Select / Special Topics in Classical Mechanics

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Module No. # 01

Lecture No. # 02

Equations of Motion

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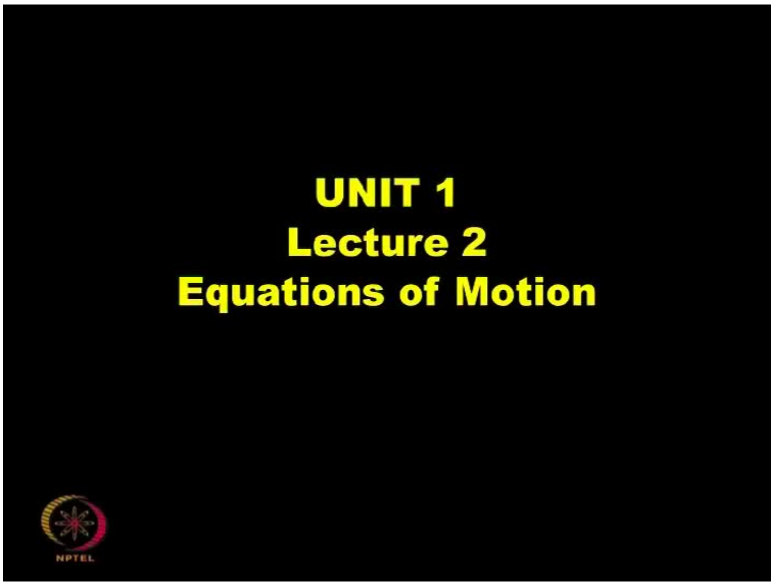

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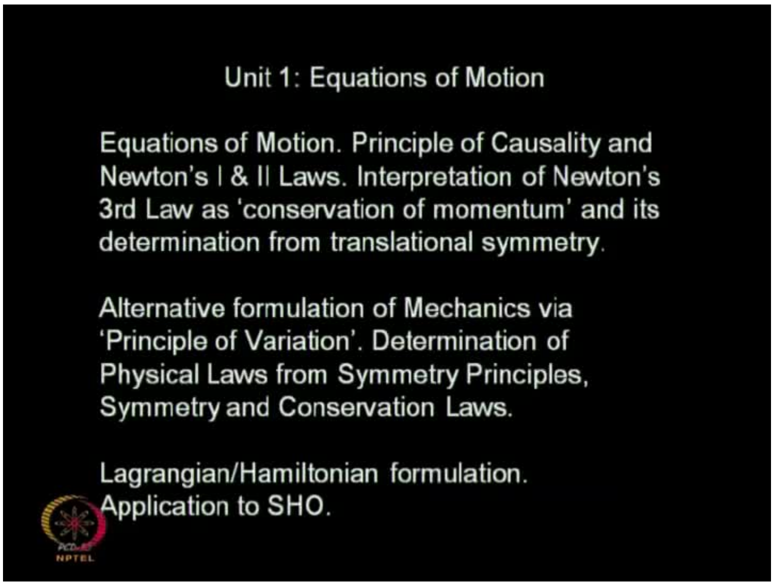

We formally begin discussion on unit one, which will be on equations of motion and these are some of the topics that we will go through. We will learn about the principle of causality in Newton's first and second law and the interpretation of Newton's third law as conservation of momentum and its determination from translational symmetry.

We will then be exposed to an alternative formulation of mechanics via the principle of variation. We will learn about physical laws, determination of physical laws from symmetry principles and the connections between symmetry and conservation laws.

We will get some introduction to Lagrangian and Hamiltonian formulation in this unit and we will see simple application to the simple harmonic oscillator. We will not be studying Lagrangian and Hamiltonian mechanics in any great detail, but we will only illustrate its application to the case of the harmonic oscillator to get some kind of understanding of the method of the principle of variation.

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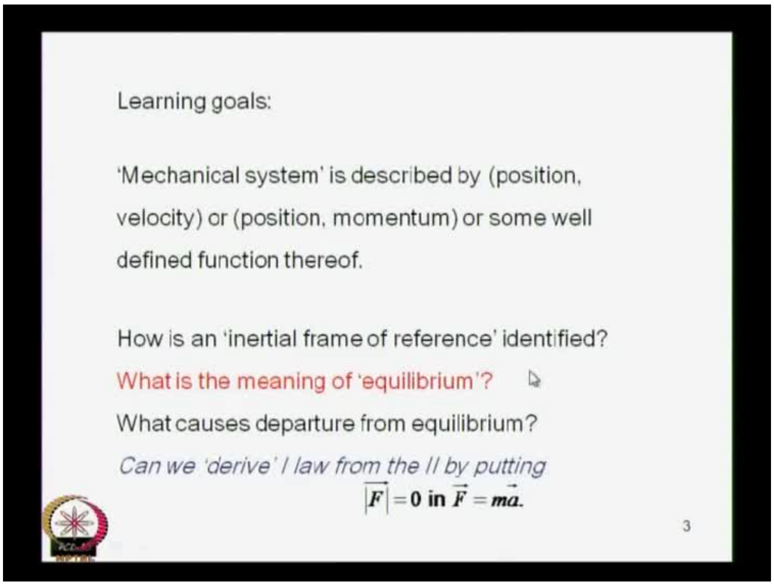

Now, what do we expect to learn in this? What are our goals? We will learn exactly how to precisely characterize the mechanical system; what is meant by characterization of a mechanical system?

We will discuss what an inertial frame of reference is; we will understand what equilibrium means and what is it that causes departure from equilibrium. We will discuss this issue: if the first law can be derived from the second law by simply putting a 0 force in this equation, which is often regarded as the statement of the second law.

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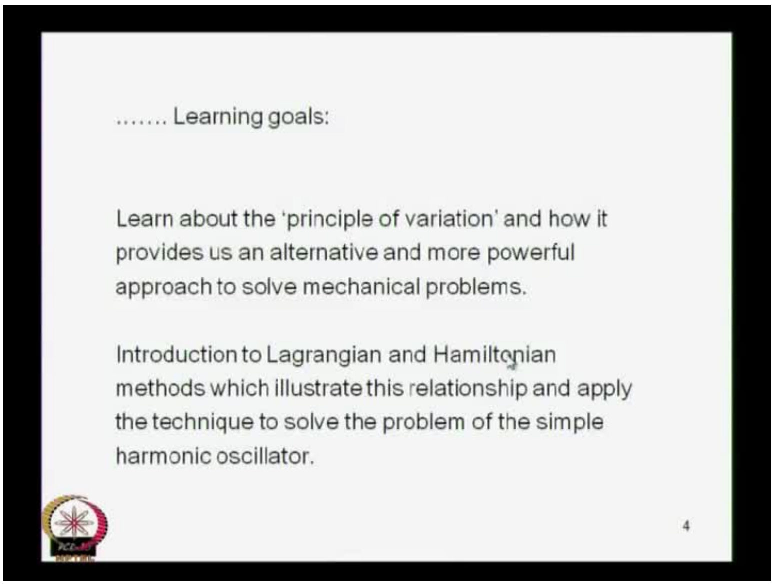

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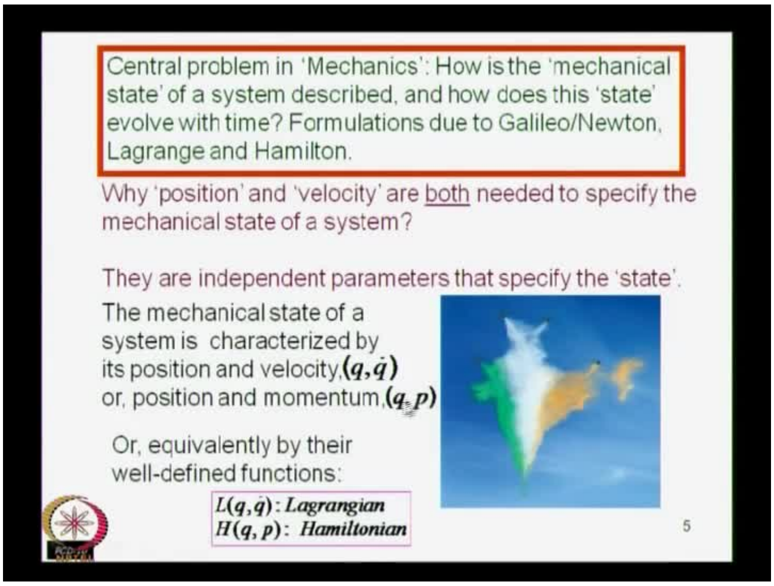

Only the introduction to Lagrangian and Hamiltonian formulation with some simple applications. Let us raise this question: how is a mechanical system described and how does this system evolve with time. Now, this is a precise statement of the question that is going to be answered and it is important to focus attention on how this question is posed- means in the context of quantum theory- Niels Bohr once explained that physics may not answer all the questions but it tells you what are the right questions to be asked and how do you go about answering these questions.

