Select/Special Topics in Classical Mechanics

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Module No. # 05

Lecture No. # 17

Real Effects of Pseudo - Forces (III)

Greetings. We will continue our discussion on the Non-inertial Frame of Reference.

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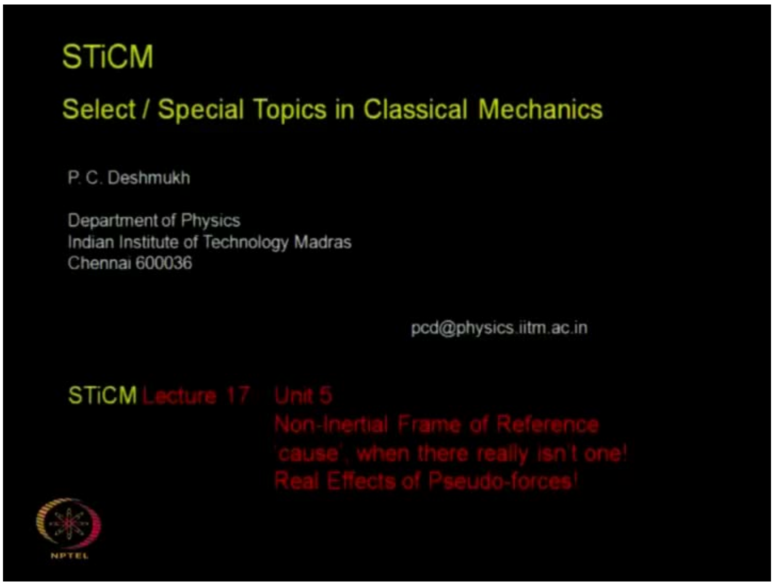

This requires us to invent a cause when there really is not one and we meet real effects of forces which do not exist. These are pseudo forces.

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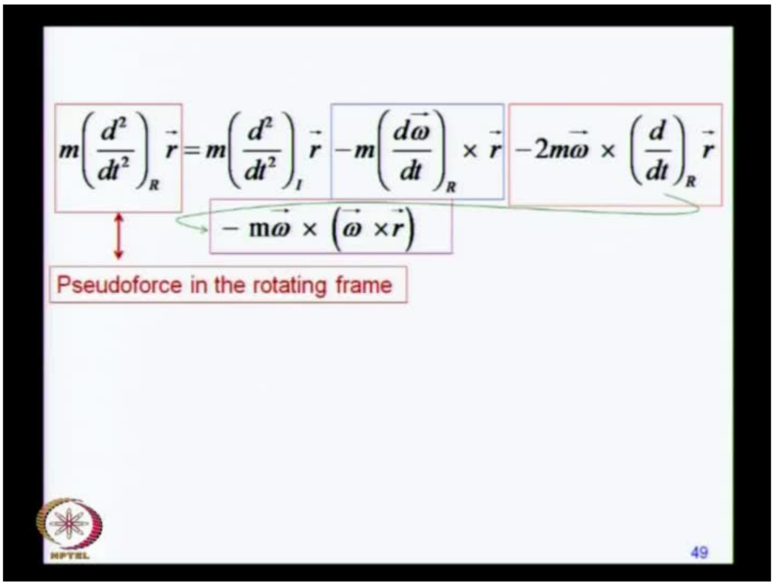

We will begin the discussion with the relationship in the rotating frame of reference for a force, which an observer would like to conceive, to explain the accelerations that he measures. To him, this acceleration is a real effect. He actually observes the position of an object; he notices that this object is no more in equilibrium; its equilibrium is disturbed and a measure of this disturbance of the equilibrium is the acceleration of the object.

Constant velocity is not what he thinks seeks a cause, because that is from his understanding of the first law of inertia. So, only when the velocity changes is he going to look for a cause which is what he will call as a force or a physical interaction. So, his perception of physical interactions is going to be completely influenced by what he measures as a departure from equilibrium. This departure from equilibrium is the second derivative of the position vector. So, this is the acceleration that he measures and he thinks that there is a cause to which this acceleration is proportional. He seeks a linear relationship between the effect which is this acceleration and the cause. He considers the inertia of the system - the mass, to be this linear proportionality and this mass times acceleration is his perception of the cause.

This is his perception of the cause which has generated the effect that he has actually seen. Then we have found through very straight forward transformations between an inertial frame of reference and a rotating frame of reference, that this left hand side is equal to the mass times acceleration in the inertial frame of reference plus a number of terms. There are three terms that we see: One in this box, another in this box, and a third in this box (Refer Slide Time: 03:15). This is the only one which is a real force. This is the physical interaction.

The right hand side has got three pseudo forces: 1, 2, and 3. These three pseudo forces plus a real physical interaction make up the left hand side which is a superposition of four elements, out of which only one is a real physical interaction; everything else is a pseudo. So, the left hand side is also as a pseudo force; it is not a physical interaction.

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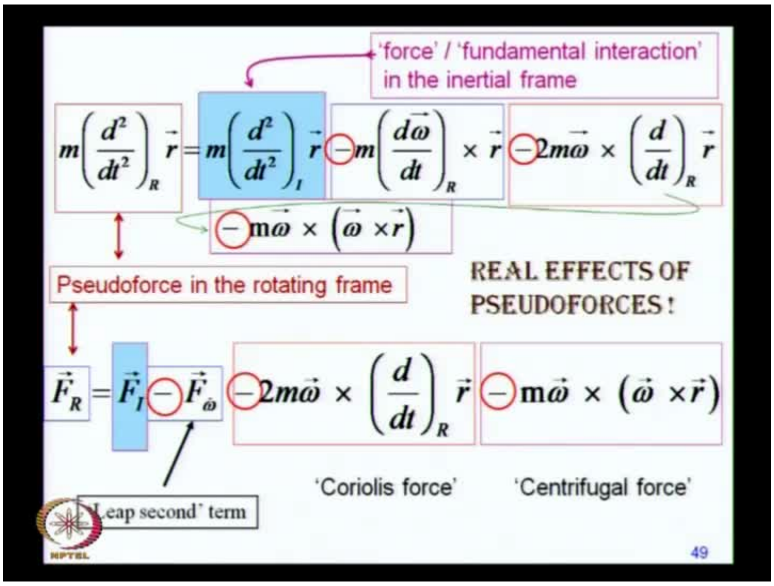

Few things that you must keep in mind: These are the names with which these terms are recognized. this one in the inertial frame of reference is a real physical interaction. this is what will belong to the category of the fundamental interactions in nature. it can be gravity; it can be electromagnetic; it can be nuclear strong or weak or it can be some unification of these forces.

This is the only term which can be reduced to a fundamental physical interaction; all the other terms are mathematical artifacts of the fact that this observer in the rotating frame is making an attempt to explain the effects that he sees, the acceleration that he measures, the departure from equilibrium that he notices in an object in terms of some additional construct. So, he invents these terms so that he can sustain the cause effect relationship as a linear response relation. So, these are the real effects of pseudo forces.

This term in this blue box (Refer Slide Time: 05:18) contains the rate of change of the angular velocity of the rotating frame. In our case, when we are on a rotating earth, this is the rate of change of the angular velocity with respect to time. So, d omega by dt. So, this is written with F. A subscript omega dot; there is a little tiny dot on this omega which is to tell us that we are talking about a rate of change of this angular velocity and this term (Refer Slide Time: 05:50) is what is called as a leap second term. I will discuss these terms in further detail in this class.

The next term, this is a next term. This is the leap second term (Refer Slide Time: 06:05). This is the next term which contains the cross product of the angular velocity of the earth with the velocity of this object in the rotating frame. So, this is the velocity of an object that this observer sees. So, in our frame of reference, now that we are in a rotating frame of reference the d r by dt for this object when it is sitting over here is 0 because this object is now at rest. This water bottle which I have kept on the table is now at rest in my frame of reference and I am in a rotating frame of reference with each one of you. This velocity is 0. So, this omega cross d r by dt term for this object is 0 in my frame at the moment.

If I throw it and if it is in motion, now during its transit, it has got a velocity and this velocity is d r by dt in the rotating frame. That I will measure and this will contribute to the omega cross d r by dt term; not when it is just at rest on the table, but when it is having some motion like this. Then, there is another term which is the last term which is omega cross omega cross r. So, this term is because of the velocity of this object when it is in transit. This is called as a Coriolis term and the last term which is a cross product of these three vectors - omega cross omega cross r. So, this is called as a centrifugal term. I will comment on these terms in this class. We will discuss these terms further.

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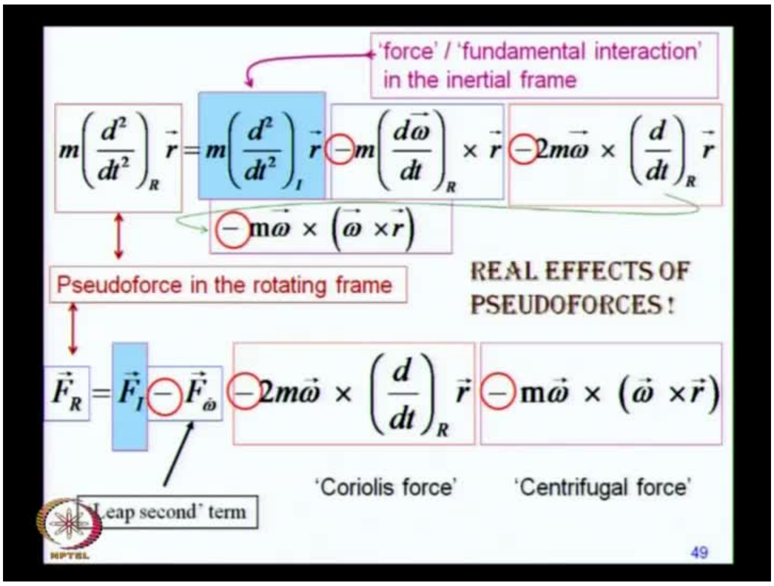

Remember that this is the only term which is a real force, which is a real physical interaction. This is the fundamental interaction. The other point I want you to notice is these minus signs over here (Refer Slide Time; 08:22). Force, of course, is a vector and if you do not keep track of these minus signs, you will have a force in the opposite direction. So, all the analysis will go wrong if you do not keep track of the sign. So, these terms - Coriolis term, the leap second term, and the centrifugal term are defined along with these minus signs. So, they are included in these rectangular boxes which I have placed around these terms.

