Select/Special Topics in Classical Mechanics

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Module No. # 05

Lecture No. # 18

Real Effects of Pseudo-Forces (iv)

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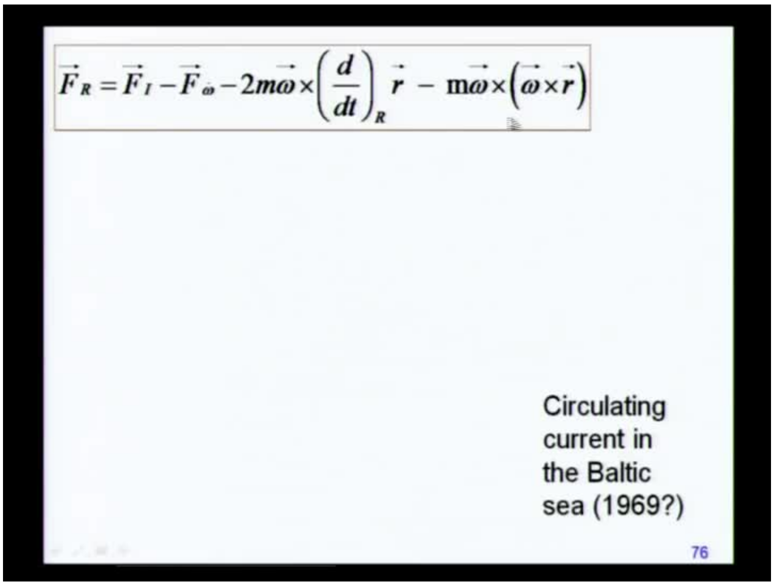

Today, we will discuss the Foucault pendulum in some details, but there are many other manifestations of the Coriolis Effect. We have already considered some of the other terms. So, let me quickly recapitulate this is the physical interaction in the inertial frame of reference with the subscript I, and the remaining three terms are pseudo forces. This is the leap second term which comes from the rate of change of angular velocity of the earth, and we discuss this in our previous class.

This is the Coriolis term; this needs to be inserted whenever we observe motion in a rotating frame of reference and analyze an object which is in motion relative to us. If it is not in motion related to us, we do not have to worry about it.

This d r by dt in our frame of reference. So, which is a rotating frame of reference? This velocity must be nonzero. If this is 0, this term obviously goes to 0, but if it is not 0, then unless we take this term into account, we will not be able to explain the dynamics of any object that we see in motion, and then, there is this last term which is the centrifugal term which is manifestly quadratic in the, in omega and omega being small something like 7.3 10 to the minus 5 radians per second. This term is rather ignorable but not 0.

It will be 0 only when the position vector r is 0, which is at the center of the earth or else when omega cross r is 0. So, when the sin of the angle between omega and r goes to 0, then, of course, it will be 0, or else where the cross products of omega with omega cross r goes to 0. So, these are some are the conditions under which this term can be ignored.

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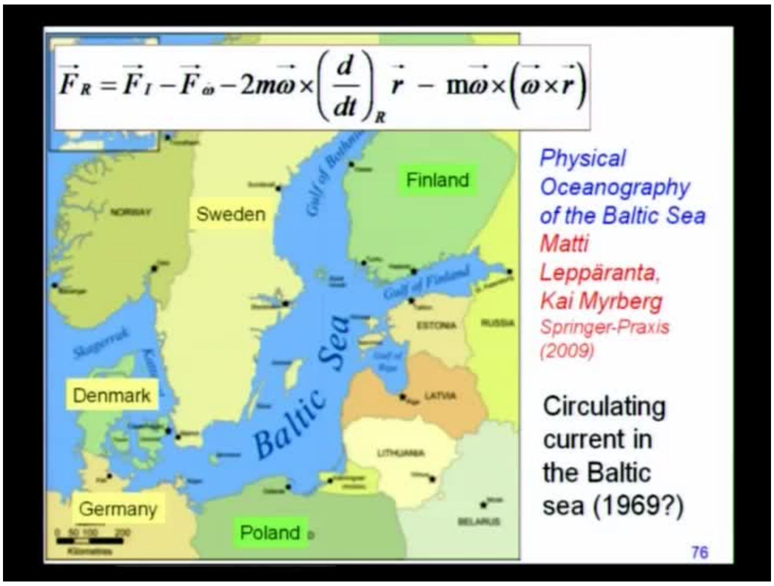

There are other fascinating illustrations of the Coriolis effect and I will like to draw your attention to very interesting circulating current that was observed in the Baltic sea, and I believe the first instance of this observation was in 1969, and this is the Baltic sea which is north of Germany and Poland and it is between Sweden and Finland. There was a circulating current and we discussed the Coriolis deflection of objects in the northern hemisphere in our previous class, and we found that in the northern hemisphere, there is a deflection of a falling object to the east.

The dynamics of this circulating current is very complex and I am certainly not going to be able to discuss that but I will refer you to a very exhaustive discussion in this book physical oceanography of the Baltic sea, in which, this has been discussed. So, those of you who are interested may read further about this particular very interesting issue, very interesting manifestation of the Coriolis deflection.

What it did is the current itself is constituted by large number of parameters. There are ocean currents, wind currents, density factors, pressure gradients, and on top of this, there is the Coriolis Effect which is not ignorable in the northern hemisphere or in the southern hemisphere. So, whenever an object is in motion and this current is a mass of water is in motion, and therefore, its dynamics will need the invention of an additional force namely the Coriolis force. If we want to understand its motion in our frame of reference which is a rotating frame of reference.

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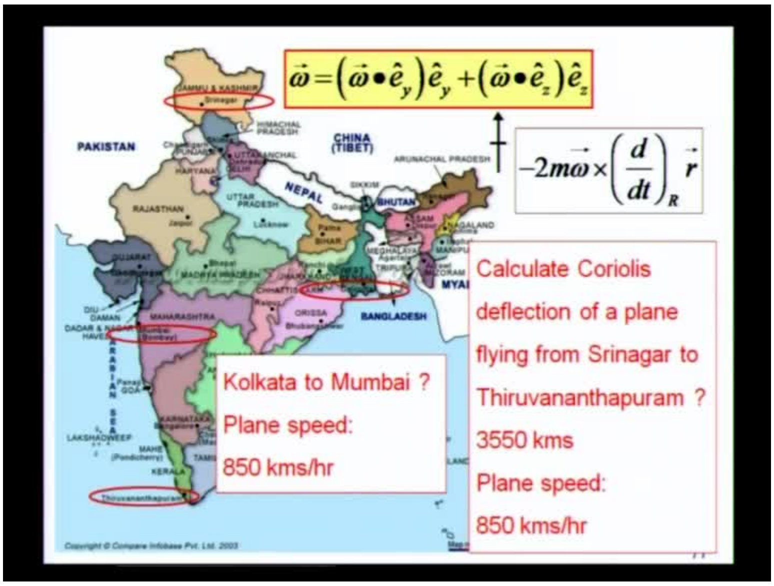

Likewise, we talked about rockets fired from the North Pole to the equator and we found that there is a deflection that they will undergo, and the deflection is quite significant. A rocket fired from the North Pole targeted at a point on equator would get deflected by as much as more than 1500 kilometers means, that is the number that we got after careful analysis yesterday.

This is of importance even for plane travel, that if, and I will leave this as an exercise that you may determine the Coriolis deflection of a plane which is flying from Srinagar which is here to Thiruvananthapuram which is over here, and you cross a distance of 3550 odd kilometers, and if the plane speed is like 850 kilometers per hour, you can see that it is travelling mostly north to south and it is a good exercise for you to work out and estimate the Coriolis deflection that this plane would undergo.

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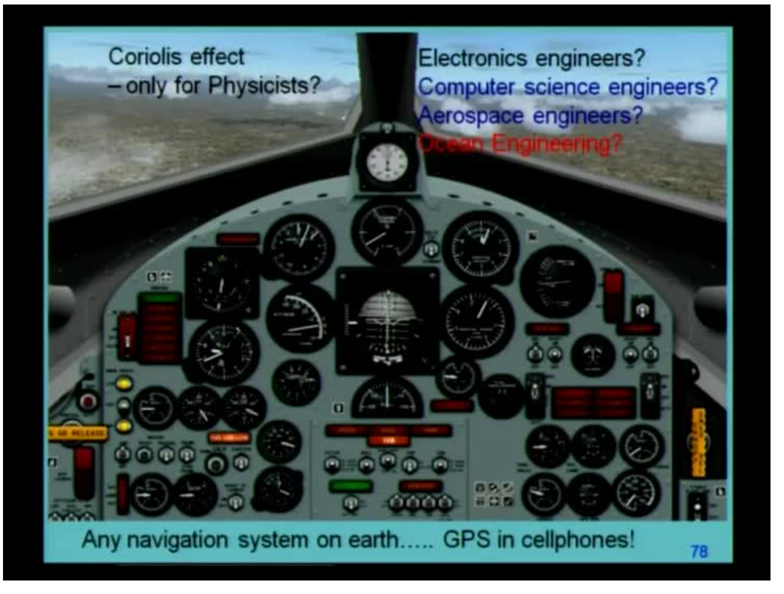

You could also do this for a plane which is flying from Kolkata to Mumbai after having some nice Misty Doi at Kolkata airport, and if you were to take this flight, then what would be the Coriolis deflection is something that you can try to determine, and it is obvious that unless you make these corrections on the flight course, you will not reach the intended target, and these corrections, therefore have to made continuously, electronically, then the navigation system must incorporate these features. The electronics is very sophisticated, and it is all there in front of the pilot, and it should quite obvious now that it is not just for physicists to learn about the Coriolis effect, but all kinds of scientists and engineers, whether you are in an electronics engineer or a computer science because ultimately everything goes into the has to be programmed to incorporate these corrections, and then, tell the pilot where he is and how to change his course.

