Select/Special Topics in Classical Mechanics

Prof. P. C. Deshmukh
Department of Physics
Indian Institute of Technology, Madras

Module No. # 06
Lecture No. # 20
Special Theory of Relativity (ii)

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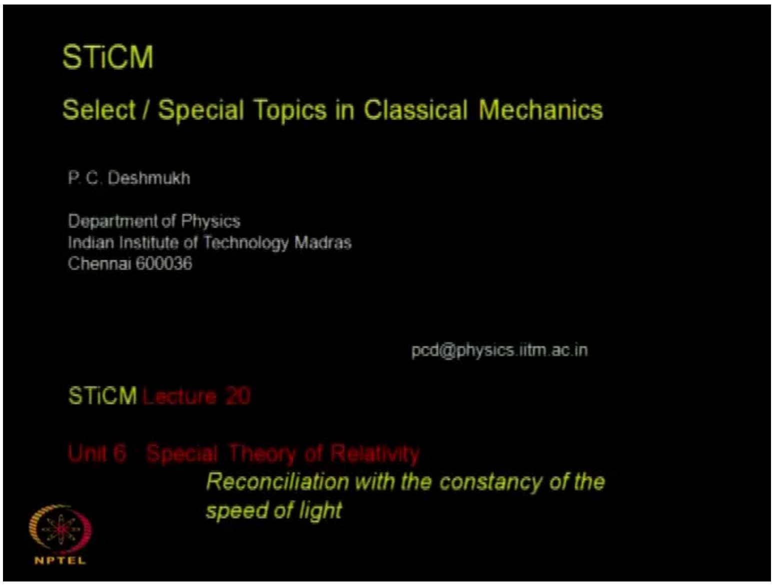

Greetings. Let us continue our discussion, and our essential task now is to reconcile with the fact that the speed of light is constant and it has the same numerical value in every inertial frame of reference.

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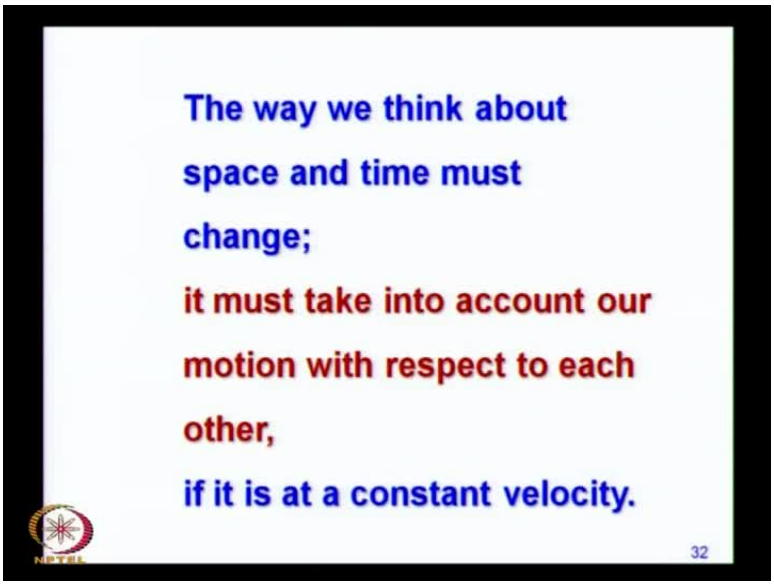

So, what it is going to force us to do is to refine our understanding of what we mean by space. What is it that we mean by time means, we already have some intuitive idea, but we need to subject these ideas to some stringent logical analysis, and from this, be prepared for some surprises because having an open mind for surprises is an essential and integral part of a scientific attitude, and if it is going to force us to come to terms with the fact that what we have so far understood by a space interval is not the same anymore, but it has some other connotations, some other implications, some other meaning.

We are going to have to come to terms with it. So, we will have to admit these sayings. It will lead us to the notion of length contraction and time dilation, and while answering the question that was raised toward the end of the last class when you asked me.

If these are separate consequences, if these are simultaneous consequences, whatever you meant by simultaneity in your mind, and I pointed out that there are a number of consequences of the special theory of relativity. There are number of consequences not of the theory but of the laws of nature. The theory is about how these laws are expressed and there are a number of consequences, mass energy equivalence, for example, as I mentioned not just time dilation and length contraction, I also mentioned the electron spin.

Yes, these are all consequences of the finiteness of the speed of light. It does not automatically mean that finiteness of the speed of light is all that is required to explain the electron spin, no, there is more to it; there is quantum mechanics. That is a separate story.

So, you need some additional things for some of these conservations, like the electron spin, but they are all, they all require this in common that the speed of light is finite; it has got the same value in every inertial frame of reference and these are automatic consequences which come as a package.

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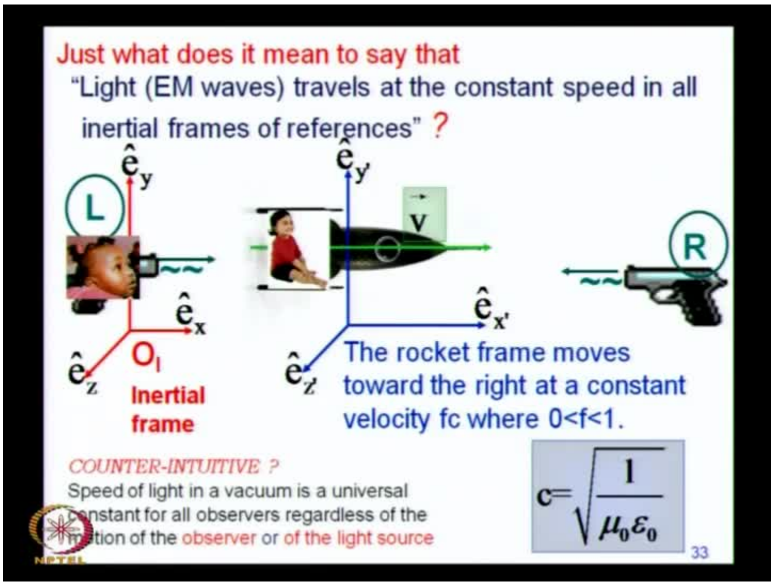

So, let us ask ourselves that what exactly might we mean by the speed of light? Being constant in every inertial frame of reference. This we have admitted in our thinking. Having accepted that Maxwell's equations are the same in all inertial frames of references that they hold good in every inertial frame of reference.

That the speed of light is given by properties of vacuum and nothing else that the speed of light should, therefore, be a constant in every inertial frame of reference is something that we have already admitted in our thinking. So, if you wish regard it as the second postulate of the special theory of relativity.

The first postulate being that the laws of physics are same in every inertial frame of reference. The second postulate being the speed of light is the same in all inertial frames of references. To call it as a postulate or not is a matter of your own choice. Essentially, what it means is that you must take fact of this point into your thinking, into your mind set, into your outlook from the very beginning. At the outset, before you do any further analysis, you admit the fact that laws of physics are the same in all inertial frames of references. You admit the fact that the speed of light is the same is in all inertial frame of references.