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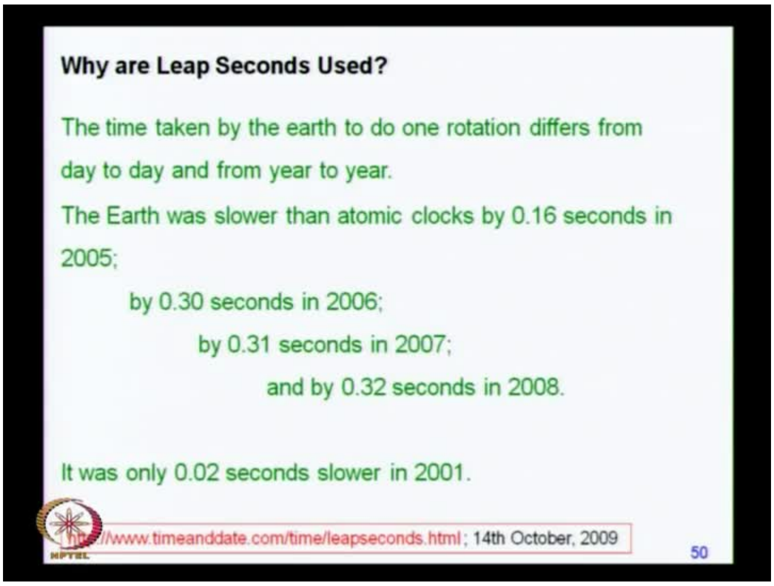

Now, let us discuss these terms one by one. First, the leap second term. now this is coming from d omega by dt which comes from the rate of change of the angular velocity now in common language, we talk about earth completing 1 rotation in a day and this is a reasonably accurate statement, but not very precise; not fully accurate.

The reason is the time which the earth takes to do 1 complete rotation through 2 phi radius - 360 degrees; it actually differs from day to day; it was different yesterday than what it is today and it will be different tomorrow. The difference may be very little, but there is a little bit of difference. The other thing is this difference is not the same every day. Like if the variation was by a certain time interval delta t yesterday, today the variation may not be the same delta t; it may be different. It does change from day to day.

If you consider the atomic clocks which are accurate time keepers, these are extremely accurate high precision time keepers and today's atomic clock technology is very advanced. It makes use of very cold ultra-cold atoms and you can measure the time very accurately using these atomic clocks. They can be used as standard for measuring time. Time being a fundamental quantity, it is obviously an important parameter to standardize so that you can do the calibration of all your clocks with respect to some standard time.

Now, with respect to these atomic clocks, the earth was slower by 0.1 seconds in the year 2005. 0.1 Second over 1 year may be not much, but then it is there; it is not 0. It was 0.30 seconds in 2006. So, it is not the same year to year, and therefore, not the same day to day. It was 0.31 seconds in 2007; 0.32 seconds in 2008; whereas in 2001, the earth was slower than the atomic clock only by 0.02 seconds. So, there is no easy thumb rule that you can use.

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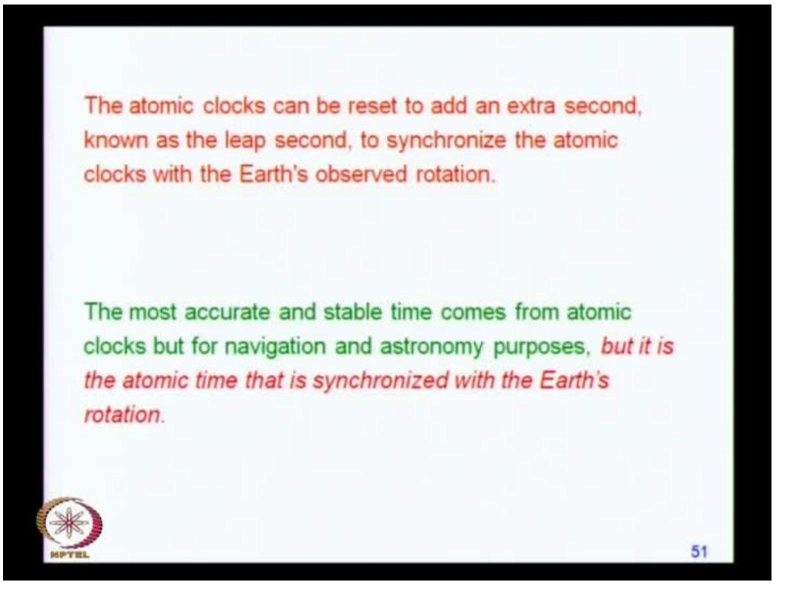

Now, what you can do as scientist and engineers, you can reprogram the atomic clock and reset. It is just the way you can take your clock and reset it whenever you think that it has gone fast or slow, you can reset it. You can reset the atomic clock so that it synchronizes the atomic clock time with the earth's observed rotation time. Now, obviously, the atomic clock is the primary standard, but it is this that you reset so that it synchronizes with the earth's rotation time because it would be very difficult to do the opposite.

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So, it is the atomic clock which is reset and this is done every once, so often. You and I do not decide when this has to be done. This is done by the international earth rotation and reference system service; this is abbreviated as IERS, but there are 2 R's in the expansion and 2 S letters at the end. This is the international earth rotation and reference system service. This agency is authorized to introduce the leap second correction and it does so, every once so often, whenever it deems it fit. The average difference between atomic clock and earth's rotation is about 1 second every 1.5 years; every 1 and a half year you will have the average this thing, but as we have already noticed that it is not the same every year. So, no big point talking about the average, but that is the order of magnitude.

The last one, the last correction which I think was made in the year 2008 on July 4th that a leap second was added to make this correction. This was in fact the 24th leap second correction that was made since this practice became common; since a year 1972. So, it is from the year 1972 that these corrections have been incorporated in time keeping and this has been done 24 times in about 44 in about 40 38 years.

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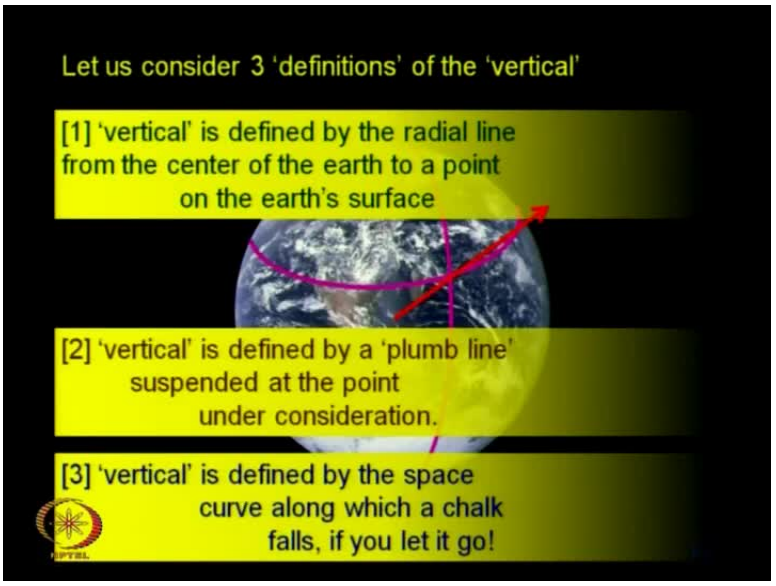

Now, in our last class, a question was asked if the centrifugal force can be a real force in some other frame of reference. I had hinted that you would get the answer if you would consider three definitions of a vertical. So, let us discuss this further. We look at the earth and then we construct a longitude and latitude. So, you have a point on the earth surface and at the point of this intersection, you want to define what a vertical is.

So, what you do is define the vertical as a radial line from the center of earth to a point on the earth's surface. So, you have got a center of the earth somewhere here and you define the vertical by this radial line (Refer Slide Time: 16:12) and this looks like a perfectly reasonable definition of a vertical.

Let us think of another definition. You define a vertical by a plumb line which is suspended at a point under consideration and Vivek has brought a plumb line, but I think it is too big. It is not too big, but anyway, we have one right here. So, we can make a plumb line just about anything which has got a thread. So, let me just pull out this mouse and then just hold it in my hand, and here we have a plumb line (Refer Slide Time: 16:52).

A plumb line is a mass which is suspended to a string essentially. If you set it in motion, it becomes a pendulum. If you just hold it and it is now let me damp it and if it remains steady, then it becomes a plumb line. I define a vertical as the line along which this plumb line is oriented. Now, this is a perfectly acceptable definition of a plumb line. One would think that this line along which the plumb line is oriented is along the radial line so that the first definition and the second definition define the same space curve.

We consider a third definition: You take a piece of chalk, hold it, and let go; you hold it and let go. You take a piece of chalk or any mass; does not matter, you can take a piece of marble, or you can take water bottle or anything that you have in your hands; let go and define the vertical as the line along which its center of mass falls; you can define a vertical as the line along which the center of mass of any object falls.

Now, the question is - are these three definitions, do they generate the same space curve; that is, a certain curve or line which is either a straight line or a curve or whatever, some trajectory? Do these three definitions generate the same space curve? is a question that we will address. What we have to remember is that we are observing this in a rotating frame of reference. You and I are not in an inertial frame of reference whether we define the vertical by the geometry using the radial line. A radius of a sphere is geometry.

So, the first definition is a purely geometrical definition. The second and the third definition involve some physical observations and that is where the physical interactions will come into play. These physical interactions will be either real physical fundamental interactions or may be pseudo forces. So, we have to remember that the earth is rotating and this needs to be taken in to account when we consider definition 2 and definition 3. So, these are the three definitions of the vertical that we considered.