When you are dealing with classical mechanisms,you must know exactly how to formulate your question and then how to go about answering it. So, the question over here is how is a mechanical system(a mechanical system is) described and how does the system change with time.

Now, it turns out that position and velocity are both needed to specify the mechanical state of a system. Sometimes a few students have some difficulty with this idea because velocity of course is the time derivative of position;

it bothers them that is it really an independent parameter and if so, what is it that makes it an independent parameter and these in fact are definitely completely independent of each other and to understand that- have a look at this lovely picture of the Indian map generated by the Indian air force; it is a beautiful spray of colors and I want you to imagine, where Chennai is; we are somewhere over here, there about if you have an aircraft, which is going from Chennai to the west at a speed of about 300 kilometers an hour, then an hour from the time it starts it would be somewhere over Bangalore; more or less, whereas if it were to start from Bangalore, it would be at the West Coast at the same speed it is still going at 300 kilometers an hour but if it starts out from Chennai or else from Bangalore after exactly the same amount of time, which is an hour it would be at different destinations; it is because the position and velocity are independent. They both need to be specified to see how a mechanical system evolves with time because you want to predict where the object will be after our time delta, t. You need to know where it is now and at what speed it is going, even if no other force was acting on it; even if it was to be moving at a constant speed.

You have to have this information about both the position as well as velocity;

you can characterize it by position and velocity, which I will indicate by these variables q and q dot, position by q and velocity by q dot.

So, the time derivative dq by dt is what is represented by this dot on q or equivalently by position and momentum will be denoted by the letter p, which is the common symbol for momentum but if we can also define this by q and q dot, then we can also define it by a well-defined function of q and q dot.

We will learn in this unit how to define it by a function of q and q dot. The function will turn out to be Lagrangian or a function of position and momentum; this function will turn out to be the Hamiltonian.

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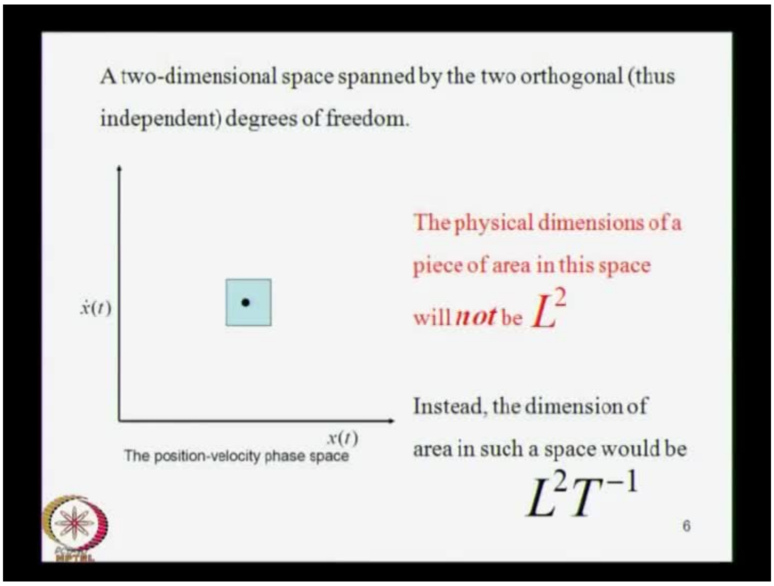

These are the 2 variables that we need; One is the position, which I plot on this horizontal axis and because these are two completely independent of each other; I can plot them along orthogonal axes. I plot the position along what we normally call as the x axis in a Cartesian geometry and the velocity along the y axis.

Now, if you look at an area in this, normally when you think of an area like the area of the table, then it is this side multiplied by that side and the dimension of the area is L square but here the dimension of an area in this space is made up of a displacement along the x axis, which has got the dimension of length, whereas on the vertical axis it has got the dimension of velocity.

So, the dimension of area in this space will be L to the power 2 T to the minus 1. Remember that area is not always L square it will have some other dimensions and it depends on what you are plotting against. It is very important in physics to keep track of the dimensions.

Now, this is what the area will look like and the physical state of a system will then be characterized by position and velocity; you specify the position and the velocity and you get a point; this is what is called as a position velocity phase space. Sometimes only as a phase space but more completely as position velocity phase space.

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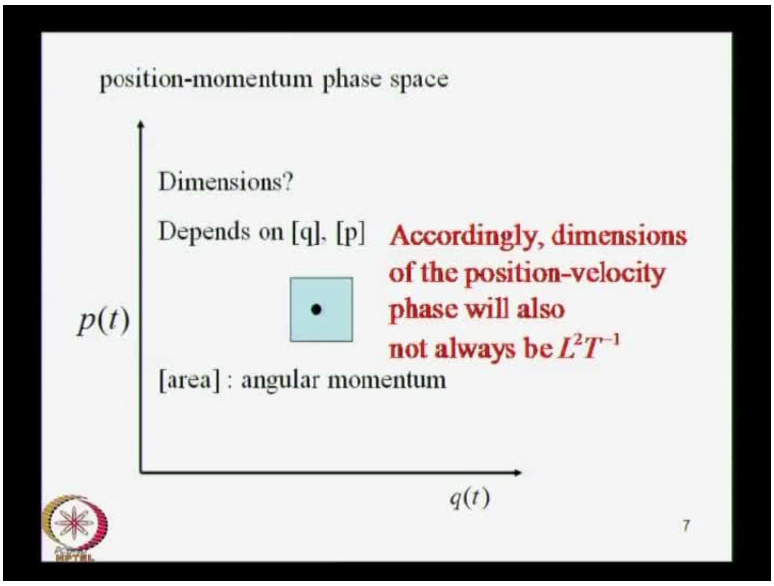

The reason of course is that you can also specify the state of a mechanical system by a point, by giving its position q and the momentum p; you plot q along the x axis and the momentum along the y axis and q and p give you the state of the mechanical system;

it is a point in a phase space this phase space is called as the position momentum phase space; phase space is a common term and sometimes it is position velocity phase space, sometimes it is position momentum phase space. From the context you know which kind of space you are using with accordingly the dimension of the phase space will also change; the a section of an area in the phase space will have the dimension of either position times the velocity or position times momentum depending on which phase space you are referring to.

When you deal with position and momentum then you know that the dimension of an area in this phase space will be that of angular momentum and the position velocity space will also have corresponding dimensions but it is possible to choose position; to have a dimension, which is not always length because depending on the context you can think of other parameters to represent the position. You can think of just an angle; For example,

if you are looking at some object and you want to look at the tip of this bottle, which is the lid and you want to indicate where it is located. if I were to be just rotating it about a fixed point in space, then if I simply indicate the angle then I would specify the position of the lid and this angle of course, is measured in radians; it is not a quantity whose dimensions are length; accordingly the dimension of corresponding phase space will also change but the corresponding momentum also will be different.

Now, it turns out and this will become clear as the discussion progresses; the dimension of the area in position momentum space will however always be that of angular momentum.