Now, given these two considerations; now explore what are the consequences of this. Now this is what we will set out to do. So, we begin with a light gun in an inertial frame of reference E x, E y, E z. Here is a gun which I label by the letter L because this is to the left of the screen, and later on I am going to put a gun on the right of the screen which I will label as R.

So, an anticipation of that this gun is labeled as L, and we have an observer who is going to measure the speed of light. Now, scientists, engineers, they are all becoming younger and younger, so, we have an observer who is young enough, hopefully, and this observer would measure the speed of light. Now, this is observed by another observer also young one, not that it matters, and this second observer who is in this blue frame of reference is moving at a constant velocity V with respect to the first observer.

This is what I call as a rocket frame; he could be in a rocket, why not? Not just children, even we would love to be in rockets. So, this is the rocket frame of reference and he would also measure the speed of light.

This rocket is moving at a certain speed which is f times c, which is a certain fraction of the speed of light, and that fraction could be anywhere between 0 and 1, and what we have admitted in our thinking is that the speed of light which is fired by this light gun that you see in this picture.

This speed of light is the same regardless of who is observing it because these two observers are moving with respect to each other at a constant velocity, and therefore, both are in an inertial frame of reference. They are in their own inertial frame of reference, and they must measure the same speed of light. That is something that we have admitted in our thinking that goes into the very plat form of the theory that we are now building.

What if light is fired from a gun on the right? If these two observers measure the speed of light which is fired by this other gun, even then, they will measure the speed of light to be the same, because the speed of light is the same in every inertial frame of reference, regardless of the state of motion of the source or the observer.

Now, this seems counter intuitive, but it is counter intuitive only because we are thinking in terms of Galilean relativity. If we keep super imposing our knowledge of Galileo's principle of equivalence, that motion in all inertial frames of reference is equivalent. With the fact that the speed of light in every inertial frame of reference is the same, then, there is no conflict with intuition. So, counter intuitive pre-supposes a certain meaning which intuition would admit. So, it is with reference to that this is counter intuitive.

If you refine your intuition to admit that constancy of the speed of light in every inertial frame of reference, then it need not remain counter intuitive any more, but that requires a refinement of intuition, and this must come from the fact that the speed of light is determined completely by properties of vacuum and it is not defined with respect to any particular choice of a frame of reference.

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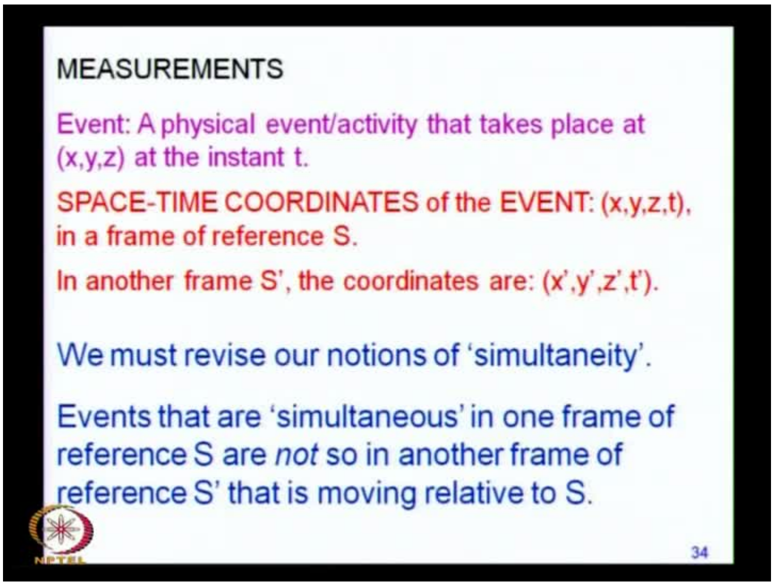

So, let us now talk about these measurements, and we will need to talk about events which take place at a certain point, at a certain instant of time. So, any physical event or activity that takes place at a certain point in space whose coordinates are let us say x y z at an instant of time will be denoted by an event which I will be referring to by these coordinates x, y, z, t and in another frame x prime, y prime, z prime, t prime.

What I am open to the consideration is that this t prime may be different from t; it is not a conclusion I have drawn as yet. We are not claiming at this point that t prime is different from t, but this is the conclusion that we will arrive at when we go through the rest of the analysis. At this point, we are only using a different label for the different frame of reference. We have not claimed yet that t prime is not equal to t, but we are not demanding either that t prime must be equal to t.

We are open to whatever t prime will turn out to be. So, we will find out what t prime must be so that we can reason out our observations in a manner that is consistent with the fact that the speed of light is the same in every inertial frame of reference. If t prime turns out to be different from t, be it so; if it turns out to be the same, well that is something that we have known from Galileo.

Now, let us be open to the fact that it may not turn out to be the same. So, let us find out what it will turn out to be. Now to do that, we must ask ourselves a fundamental question, as to what is it that we mean by simultaneous events, and we all have some sort of a notion, and I am not going to refer to any dictionary as such to give you the meaning of what is meant by simultaneous, but I am going to ask you to think for yourself as to what is it that you mean by simultaneous, what does this term suggest to you, when do you consider two events to be simultaneous.

A coincidence of two events is something which will come to your mind, that when there is a coincidence, not that there is a cause effect relationship between them but you often suspect that they are connected, like in Marathi, there is a saying which loosely translated would mean that, if a crow comes and sits on a tree and the branch breaks. That is what they say right.

That, if a crow comes and sits on a branch and the branch breaks, does not necessarily mean that the branch has broken because the crow has sat on it, but it might just happen what you would call as simultaneously, and this is the kind of meaning which comes to your mind, when you think of simultaneity and you all have the same meaning in your minds.

Now, if Milind were watching this happen sitting under the tree, and if I were to watch this a when I am walking at a certain speed, or going in a car at a certain velocity - at a constant velocity – so, I am also in a inertial frame of reference to him. Would we still call it as a simultaneous event? By and large, our perception of simultaneity would not be different in most cases, which is what we mean by simultaneity. However, when you are observing light, these ideas have to be refined, and let us see why it has to be so.