Or if you are an ocean engineer, or in aerospace engineer, no matter what, in every branch of physics and engineering, mechanical engineering as well, one must study these terms in considerable detail so that you can get a good handle on the science and engineering applications of this particular phenomenon.

Navigation system, the GPS, which is there in your cell phone. So, even if you are not a scientist or an engineer, you are still using these corrections in some way because you are using the cellphone and the cellphone must incorporate all of these corrections. The satellites which are providing this information into the cellphones have to incorporate these corrections. So, it is a very complex phenomenon, and whether or not we are aware of it, our lives are intimately connected with these physical phenomena and their detailed mathematical analysis.

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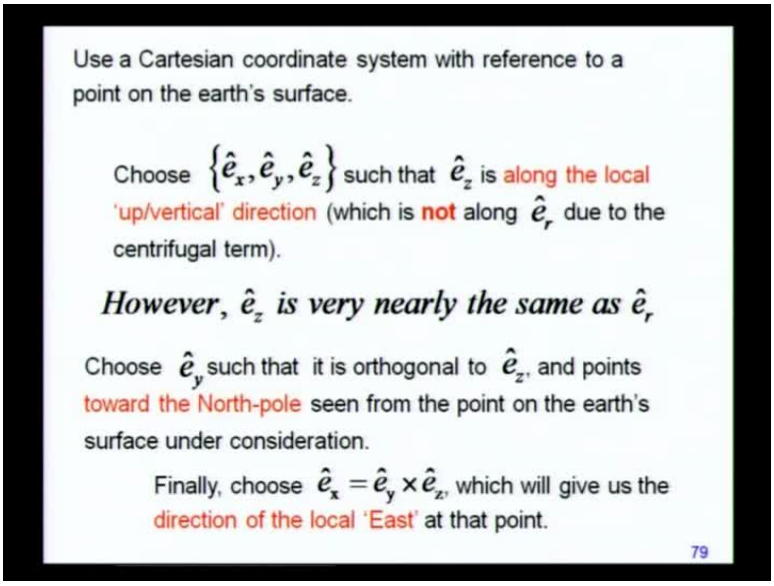

So, to do this analysis, we construct a Cartesian coordinate system, and I mentioned this in my previous class, but I will do this again because this is important; it is very important to recognize the coordinate system that we are going to be making use of. You choose a Cartesian coordinate system with unit base vectors e x e y and e z.

The first thing you do is to choose the e z; the z-axis of this coordinate system along the local vertical, and we agreed after our discussion yesterday that the local vertical is not necessarily the radial direction, because we are observing the vertical on a rotating earth, and therefore, we must take into account the centrifugal term which is.

So, we choose a Cartesian coordinate system spanned by these unit base vectors e x e y e z. The first thing we do is to choose this direction of the z-axis, and it is chosen along the local vertical pointed upwards - up is toward the sky from where ever you are standing - and it is the local vertical which is not necessarily the radial direction from the point of the centre of the earth surface to the point on the earth surface that you are standing.

Because there is a centrifugal term that you must take into account since you are on a rotating frame. So, it will be directed slightly away from the earth’s axis along the cylindrical polar radial direction, orthogonal to the earth’s axis. It will be slightly away not very much, almost the same. The effect is not very significant; it is quadratic in omega. So, we are eventually going to ignore it, but in principle, there is a small departure.

The next thing you do is choose the e y axis. So, from where ever you are standing, you can pick the direction to the north and that will be the direction of the y axis, and having chosen these two directions, e x automatically selects itself as e y cross e z. So, there is no ambiguity with respect to that and that will be your identification of the east direction. That may not be necessarily the direction along which the sun will rise by the way.

So, you may continue to say that the sun rises in the east, but it may not necessarily coincide with this. As a matter of fact, if you watch that the direction from which the sun rises, and if you see the sun rise from the window of your room, of your bedroom every morning, the only bad thing about it is that you will have to wake up before the sunrise, but you see the window to be a certain frame, and you see the sun to be rising, let us say from this direction, and then, the next day it will not be the same, and if the difference will be marginal, but from summer to winter, the difference will be dramatic. I do not know if you have ever noticed this, but the difference is absolutely dramatic.

So, sun light falling in one part of the room through the window, and if there is a narrow hole, narrow hole is face through with the sun light is coming, and it is hitting one part of the floor, it will go in some completely different direction, but that is because the earth is, earth’s axis is tilted with reference to the equatorial plane, and then, as it goes round the sun, this changes, but that is, that is a different phenomenon, that is not what we are talking about over here.

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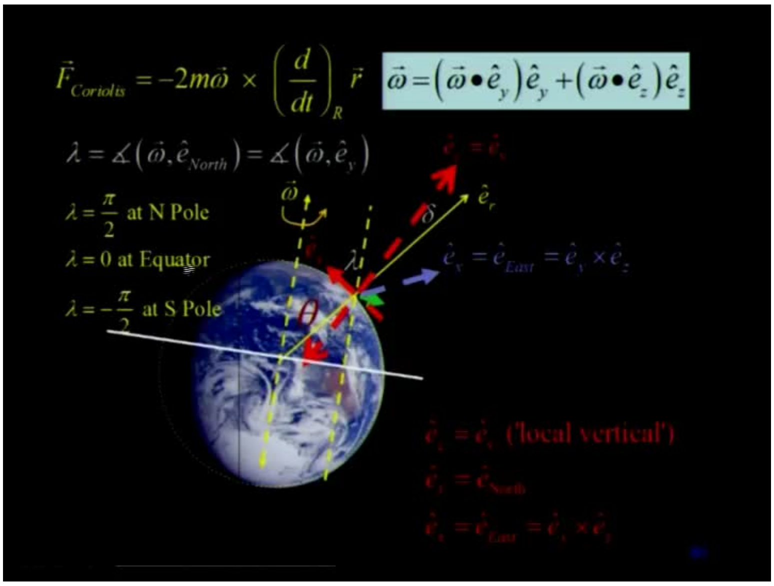

Here, our description of east is the cross product of e y with e z, because this cross product cannot point in other direction than the direction which is orthogonal to both e y and e z. There is only one such direction and this direction does not care where the sun is rising from on that given day. So, this is your direction of the local east; this is the picture that you get; and this is the Coriolis term that you need to investigate. (Refer Slide Time: 13:28) Omega as you seen this frame of reference will be in the plane of e y and e z. It will have no component along what you have now defined to be the east.

This angle delta is rather small; this is coming because of the centrifugal term: omega cross omega cross r. By large you are going to ignore this because it is coming from the quadratic term, and the angle lambda which is the angle between omega and the north. So, this is omega; you draw a line parallel to omega which is this yellow line at the point at which you are doing this analysis. The angle between this omega and the local north is the angle lambda and you can easily see that this lambda is pi by 2 at the North Pole; it will be 0 at the equator; it will minus pi by 2 at the southern pole. You have to be careful because we are going to really mix various coordinate systems for this discursion, because it makes the discursion simpler by referring to different coordinate systems.

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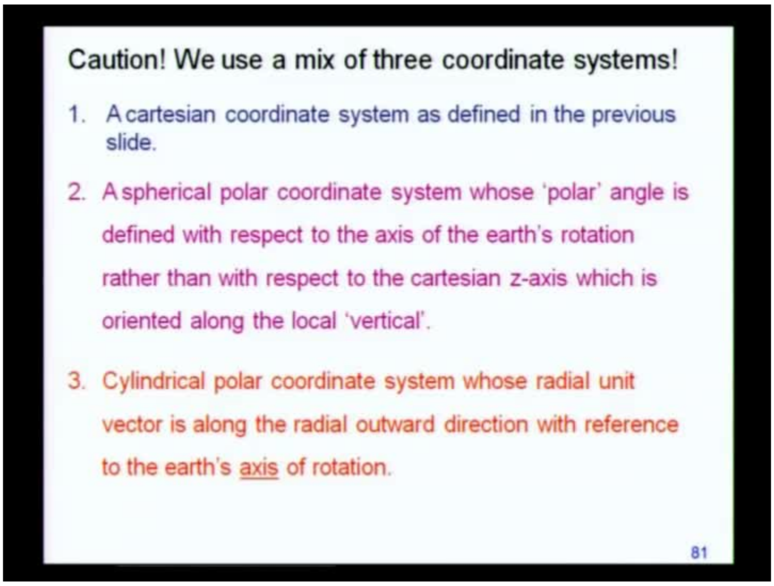

So, we first use a Cartesian coordinate system the manner in which I have defined it earlier, but I will also use parameters which belong to the spherical coordinate system, because I will talk about a polar angle which is defined with respect to the earth's axis of rotation. The earth's axis of rotation is not the z-axis; the z-axis we have chosen to be along the local vertical. So, we have really mixing these things.

And then, we shall also be using a cylinder polar coordinate system because we will talk about the centrifugal term which is along the radial line, which is orthogonal to the earth's axis. So, this direction is best described in terms of the cylindrical polar coordinate system with the earth's axis of rotation to be considered to be the reference axis.