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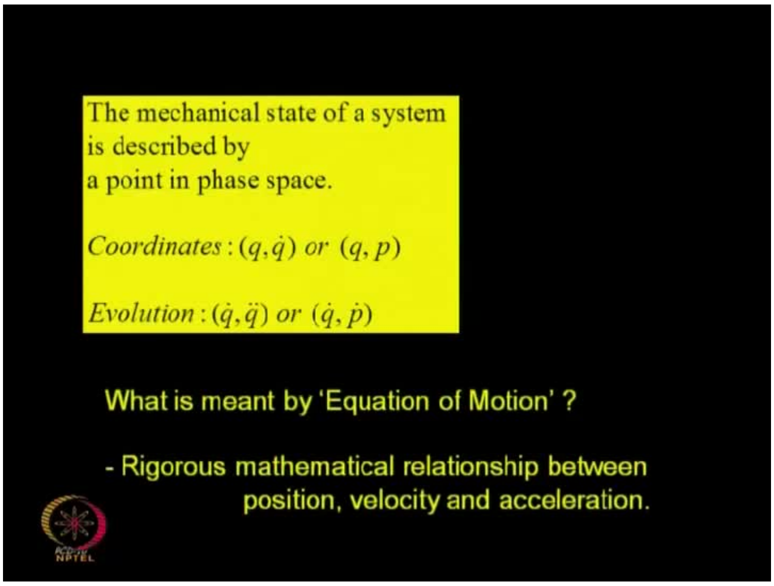

Now, this is our question:

Sir

Yes

Sir, you said that the position and velocity necessarily are required to describe the state of a system

Right

They are necessary things to describe

Yes

We cannot describe with position or velocity alone

No

You need both

You need either position and velocity

Or

position and momentum,

but how do you say that they both together suffice or that is they are sufficient, maybe there is something more that is required. How to come to the conclusion that only two is enough.

See, the interest in mechanics is to characterize the mechanical system and then to ask how does it evolve with time; that is the right question to be asked. Now we must ask ourselves what is that we need to describe the evolution of the system;

you are looking at q and q dot as to describe the mechanical state of the system over here or q and p and the time evolution given by q dot and q double dot;

here you have first derivative and the second derivative involved over here; this is what will give you the velocity, this is what will give you the acceleration.

Now, if this were not enough you will need some additional information and that is what your question is about; if you were to need any additional information, one could ask if the equation of motion could in fact be a third order differential equation rather than a second order differential equation and an attempt has been made to define the time derivative of the acceleration; it is sometimes called as jerk in some books but it is not a very useful quantity but it has been defined.

The reason is, it turns out that you do not get any extra information about the mechanical system.

You can completely predict where the object will be because all you get is either a second order differential equation, which gives you the complete time evolution of the system and it requires only two constants of integration; you do not have a third.

That is the short answer but it will become clear as we discuss this further. You need only two unknowns, which are the two constants of integration. Once you pin them down there is nothing unknown that remains in the equation of the motion;

this is what the system evolution requires, you need either the time evolution of q and q dot which will be q dot and q double dot, which is the second derivative or else position and momentum and their respective time evolution which is q dot and p dot.

This is the information that you are seeking; in describing mechanic, the evolution- evolution means how the system evolves as a function of time; that brings us to what is exactly meant by an equation of motion- an equation of motion is a mathematical relationship; it is a rigorous relationship between position velocity and acceleration. Once you have this relationship established and this equation of motion does not require anything more than that because the equation of motion turns out to be either the second order differential equation or to first order differential equations but in neither case do you need more than two unknowns.

The two unknowns are the position and velocity or the position and momentum and you do not require any additional information;

this is the minimal set, it is also the complete set; it is like King Lear’s daughter, who told him that I love you as much as a daughter should love the father no more and no less.

So, these two parameters are what you need, you do not need; anything more than that and you cannot do with anything with less than this it is just right.

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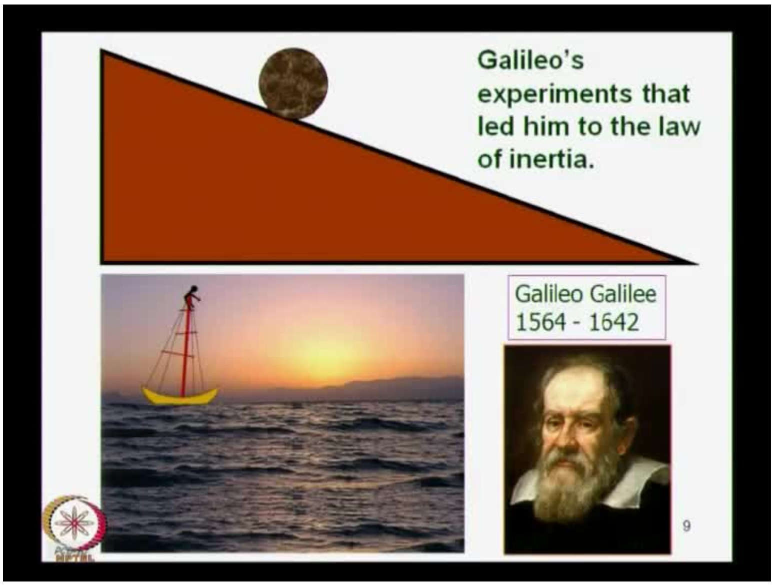

We need to understand first what is meant by equilibrium and this understanding really comes from Galileo and he was great experimentalist known as the father of experimental physics and what he did was to carry out very fascinating experiments. He would take various wooden balls and roll them down in inclined planes with different angles and then observe the motion and how this motion would change depending on the angle of inclination; he carried out some experiments by dropping objects from the top of a mast of a ship and you can think of this experiment; you would have done this experiment may be not by climbing the mast of a ship but perhaps in a train which is again moving with respect to the earth; if you are travelling in a train and you take a piece of orange and drop it and it does not drop at the previous railway station but it drops right where it would if the train were not in motion at the same corresponding point, which means that there is a certain equivalence between the state of motion of rest or of uniform motion because if this ship were stationary and you go to the top and drop the ball it would fall right at the bottom right below the point where it is dropped and if the ship were in motion, even then it would drop at the same corresponding point in the ship if it were moving at a constant speed.

Now, what this really means is that the state of motion with respect to either rest or uniform motion, these two are completely equivalent and they both describe what is equilibrium and this is a discovery which Galileo made by carrying out very fascinating experiments and this in fact is what gets to be known as Newton's first law- that a body remains in a state of rest or of uniform motion unless and until acted upon by an external impressed force;

that is the statement of Newton's first law but the first law in fact was discovered by Galileo. Subsequently, it was incorporated in the Newtonian scheme of mechanics by Newton. It gets to be known as Newton's first law but the credit for discovering the first law rightly goes to Galileo.

Uniform motion does not require any explanation; that is something which you expect to happen. You keep an object, it is going to stay at rest; nothing is going to happen, the equilibrium is not going to change. Something would happen only if you push it pull it or you apply some force on it.

No further explanation is required to explain this state of equilibrium. Now, same thing with uniform motion and this is really amazing because it is in fact counterintuitive in some sense. Let me come to this slide may be a little later. Let me share this excitement with you that the discovery of first law in fact was a huge step because it is almost counterintuitive;

if you take any object, here a piece of chalk and I roll it and I do not apply any force on it. Now, I have done but until that happened I am not conscious of any force which was being applied on the chalk; I just set it rolling. Would I expect it to keep sustain its state of motion, since I am not exerting any force on it or I do not see anybody else picking it

but then you know that the chalk would roll and eventually come to a halt because of frictional forces. After Galileo and Newton, we know how they operate but when these laws were discovered, if you place yourself back in time when friction was not known or it was not conceived then it would be very strange to understand the first law because your usual observation is that an object, if you take a piece of chalk and set it rolling, you will need to continuously push it and have a continuous force acting on it to keep it moving at a uniform speed.

The first law is in fact absolutely counterintuitive; it was a great discovery that Galileo made and this requires no explanation this is how you expect objects to behave under mechanical conditions. When no external agency is acting upon it when there is no interaction with any external field and what you need to explain is only when there is a departure from equilibrium.