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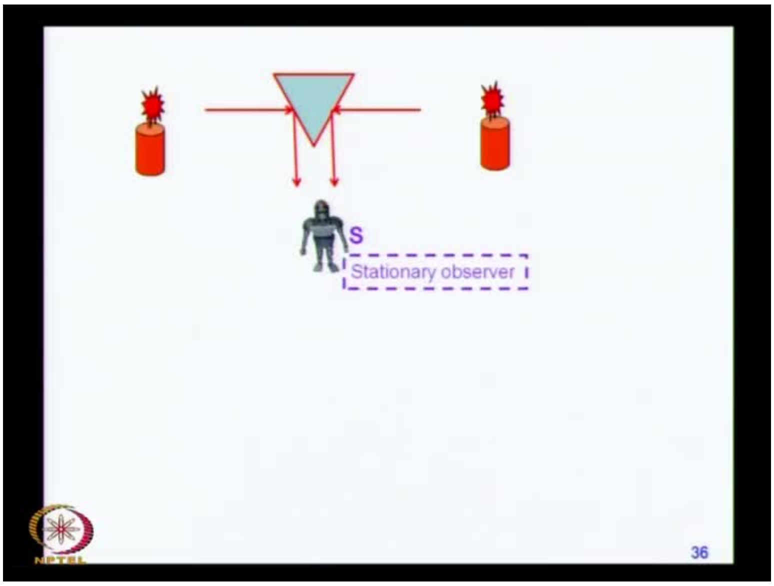

So, let us look at light. So, there is a firecracker and you light it, and that, now stationary observer will see the firecracker light. He is going to observe it. Here is a stationary observer and he will observe the burst. Now, this is the experiment that we are going to discuss that the firecracker is lit and the stationary observer observes the light burst. What we will do is we will set up an arrangement so that there is some kind of a mirror here; there is a mirror of this face. So, that the light coming from this burst will get reflected, and then, get reflected so that the observer can see it from here.

The reason I set up such an arrangement is because I could then have a firecracker burst to the left of this stationary observer, and this observer could see the burst from both the firecrackers without turning his neck to the left or right or anything right. So, if there is another cracker, firecracker on the left of this observer and both of these fire, then he could observe both of them together, so that you do not have to worry about how is he going to observe the one from the left, if he is looking toward the right, and vice versa. So, we do not worry about it.

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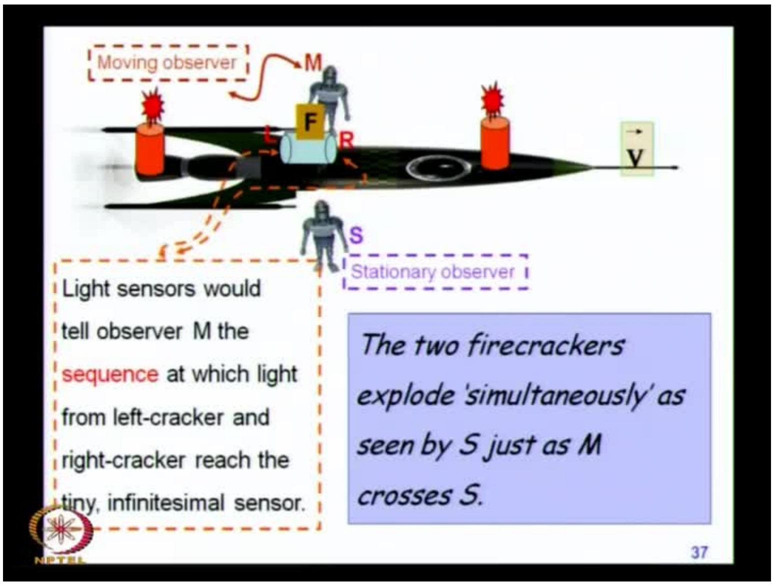

So that complexity is eliminated from our discussion at the very beginning. In other words we have set up an arrangement so that, if the two firecrackers burst simultaneously, this stationary observer can see, and now, we throw an another observer who is in the rocket frame which is moving at a constant velocity V with respect to the stationary observer, and there is another observer on this rocket. So, there is a moving observer M; there is a stationary observer S. The moving observer is travelling from left to right on the screen at a velocity V with respect to this.

And these two firecrackers burst simultaneously as seen by the stationary observer S, is the picture clear in your mind? And the burst takes place exactly when the moving observer is right in front of the stationary observer.

So, the stationary observer is right in the middle; he is equidistant from the two sources of burst; he is exactly in the middle; he is going to see the two bursts simultaneously, and now we have some idea of what is meant by simultaneity. At the simultaneity that we have described is with reference to the stationary observer. Let us not pretend that this simultaneity applies also to the moving observer, something to be tested. It may turn out to be the same; it may not. So, let us investigate this with an open mind.

So, we have a certain element of understanding of simultaneity in our minds, and with respect to that understanding, these two firecrackers burst simultaneously, further stationary observer, just when the moving observer is right in front of him, but he is moving at a constant velocity, so, he comes from the left. He is in front of the stationary observer for a moment, and then, he keeps moving. His motion is always at the constant velocity V.

Now, he also has some device with him so that he can ask himself the question: did the two burst take place simultaneously? The stationary observer has concluded that the two burst took place simultaneously. The moving observer wants to know, if the burst took place simultaneously, so, he needs to observe both the burst.

So, just the way the stationary observer had this device with these two mirrors. The moving observer also has some device, some similar device, and this is his device in which there is a sensor on this side and there is another sensor on the left side, and depending on, which sensor gets triggered first, the right, or the left? The moving observer will be able to decide as to which burst took place before the other or did they take place at the same time simultaneously in his own frame.

So, that is the question that he is going to address. So, he has some sort of a device with him. Our stipulation is that the stationary observer certainly finds that the two firecrackers exploded simultaneously. We want to know what is the inference that the moving observer will draw.

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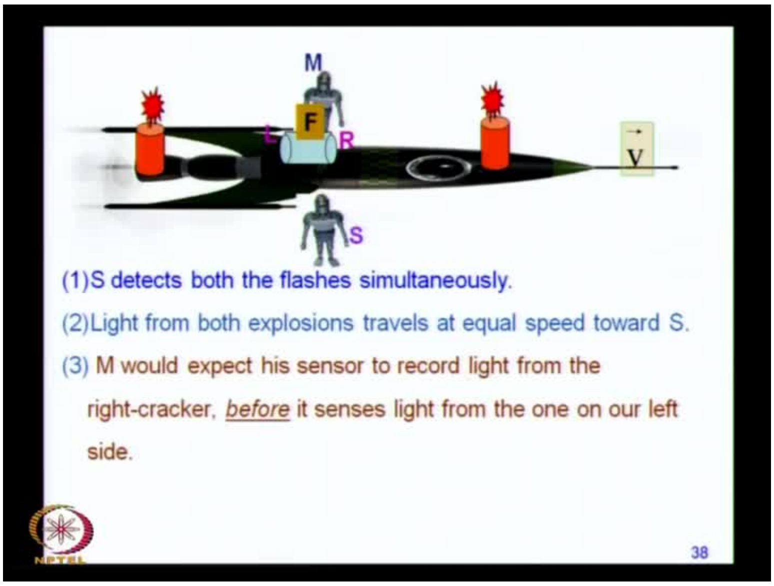

So, here is our first remark that the stationary observer detects both the flashes simultaneously. We know that light from both the explosions travel at equal speeds. So, this, from this explosion, toward the middle, and toward, from this explosion, toward the middle. In both the directions, light would travel at the same speed. That speed is not infinite; that was recorded by Romer, then by Michelson and Morley. We know its value; it is a finite value. So, at that finite value, while the light travels from one burst to the moving observer, and from the other burst to the moving observer, the moving observer is moving, his rocket is in a state of motion; so, he is moving from the left to right in the screen.