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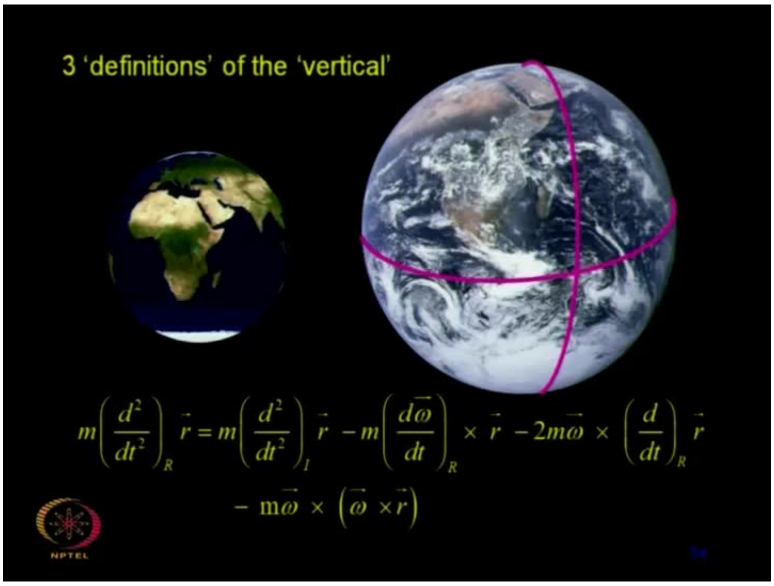

When we look at any object whether in our frame it is at rest as this bottle is at the moment or this chalk is at this moment as long as I am holding it, it is in equilibrium in my frame. But when I let it go, when I let the chalk fall or throw it, any which way, this way, that way, slowly or I throw it hard, the moment it has got a velocity, it has got this term which is d by dt in the rotating frame of the position vector of this object and I must consider this term (Refer Slide time: 21:23).

Whereas this term - the last term omega cross omega cross r, I must take into account not only at each instant of time if the object is in motion, but even when the object is not in motion; like I am holding this chalk; it is in equilibrium the velocity of this chalk in my frame and the moment is 0, but even then the omega cross omega cross r term must be taken into account. So, geometry is not bothered about any of these terms.

Geometry is just the radial line from the center of this sphere to a point on the surface of the sphere; that is a radial line; that is geometry. Geometry does not care for whether or not there is any physical interaction; lesser still if there is interaction, it is real or pseudo; nor does geometry care if the two of masses at all; geometry just connects a point at the center of the sphere with the point on the radius that is a radial line - that was our definition 1. But an object anywhere on the surface of the earth, because it has got a certain position vector with reference to the observers frame of reference, you must always take into account the omega cross omega cross r term, even if this object is not in motion relative to you. Without that, you cannot explain the acceleration of this object even if it is 0 in your frame of reference. Because the acceleration of the object in your frame of reference which is the second derivative with respect to time in the rotating frame of reference of the position vector in the rotating frame of reference; this acceleration even if it is 0 can be explained correctly not just by the first term alone, but by a combination of the rest of the terms. So, even if this term is ignorable, the rate of change of angular velocity, the leap second correction even if you want to ignore and even if this object is at rest as this chalk is when I am just holding it, you must take into account this term (Refer Slide Time: 24:02).

In other words, if I look at this plumb line, let me make a plumb line out of this little rat; that is why it is called a rat. Now, this plumb line if I consider, if I believe that this object is in equilibrium in relation to me because in my frame of reference, it is not accelerated. I can explain its equilibrium only when I consider all the terms that you see on the screen; that includes not only the gravity which is the first term, which is the term recognized in the inertial frame of reference, which is the real physical interaction, but then, by the remaining three terms in which if I were to ignore the angular, the leap second term, and the next term which involves the velocity of this object omega cross d r by dt. If this term goes to 0 because this object does not have any velocity in my frame of reference, I still cannot ignore the last term which is omega cross omega cross r.

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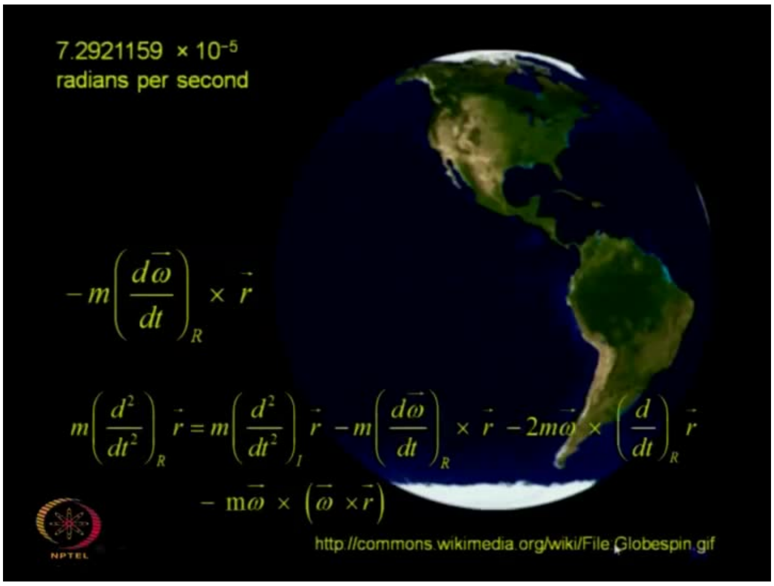

So unless I take that term in to account I will not be able to explain the fact that this object is in equilibrium in my frame of reference. Let me connect the mouse back. So, the plumb line now becomes a mouse. So, this term (Refer Slide Time: 25:46) will always have to be taken into account. This term has to be taken into account if and when the object under observation has got a velocity.

So, when I am holding the chalk, you do not have to take it into account because the chalk is at rest in your frame of reference. When I let go, when it is moving in my frame of reference, no matter what its velocity is – constant, changing, whatever. The moment this object has got a velocity the omega cross d r by dt term must be taken into account.

Now, the leap second term correction is important and it is for accurate time keeping. The atomic clocks are reset; that is certainly true, nevertheless the correction is marginal for most of the other applications where standardizing time is not the most important thing under consideration. For other applications, the rate of change of the angular velocity of the earth is ignorable because roughly speaking the earth rotates through like 7.292. This is a fairly accurate number as you can see; 10 to the minus 5 radians per second. So, this is a fairly small number. It does not change a whole lot. Whether you talk about its change or its remaining invariant depends on how accurately you want to do time keeping; that is of course important when you want to come up with time standards.

So, in the context of resetting atomic clocks, it is important but most of the other applications it is not so important. So, in general you can ignore this term d omega by dt.

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Now, let us look at some of the other terms. To do that, let us take a look at the picture of the earth and the moon. This is a beautiful picture this is the first picture of the earth and the moon ever taken from another planet and this was taken by the probe which was launched several years ago. This picture was taken in 2003 on the 8th of May by the Mars Global Surveyor - the MGS that is called. MGS has taken this picture. This is the earth that you see in this picture and this is the moon taken that you see in this picture and here are these two masses and they are obviously attracting each other (Refer Slide Time: 27:34)

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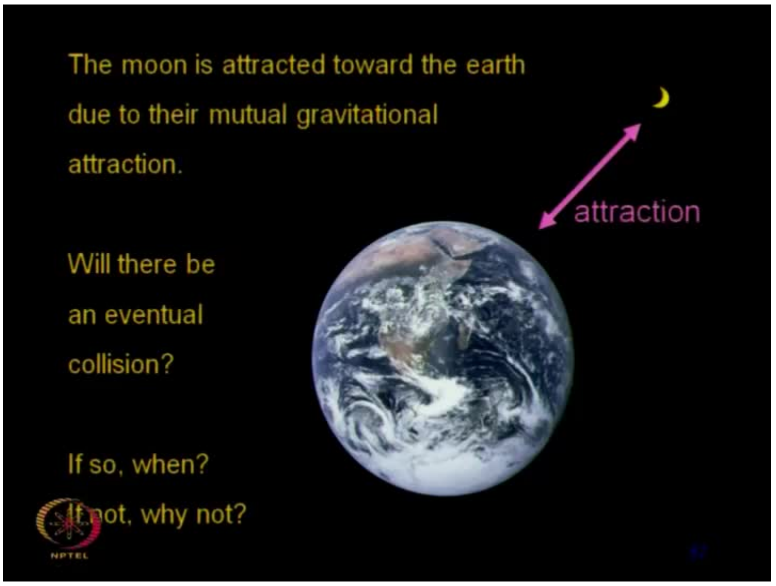

Here is a mass. This is a moon. This is the earth. There is mutual attraction. This is gravity. This is G m1 m2 by r square. What do you expect between two objects which are attracted towards each other? Here is an object. Here is another object. These are attracted to each other. The force of attraction will accelerate them towards each other and they would collide eventually.

So, will there be an eventual collision between the moon and the earth? Scary thought. You cannot switch off gravity. If it is an electromagnetic interaction, you can put some insulators. What do you do for gravity? There is no insulator gravity.

Will there be an eventual collision? I have some plans for tomorrow and also for day after. So, when will this collision be? If there will not be any, what is the reason? When I raise this question in my regular classes, I always get one answer – No. Do not worry about it; it has not happened for millions of years; it would not happen at least in the near future. So, that is the first answer I get which is consoling, but the most common answer that I get is that there is a centrifugal force which is going to keep the moon from falling in to the earth.

Now, who is exerting the centrifugal force? We know that gravity is pulling these two objects toward each other. So, there is an agency namely Gravity. This agency results in a force of attraction between these two masses, as a result of which these two objects will be attracted to each other and fall into each other. If centrifugal force in whatever frame of reference you want to consider is a real force, if it is a physical interaction, if you suspect that there is some other frame of reference in which the centrifugal force will be a real force, if you were to believe it, then you need to ask yourself - what is the physical agency which generates it? It is obviously not gravity because gravity is actually pulling things toward each other rather than preventing the collision. Gravity is acting in just the opposite direction. It is of course, not electromagnetic interaction; it is of course not nuclear strong or weak interaction; what else is it?