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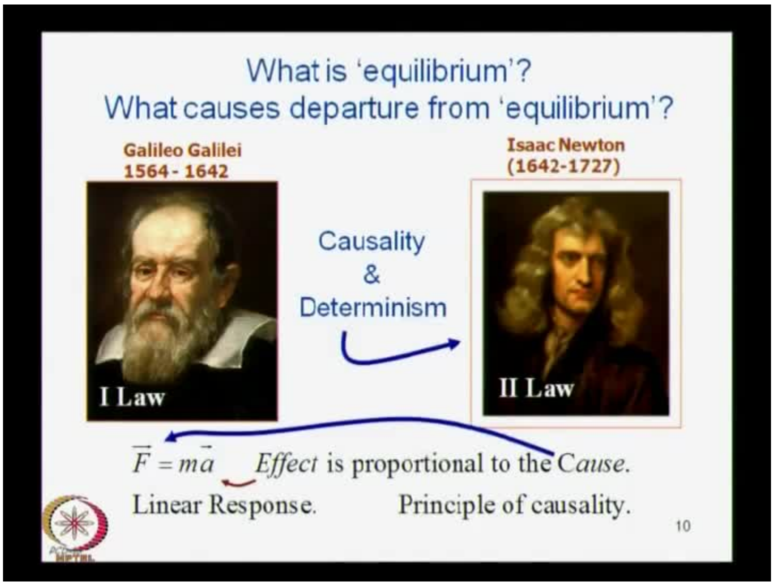

As long as an object is at rest you do not have to offer any explanation; it is self-evident but if it changes its state of rest then you have to look for a cause and then this cause would then be responsible for changing the state of motion. You have a cause effect relationship and it is this departure from equilibrium, which Newton explained and this is called as the principle of causality and determinism.

In fact, it is a rigorous mathematical statement that the cause is: what is called as the force, the effect and the acceleration. The acceleration is a measure of departure from equilibrium; in the absence of acceleration the body is in equilibrium. When acceleration is 0, the velocity is constant and the body is in equilibrium that requires no cause when you change the velocity. You look for a cause and there is a linear relationship, the acceleration is directly proportional to the force; there is a constant of proportionality this is a linear equation and this proportionality is what we call as inertia, this is the mass of the system

and this is the linear response theory, which Newton formulated and it is at the heart of Newtonian mechanics. This is what we call as Newton's second law.

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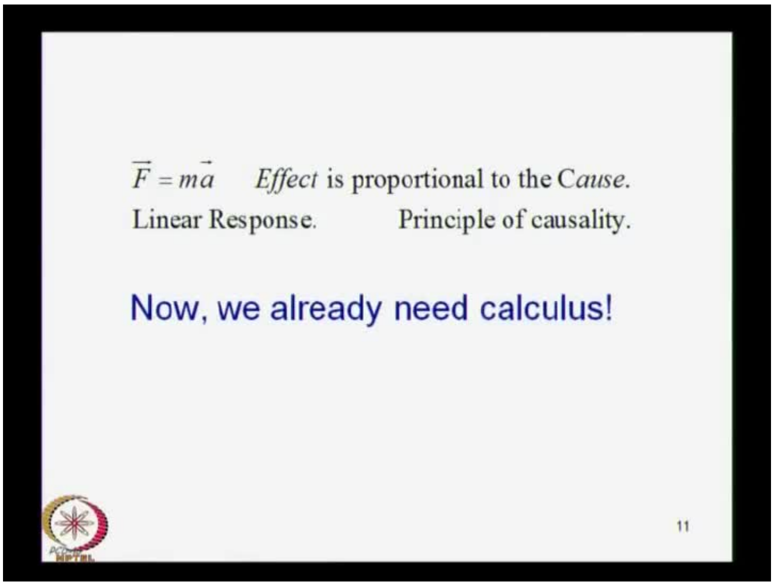

Now, what is this acceleration- this is the time derivative of velocity; the velocity itself is a time derivative of position. We have a second order differential equation and we already need calculus. We will quickly go through some of the essential steps in calculus because developing mathematics as we need is an integral part of this course; we will do it as we discuss this further.

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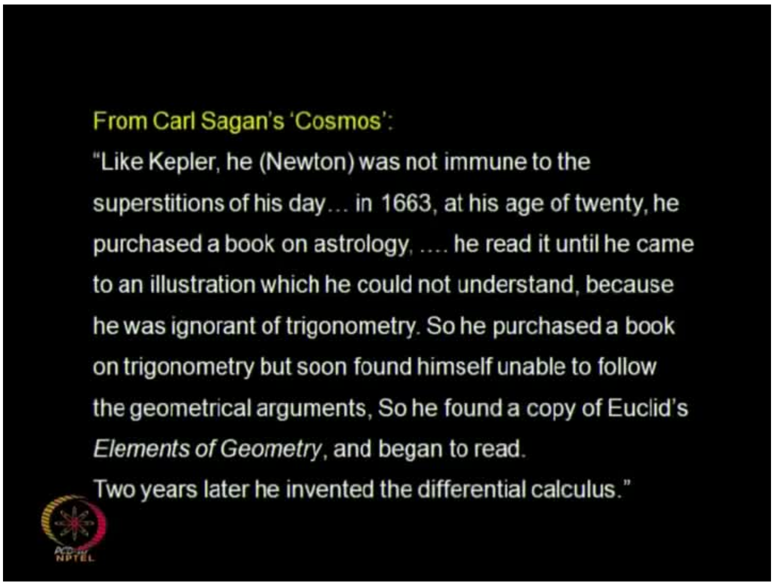

Now, it is interesting because you would have learned calculus in your high school and you know other classes and other courses that you have taken in school and colleges and so on but nobody taught calculus to newton.

He was in fact means….. I will quote from Carl Sagan's book Cosmos, in which he points out that just like Kepler, Newton was also not immune to superstitions of his day and when he was about 21 years old, which is about the age that some of you must be, most of you must be, he in fact was interested in astrology of all the things in the world, which is almost embarrassing

but then he got a book on astrology and he read it and then he could not understand it; he needed to learn some trigonometry because he needed to track the positions of different planets and at what rate their angular orientations change. So, he needed to look for some quantity which we would now call as d theta by d t but this quantity did not exists because idea of a function did not exist the idea of a limit that you take the value of the function and find what this value is if you led the function or if you change the independent variable; approach it from the left and from the right and if it is equal to the value of the function at the particular point then you have the idea of continuity and when you have continuity you can take a derivatives. All of these ideas we learnt in our high school and college courses; these ideas were not developed and Newton did not have this. So, he could not understand all this and he ended up actually inventing calculus and this is when he was just about your age means 22 or 23.

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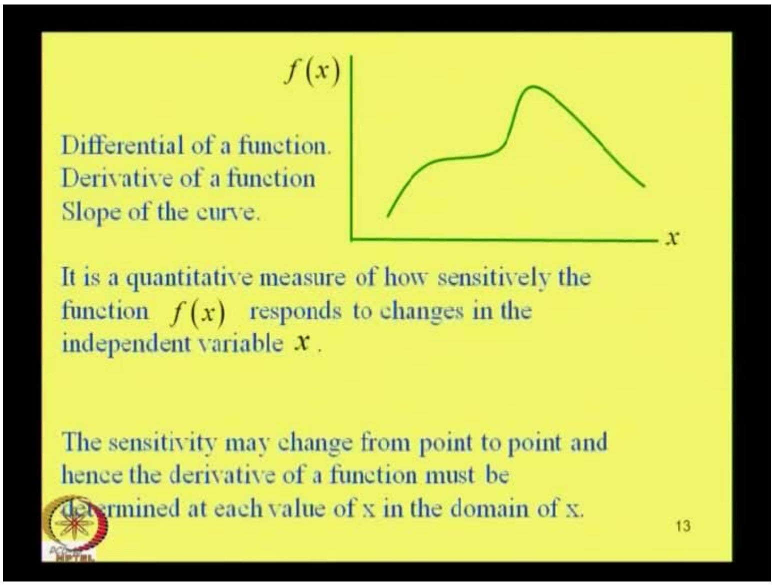