So, even as that light is coming this way, this fellow is moving, right. So, what does he expect? He expects that since he is moving from left to right in the screen, he expects his sensor on the right to get triggered before the sensor on the left will get triggered, because even as the light is travelling from both the directions toward the middle, he is moving, and he will therefore expect that his sensor which is toward our right to his left, because he is standing on the other side in this picture.

So, his sensor toward our right will get triggered before the other sensor. His conclusion therefore is that the two burst are not simultaneous. The conclusion of the first observer - the stationary observer - is that the two burst are simultaneous, but the conclusion of the moving observer is that these two bursts are not simultaneous. So, this is the first consequence we must reconcile with. That simultaneity is not absolute; what is simultaneous for one observer cannot be simultaneous for another observer even if he is moving with respect to the first one at a constant velocity.

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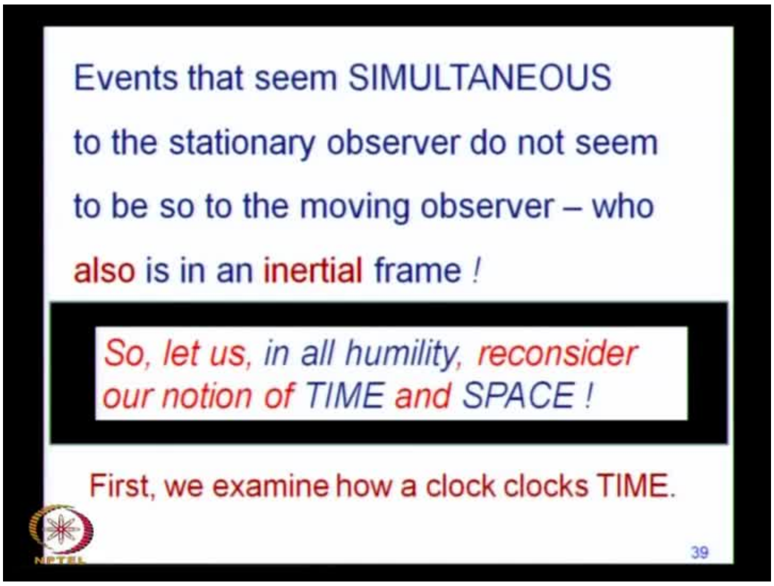

So, both the frames of references are inertial. The moving observer is moving at a constant velocity; it is not that he is accelerated or anything; no pseudo forces or anything of that kind is coming into the picture, but the meaning of simultaneity is different, and we have got to accept that. Having done so in all humility, because our notion of simultaneity was connected with how information travels from the source to the observer. So, it has this consideration of distance and time involved right.

Because what we regard as simultaneous events is coming from our understanding of how information propagates from the event, the source of the event to the observer. That if simultaneity is different, then may be our understanding of space and time also must be refined, and what is space interval and time interval for one observer may not quite be the space interval and the time interval for another observer.

So, let us be open to that consideration, and we will see exactly how these are related, and we will do it by asking us how clocks clock time or a clock clocks time. It is the same thing. So, we will ask ourselves how it is that time is measured and you can do it anyway that you want. You might want to suspend a pendulum and measure its oscillation time, right, and then, measure the time period; you can have your own clock, or you can measure time the way Galileo did by measuring your pulse, or you can have any device, it does not matter, but you need some periodic phenomenon which will be repetitive at a regular interval in your own frame of reference and that will give you your calibration for time.

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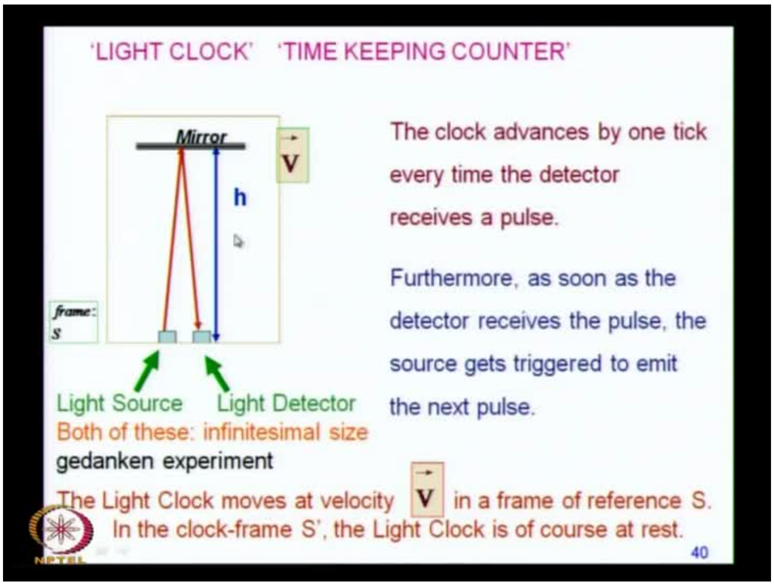

So, let us see how a clock clocks time. So, we will have a time counter enough. We will consider a frame of reference s, in which, there is a clock which is moving at a velocity V in this frame. So, this rectangle is where I am going to mount the clock. The clock will be in a frame of reference which is moving at a velocity V with respect to some frame of reference which I call as S.

So, there are two frames of references in our discussion: one is this frame of reference S, the other is the clock frame of reference which I will call as S prime. In the clock frame of reference, the clock is at rest; it is in the frame of reference s that the clock is moving.

Now, what is this clock? Now you have got a light source in this, this source of light it will fire a pulse of light and mirror reflects it and the reflected light is detected over here by a detector, and the time taken for this to happen is the unit of time measurement. This is the unit of time measurement. This is what we will consider as the unit of time measurement, and we will compare how this unit is related, when it is reference to the frame S with how, with what it turns out to be in the frame S prime.

Now, this is a thought experiment; this is what Einstein called as Gedanken experiment. So, you do not really have to worry about the size of the detector or the size of the source; you can consider to be infinitesimally small. All those parameters are irrelevant; you can throw them from your disk analysis completely.

So, we consider the source and the detector to be infinitesimally small, and the events we are looking at is a pulse of light which is emitted by the source gets reflected by this mirror, detected over here. This is one cycle, and as soon as the light is detected over here, the source sends the second pulse, and this is a repetitive phenomenon and the clocks clock keeps ticking; that is the clock.

So, there is a periodic phenomenon. The clock advances by 1 tick, every time the detector receives the pulse, and immediately, the source gets triggered and sends the next pulse. Now we have a periodic phenomenon; we have got a clock. Now the distance between the mirror and the source, this is h, this is this distance here.