It is not a force that you can reduce to any one of the fundamental forces in nature; 4 or 3 or whatever unification level you talk about. It is not a force that you can reduce to one of the fundamental interactions in nature. There is no frame of reference in which this can attain the status of a fundamental interaction. Your interpretation of a fundamental interaction may however change in an accelerated frame of reference. So, you may associate accelerations with gravity and that will change your notion of geometry of space and time.

These are subtle and deep issues which I will not get into, but as you can see, this is not a real force; the centrifugal force is not a real force. There is no physical agency which is generating it. It is only a mathematical artifact of the fact that we are trying to interpret our observations in a rotating frame of reference, in a frame of reference which is accelerator.

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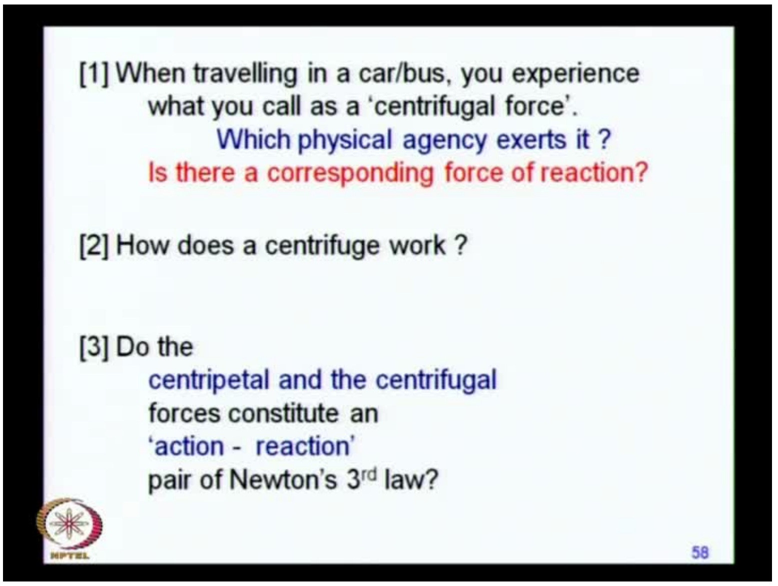

So, let me leave you with these few questions: When you are travelling in a car or a bus you experience what you call as a centrifugal force when the car is turning. I recently had an experience when my car skids. The question is which physical agency exerts it? If there is a force of action, is there a corresponding force of reaction?

You should also ask yourself - how a centrifuge works? A good number of students over here will be doing experiments in the chemistry lab in which you make use of a centrifuge. Try to explain how a centrifuge works and how does it result in the separation of various objects.

Explain this carefully without using the idea of a centrifugal force because a centrifugal force is after all a pseudo force. The separation is real. So, how would an observer in an inertial frame of reference explain the separation of objects in a centrifuge? Give it a thought and this is a good exercise to do.

The third question, I in fact already answered it, but very often when I raise this question in a class, If a centripetal and centrifugal force constitute an action reaction pair? The answer I get is yes, but we already discussed it that it is not so. It is obviously not so. They do not constitute an action reaction pair. Action is a force by one object on another and reaction is a corresponding force generated by the other object on the first. This being a mutual interaction, it is a mute point as to which object you call as a first object and the second, the other object is a second object; that is a mute point. So, it is a mutual interaction between two objects one on the other and by the other on the first, and they obviously do not constitute an action reaction pair. You need to understand this.

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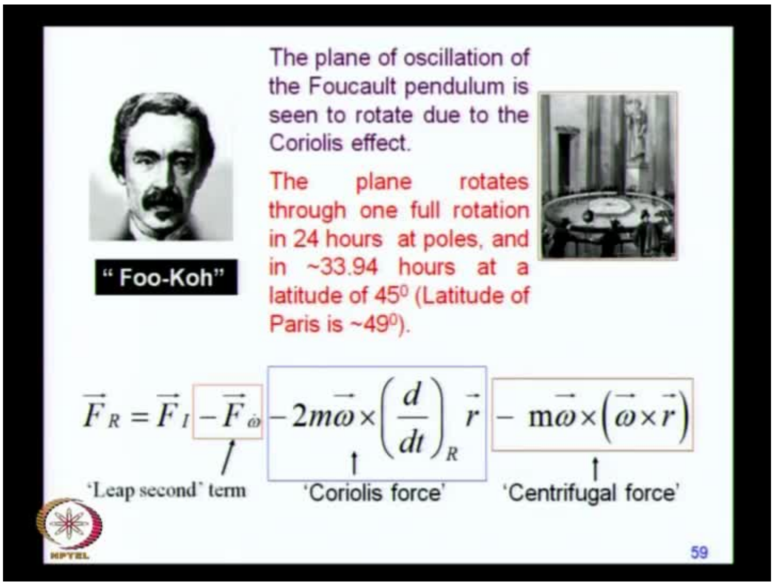

We will continue our discussion on these terms. This is the leap second term which we will ignore for most purposes. This is the Coriolis term as it is called. This is coming from the velocity seen of an object in the rotating frame. so this chalk has got a velocity which is 0 at this moment, but if I throw it, if it is in motion any motion whatever it is, this term must be taken into account and the last term is the centrifugal force.

Let us complete the discussion on this moon earth business because I think it is important because if the moon were to hit us tomorrow or day after002C I think our plans to go for a movie this weekend will have to be abandoned, which is the most tragic thing which even Shakespeare would not have imagined. So, if the centrifugal force does not exist, and the moon is not going to hit the earth, hopefully and thankfully so that we can all go for the movie this weekend.

Why is not the moon coming? There are these two forces. There is gravity between these two masses and the question remains to be answered; why is it that these two masses, if they are attracted towards each other, why do they not get accelerated towards each other and eventually collide? Why does that not happen? That can be answered very well by throwing pieces of chalks in the classroom which I always do to illustrate this point. I am going to throw this chalk. So, I hope that the camera can shoot this. I am going to throw this chalk toward Vivek and he is sitting here on the first bench, but I may throw it behind him as well.

So, let us see if you can shoot this. So, I take this piece of chalk and throw it towards one. One more you need to dive Vivek. Alright; not yet alright. Gagan alright. So, let me throw it one back there no it hit the sealing alright now Preeti you missed it could you get this on the camera all of them you want one more let us throw it toward Vivek and now Gagan and Prethi no alright further back are what is your name behind Preeti.

Oops I am sorry I did not I did not need to hit you I am sorry

Anyhow what do you notice? Depending on how hard I throw, it either reaches Vivek. If I throw it harder, it goes beyond him and gets to Gagan. If I throw it harder, still it gets to Preeti and if I throw it harder, still it gets to your name Uma. So, depending on how hard I throw, it goes that much farther. That was the idea I wanted to illustrate. Now, think about the earth being a sphere and you have an object; you throw it and it falls here. Now, you throw it harder, it falls here. From here you throw it again, but this time harder and it falls here. You see how it is travelling further and further, and if you throw it harder still, then it will fall here. Keep falling, keep falling, but it has missed the earth surface. So, it keeps falling, it keeps missing the earth surface; keeps falling keeps missing the earth surface; keeps falling misses the earth surface and keeps going round and round, and round and round, and round and round, and round and round for ever and ever and ever and ever again.

So, do not cancel the tickets for the movie if you have booked and if you have not booked, please book them now.

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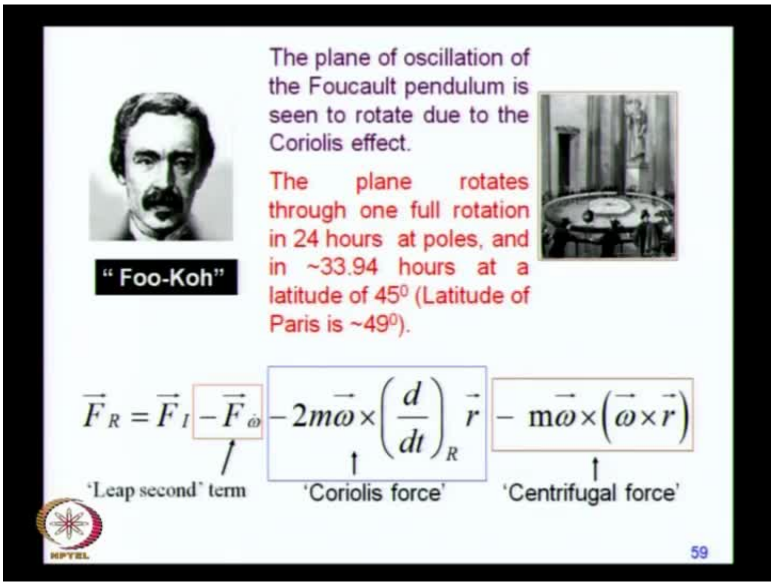

So, in explaining why the moon does not fall in to the earth as a result of gravitational attraction, we did not make use of any centrifugal force.

So, the centrifugal force is not responsible for keeping the moon from falling. Its initial condition prevents it from falling into the earth. It keeps falling, but keeps missing the earth. But if you and I want to explain this in our frame of reference, now think of an observer who is here and he is in a rotating frame of reference and he sees the moon over here and the moon is accelerated toward this, but then the moon is not falling because its motion is determined by the initial condition. So, it is keeping round like this while it falls, but if this observer is also moving constantly and if his rotation is synchronized with the rotation of the moon, then what is he going to see? He is going to see the moon there at the zenith. Let us say right above his head. At some point, he is a student and therefore, awake at night and sees the moon at the top at the zenith.

He sees that the moon is not falling and as the moon keeps falling, but keeps missing. It has crossed, but this fellow is also crossed. So, he continues to move. See it at the zenith all the time and then he is going to say that gravity is attracting it. I know because I know Newton's law G m1 m2 by r square. But I know it is not falling towards me. I always see it at the same point. So, he will invent a cause which he will call as a centrifugal force. So, he has to invent a cause which does not exist. The effect that he sees that the moon is not falling is a real one, but he can explain this only by inventing an unreal cause. So, this is the centrifugal force which is a pseudo force.

Then we need to discuss another pseudo force namely the Coriolis force which is a force that comes into play when the object - this chalk, which I was throwing when this was having a velocity, this must be taken into account.