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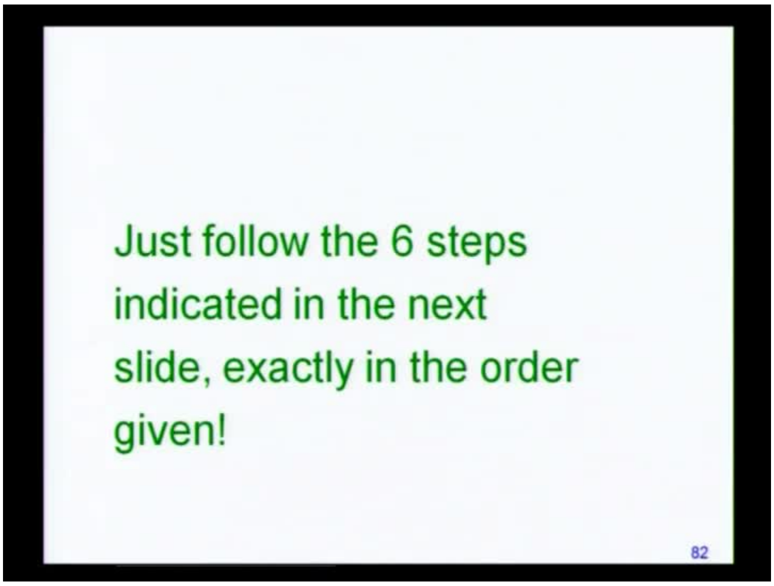

But there is no big deal about it, because as long as we know about what angles and distances we are talking about, we keep track of that. Then we can use the appropriate coordinate system. All we need to do is to follow these 6 steps, and follow these 6 steps exactly in the order in which I will show them.

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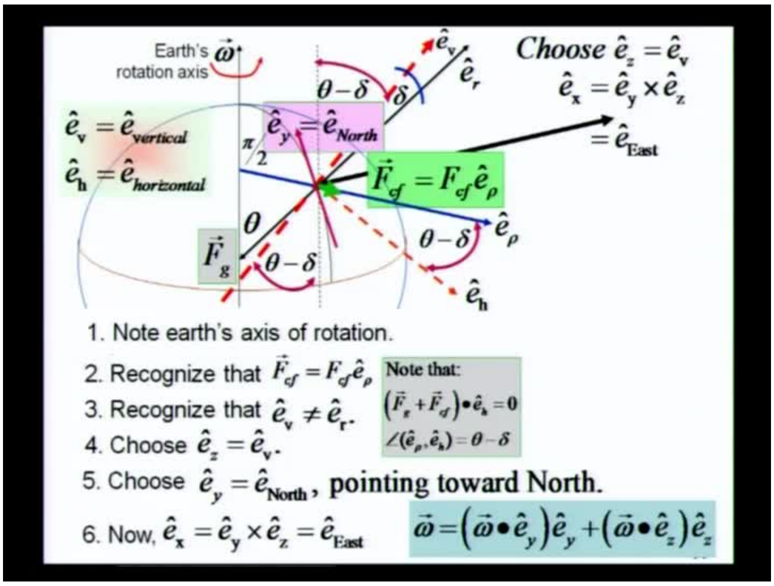

Note the earth's axis of rotation which is this; this is the radial line; this is the radial line along which the gravitational interaction will take place, because that is where the centre of mass of the earth is. So, an object on the earth surface will be attracted toward the earth, so, it will be along this line. So, this is the line along which gravity would act. That is the real physical interaction. Then you recognize that there is a centrifugal term which is orthogonal to the earth's axis and directed away from the earth's axis. So, if you define a plane which is perpendicular to the earth's axis, it will be in that plane in a direction which is radially away from the earth's axis, and this is what we call as e rho in the plane polar coordinate system.

So, this will be the direction of the centrifugal term. The local vertical will be given by the direction determined by the combination of gravity under centrifugal term. So, gravity acts along this line; centrifugal term along this tiny green arrow; the resultant of this will go along this red line; and this will be the direction of the local vertical. (Refer Slide Time: 17:18)

Eventually, we will ignore the difference between the local vertical and this e r, but it is important to keep track of the difference. So, you recognize at e v, the local vertical e v is a unit vector along the local vertical. This is not exactly equal to the unit vector e r of the spherical polar coordinate system which is along the radius.

Now, we choose a Cartesian coordinate system with the e z which is along the local vertical. Now e v has been selected earlier, and this enables us to select a Cartesian coordinate system with its z-axis along the local vertical. So, choose e z equal to e v.

Now, you choose e y along the north as seen from that point. This is no problem; once you know where you are, all you do is point your hand, point your finger toward the North Pole. Once you do that, so this direction can be identified unambiguously. Having fixed e y and e z, e x automatically fixes itself as the cross product of e y with e z. That is your identity of the local east.

So, this is the coordinate system in which we mix various terms coming from different coordinate systems, and it makes the discursion very easy as you can see. In this figure, you find that omega has got a component along e y and also along e z, but of course, none along e x.

So, omega will be the projection of omega along e y times a unit vector e y plus the projection of omega along e z times a unit vector e z. So, these are your descriptions of vertical and horizontal. A horizontal direction is any direction which is orthogonal to the vertical. So, that defines your horizontal plane, and the horizontal plane is not necessarily orthogonal to the unit vector e r but it is defined as orthogonal to the unit vector e v. So, keep that difference in mind. So, it also tells you that e v is not along the e r, nor is the horizontal orthogonal to e r.

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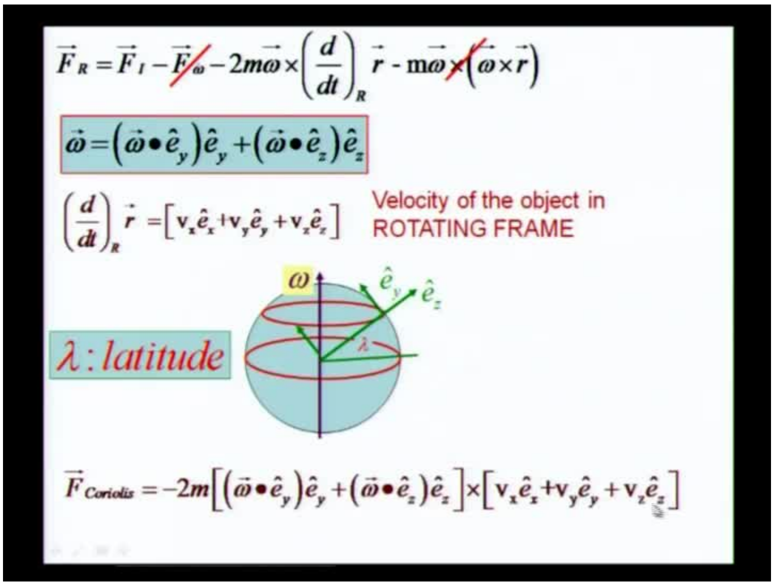

So, now, let us look at this complete expression. We will now throw off the leap second correction; we will throw off the centrifugal term; so, I strike them out, and once you strike this term out, notice that the angle lambda between omega and e y which is the direction of the local north. This angle is practically the latitude at that point. So, omega dot e y will contain the cosine of the angle between omega and e y, and that will be cosine lambda, where lambda you can recognize to be essentially the latitude of the point where you are carrying out this observation. So, it will change from point to point along a longitude.

The velocity you can always resolve along the three components e x, e y and e z, and depending on what kind of velocity means, I asked you to find the Coriolis deflection for a plane which is flying from Delhi to Chennai or from Srinagar to Thiruvananthapuram. You need to plug in its x component, y component, and z component, and then, you can construct the cross product of omega with this velocity and find the deflection. It is a very simple exercise.

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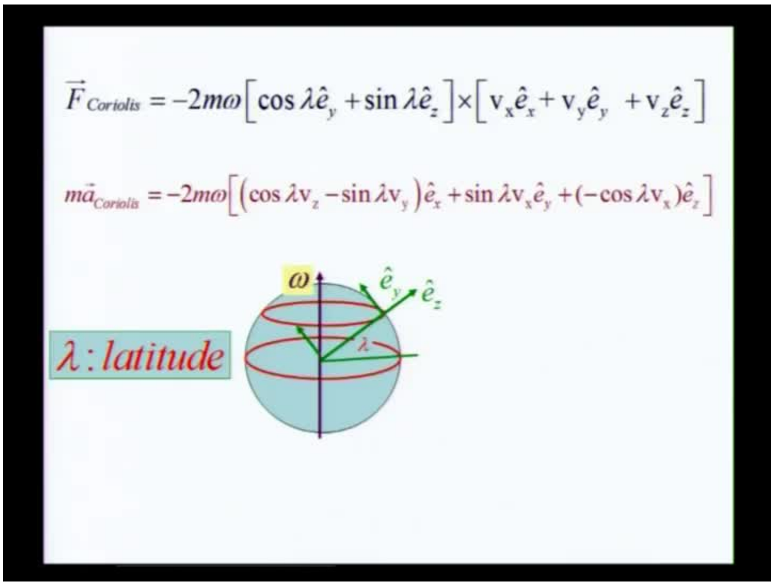

So, let us construct this cross product now. The Coriolis force is then given by this minus 2m omega cross v. So, this is the cross product we have to construct, and then, of course, terms like e y cross this e y, and this e z cross this e z, they will automatically go to 0. So, you will get only the remaining terms, and you get these three terms, once you carry out this cross product, you get a component along e x, a component along e y, and a component along e z.