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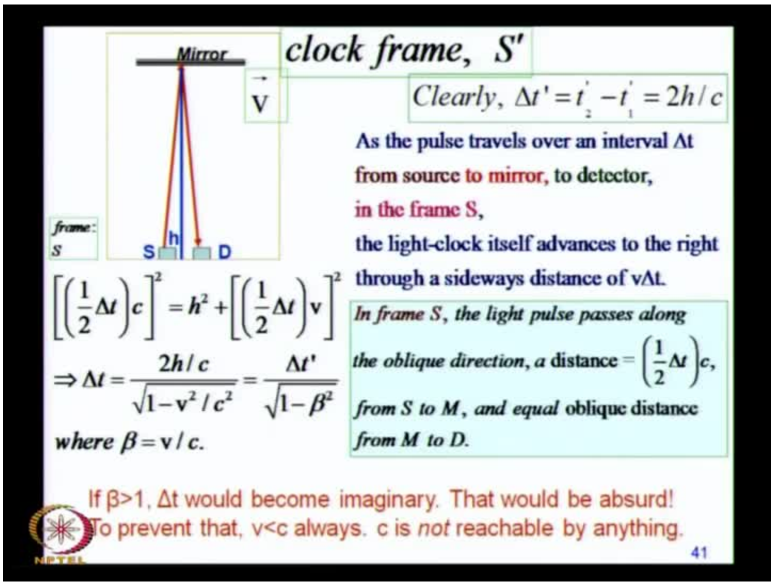

Now, let us look at this in the clock frame. So, in the clock frame, light goes straight up and down. The source and the detector are of infinitesimal size, point size. The source and the mirror, both are moving together the source, mirror detector, everything, the whole assembly is moving together.

So as within this frame of reference, light just goes up and down, there is no transverse motion in this frame of reference, absolutely no transverse motion. So, the time for this to happen is obviously the total distance divided by the speed of light, and the total distance is twice h - h to go up and h to go down. So, that the total distance traversed is twice h, and this time interval in the clock frame, in the frame s prime, for which our time variable is t prime. We have not claimed as yet that t prime is different from t but I think we are already anticipating that.

So, this delta t prime which is the time interval between these two events. When the pulse is detected, and the event, when the pulse is emitted by the source, and this time interval is clearly 2h by c. What about the same time interval as seen by an observer in S? So far as observer in the frame S is concerned, light is emitted, a pulse of light is emitted by the source, and as light moves from this source to the mirror which it would do so at a finite speed, not at infinite speed.

During this time, this whole assembly of the source detector and mirror is moving. So, an observer in the frame of reference S would see the same pulse of light to go along an oblique direction, not straight up, what we earlier called as up is not the direction along which an observer in the frame of reference S will see the light to travel. He will see that the light has gone from here to here, and it has gone up, but there is also a little bit of transverse motion.

How much is this transverse motion? In the full time interval, this rocket is moving at a speed of light, at the speed of v with respect to S. So, if the total sideways distance that this moving clock would traverse between the two events of the emission of light to the detection of light. The transverse distance through which the rocket would have moved, or the clock frame would have moved will be v into a time interval delta t which is the time interval in the frame S, right. So, he may notice a sideways transverse motion.

So now, this is a very simple geometry to work with, because you can use the Pythagoras theorem that as the light travels from the source, to the mirror, to the detector. There is a certain oblique distance that the light would have traversed, and while coming back again, there would be a transverse displacement, and that corresponding oblique distance will be the same as the first oblique distance, because the light clock, the rocket is moving at a uniform velocity V with respect to the observer in the frame S.

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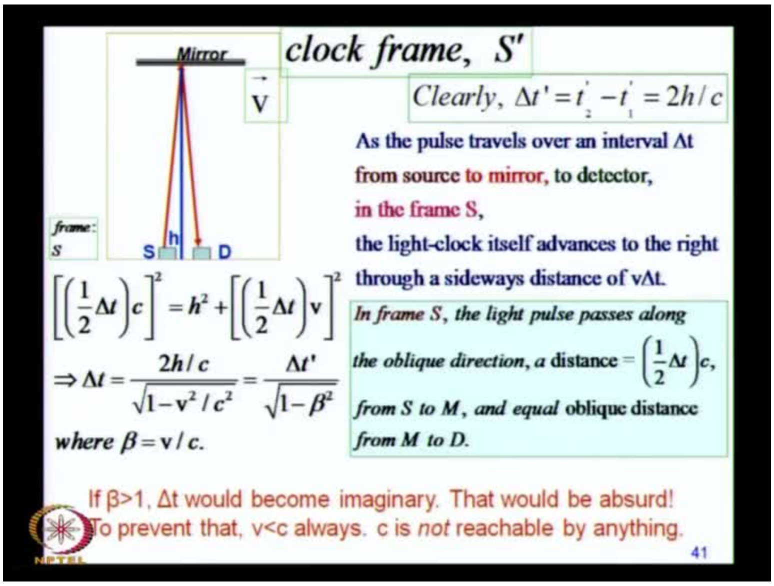

So now just make use of the Pythagoras theorem. The square of this side this side will be time into distance, time will be half delta t, right, delta t is the total time between the two events, half of that is the time from the source to the mirror. So, time into the speed of light and the speed of light is the same in every direction; it has nothing to do with direction.

So, the square of the distance of this oblique side of this right angle triangle is half delta t times c and the square of it. This is equal to the sum of the squares of the other two sides, and this side is h, and this side is half the lateral displacement. So, it is v into delta t times half and the square of it. So, all we have done is to make use of the Pythagoras theorem.

So, here you go. So, delta t is equal to, if you just solve this equation for delta t, this comes straight out of this equation. The delta t turns out to be twice h over c over square root of one minus v square by c square. There is nothing in this which is not in the Pythagoras theorem. This is no new hypothesis or anything, but what pops out of it is an amazing reality. The twice h over c is what we agreed is the meaning of the time interval.

It is a unit of time in the moving frame of reference, in the rocket frame of reference, in the clock frame of reference.

This is delta t prime which is twice h over c which comes in the numerator here. So, this is equal to delta t prime, and we find that delta t is not the same as delta t prime. That the time interval for the clock frame of reference is not the same as the time interval in the s frame of reference. We are not making any postulate; we have concluded this by simply using one fact that the speed of light is the same in all inertial frames of references. Other than that there is nothing else that we have plugged in.

We have to reconcile with the fact that the time interval in the clock frame of reference which is a unit of time. He will measure his time, his life, the events in his life, what would happen or what would have happened in his life from yesterday to today to tomorrow; from birth to death; his aging; his biological clock; everything will go according to his own measure of time, and this measure of time for the observer in the clock frame, it turns out is not the same as the measure of time for the observer in the frame of reference S.

The difference is because of this beta not being 0, beta is a ratio of v over c, beta would be 0, if v is 0; if the two observers are not even moving with respect to each other, then there is no change, fair enough. Beta would be 0 also if the speed of light was infinite, but that is not the case, and therefore, delta t and delta t prime are different.

Now you can immediately see from this that if beta were to exceed the value 1, then you will get the square root of a negative number and that will make this time interval imaginary. So, that would lose sense; which puts a limit on beta that we will not exceed c. So, you cannot have anything which will cross the light barrier. So, that is what is sometimes referred to as light barrier.