Now, this was demonstrated by Foucault in what is known as a Foucault pendulum and this was a wonderful experiment that was done in France. What this experiment did was to demonstrate that the earth is actually rotating because you and I do not see the rotation of the earth. We just sit here and we see the rotation of the sun and the moon around us. Only if we were to stand outside, we would see the rotation of the earth. So, we do not really see the rotation of the earth.

So, why would you believe that the earth is rotating? I want you to do an experiment sitting in this room and convince yourself that the earth is rotating.

If you were asked this question and this is a question which needed to be addressed. Foucault demonstrated that the earth actually rotates by carrying out this experiment in which he had what is called as a spherical pendulum which can rotate in the x y plane and the plane orthogonal to that as well. What he found is that the plane in which the pendulum oscillates; like if I take a pendulum, if I take a plumb line and set it in motion, it becomes a pendulum. Here, I have set it in motion in this plane. I can also set it in motion in this plane. I can set it in motion in some other plane.

Now, you get the idea what I mean by the plane of oscillation of a pendulum and this plane of oscillation you can see from where you are. You are able to see this in this lab, in this room, in the classroom. You do not have to go outside the earth to see this.

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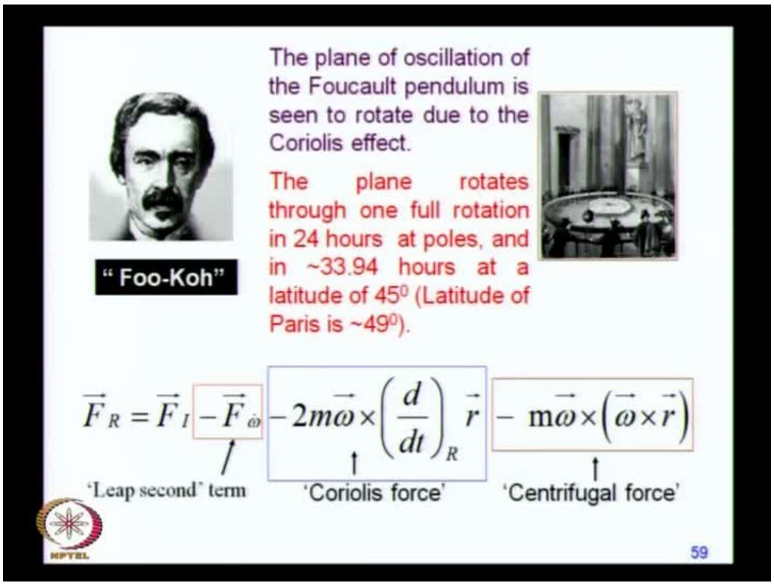

What Foucault did was to demonstrate that the plane of oscillation of this spherical pendulum actually turns. It does not remain steady. It does not remain constant and whether it is this plane or this plane you can observe, notice and judge right here. He could use this to demonstrate that the earth is actually rotating. This is coming from the fact that when the pendulum is moving, this is a plumb line which is not moving as long as it is steady, but the moment this is set in to oscillation, the plumb line becomes a pendulum and the plumb line is not in motion. It is steady, but the pendulum is in motion. So, the speed of this, the velocity of this object in my frame of reference, in the rotating frame of reference d r by dt subscript r which is the term that you see over here.

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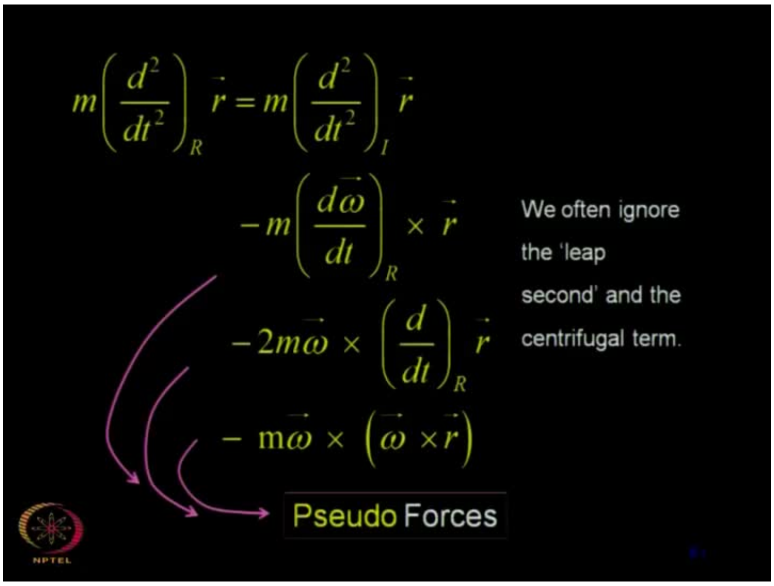

This is d r by dt. This is the velocity of this object and when this object is oscillating, it has got a velocity. When it has this velocity, the Coriolis term must be taken into account to explain its motion. This term is responsible for the fact that the plane in which the pendulum oscillates, rotates over a period of time and the physics of this is explained in terms of the Coriolis term who lived between 1792 and 1843. The experiment which demonstrated the earth's rotation using a term, which is a Coriolis term was performed by Jean Foucault. These are the various terms that we are going to talk about, of which these three terms are pseudo forces. This is the only physical interaction on the side.

Let us just ask ourselves if a pseudo force is a pseudo vector. No. It is not; it is a polar vector; like all the other vectors, you do not add a pseudo vector to a polar vector. It is a pseudo force. It is a force and a vector and a polar vector, but it is a pseudo force because it does not come from any physical interaction. So, the word pseudo has different connotations in different context and you have to keep track of that.

We ignore the leap second term and the centrifugal term. Now, why do we ignore the centrifugal term? The angular velocity if you remember was 7.29 something; 10 to the minus 5 radians per second; so, 10 to the power minus 5 and 7.3. It is a small number; square of it will be smaller still. So, in the centrifugal term, you have a quadratic term in the angular velocity. It is omega cross omega cross r. So, when you expand this vector triple product, you will have quadratic terms in the angular velocity omega square; whereas, in the Coriolis term, you have only a linear term. So, no matter how weak this is, it is certainly more important than the centrifugal term because the centrifugal term is quadratic in omega.

The Coriolis term is linear in omega. Omega being small, very often you can ignore the centrifugal term along with the leap second term. But for objects in motion which have got velocities, the Coriolis term is the dominant of the pseudo forces.

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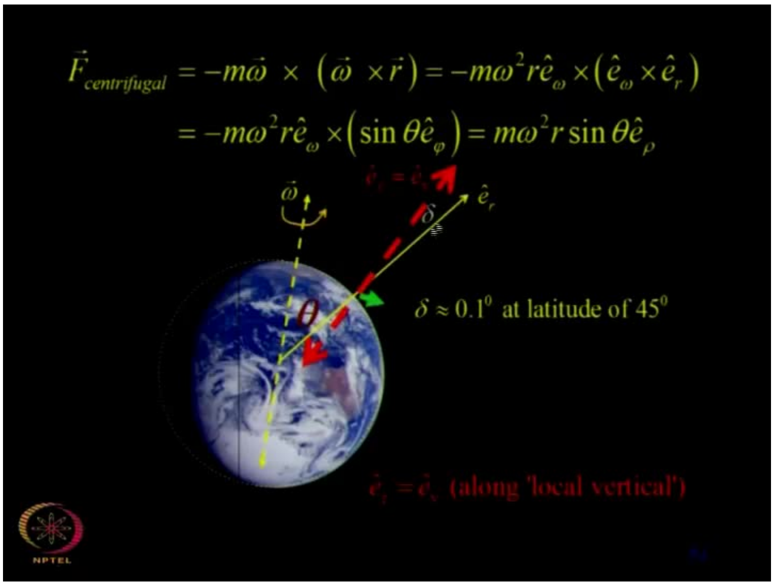

So, this is the centrifugal term. We have agreed that it goes as omega square. You can see it. It goes as quadratic in omega. You can also see that it is along the radial unit vector. If you draw a radial unit vector of the cylindrical polar coordinates with the earth's axis as the polar axis, then the centrifugal force will be along the e rho of the polar coordinate system. If you think of this geometry, the angular velocity is along the earth's axis and you have got a radius vector. This is the geometrical line and this e rho is perpendicular to the earth's angular velocity direction. So, it will be along this tiny green little arrow. So, this will be the direction of the centrifugal term.

The centrifugal pseudo force will be directed away from the earth's axis, as a result of which the object the plumb line will be seem to be oriented along this red line rather than the radial line because the plumb line which is steady, which is static in your frame of reference must take into account the centrifugal term because it has got a position vector in your frame of reference. So, omega cross omega cross r you can determine explicitly. It will be along e rho and as a result of this, the direction of orientation of the plumb line will be given by the resultant of gravity which is along this. This is pointed towards the center of the earth and this centrifugal term which is away. So, it is given by the triangle law of addition or parallelogram law of addition; one along this yellow line which is gravity and the other along this little tiny green line which is a centrifugal term. The resultant will be along this red line and there is a little angle delta between these two which is about 0.1 degree at latitude of 45 degree. It is not a whole lot; it is not 0.

So, depending on how accurately you want to do this analysis, you will have to take into account. But if you do not need that level of accuracy, you can ignore it. For moving objects, it is still ignorable compared to the Coriolis term because a Coriolis term is linear in omega that centrifugal term is quadratic in omega. This is therefore ignorable even if it is not 0, but ignorable compared to Coriolis term when the Coriolis term has to be taken into account.

