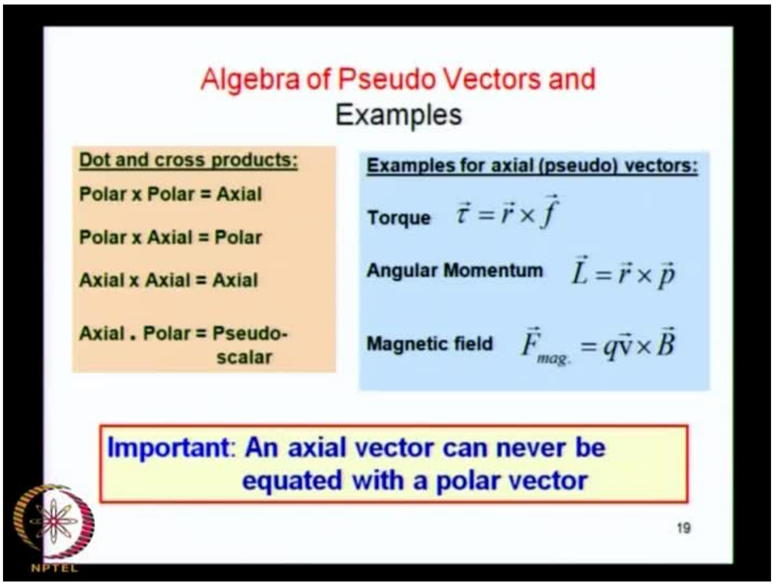

So, these are certain simple rules that one can keep track of. That if you take the cross product of polar vectors, you get an axial vector, but if you take the cross product of a polar vector with an axial vector, then you get a polar vector. Likewise, if you take the dot product of a polar vector with a polar vector, you get a scalar, but if you take the dot product of a polar vector with an axial vector, you get a pseudo scalar.

So, when you talk about the vector triple product of three vectors a dot b cross c, then b cross c would give you an axial vector, if all a b and c are polar vectors, then b cross c will give you an axial vector. Its scalar product with another polar vector of that is that product of an axial vector with a polar vector, what you get is really a pseudo scalar rather than a scalar. So, the volume which comes after the scalar triple product is actually pseudo scalar.

So, these are some of the physical quantities. Angular momentum is an axial vector, B is a magnetic field which is an axial vector, but the cross product with the velocity times of scalar is again a polar vector, force is always a polar vector.

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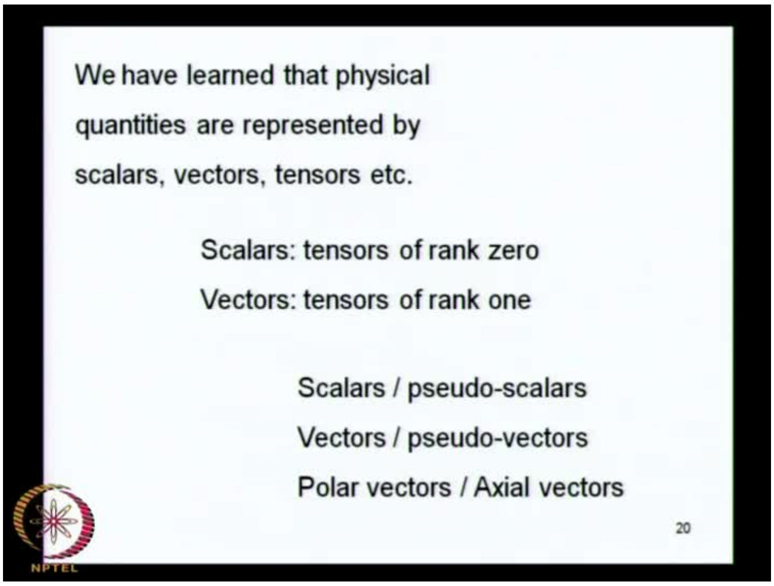

So, any how we have now learned that physical quantities are represented by scalars, vectors, tensors, etcetera, in the larger sense. We recognize scalars as tensors of rank 0, vectors are tensors of rank 1, we have got a little bit of introduction to the difference between scalars, pseudo-scalars, vectors, pseudo-vectors and etcetera.

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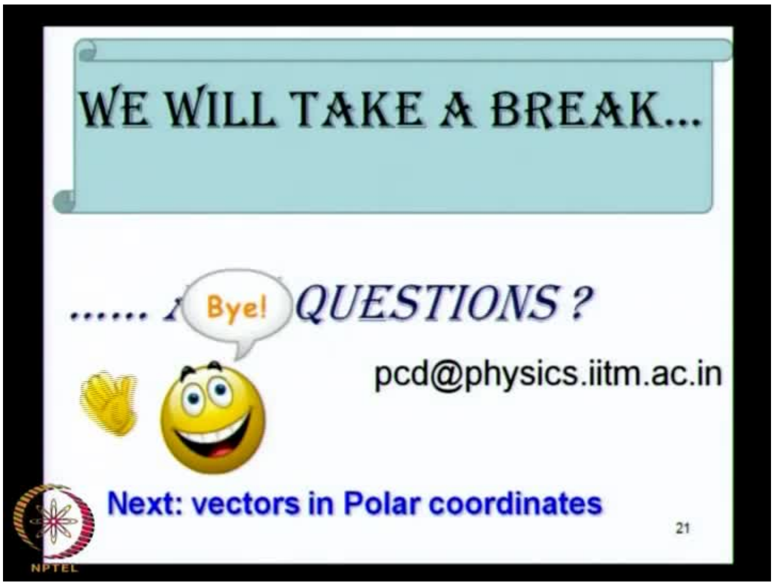

We will take a break here, if there are any questions I will take it. Then, in the next class, we will describe these in appropriate coordinate systems, when we will start exploiting the symmetry, which I mentioned earlier. That depends how the configuration is laid out in front of you at different symmetry, will suggest to you what is the most appropriate coordinate system you can use. So, I will take a short break, we will resume from this point.