And there are weight factors which depend on the latitude. So, you get the cosine lambda and the sin lambda terms, and then, of course, there is a v x, there is a v y coming over here, and a v z coming over here. So, these are the parameters which go into the net determination of the Coriolis acceleration.

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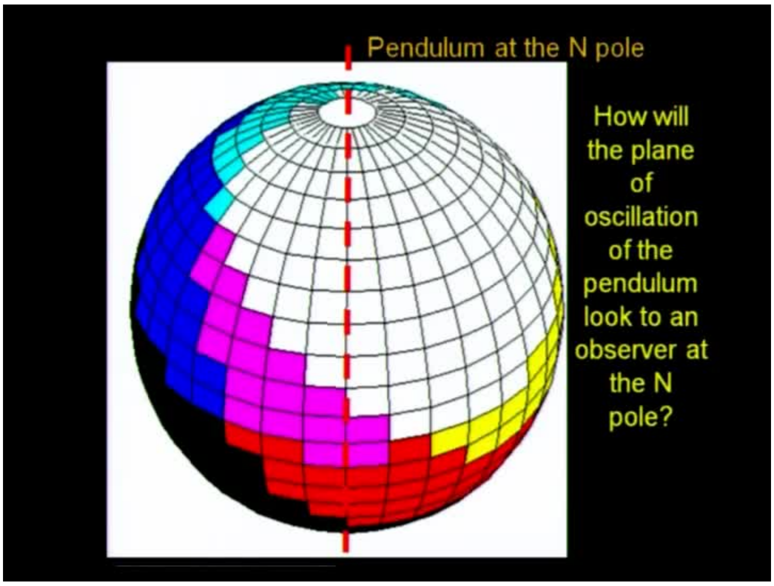

So, now, let us get a pendulum, and I think Vivek has got a nice pendulum; Vivek can you bring it up. So, this is the picture of the earth, and you can see that, you have got nice great circles, come Vivek. If you just stand by over here and show this pendulum, and let us say that Vivek is holding this pendulum. This is, when he has not set it in motion, it is actually a plumb line, and once he set it in motion, it becomes a pendulum, and let us say, Vivek holds it. Vivek smile.

This will be, let us say Vivek goes to the North Pole and does exactly this experiment. Now, we will make sure that he wears enough jacket and so on, so that he does not feel cold. He sets it in motion, and the question is, if he sets it in a motion like this, you see that the motion is taking place in this plane, the plane of the palm of my hand, the broadly speaking. Now, understanding the details, this is just an approximate demonstration, this is not an accurate experiment. Vivek is not even at the North Pole, he is here in Chennai at latitude of 13 degrees so well away from the North Pole closure to the equator as a matter of fact.

However, if he were to do this and if he sets it in motion in this plane and think that he is looking at it at the North Pole. Now, will this plane which is now like this and this plane is perpendicular to the edge of this table; it is parallel to the edge of this table. So, do you think that this pendulum will always be seen to be moving in a plane which is parallel to the edge of the table, if he is standing at the North Pole? The earth below him is rotating and if we give him a platform like Trishanku, but we will make sure that you are stable, do not mind.

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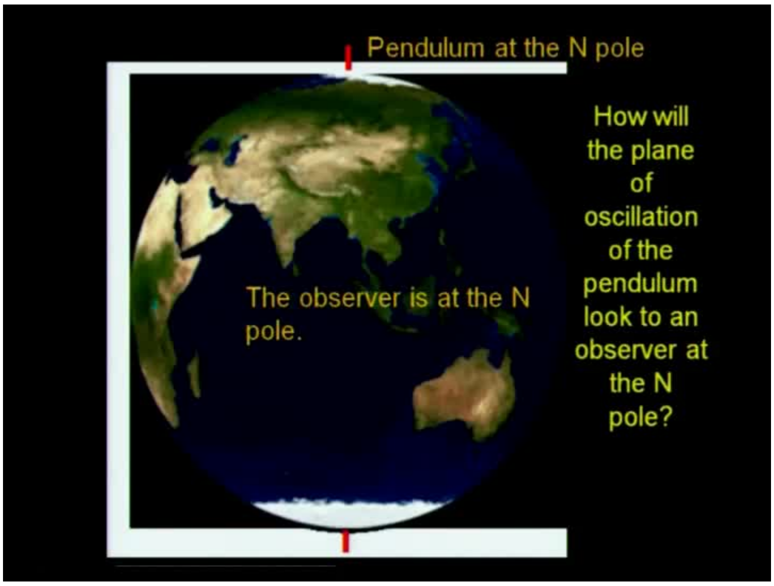

But if he is standing outside the earth and he will looks at the earth rotating under him. He is going to see the plane of oscillation to be actually rotating. That is what he is going to see, and this is what would happen if he considers the earth to be rotating under him.

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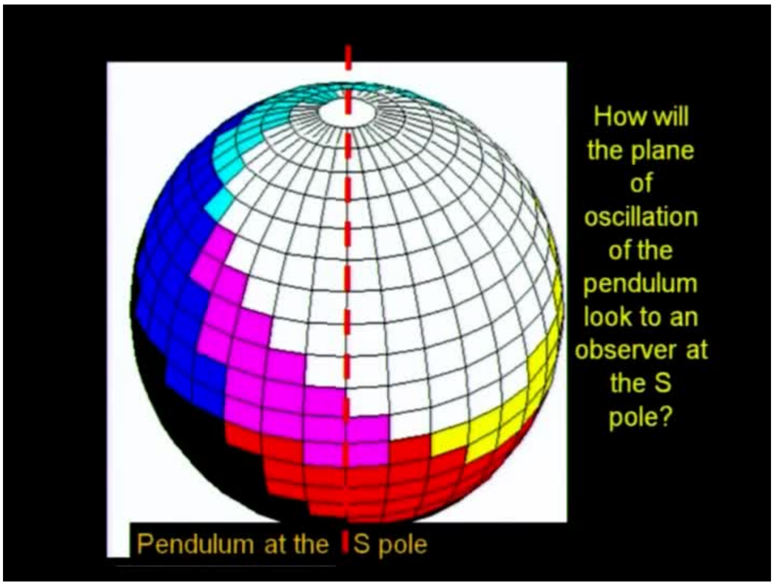

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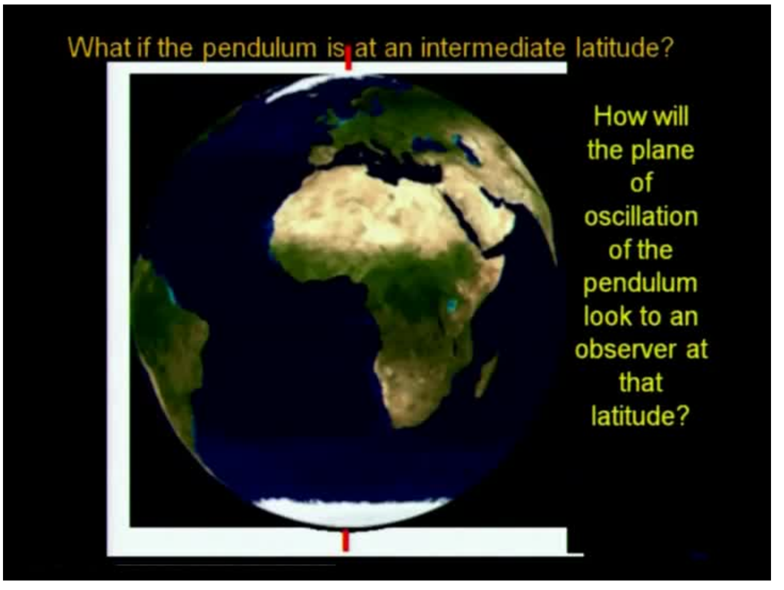

Now, what would happen if he does this experiment at the South Pole? Now, this is what you call as the thought experiment. So, at the flash of a thought, we can place Vivek at the south pole from the North Pole without any travel involved, and then, you again allow the earth to rotate, and he will again find that the plane of oscillation is rotating but in the opposite direction to what he saw when he was at the North Pole, because, now, he is standing at the South Pole. So, he will still find that the plane of oscillation is rotating but in the opposite direction. So, it was clockwise from the North Pole; from the South Pole, he will think that the plane of rotation is rotating anticlockwise.

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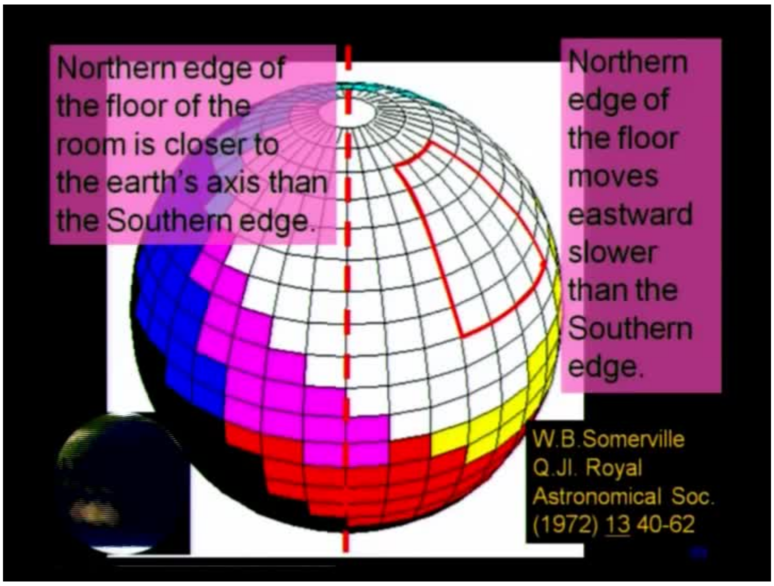

Now, what about some point at some other latitude, and you can sit down Vivek, thank you very much. If this experiment were to be done at some other point, at some other latitude not at the North Pole neither at the south pole, but somewhere in between. Then Vivek is going to be standing on a floor somewhat like this, or if you look at an exaggerated picture just because it will dramatize the effects, if the floor is something like this, it could be a live floor, it will, could be kings mansion.