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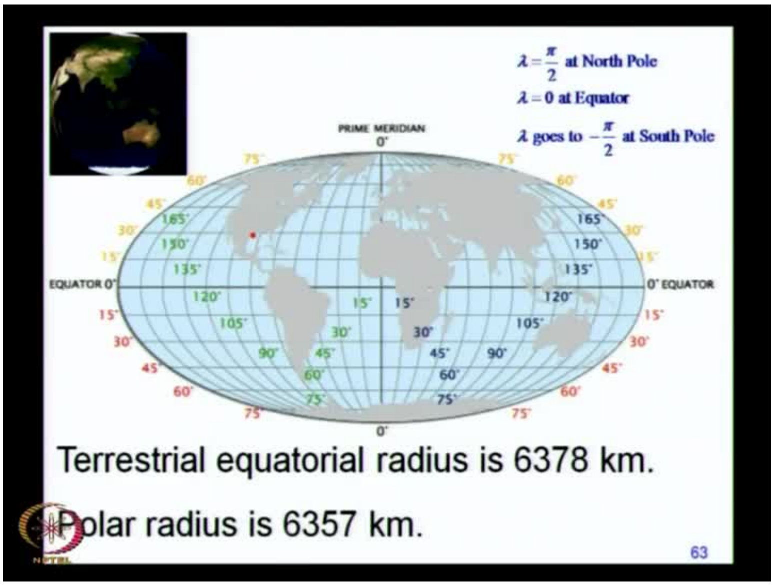

What the centrifugal term does is just the way a centrifuge works. It does this. It throws, it swings objects around in your frame of reference, and it does result in a certain bulge. This is consoling that there are other objects in the universe which have got a bulge in the middle; it is not just me alone. But the bulge is not very much over here unlike me perhaps because a terrestrial equatorial distance is 6378 kilometers which is this; whereas, a polar radius is about 6357 kilometers. So, there is a little bit of difference but it is not a whole lot; means if you look at the difference between 6378 and 6357, how much is it? It is about 21 kilometer and 21 kilometers in 6000.

So, how much does it come to? 21 in 6000. So, in a 1000, it is 6 of that which is a little more than 3. Then in a 100 it will be about one-tenth of that. So, it will be about 0.3 or something. So, it is like less than half a percent. So, it is less than half a percent difference between the polar distance polar diameter and the equatorial diameter. So, it is not a whole lot and that is part of the reason that you can ignore it in good number of cases.

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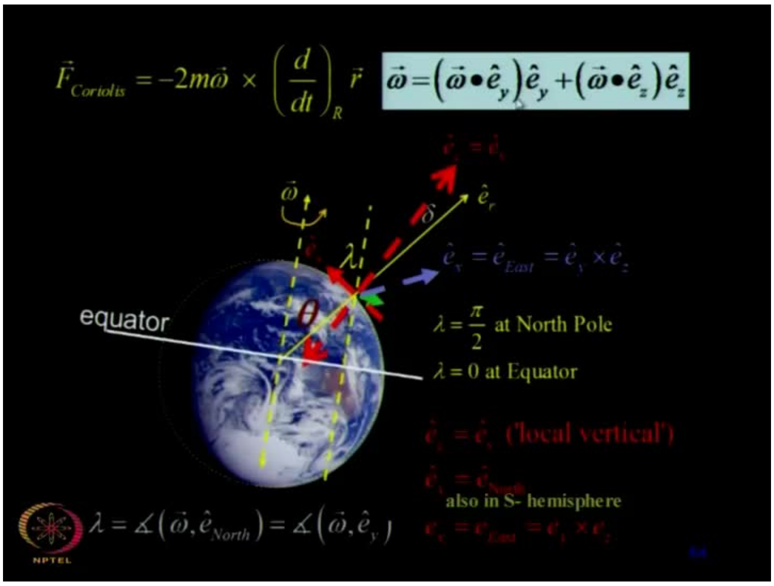

The Coriolis term, of course, will have to be taken into account. Let us see how to do that. So, you have the earth rotating like this as shown in this figure that you see on the screen and at the point at which you are carrying out this observation, let us draw a line which is parallel to the earth's axis of rotation. So, this is the line that I draw parallel to that and this is the direction which is a resultant of gravity. The centrifugal term there is a little departure of the angle delta which we have seen already. What you do is that at this point, you point over the north and you can always do that.

Where? From wherever you are sitting. If you are asked this question, point in the direction of north. Can you do that? You know where the sun rose from this morning and you have your east and if you are facing towards the east and you stretch your right hand and you stretch your left hand, then you know where the north is and where the south is.

So, wherever you are sitting, you can always point toward the north and this will be the direction of the north which is orthogonal to the z axis, which I choose to be along this red line; not along the yellow line.

So, I am using a Cartesian coordinate system in which e z is along the red line and e y axis of a coordinate frame will be of course perpendicular to that. So, it will be perpendicular and pointing towards the north. So, this will be your left hand pointing toward the north if you are looking at the east, when the sun is rising in the east. You can find out the north even in the evening, but that is very easy. In fact this was the remedy which was given to the Pandavas when they were in this [fl] for 1 year. You remember that story and Vidur gave them this advice that you will be travelling in the jungles. You will lose your direction. You will lose your way, but the sun and the stars will show you the way. They will tell you where you are and how to get back.

So, you can always locate the north and you choose an e y axis which is pointing toward the north. The angle between omega and the north; this is the direction of omega. This is the red arrow which is a local direction of north; it will change from point to point. This angle is what I call as lambda. This is the angle between omega and e y; e y is a unit vector in the direction of north at the local point. You can easily see that lambda is equal to pi by 2 at the North Pole. Lambda will be equal to 0 at the equator.

You can easily see that from this figure and in fact the angle delta being ignorable between this; red line being almost along this yellow line; you will notice that this lambda is pretty much the latitude at the point at which you are constructing this frame of reference. It is pretty much the latitude and what you do is having defined your e z and e y e x. It selects itself as a cross product of e y with e z. So, your Cartesian frame of reference gets fixed.

You choose e z along the local vertical. Choose the e y along the local north and e x selects itself as the cross product of e y with e z. The lambda which is the angle between angular velocity with the earth and the local north which is e y is almost equal to the latitude of your point and for all practical purposes it can be taken as a latitude.

Now, in this geometry, in this frame of reference, the angular velocity of the earth will not have any component along e x. It will be in the y z plane and therefore, omega will have a component along e y and along e z.

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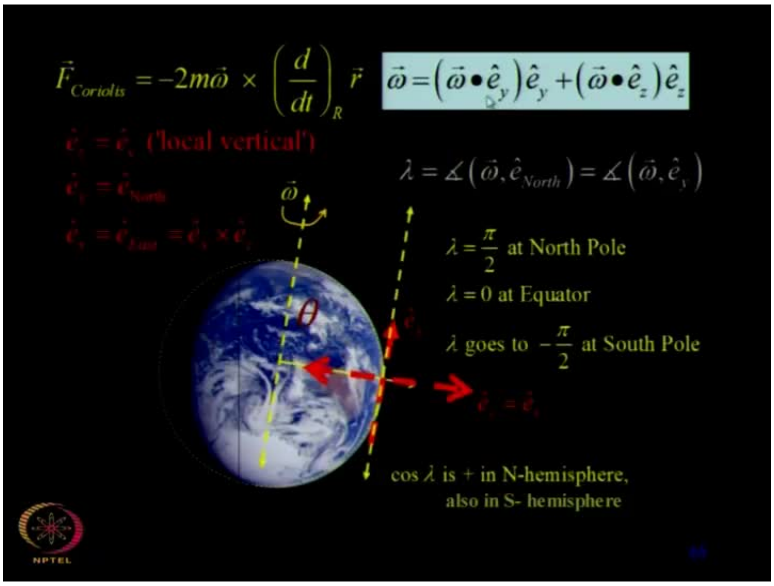

Omega dot e y will contain the angle lambda which is the latitude and this cosine term depends on the angle because the cosine of an angle between 0 and pi by 2 is positive.

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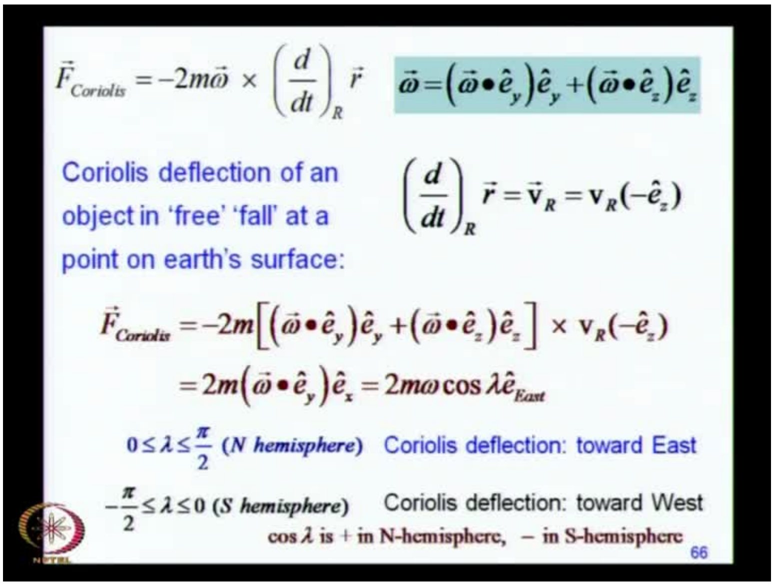

So, now it is very easy to estimate the Coriolis deflection because the Coriolis deflection is given by this term. We have included this minus sign. Remember, I had put a red circle around this minus sign, earlier in this class. So, it must be defined with this minus sign; otherwise, it will give you the wrong direction. So, it is minus twice m times omega cross the velocity of this object. So, if I have an object and it is in a state of free fall like this, then it has got a velocity which is falling along the local vertical. So, its direction is this is my e z; this is my minus e z. So, an object in free fall has got a direction along minus e z. So, this is the local velocity. v R times minus e z omega is this which we have already found. It has got a component along y and z.

So, the Coriolis deflection of an object in free fall will be given by the cross product of v R times minus e z and this. Now, that is very easy to determine. In your geometry, you just construct the cross product of this angular velocity. With the velocity of this object in free fall, e z cross e z will give you 0. So, this term does not contribute. It is only the first term which will contribute and here you have got a minus sign over here. So, minus e y cross minus e z. So, minus e y cross minus e z will be equal to e x which is your local east. So, the Coriolis deflection will be along local east as long as cosine lambda is positive.