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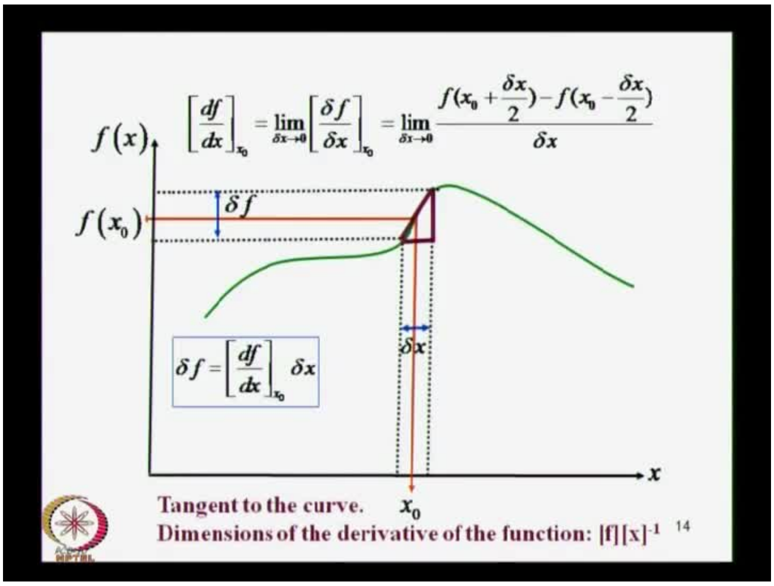

Let us see what a derivative is- you have a function, this is how a function is plotted; you have function of a single variable here and what it really does is, it is measuring. .It is a statement of how sensitive a certain variable is to another variable on which it depends, it is independent of, it does not change or if it changes, how does it change as the independent variable changes from one to the other; it may change a little, and then maybe there would be a range where it does not change very much or it could change very rapidly, its value could increase or decrease and so on. This is the measure of the sensitivity of the function and what you want to really find out is to develop a quantitative measure of the sensitivity of the function f, to the independent variable x, it is this sensitivity. How sensitive it is to x what you want to determine, what you want to ask is that at a particular point when x is equal to x naught and the value of the function is f at x naught; this is the value that the function has at this point and then if you look at some neighboring points about this x naught then through what quantity has the function f itself changed;

that will be a measure of the sensitivity of the function. So, obviously you can construct a triangle over here and if you take the ratio of this change divided by this you get a measure of how rapidly the function is changing with respect to the independent variable; that is what the derivative of the function gives us and we will be using this extensively in our discussion. You can play with this equation and then write an expression for this numerator over here, which is the change in the value of the function. At two neighboring points, which are fairly closed to each other in the limits, they will be infinitesimally close to each other; that is the limit that you must seek and you will find that this numerator is nothing but the derivative multiplied by this denominator. So, you can actually get an estimate of how much this function changes.

In fact, the more correct expression will have many more terms but this is the leading term as you get; this is the idea of a derivative- it is a measure of the slope of the function, it is a tangent to the curve as you can see from this figure- from the geometry and this derivative is of course a quantity which has got dimensions and it will have the dimensions of the function divided by the dimension of the independent variables. You should always write the dimensions along with the derivative or the tangent to the curve, do not ever write it as dimensionless quantity.

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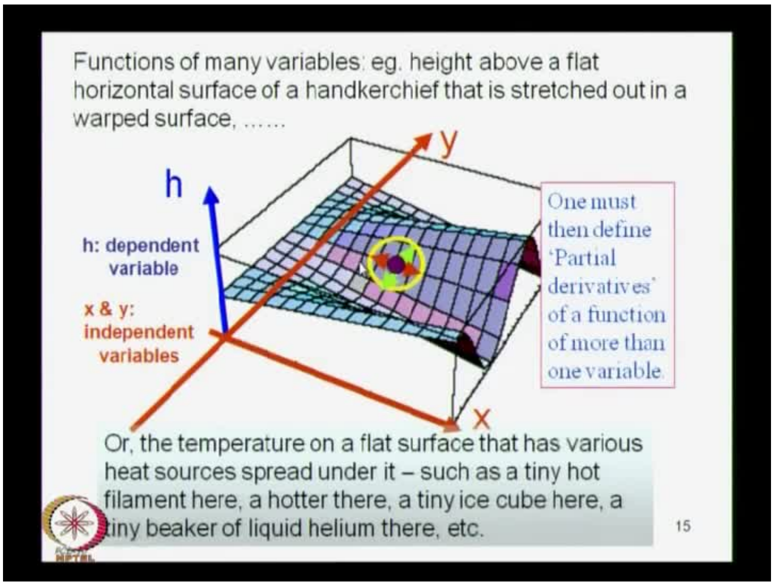

You may also have functions of many variables because if you take some piece of cloth or some paper and you hold it in some arbitrary shape not necessarily flat, then how much it rises above the level of the surface of the table, it depends both on the x coordinate as well as on the y coordinate; this is a function of two variables or you can have either situation or you can even think of a flat surface but you ask yourself what is the temperature at each point on this surface and this need not be constant because I can have a source of heat over here, I can have a source of ice cube over here or I can have liquid helium over here and depending on what I keep beneath this, the temperature even on this flat surface will change from point to point and it will be different along the x axis and along the y axis.

So, here is a function of two variables and in this case you will have to define what are known as partial derivatives and these are mathematical quantities of great importance. You will have to consider how much the function changes if you consider displacements either along the x axis, which is shown by this red arrow over here or by this green arrow, which is the displacement along the y axis and depending on which direction you are seeking, the change on the independent variable the function itself. You are plotting the property that you are examining, which is a physical quantity which could be the height of some point on this surface or it could be the temperature on a flat surface or any other function, which really depends on two parameters- two independent parameters- will be shown by this:

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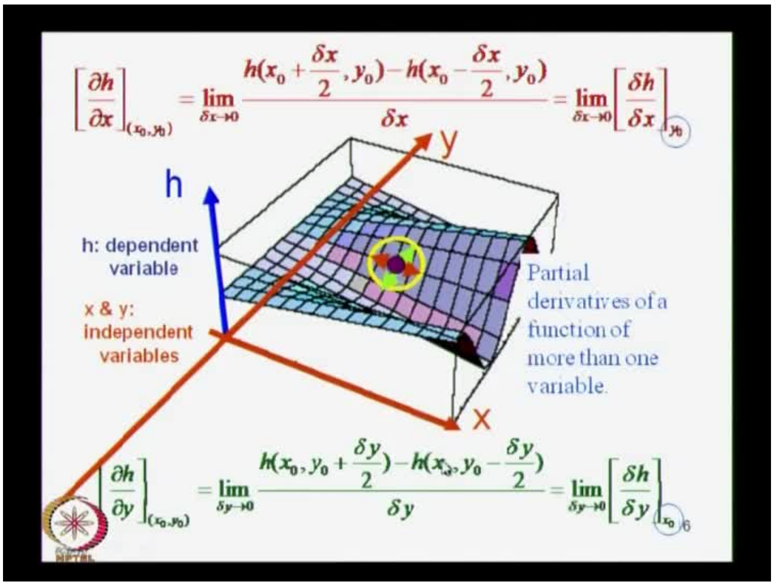

You will have to determine what are called as partial derivatives rather than the total derivative and here you determine the derivative in the height h as a function of x and you determine this by taking displacements along the x axis over a short interval in which the limit must become infinitesimally small and at the same value of y. So, you take some point x equal to x naught and y equal to y naught and take displacements around this point along the x axis so this is point is at x naught plus delta x by 2 and this is x naught minus delta x by 2; take this ratio and seek its limit in as delta x goes to 0 you get the partial derivative of h with respect to x when y is held constant at y naught. So, the other variables must remain constant.