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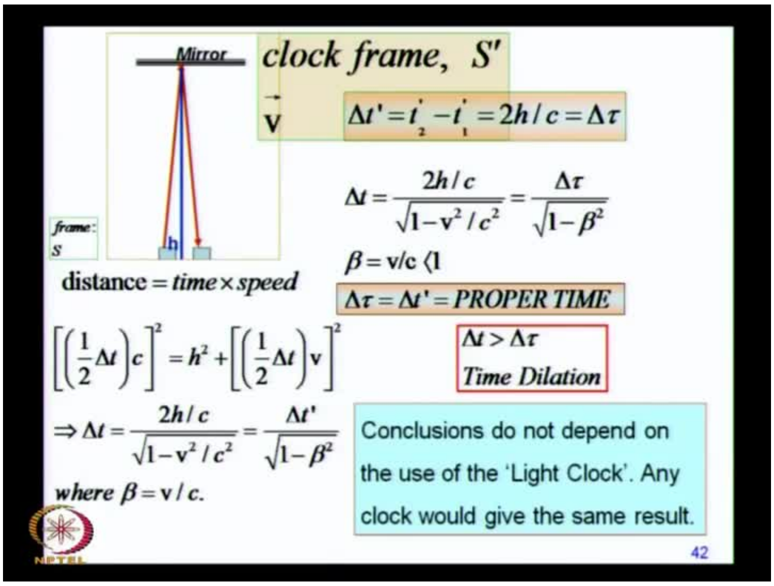

So, these are our conclusions. So, this time interval, this is the unit of time, this is the time between two events that a pulse of light is emitted from the source and it is detected at the detector. These are the two events and the interval between this is a unit of time in which an observer in the frame S prime measures his time.

This is what we can call as his own time; I might want to use eigen time for him, his own time. This is what is called as a proper time. In the context of the special theory of relativity, this is called as the proper time; this is the proper time for the observer S prime, because in his frame, the clock is stationary, so, this is the proper time.

The relation between delta t and delta t prime, so, delta t prime, since this is proper time, I use a different symbol now, delta tau, just to highlight the fact that it is a proper time or the eigen time for the observer in the frame S prime. Delta tau is the same as delta t prime; it is equal to 2 h over c and the relation between delta t and delta tau is given over here and you can immediately see that since beta is less than 1 delta, t must be greater than delta tau.

This is what is meant by time dilation. That the measures of time are not the same for two observers, even if they are moving with respect to each other at a constant velocity. The time measures the time scales, they must be different; this is not a postulate, this is the consequence of one single fact that the speed of light is finite.

We have not done anything else. We just began with that and we asked ourselves what will be the implication of this on our clocks, on our measure of time, on our unit of time, and we find that the two observers one in the frame S and the other in the frame S prime cannot agree with each other on their unit of time. They must differ, and they must differ exactly by this relation. This is a quantitative relationship between how their time intervals differ.

It can be any clock. You can measure time anyway you like; you can use a pendulum if you like; you can measure your pulse rate. It does not matter, but the unit of time, finally what it boils down to is that what you consider as a unit of time; time is measured in units of what, that unit of time is not equal for the observer S, and the observer S prime.

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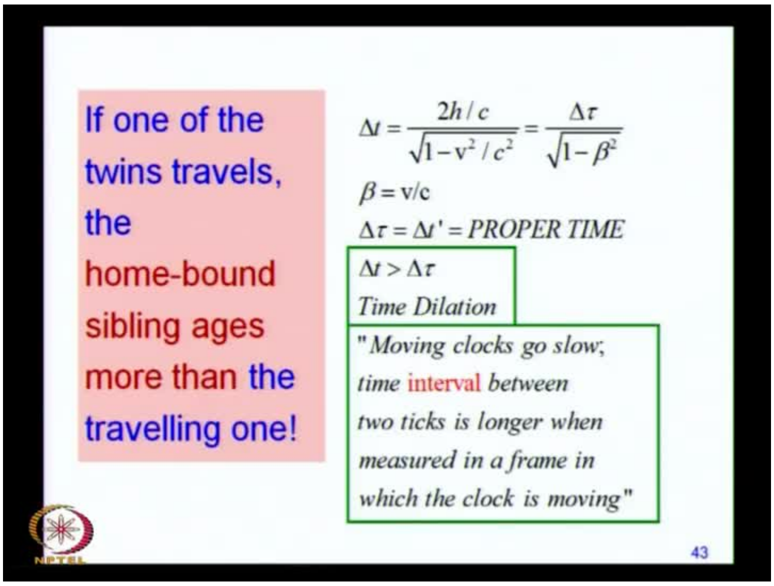

So, this is what is meant by time dilation. What it means is that the moving clocks go slow. The time interval between the two ticks is longer when measured in a frame in which the clock is moving. These ideas become very clear when we discuss some application and the most fascinating application to discuss in this context is the twin paradox which I will be discussing that will make all these very clear.

The reason that paradox becomes so important of this context is because it explains the idea of time dilation very nicely. A moving clock goes slow, and you have twins: one is a home bound twin who stays at home, and the other one who is a travelling twin. Then their measures of time must be different and their aging will be different, sounds strange, but that is exactly what it will mean, and this problem is what is posed as the twin paradox. So, we will discuss it, but before we do that, let us continue our discussion on the perception of time in these two frames of references.

Now we have reconciled with the fact that the measure of time is different. Our framework is the fact that the speed of light is the same, and speed being distance divided by time. Time is different; speed is the same, what happens to the distance? We need to refine our ideas about distance as well. So, let us do that.

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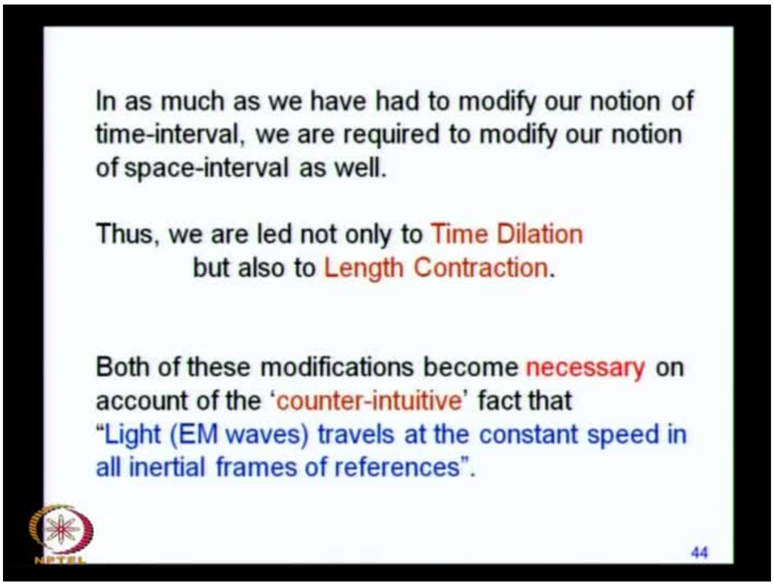

So, the space interval also what is our perception of space interval, and you take any rod and you look at the distance between the two tips of the rod and hold it in your hands and ask your friend to sit in a car and go at a constant velocity with respect to you, and ask him what is his perception of the length interval between these two end points of the rod.