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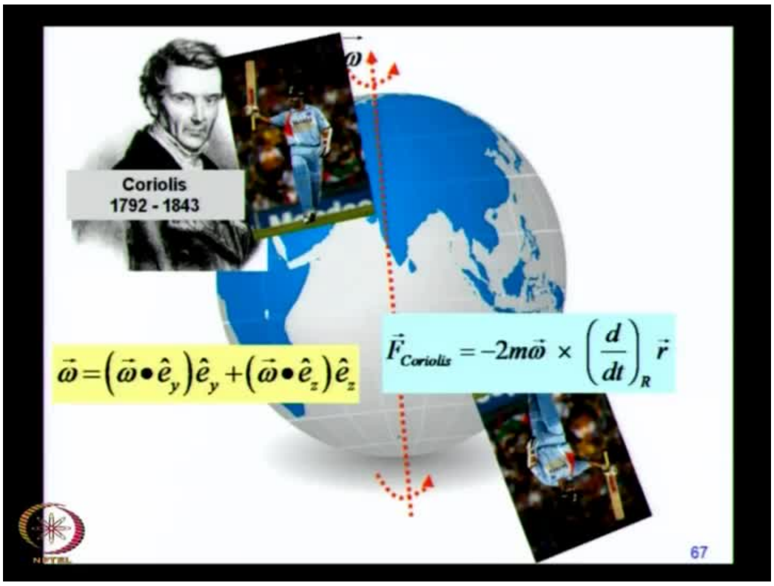

It may not be very important to find if Sachin Tendulkar hits a sixer and if he is playing in Europe in the northern hemisphere or in Australia in the southern hemisphere. How will the trajectory of the ball hit by him? It is a long boundary and he is not a physicist. So, he does not care, but you might. Will there be a Coriolis deflection of this ball? Because this ball is in motion, it has a velocity and yes it will undergo a Coriolis deflection; may be minor; be 0.

As a result of this deflection, these cyclones for example, the whirlpools that you see in northern hemisphere, in the southern hemisphere, one goes clockwise and the other goes anticlockwise for exactly this reason. So, it has important effects. At latitude of 60 degrees, an object falling through a 100 meters undergoes a deflection of about of 1 centimeter. So, it is quite significant and that is a reason for the cyclonic storms and all this.

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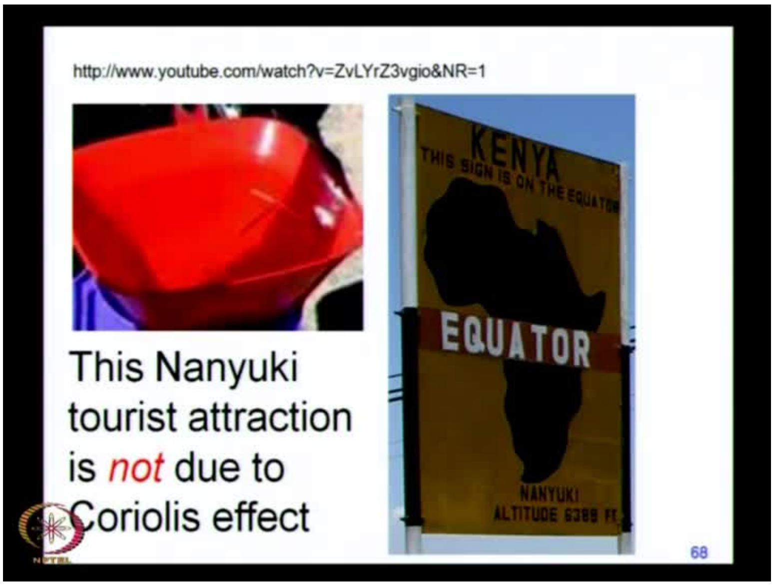

What about the equator? You already know that north of equator will go one way, south of equator it will go the other way. There is a tourist attraction in Kenya. There is a city called Nanyuki at an altitude of 6389 feet. A major tourist attraction there is that they keep get a funnel there and you can watch this on the YouTube at this link. You do not have to note down this link with all this Z and v and L and Y. If you just Google it, you will get it.

There are these people who will demonstrate what they claim is a Coriolis effect. So, they put water in a funnel and there are two small pieces of straw over here; one is I do not know if it is clearly visible over here; there is a piece of straw over here, another over here, and you actually see these pieces of straw going around in circles. What this guy does? He takes you there. There is a street which crosses at the equator. So, he takes you to one side of the equator and does this experiment and shows you (( )) this straw going around the whirlpool in one direction then he will charge you may be 50 or 100 dollars. I do not know how much he charges, but he will take you to the other side of the street make some money out of you and show that it revolves the other way around, but this is not Coriolis effect.

The Coriolis effect of course, it is a real effect. Although it is due to a pseudo force, it does result in visible perceptible deflections like cyclonic storms and so on, but it is not responsible for the motion that you see in the wash basin. In the basin, there are these vortex currents that you sometimes see and people claim that this is due to the Coriolis deflection. It is not the case because of the local geometry. You know the way these funnels are made. It is so crude or even a wash basin and the vortex and so on. It is really so crude. It is not at all sensitive to the Coriolis deflection.

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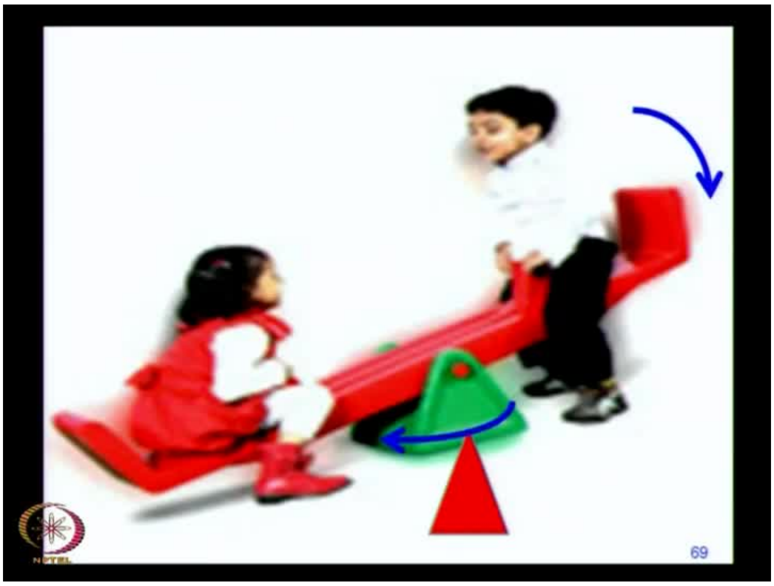

So, that is not the Coriolis effect. So, if you go to Nanyuki, save your 50 dollars or 100 or 10 or whatever, but people actually make money from the Coriolis effect and this is not only a pseudo force. In this case, it is even the effect is a pseudo effect; not just a force.

So, anyhow, now if you do not see this picture very clearly, it is not because you are very sleepy at the end of this class. It is because it is not in focus and you have two lovely children. This beautiful girl who is even having a matching ribbon and if you mount this see-saw like this and set it in rotation and watch the trajectory of a ball which one of these children throws at the other, takes a ball throws at the other. These children are in a rotating frame of reference. When they are going round, the see-saw is turning around and a ball is thrown and this object will be seen by these observers. To them, the trajectory of the ball which one child throws at the other is a real trajectory; its real effect.

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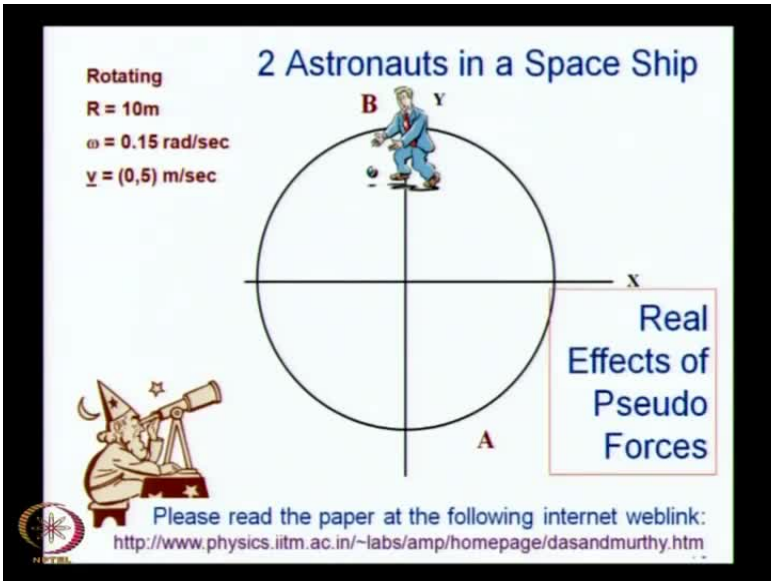

If you want you can think of two astronauts doing this experiment in a satellite and the satellite is given a little bit of rotation so that you know it generates an angular momentum that is usually given for stability, but we will not get into those things. This is a software which was developed by two of my students which is available at this link and you can read about it. They consider two astronauts one over here and astronaut A over here and the astronaut B over here. One fellow throws a tool at the other; could be a spanner or something. Then you ask the question - what will be the trajectory of this tool? or that ball which the two children are throwing at each other on a see-saw which is on a merry go round and the trajectories are amazing.