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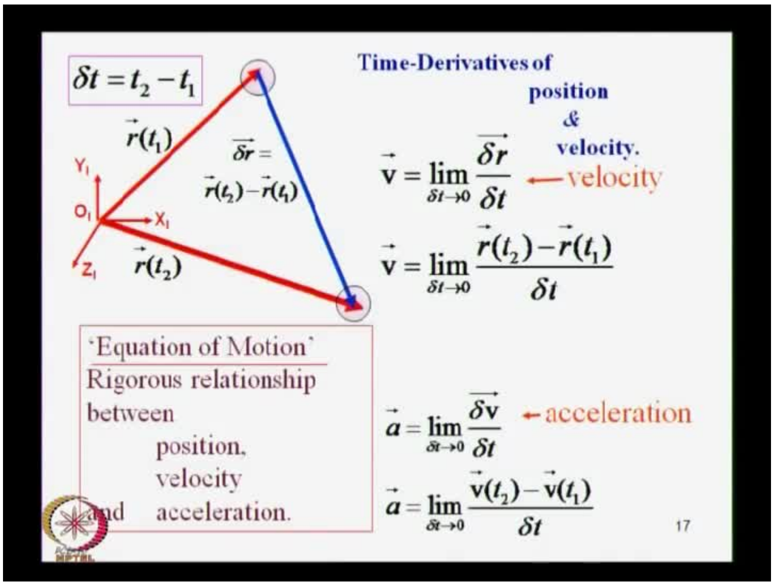

and then for the other partial derivative you keep x naught is constant; so x naught has got the same value in both of these terms and then you take the difference of the corresponding terms. Take the ratio in the limit delta y going to 0 you get the partial derivative with respect to y. Now, we know what the derivatives are; we can extend that idea to get the time derivative of position; you have a position of an object at a certain instant of time t 1 and at a later instant of time t 2 the position of that object is over here; this is the displacement vector shown by this blue arrow over here which is the difference vector r t 2 minus r t 1 so this is the difference vector and if you divide this difference vector by delta t take the limit delta t going to 0 you get the velocity and if you define corresponding quantities, if you construct a similar diagram for the velocity vectors you will get the acceleration. These are the quantities which are of importance in mechanics: the position and the velocity and their time derivatives- the time derivative of position being the velocity and the time derivative of the velocity being the acceleration.

Now, what you are going to look for is a rigorous relationship between position, velocity and acceleration which will be the equation of motion.

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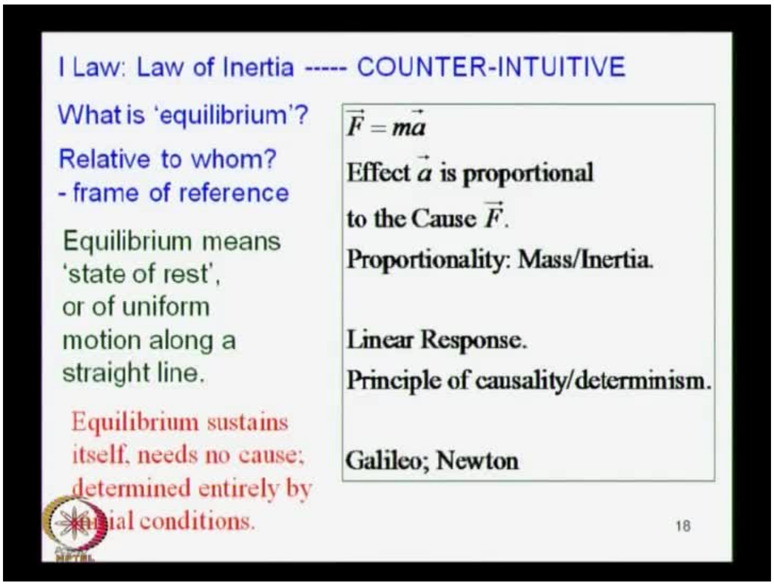

This is our frame work. Now the first law of inertia, which I emphasized, is counterintuitive; it tells us what equilibrium is. We have to define equilibrium relative to whom and that connects as to the frame of reference. We recognize by equilibrium- a state of rest or of uniform motion along a straight line both are completely equivalent. We recognize that equilibrium requires no cause, it is self-sustaining. We recognize that F is equal to m a, is the cause effect relationship, which explains the departure from equilibrium that whenever equilibrium is changed there must be an acceleration.

So, the momentum must change and if it does so it will have to change at a certain rate, which is d p by d t and this rate of change of momentum is nothing but the force itself; so this was the main contribution of Newton in developing this principle of causality as a linear response theory.

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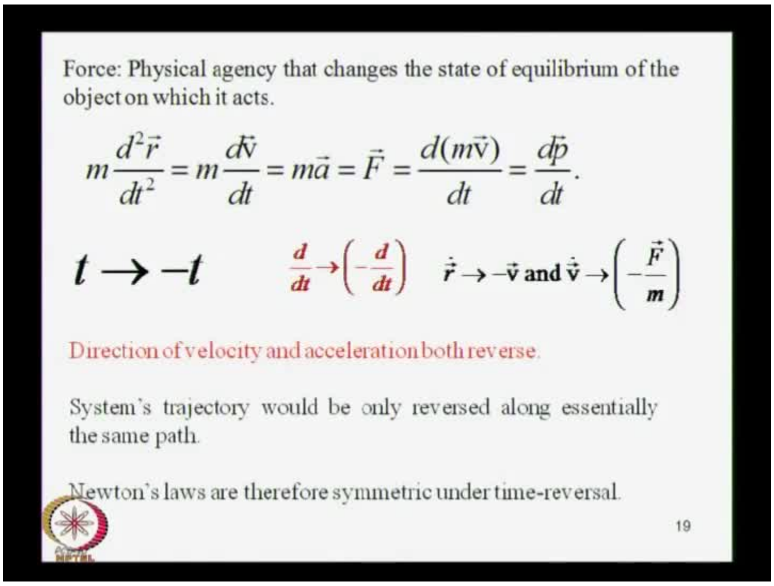

Force is now identified as the physical agency, which changes the state of equilibrium of the object on which it acts. Now, let me bring to your notice that in this relationship if you consider time reversal, instead of time going forward you consider time going backward. So, do not examine, where the object will be in the future but you ask where was it in the past and how did it come here, how did it get here. We are enquiring the time evolution not from the present to the future but from the past to the present and you can ask this question by examining the laws of motion under what is called as the time reversal.

So, you are letting t going to minus t in the equations of motion. Now, when t goes to minus t, the time derivative operator d by d t would go to minus d by d t and r dot, which is d r by d t will go to minus v and v dot, which is d by d t operating on the velocity will go to minus F over m. So, the direction of velocity and the direction of acceleration both reverse.

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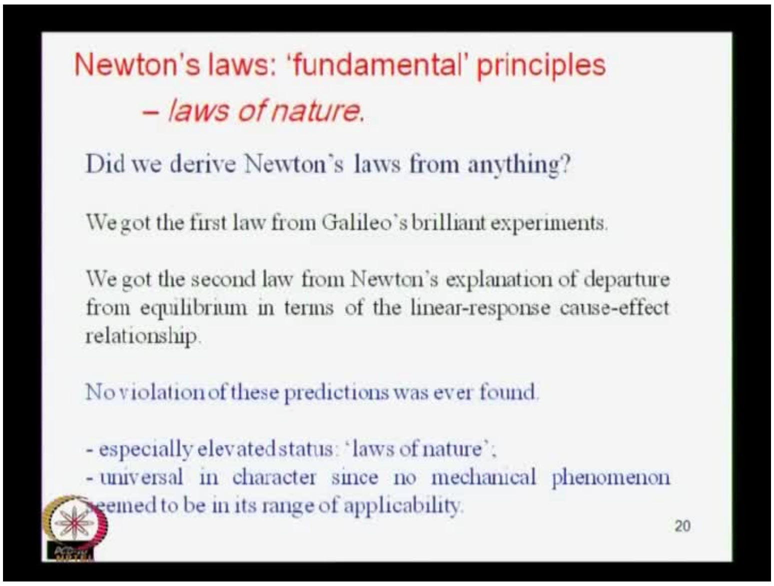

So, the motion itself is reversed under time reversal and the Newton's equations are completely symmetric under time reversal; there is no change and that is the reason of course the time derivative operator d by d t comes into play twice in the equation of motion; The twice occurrence of the operator d by d t makes the equation of motion completely symmetric under time reversal. Another point that is of importance, is to ask ourselves if we have derived Newton laws from anything and you should remind yourselves as to how it is that we have arrived to Newton's laws. We discussed only the first two laws. The first law, which is the description of the state of equilibrium, an object requires no application of force to sustain its motion along a straight line in uniform motion or in a state of rest.