In which case notice that the northern edge of the floor is closer to the earth's axis of rotation than is the southern edge. So, the northern edge would seem to be going eastward somewhat slower than the southern edge which is farther away, because to keep at the same angle, this will have to go faster, and this is very nicely explained in a paper by Somerville. I strongly recommend this; it is available on the internet from the quarterly journals of the royal astronomical society in 1972, volume 13, and without getting into too much of math, these explanations are very well provided in this article.

And we have already seen that as a result of the earth's rotation, the plane of oscillation is actually seen to rotate by the observer, but there is no physical interaction that the pendulum is subjected to which is causing it to rotate. So, if you observe the pendulum from an inertial frame of reference, which is outside this, if Trishanku were to observe it, and he does not care to look at the earth or anything; he just looks at the plane of oscillation and he will find that it continues to oscillate in the same plane.

So, it is only for the observer in the rotating frame that he has to think about what is causing this plane of oscillation to rotate, and he will ask - does the plane of oscillation rotate like this and how much time does it take to come back to the earlier orientation because it will rotate, it will spin, and then, he will ask what is the period of oscillation of the plane of rotation of this Foucault pendulum. So, what we will discuss today is how to determine this time period.

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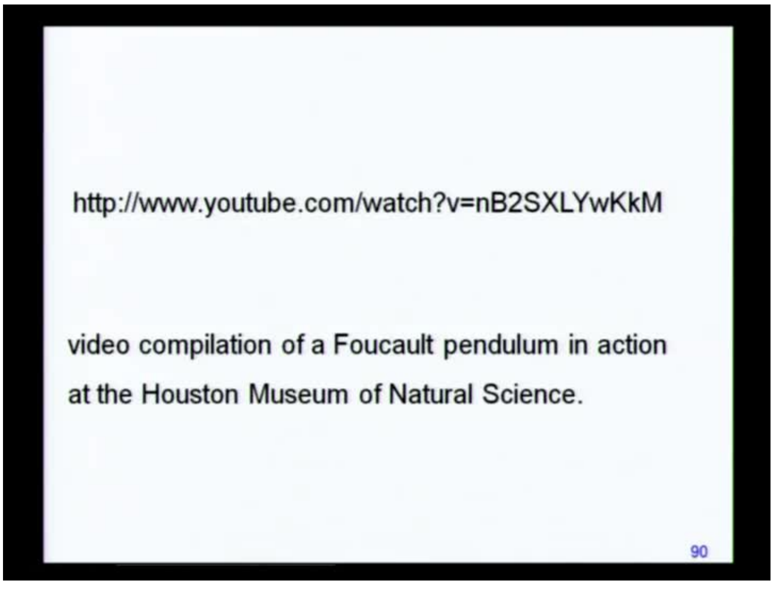

So, this becomes a pertinent question as to how much time it takes for the plane of oscillation of the Foucault pendulum to rotate, and you can watch some of these things on the YouTube. There are several Foucault pendulum which you can watch on the YouTube. There is a very nice one at the Houston museum of natural sciences. So, if you just Google this, you will get it. You do not have to write down the link, but if you Google it, you will easily get it just look for the Houston museum natural science Foucault pendulum.

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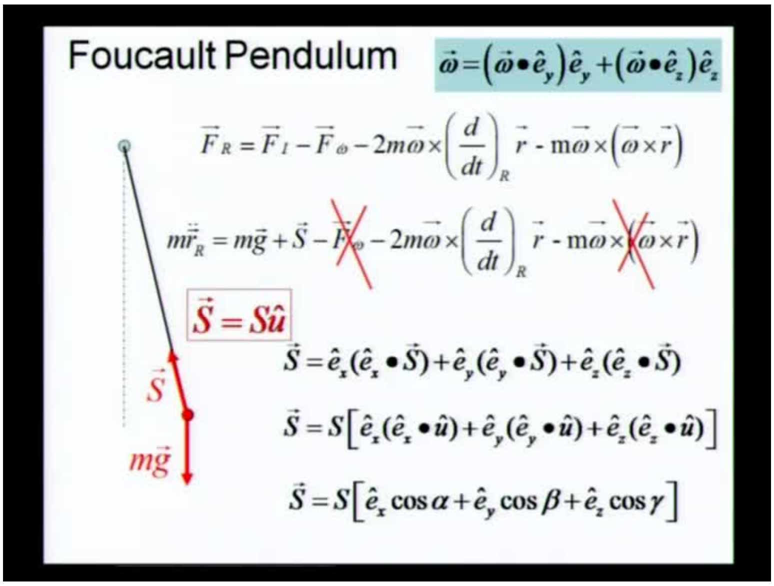

What we will do is to see how the time period for the rotation of the plane of oscillation of the Foucault pendulum is to be determined. So, we have got these three terms and this pendulum now is subjected to not just the physical interaction generated by gravity. This is something that we always took into account, but it is suspended this plumb line which Vivek set into motion was held by a string, and there is, therefore, this tension in the string and which is the real physical interaction solely.

So, this tension must be taken into account. We will ignore the leap second term; we will ignore the centrifugal term; and we consider a unidirection for the tension. So, the tension is equal to the magnitude of the tension S times a unit vector along which the tension acts, and you can decompose the tension along the coordinate system that we have earlier chosen. As long you can choose any coordinate system to resolve a vector. So, we choose the e x, e y, e z that we have chosen earlier, and this is your coordinate system, and you factor out the magnitude S from each term. Essentially, what you have is that this tension has got components along e x, e y, and e z, and the magnitudes of these components are given by the direction cosines of the unit vector u. So, these direction cosines will play a role in our analysis; these direction cosines is what I will represent by cos alpha, cos beta, and cos gamma.

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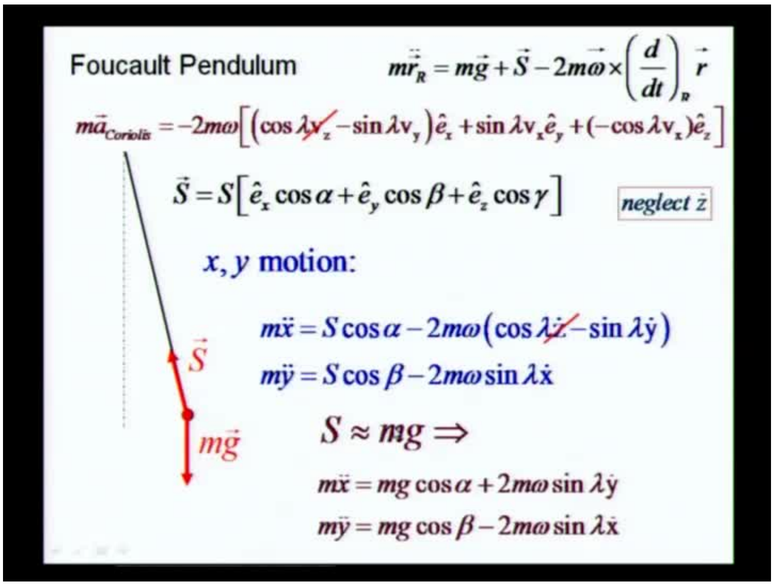

So, let us do this analysis further. What we will do is actually a detailed solution to this problem is quite complex, and it is not my ambition to give you a very rigorous solution to this problem, but I will give you a solution which is reasonably good, and for which we will have nevertheless to make a few approximation. So, we will neglect z dot the velocity of the object along the z-axis which is obviously not a bad approximation for this experiment you are analyzing.

So, z dot vanishes, which means that this v z terms goes, and then, we will look at the motion in the x y plane, that is a one of interest. So, this is the mass times acceleration, and we resolve this along the x and y axis. So, in the x y plane, in the x y motion, mass times acceleration of the x coordinate will be given by s cos alpha coming from this term, and then, whatever else that has got the x component. So, there is a sin lambda times v y with a minus sin which must come, which is here, and then, the rest of these are y and z component. So, they do not come in this equation, and likewise, there is the mass times acceleration for the y component.

So, for the y component, you have got this term cosine beta which is along e y, and then, there is this term, and this term, we do not worry about; we are analyzing motion in the x y plane. So, these are the two equations that we get. We will make a further approximation that by and large s is nearly equal to mg. It is again not a bad approximation.