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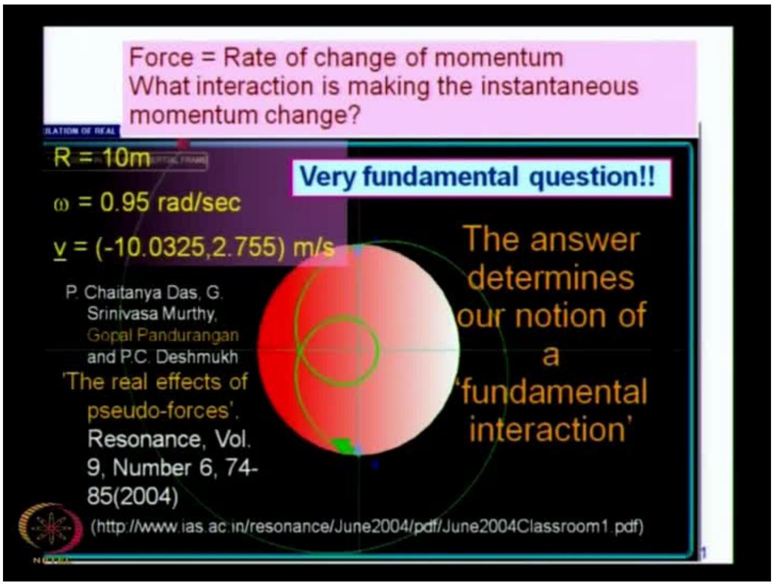

Look at this one for example, now this is software which was written by Gopal Pandurangan first and updated by Chaithanya and Shrinivas Murthy. If it is shown what this software allows you to do is to select the angular speed of rotation and also the angular velocity. So, it sits on x y frame and in the x y frame you can select the x component and the y component. Here is the result when they choose their x component to be point minus 10.0325 meters per second and the y component to be 2.755 meters per second and angular speed of 0.95 radians per second.

You can just play with this on the software and key the numbers whatever you like. So, you will get different trajectories and once the object is thrown, then of course in the plane there is no further physical interaction in which this object is involved. So, it must continue to go by inertia in one direction unless and until acted upon by some external impressed force. The object must sustain its state of equilibrium and it cannot to be seen to be changing. But what these two observers see is that because they are in a rotating frame of reference, this object one fellow throws this towards the other, but during the transit, these two observers are going round.

So, they do not see this object to be going straight. Depending on the speed at which the velocity with which this object is moving and depending on their angular velocity, they see strange trajectories. Look at this one here. The object actually seem to go through a loop inside and then reach the other fellow.

In other words, if you are with another astronaut in a satellite and you want pass on a tool to the other person, you will have to select your throw conditions very carefully. You cannot just look at the other guy and just throw it; maybe you can, but it really depends on the details and here is a simulation which is an exaggerated effect because on the software you can key in any angular velocity, any velocity for the initial throw, and you will get different answers.

Now, these are important issues to be considered because what do you see? These observers who are in a rotating frame of reference is a real swing in the trajectory. So, they see that the momentum which is going like this. Now, goes in this direction; now goes in this direction. So, they see the momentum changing continuously. If the momentum is changing, they must ask if the momentum is changing. It must do so at a certain rate which means that there is a dp by dt if p is a momentum. There is a dp by dt and dp by dt they know is force. So, what is a force which is causing this? It is not gravity; It is not electromagnetic interaction; it is not nuclear strong, nuclear weak; it is none of the fundamental interaction. This understanding between a real physical interaction and what is only an observed real effect in your frame of reference is a real consideration.

Here is an example of a tool thrown by an astronaut toward the second astronaut who is diametrically opposite with, but the satellite is spinning and you consider only the motion in the latitude in the horizontal plane - in the plane of the vehicle, or in terms of the ball thrown by one child towards the other. We consider only the motion in the horizontal plane. I am not talking about the parabolic trajectory in that. It is controlled by gravity. This is controlled by the throw. The vertical motion will be determined exactly by the local gravity. So, let us not worry about it.

So, here you see that depending on certain initial conditions, the trajectory can seem to be quite weird and this software developed by Gopal Pandurangan and Chaithanya and Srinivas allows you to choose the parameters for different angular velocities and different velocities of the initial throw. You can see very strange trajectories and as you see this is an extremely fundamental question because if you observe that the trajectory of this object is changing, you see that the direction of motion is changing; you see that the momentum of the object is changing; you conclude that the momentum is changing and therefore, it does so at a certain rate which is d p by dt. You would ask what is the force which causes this? There is no real force which causes this. There is only the effect that you see in your frame of reference because you are rotating. It is your problem; nature is not contributing to it.

Laws of nature are not contributing to it. You are in a state of frame of reference in which you see an effect which nature has not caused. Nature generates only the fundamental interactions, gravity, electromagnetic interaction, nuclear strong, nuclear weak or electroweak interaction, unification, grand unification, whatever. It is only the fundamental interactions.

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What will be the effect on rockets or on intercontinental ballistic missile.

Now, I do not know if you are worried about it, but someone does recognize this lady. No. At least my [fl] friends should. She is the missile woman of India and she is incharge of the [fl] program and quite recently just about few days ago on 17th may 2010, the [fl] 2 was test fired. This has got a range of more than 2000 kilometers and doctor Tessy Thomas is incharge of this project. Wonderful scientist and the project will develop into [fl] 5 which will have a range of 5000 kilometers which will bring targets in Pakistan and China in your range. I do not want you to use it.

But I want you to have that power and that power will not be potent if you do not take into account the Coriolis effect because here this is not just a cricket ball which is hit over like 80 meters or 90 meters for a big 6, but here you are talking about objects going over a range of 1000 of kilometers 2000 kilometers 5000 kilometers.

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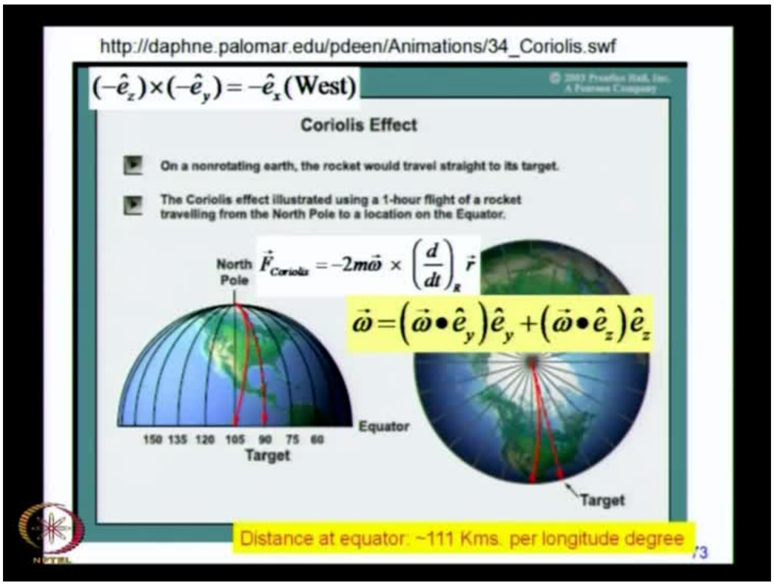

There is a nice software which you can actually play and this is available at this palomar dot edu website. So, you can just click and it will show you that if you fire a rocket from the North Pole and the designers of the software have taken care that they do not have any target on land. So, they hit it into the sea because even in imagination, you do not want to hit anybody. So, their intended target is over here at the equator, but during the trajectory of the rocket, the earth has turned and this term must be taken into account. This is the Coriolis deflection.

You know that the angular velocity is given. It has got these components e y and e z. What will be the component of this rocket? It is fired from north towards south. We chose e y to be directed toward North. So, this is pointed toward the South. e z will not matter because e z cross e z will give you 0. So, what will be required to be determined is this minus e z cross minus e y minus e z coming from here minus because of this sign.

Then there is a minus e y because it is pointed from North to South. So, there will be a direction toward minus e x which is in the direction of west. So, instead of landing at this point which is the intended target, this lands over here and this is quite a bit this is 90 degree longitude, this is 105 degree longitude, and what is the distance between these two points on the equator - the distance between a 90 degree longitude and a 105 degree longitude? There is a longitude difference of 15 degrees. This longitude difference of 15 degrees results in a lateral distance between the two points. That distance will be 0 at the North Pole, of course, because that is where they start launching out, but by the time these lines these longitudes get to the equator, there is a 15 degree difference at the equator. Equator is how much? Means equator is a parameter of the earth's radius which we just estimated to be a little more than 6000 kilometers.

So, 6000 twice by r is the parameter so 6000 into 2 12 times 3.14 or whatever is the pi. So, its three times - 12000. So, it is more than like 36000 kilometer. So, just order magnitude and 36000 kilometers over 360 degrees. So, that gives you almost like a 100 kilometers per degree. Actually, it is more than a 100 kilometer per degree. If you do it more accurately, it turns out to be almost a 111 kilometers per longitude degree and there are 15 degrees; so, 15 into 100. So, it will be more than a 1500 kilometers distance.

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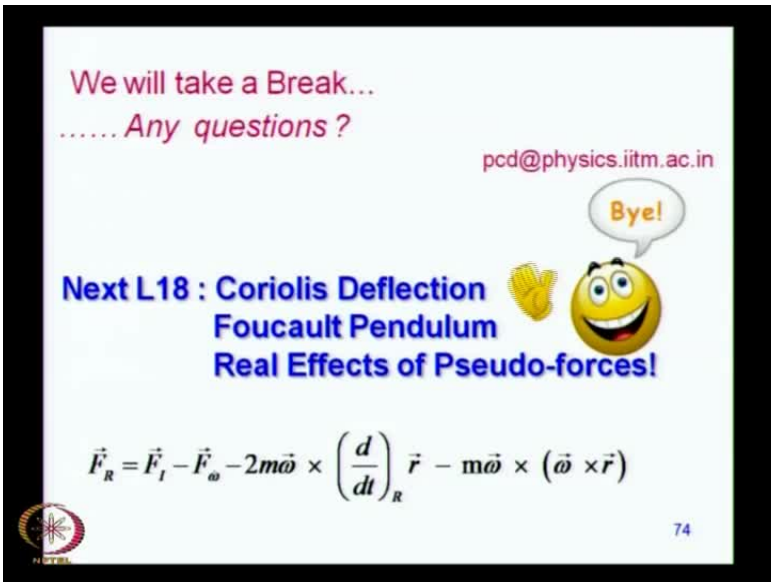

Now, you see how important it is to take into account the Coriolis Effect. 1500 kilometers will be the lateral shift. At this point, I will take a break. If there any questions, I will be happy to take; otherwise, we continue in our next class in which we will discuss the Foucault pendulum, further real effects of pseudo forces; I will continue discussion on that.