Our Galilean outlook is that the two perceptions are identical, but now, we must be open to the possibility that it is not quite so. In fact, we have already reached a point that we must actually expect it to be different, because we have reconciled with the fact that the speed is the same; we have reconciled with the fact that the time intervals are different. So, we must expect the space intervals to be different.

But we must establish a quantitative connection between this and the difference in the space intervals manifest as what is called as length contraction also called as Lorentz contraction. So, these are in the certain sense counter intuitive but counter intuitive only if we allow our intuition to be dictated completely by Galilean relativity. If we admit the fact that the speed of light is the same in all inertial frames of references into our intuition, it need not remain counter intuitive any more.

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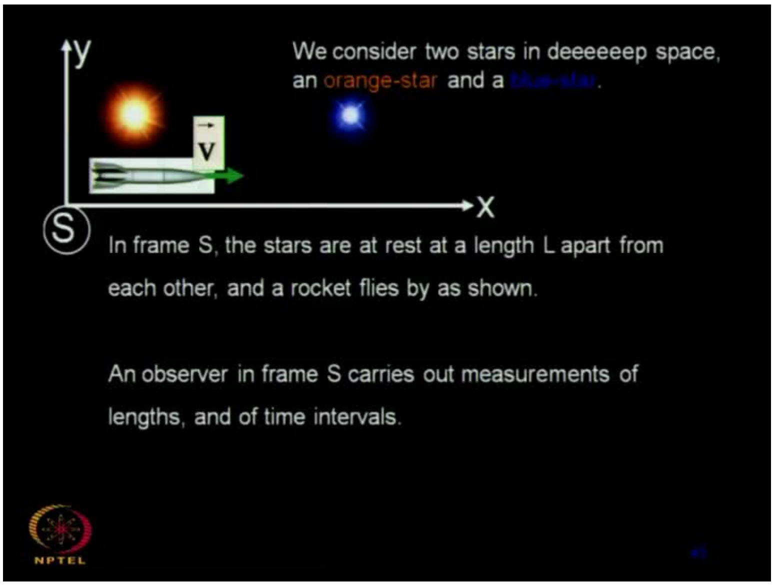

So, now, let us measure distances. So, let us measure the distance between two stars which are deep in space. There is an orange colored star and a blue colored star and you observe it in a certain frame of reference, in which, the stars are at rest. In your frame of reference, in which, the stars are at rest. The two stars at a certain length apart, so, that is your perception of length in your own frame of reference. In this frame of reference, you have your own perception of time interval; you have your own measure of time.

And with reference to these measures, measure is a unit in your units of distance and your units of time. You find a rocket which is moving from the orange star to the blue star and in your frame of reference with your measure of distance, with your measure of time, you find that this rocket is moving at a velocity V; now, that is a scenario, and you will measure the distance and time as also an observer on the rocket, and we will find out how do his conclusions compare with your conclusions. Of course, if you want to choose, since this is a thought experiment anyway, you might choose yourself to be the observer of the rocket. It will be more fun I think.

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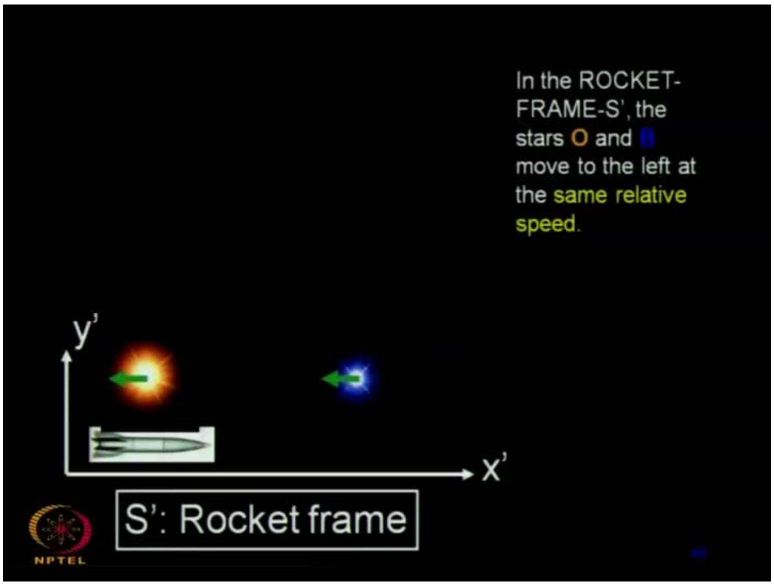

So now, the observer in the frame of references measures length and time, and if you are the observer now in the rocket, the rocket of course is not moving in your frame of reference, because you are in the rocket frame of reference. It is the stars which are moving with respect to you, and the stars are moving from right to left in this figure as the rocket is moving from left to right in this figure.

So, in the rocket frame which is the frame S prime, what will be the relative speed at which these stars will be seen to be moving, will this speed be different from the speed that the earlier observer had measured for the rocket, it has to be the same because it is with the same speed that these two stars will be seen to be moving from right to left to the observer in the rocket, except that, it will be in the opposite direction.

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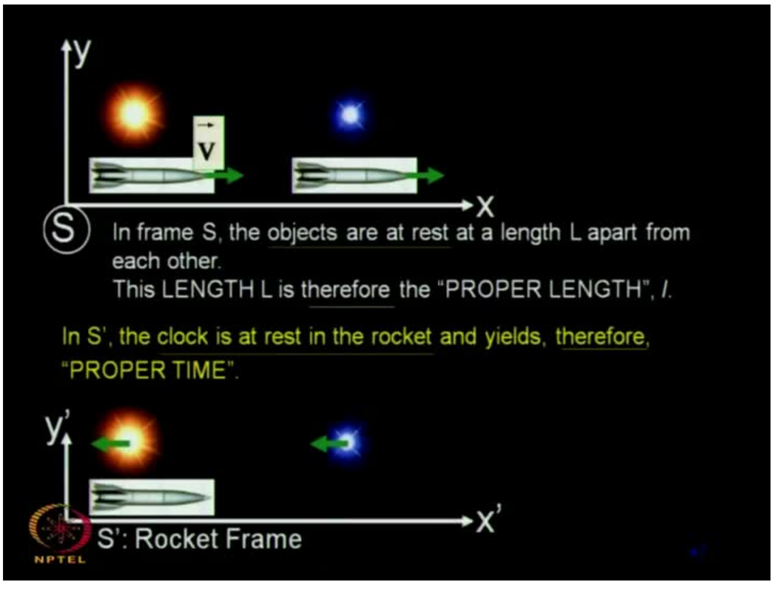

So now let us put all of these together. So, here, the first observer who is in the frame of reference S, in whose frame of reference the two stars are at rest, and the rocket is moving, it was here earlier; it is moving at a velocity V; it keeps moving at a later instant of time, after good amount of time, it gets to this point, and the propagation of this rocket in this frame of reference is the velocity V, and he measures the length and this is the length that we will call as the proper length. This is the proper length between the two stars.