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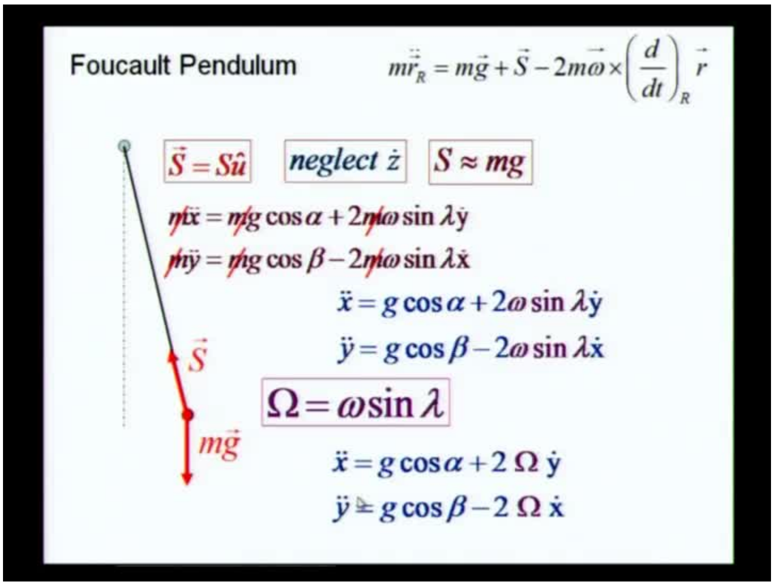

So, for this s, I substitute mg. Having done this, we have neglected z dot; we have assumed that the magnitude is nearly equal to mg throughout this motion, and then, we strike out the masses so that we get expressions only for the accelerations, because the mass is a common term, in every term. So, we got this mass time acceleration, and we just strike out the mass which is there in every term.

Look at the differential equations for x and y. These are second order differential equations with respect to time, and the differential equation for x includes a term in the y velocity, and the differential equation for y includes a term in the x velocity. To write these equations a little simpler, because both of these have got the term in omega sin lambda. Lambda, we know is the latitude, we recognize that in our earlier discussion.

So, there is a latitude which is playing a significant role here, not surprisingly because we already knew that it would, but omega sin lambda, instead of writing it every time as omega sin lambda, this will have the dimensions of angle of frequency, because sin lambda is dimensionless. Omega, this little lower case omega has got the dimensions of angle of frequency. So, this upper case omega will also have the dimensions of angle of frequency. So, we define a new angle of frequency this upper case omega which is the lower case of omega which is the angular rotational velocity of the earth radians per second times the sin of the latitude. So, both of these equations for x and y got this omega sin lambda. For which, we will write this upper case omega.

So, x double dot equal to g cos alpha plus twice omega y dot, and this is y double dot, the second a dot denotes differentiation with respect to time double dots represent this done twice. So, d 2 by dt 2 that is our operator.

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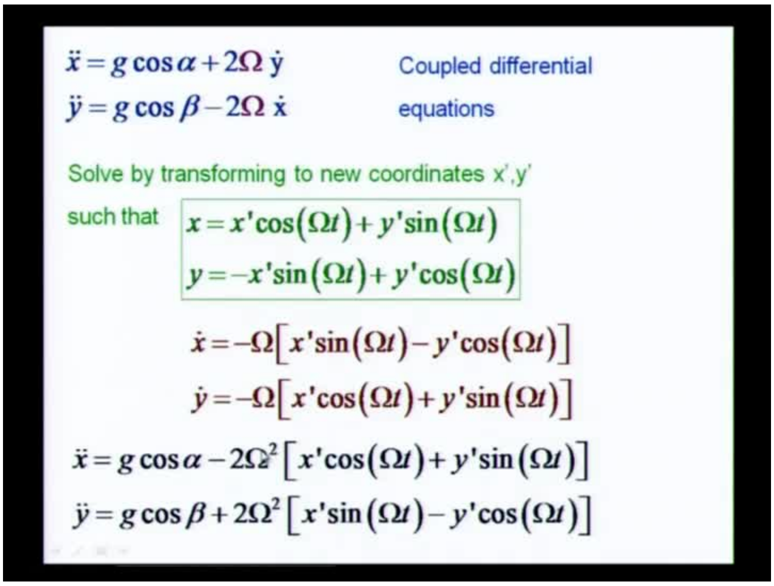

So, these are your differential equations, and these are obviously coupled differential equations, because of y motion and x motion are connected. The acceleration along x includes of velocity along y, and the acceleration along y includes of velocity along x. So, these are coupled differential equations and you have to find some means of decoupling them.

So, one can do this using various tricks in differential calculus, and I am not going to get into all the detail, but suggest a simple trick that you can do which is to transform to a new coordinate system. If you do that, the equation separate out at least to a reasonable degree of approximation, which is all that I am aiming for.

So, this is a very fruitful transformation. From x and y, you transform to x prime and y prime with this as the relationship. You define x prime and y primes, such that x and y are given by these relations over here, in which I plug in the angular frequency, which is this omega sin lambda, which we defined in the previous line.

So, this is the transformation that I will make use of, and you will see that this transformation x equal to x prime cosine omega t plus y prime sin omega t and a similar expression for y gives you, from here, you differentiate this with respect to time. So, you get x dot; so, you differentiate the right hand side with respect to time. So, from differentiation of cosine, you will get minus sin omega t which is over here. So, sin omega t here; the minus sin is here. You will multiply also by an omega, so, that omega is over here. So, you get this from differentiation of this term, and from the differentiation of the second term, you get y dash from the differentiation of sin, you get a cosine, but this time you do not change the sign, because there is differentiation of sin is plus cos theta. So, this is originally plus; so, this minus into this minus is plus and the omegas sits out.

So, you get the velocity along the x component and the velocity along the y component. This is what you have over in these two terms and you can substitute this form over here and you find that the differential equations, the coupled differential equations separate out at least in an approximate way, not exactly, and I will tell you what the approximation is. So, first of all, what we do is to substitute this x dot and y dot in this differential equation. There is nothing in this equation; all you have done is to plug in this, over here, and there already was an omega over here, and with this omega, you get a term which is quadratic in omega and we will exploit that.

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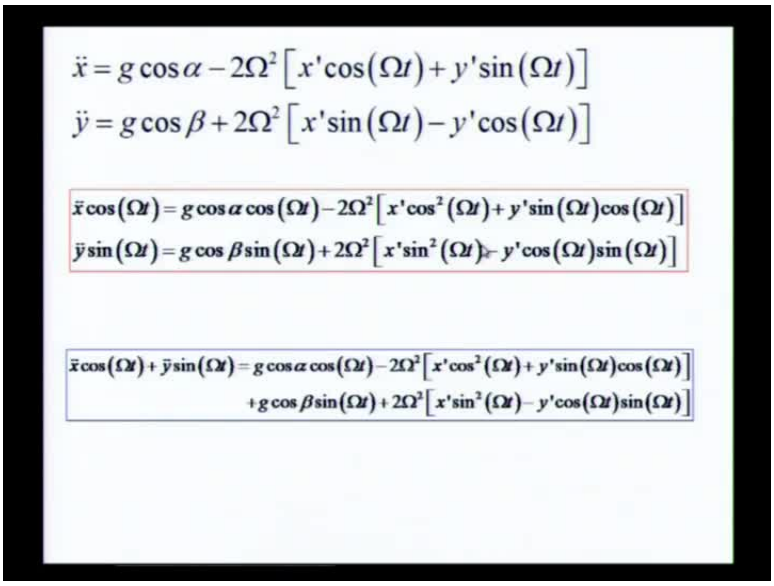

So, you get a term in quadratic in omega, and then, just for convenient interpretation of these mathematical forms, I multiply the first equation by cosine omega t. So, the first equation is for acceleration, I multiply this entire equation both the left hand side as well as the right hand side by cosine omega t. So, there this term becomes g cos alpha times cosine omega t and there is a cosine omega t multiplying these two terms as well.

Likewise, I multiply the second equation by sin omega t, and then, what I am going to do is to add these two equations, I get these two equations. This is what I call as the first and this is the second, and if I add these two equations, I get x double dot cosine omega t plus y double dot sin omega t on the left hand side, and the right hand side is just a sum of everything that you find on the right hand side of these two equations. So, you really do not have to write down these equations; all you have to do is to add them up, when you work it out for yourself.

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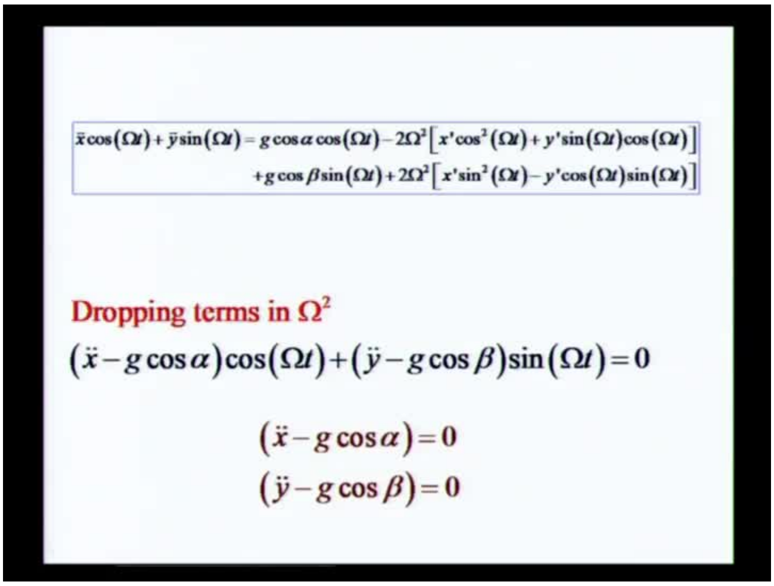

So, this is the form that you get, and what we will do is omega we know is small. We are already working in an approximation scheme in which terms which are quadratic in omega are ignorable. This is quadratic not just in upper case omega but obviously, correspondingly also in lower case omega, and lower case omega is like 7.29 whatever, ten to the minus 5 radians per second. So, this will be a very small term and you can ignore it. So, it is an approximation but not a bad one.