We have not derived it from anything. Now, at this point it is a good idea to just discuss if it is possible to derive the first law from the second

because if you look at f equal to m a, as the second law and in this you put F equal to 0, then what you get is m a equal to 0; mass is not 0. The acceleration is 0 and if the acceleration is 0, velocity would be a constant and may I then claim that I have derived Newton's first law from the second; may I do so, it is very tempting and many students tend to say yes and their answer is wrong; its better be wrong because if the first law is easily derivable from the second law by doing one simple integration and you do not have to do any advanced calculus or any number crunching to get this.

why would you even bother to call it as a law of nature. Why would you elevate it to that status. You cannot do that -you just cannot do that- you cannot touch the second law unless you identify for yourself what is called as an inertial frame of reference and the first law is what gives you the mechanism to recognize what an inertial frame of reference is because it is sometimes defined as a frame of reference in one, as one in which Newton's law hold but this definition really goes into a cyclic loop because Newton's laws again hold only in inertial frame of reference.So, you really have to break this loop and ask yourself how you recognize an inertial frame of reference

and the way to do it is to define it as one, in which motion is self-sustaining; no external agency is required to sustain that state of uniform motion and if you identify such a frame of reference, then you recognize it to be an inertial frame of reference, then you know what equilibrium is and then you ask that in this. Frame of reference is equilibrium changing at all if it is not changing you do not look for any cause it is self-evident and everything is explained by first law, which is Galileo's law; you do not have to carry mechanics any further. However, if you observe that the state of equilibrium has changed then you would ask what is that it caused it and that is a separate question which is answered by second law.

You cannot derive first law from the second law; both are independent statements of laws of nature. We have not derived either from anything else; the first law we got from Galileo's brilliant understanding of the experiments that he carried out. It was a statement of his experience; it was not a derivation from anything else. The second law also was not derived from anything else; it is a statement of Newton's interpretation of what is it that changes equilibrium.

Newton explained that when equilibrium changes it has to have a cause and the cause manifests itself as acceleration and this acceleration is directly proportional to the agency which cause this acceleration. It is a statement of Newton's explanation of what is it that causes departure from equilibrium.

So, we have not derived it from anything else -these are not derived from anything else- these are in fact themselves they provide us the platform from which everything else in mechanics is derived; this is how we get the 2 laws of Newton.

The first law is Galilean's, it comes from Galileo's experiments; the second from Newton's explanation of the linear response relationship between the cause and the effect force being the cause acceleration being the effect.

Now, no violation of these predictions was ever found and this is within the domain of the classical mechanics. We have to refine these ideas a little bit as we go further in quantum theory but I will not get into that at this point but within the realm of classical mechanics you do not observe any exception absolutely none and that is why you can call it as a law of nature; you call it as the law of nature, you call it even as a universal law of nature because you expect it to operate throughout the physical universe.

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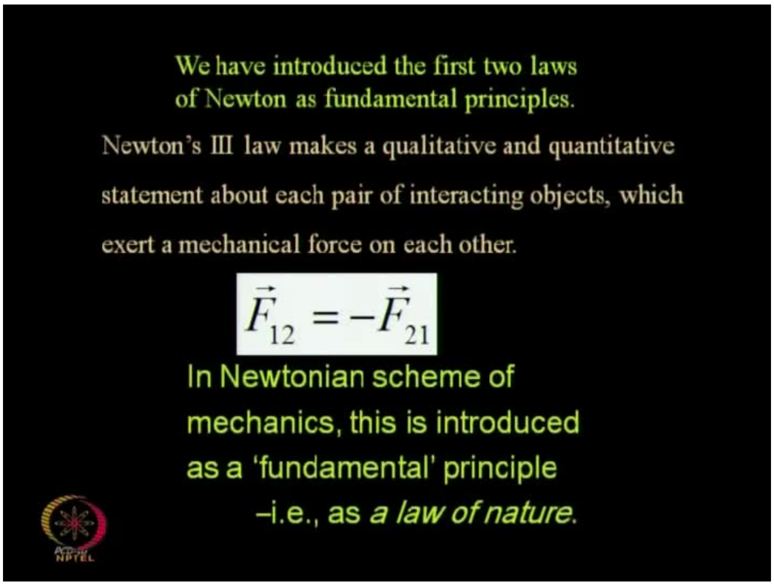

Now, these laws, the first two laws were introduced as fundamental principles recognized by Galileo and Newton respectively and the only interactions that newton had to worry about were the interactions between objects because you know fields were not understood at Newton's time; not the electro dynamic field for sure gravitational field of course, Newton was quiet conscious. In fact he used it to explain planetary motion but did not recognize it in terms of the field theory that we know it as of today.

The forces that newton was dealing with were forces between objects between particles or stones or masses or planets or a planet and a star like the sun and the earth or the sun and the Jupiter or the moon and the earth and there were two body or three body problems and so on….

These were the forces between pairs of objects and what Newton figured out is that the force by 1 on object 2 would be equal and opposite to force by object 2 on object 1. This is the statement of Newton's third law.

Again this is not derived from anything; this was Newton's understanding of how pairs of objects interact with each other and he conjectured that the relationship of these forces between pairs of objects is equal and opposite; this is what we say very simply as action and reaction as equal and opposite it does not matter, which force you call as action and which force you call as a reaction but this again emerges as the fundamental principal; it is not derivable from anything else as we find it in the Newtonian scheme of mechanics.This is also a fundamental law of nature and this is what we call as the third law of classical mechanics.

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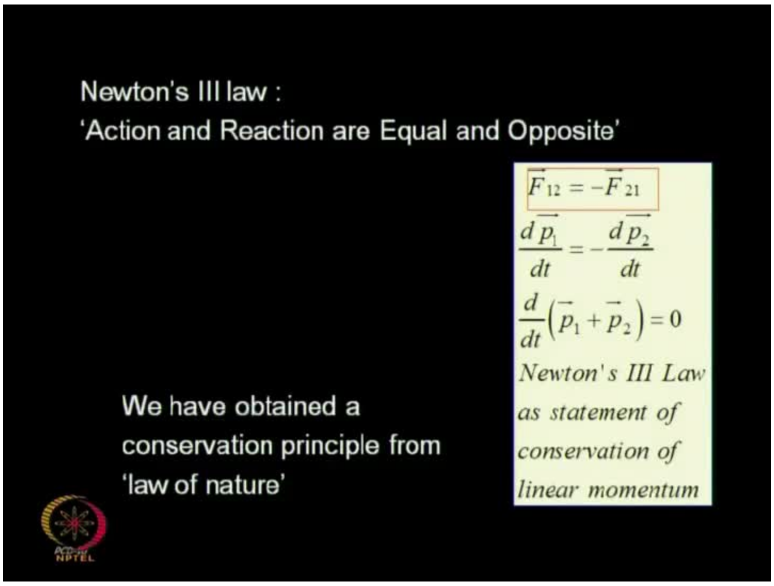

Now, let us have another look at Newton's third law it is written over here in this red box f 1 2 is equal to minus of f 2 1 but we already know from second law that this force on 1 is a rate of change of momentum of the object 1 and this force on object 2 is the rate of change of the momentum of the object 2; it is d p 2 by d t.