Rest of the analysis has already been done at that level of approximation. So, we are doing an approximation which is completely consistent with other approximation that we have employed. This is an important point to keep track of because, whenever you ignore terms, because they are small; you cannot retain one term of one order and ignore another of the same order. So, whatever you ignore must belong to corresponding orders or lower orders but not higher. So, as long as you keep track of that and we are quite with that. We confirm that is what we have done.

Now, we ignore the quadratic term, and then, we get, I move this term everything to the left. So, I get x double dot minus g cos alpha times cosine omega t from here, and then, y double dot minus g cos beta sin omega t equal to 0, and this obviously has got a solution, when the coefficients of both are 0. So, that solution is always present.

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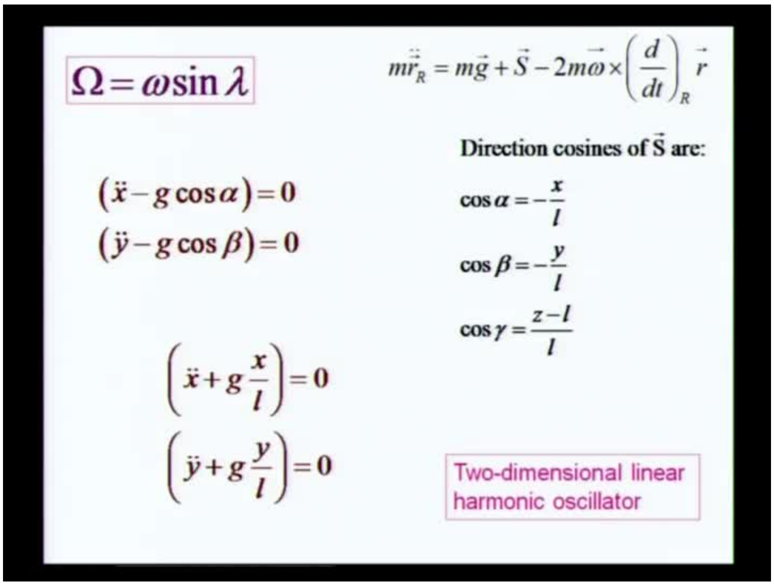

So, these two equations, and now, you are okay, because you do not have the coupling between x and y anymore. All you have to do: g is the acceleration due to which you know; cos alpha and cos beta are the direction cosines of the tension if you remember. So, if l is the length of the pendulum, then minus x over l and minus y over l will give you the cosine alpha and cos beta, that comes straight from the geometry, and by putting minus x over l for cos alpha, this minus sin and this minus sin gives you x double dot plus g x over l equal to 0 and y double dot plus g y over l equal to 0, and essentially, you have a two dimensional linear harmonic oscillator.

You know the linear harmonic oscillators very well. Acceleration is always directed toward the equilibrium; it is always proportional to it; the proportionality is the spring constant; this is just the Hooke's law, the simple harmonic oscillator, and you have a two dimensional linear harmonic oscillator whose solutions are very well known, but this will be a superposition of those two solutions.

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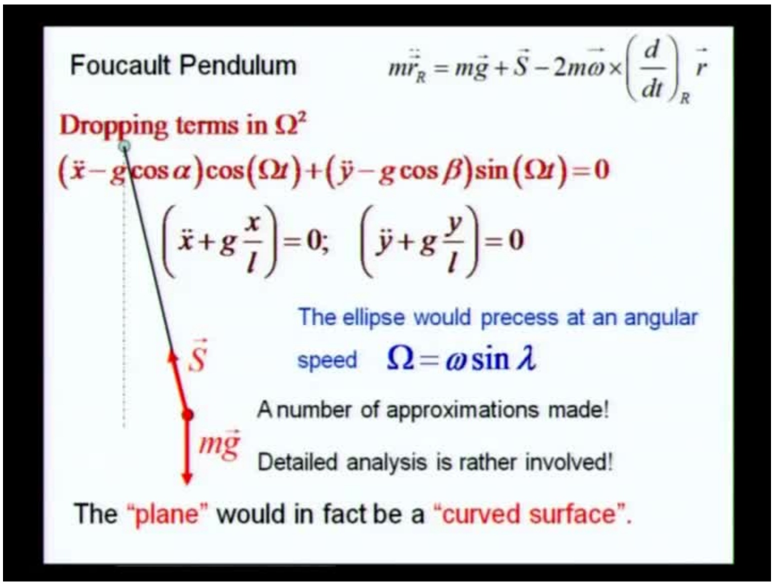

So, if you have a sinusoidal solution along the x axis and a sinusoidal solution also along the y, this will be a superposition of those two and this will generate Lissajous type of patterns; you will get an ellipse. So, the solution is an ellipse. This is the two dimensional linear harmonic oscillator. In your frame of reference, this will be an ellipse, but mind you there is a cosine omega t and a sin omega t here.

So, this ellipse which has got a certain major axis will be seen to precess at a time period given by this upper case omega. That will essentially be the precision of the plane of oscillation of the Foucault pendulum. So, the Foucault pendulum will precess at an angular frequency which is this omega sin lambda and it is determined completely by lambda, and of course, the earth's angle of frequency.

The earth's angle of frequency is the very cause of this and depends. The latitude at which you conduct this experiment determines its actual magnitude. We have, of course, made a number of approximations. We considered, we ignored the quadratic terms in omega; we considered S to be nearly equal to mg; detailed analysis is rather involved, but as a result of this approximation, there are some implicit consequences. The plane of oscillation is not exactly a flat plane; it is not like the flat plane you want like to see on your television screen when they advertise and sell flat screen. So, it will not be a flat plane, it will be actually warped; it will have some kind of a curvature.

So, it will not be exactly flat plane, but it will be, if I cup my palm, I can still say it is very nearly flat. So, I can still talk about a plane of oscillation, but if the oscillation is along this cupped kind of shape of my palm, you really cannot very accurately say that it is a plane of oscillation which is precessing, it is some very complicated surface, and the detailed analysis is really very involved and that is not the purpose of our discussion here.

But what we have is the angular frequency of rotation of what I now call as the plane of oscillation within the approximation that we are talking about. I will continue to you to talk about the plane of oscillation, but we know that it is not very rigorously a flat plane.

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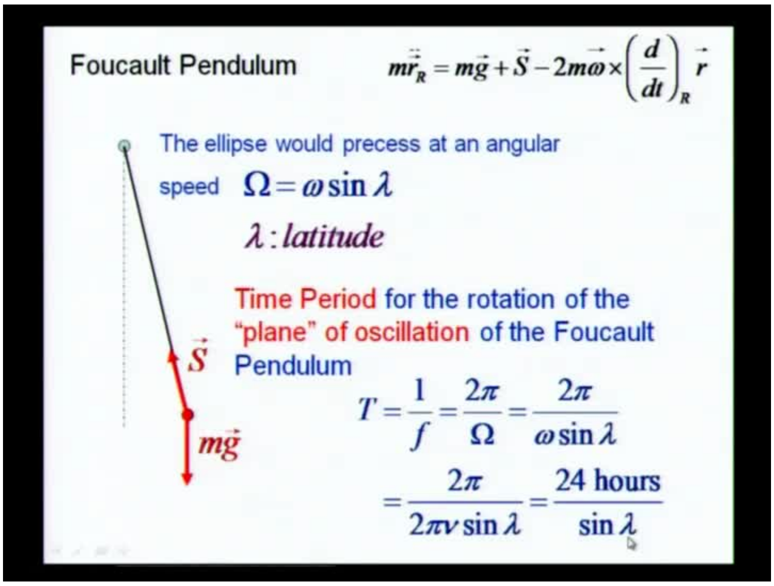

So, it is this frequency is determined by the latitude and the time period is just the reciprocal of frequency. This is the angular frequency, so, the time period will be 2 pi over omega, and you have omega sin lambda, and 1 over omega is this 2 pi nu, and if the earth is considered to do 1 rotation in 24 hours, which is again an approximation because you know there is a sidereal day and so on, so, I will not go into those details, it, so it is like 23 hours and so many minutes and so on, it is very nearly 24 hour. So, it is a fairly very good approximation again.

So, 1 over nu is 24 hours that is the time period of rotation of the earth. So, the time period for the rotation of the plane of oscillation is given by 24 hours divided by the sin of the latitude in units of hours.

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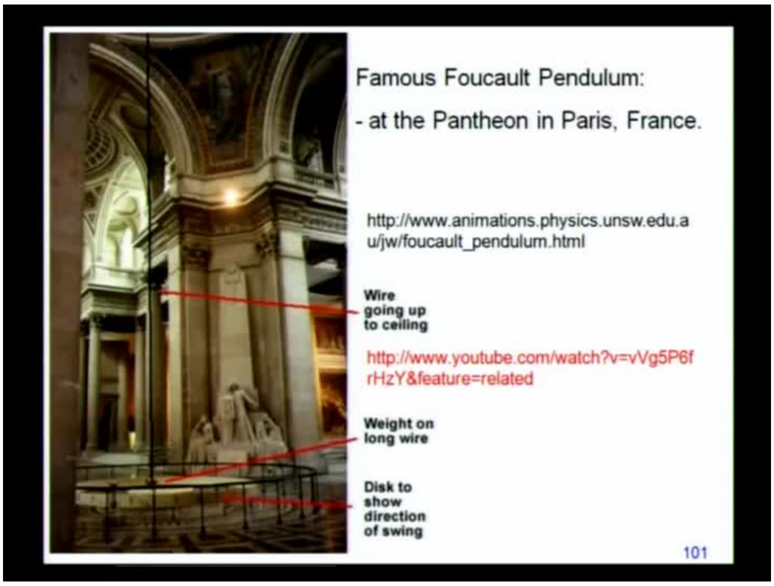

There are some very nice Foucault pendulum which have been set up in different parts of the world, and there is one which I know is being constructed at Indore where they had some difficulty, because I think their time period of oscillation is not quite stabilized, because there are frictional losses and other thing that one has to overcome and it is really not trivial to construct a Foucault pendulum, but it is possible. This is one of the better known of the Foucault pendulums; this is at the pantheon in Paris.

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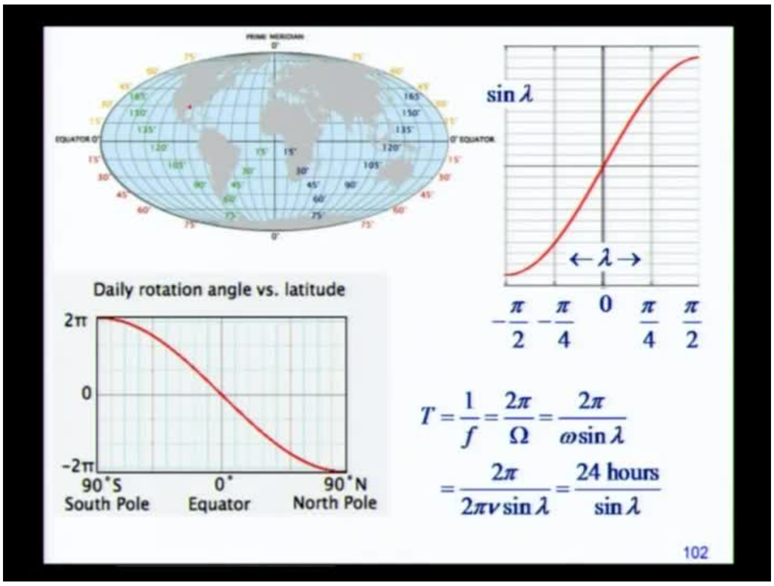

This is our time period in determined by 24 hours; it is determined by sin of the latitude, and depending on where you are on the earth at the equator, the latitude is 0. On the North Pole, the latitude is 90 degrees.

The range of sin theta is from minus 1 to plus 1. So, when the latitude is pi by 2 at the North Pole, sin pi by 2 will be 1 that is the maximum value that the denominator can take, and that will be the least time that the Foucault pendulum will take to go through 1 procession. That is the one that Vivek would have seen if he were to set his pendulum into oscillation, if he were standing at the North Pole.

At the South Pole, it will be just the opposites; he gets the same number with a an opposite sign. It does not mean the time becomes negative, but it only means that the corresponding rotation is in the opposite direction. So, we already discussed this that if it was clockwise at the North Pole, it would be anticlockwise at the South Pole.

At intermediate latitude, let us consider the equator, at equator the latitude is 0, or a sin 0, so, the denominator goes to 0, the time period goes to infinity, which means that it is not going to rotate at all. So, at the equator, it will remains steady even for an observer on earth. At any intermediate latitude, it will be determined by the sin function which goes from minus 1 to plus 1 over the latitude range minus pi by 2 to 0 in the southern hemisphere and from 0 to pi by 2 in the northern hemisphere, and accordingly, the daily rotation angle versus equator goes from 2 pi at the North Pole, at the South Pole, and then, it goes to minus 2 pi at the North Pole.

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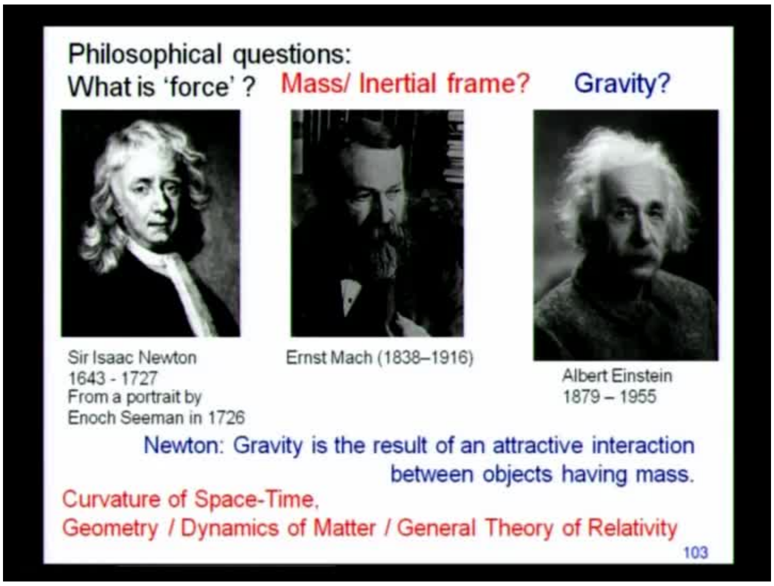

Now, finally there are some philosophical questions which I will not discuss, but I will mention these only for some of you to read further on, and with that, we include this unit. What is it that is, the cause of departure from equilibrium. Equilibrium itself seeks no cause; it is determined by initial conditions. This was recognized by Galileo and incorporated in Newtonian mechanics as the first law of inertia.

It is the departure from equilibrium which needs to be explained. Why is it that there is a departure from equilibrium? And if there is one, what is it that accounts for it? And to do this, Newton invented calculus, he learnt how to get the derivative of a function, derivative of position gave him velocity derivative of the velocity gave him acceleration, and he said that this acceleration is proportional to the cause which we now identify as the Newtonian force. So, that is the physical interaction that we talk about, this is in the Newtonian scheme.

Variational calculus addresses this using a different principle, which is the principle of variation, which is an integral principle that motion takes place not because there is a physical interaction which changes the equilibrium, but because, whatever action integral is ought to be an extremum.

So, that is the foundation principle on which one understands the evolution of the mechanical state of a system, and Newton dealt with this question, so, what is force, and he is then said that it is, that agency which generates an acceleration, which generates the effect that you see in your frame of reference, but then, what is mass? And what is an inertial frame of reference? And we mentioned that there is some kind of poetry; there is some kind of romance which is involved in Newton’s perception of an inertial frame of reference.

Newton envisaged that in inertial frame of reference situated deep amidst stars. Very romantic idea, it explains a lot of things, but there is something funny about this idea, and Mach challenged this idea; Mach wanted to understand this, and he needed to analyze this in great details and he was not very happy with this conception of an inertial frame of reference situated deep amidst stars and he tried to explain dynamics with reference to this stellar motion and so on, so the, I will not be going into those details.

But these issues are fundamentals because they inspired Einstein to think about these issues further as to what is gravity, and then, gravity which Newton interpreted as a result of an attractive interaction between two masses which he admitted already in his analysis in the stimulus response linear relationship - f equal to ma and when this is due to gravity.. Newton recognized the gravity to be the force of attraction between two masses

But then, Einstein interpreted this in terms of curvatures of space time and so on. So, these ideas clearly go beyond the scope of this course, and they belong to the steady of the space time continuum, the curvature of space time, Riemann surfaces, and there are fairly complex ideas which are involved in this. One really has to get into the general theory of relativity and that is clearly beyond the scope of this course, but even before we can talk about the general theory of relativity, we should at least get to the special theory relativity which is in fact a special place of the general theory of relativity, which is why it is called as a special theory of relativity.

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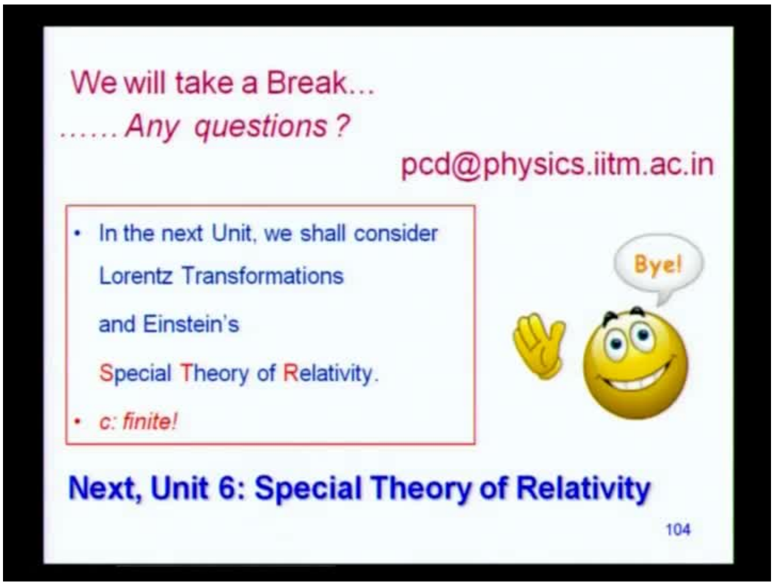

So, we should at least do the special theory of relativity, and that is what we will do in our next unit which will be the unit 6. This will be on special theory of relativity and with that I conclude unit 5. There we will meet Lorentz transformation, so the relativity will be essentially Lorentzian and not Galilean, and this has some very fascinating consequences. We will talk about length contraction; we will talk about time dilation; we will discuss the twin paradox. So, there are some interesting applications of the special theory of relativity, but we will, of course, not get in to the general theory of relativity, and then, we will continue our discussion on classical mechanics in unit 7.

So, thank you all very much. If there is any question, I will be happy to take.