Now, if you just take, bring this term on the left you get a very simple result that the time derivative of the total momentum must vanish. Actually, I should have an arrow on this 0 with corresponding to the null vector because the left hand side of this equation is a vector; so, you have the rate of change of the total momentum which must vanish, in other words the total momentum is conserved and Newton’s third law is therefore a statement of conservation of momentum. Do you recognize that, do you see that conservation of momentum comes straight out of Newton’s third law.

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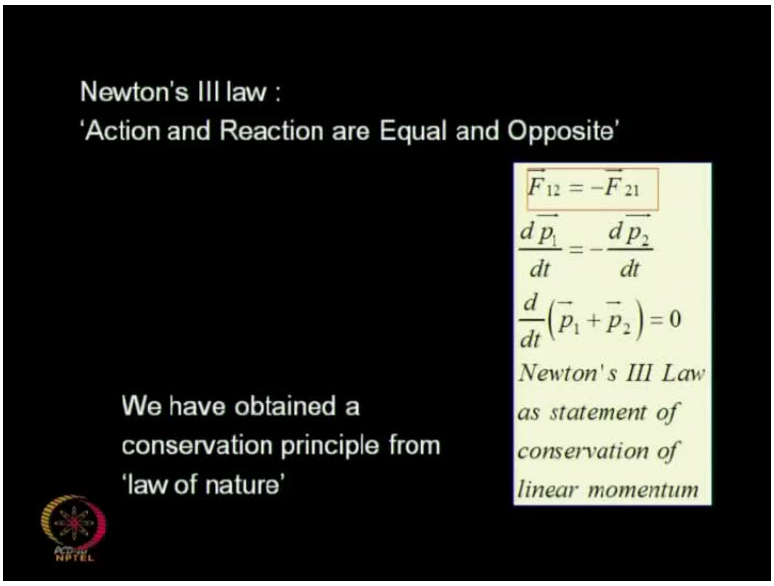

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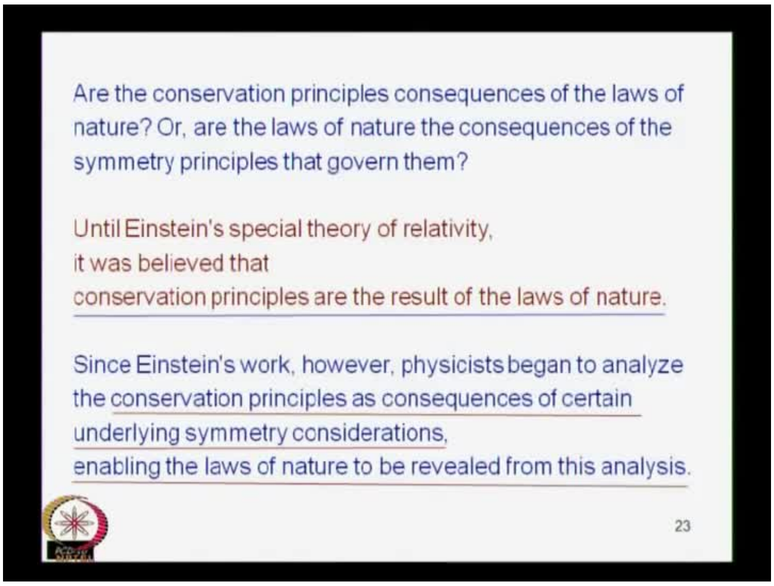

This really raises a very interesting question that you have got a conservation principle, namely the conservation of linear momentum from a law of nature. The third law, which is on the same footing as the first law and the second, which is what you would call as the law of nature and you can ask this question, are the conservation principles consequences of the laws of the nature or, are the laws of nature the consequences of the symmetry principle that governs them.

Now, this a very exciting question. Now, it turns out that until the special theory of relativity was formulated by Einstein, which was in 1905 the way physics was discussed and understood and debated and comprehended was that the conservation principles are the results of laws of the nature; just as we saw in the previous slide that the conservation of momentum followed from a law of nature.

This was the perception of physical laws and this was the situation until Einstein's theory of relatively but Einstein's thinking gave it a different turn altogether and physicists began to analyze conservation principles as consequences of some underlying symmetry considerations.

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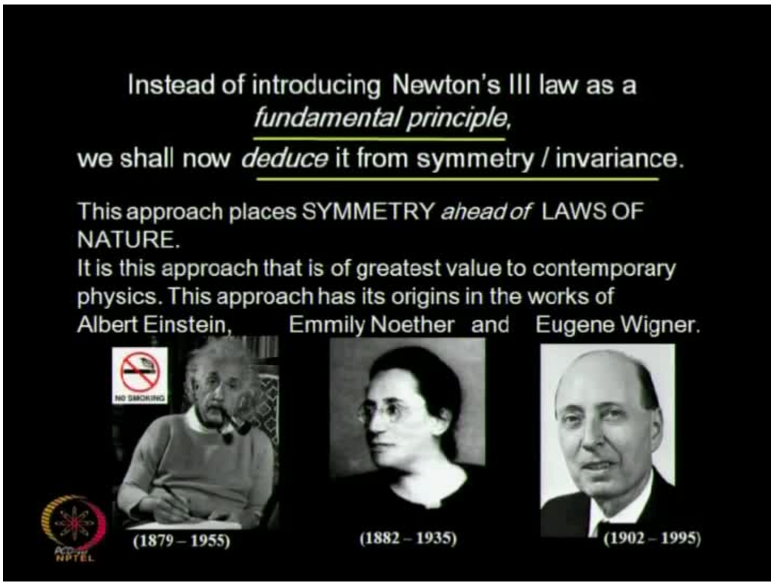

This is something which we have to discuss in some details now. There are some underlying symmetry considerations, which enable the laws of nature to be revealed from this analysis; this is how laws of nature can actually be deduced and what we are going to do is to illustrate how Newton's third law can actually be deduced from symmetry principles; that is something that we will do to get some kind of an understanding of this particular approach and this approach actually places symmetry ahead of the laws of nature and this approach is very fascinating; this approach began with Einstein it is enunciated in the statement of what is known as Noether's theorem

How is the symmetry principle different from law and nature

They are intimately connected. Actually, this is going to be the statement of Noether's theorem, in fact it is following or it is a popular statement means the exact mathematical statement comes from field theory; I would not, I will not get into that but generally speaking it can be stated that for every symmetry principle there is a physical quantity, which is conserved and for every physical quantity that is conserved there is a corresponding symmetry principle. you will start beginning, you will start to begin to see this relationship as we discuss this further and what I am going to do is to deduce Newton's third law, which is the statement of the conservation of the momentum, which will come from a symmetry principle namely symmetry in translational space that if you have got a space, which has got the same properties whether you consider the properties of that space over here or here.

Now, this displacement- that we are talking about from when you go from here to here to here, you are talking about translational displacement, you are not talking about any physical quantity, which is conserved but you are talking about symmetry; this is symmetry now, when you have such a symmetry, what is conserved is the linear momentum and this is what we are going to show as the discussion progresses:

Why cannot a symmetric principle be a law of nature

Well, it is a matter of vocabulary, it is a matter of semantics law of nature; it is traditionally used to describe what is a mechanical system and how does it evolve with time when you make a statement of this situation, you are talking about a law of nature, what is the physical law which governs the time evolution of the state of the system.

So, in classical mechanics this statement comes from Newton's laws or Lagrange's equations or Hamilton's equations. Those are the laws of nature in quantum theory; it is a Schrodinger equation; that is the law of nature, it tells you how the wave function develops as a function of time; when you talk about how a physical system evolves with time that is when you say you are talking about a law of nature;

that obviously requires first a statement of how you describe the physical system at all and in classical mechanics it is described by position and velocity or position and momentum.

In quantum theorem it is a state vector; this is what we understand by a law of nature- symmetry is when you consider certain operations like: when you consider properties of space from here to here you are talking about displacements and these are symmetries.

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So, this is at the heart of Noether's theorem and this was very nicely elucidated by Eugene Wigner, using arguments which required group theory and it is a very fascinating formalism. Now, physicists understand this particular approach, which began with Einstein and developed through the work of Noether and Wigner and we will take this up in our next class.