Why is it proper? Because in the frame of reference of the observer S, the stars are at rest, this is proper length. In the rocket frame of reference, it is the time which will be in the proper time because the clock will be at rest in that frame of reference, not the stars.

So, S measures the proper length, and S prime measures the proper time, but of course, they are related. So, in the rocket frame of reference, these two stars are moving from right to left. So, he will not measure the proper length but he will measure the proper time. Regardless, both will measure a certain space, certain length, and a certain time and we are going to compare the two.

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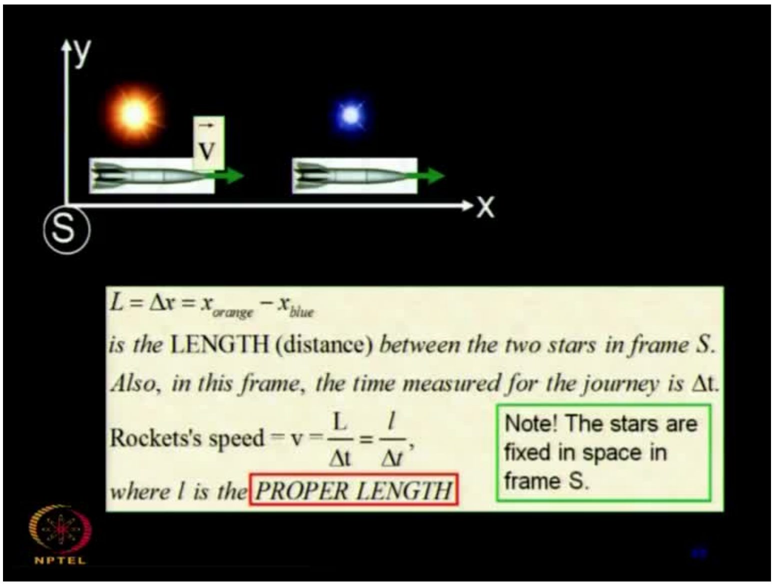

So, what is this length? Now, for the observer, in the frame of reference S, this is the proper length. The stars are at rest. The length between the two is the difference between the x coordinate of the blue star, and the x coordinate of the, of the orange star, so, it is x orange minus x blue, right. It is the difference in the two coordinates. I am thinking only of the models, just the magnitude. I have got written it as x orange minus x blue, but I am only thinking of the length.

So, this is the length measured by the observer in the frame of reference S, and since in his frame of reference, the rocket is moving at the speed v. He finds that if the rocket takes a time interval delta t to go from the orange rocket, orange star to the blue star, then the speed of the rocket is L over delta t, fair enough, no contest. The stars are fixed in space in the frame of reference of the observer S. It is the rocket which is moving. The rocket takes a time delta t in this observer's frame of reference. According to his own measure of a time, according to his own clock, he measures in his clock a time interval of delta t, and to him, the rocket speed is the distance between the two stars which is the proper length in his own frame divided by the time taken which is his own measure of time, which does not agree with the other fellows measure of time, but so what and he measures the rocket speed to be L over delta t. So, this is his relation.

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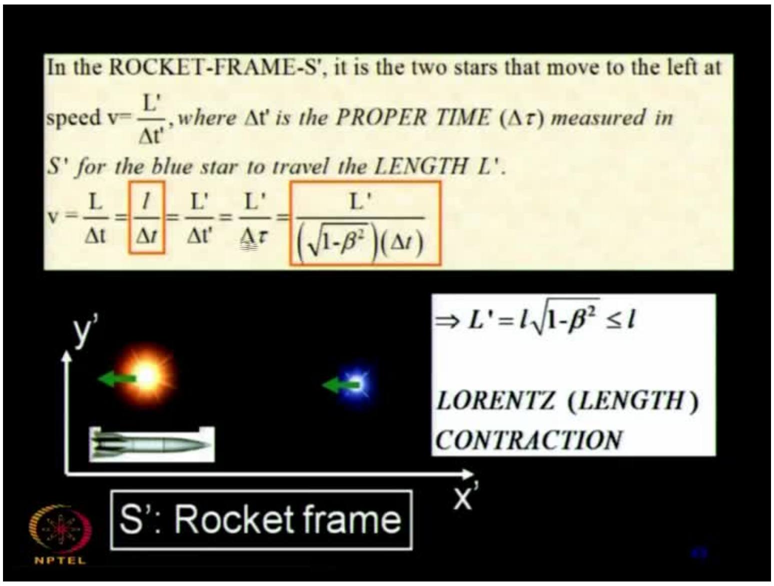

What about the other observer? The other observer moves these two stars to be moving in the opposite direction at the same relative velocity. So, it is the ratio of the distance between the two stars which is L prime, which, let us not assume is the same as L divided by the time interval that the rocket takes to go across the first star to the next. According to his own measure of time, that is the prime t, which is the proper time in his own frame, because he is in the clock frame of reference.

So, he measures this velocity to be L prime over delta t prime. Delta t prime is the same as the proper time delta tau which we know is related to delta t by this relation. We have already determined this relation. We have only used it here.

So, this relation between the time intervals in the clock frame, and in the frame S, the relation between delta t prime and delta tau is what allows me to write for this denominator the square root of 1 minus beta square times delta t, where this is the time interval in the frame S and beta is a ratio v over c, but earlier we found that this v was nothing but L over delta t. Now all of these quantities are equal to each other, right? This is the same relative speed.

So, all of these quantities are equal to each other. So, if you now focus on what is inside these rectangles, you can equate this quantity in this rectangle, to this quantity in this rectangle; strike out delta t in the denominator which appears in both and you immediately find how this proper length is connected with L prime, right. They are not the same; they would not be the same only if beta is 0. Beta would be 0, if v is 0, or if c is neither infinite nor otherwise.

But beta not being 0, L cannot be equal to L prime, and L prime must be less than L, because beta which is v over c will always be less than 1. So, this is length contraction, okay. Is this a partial length that we have proposed? No; it has come from a simple substitution simple recognition of the fact that this L over delta t, the ratio, the speed at which this rocket is moving must be the same with respect, at which to an observer in the rocket frame, the stars will seem to be moving but in the opposite direction.

It has come from the fact that we have factored in the time dilation because delta t and delta tau are related through the time dilation relation that we derived earlier, which was not a postulate, it came as a consequence of the fact that the speed of light is the same in all inertial frames of references.

So, both length contraction and time dilation come as a natural consequence of one single reality that the speed of light is the same in every inertial frame of reference; both are concurrent consequences, they come as a package, you cannot keep one and do away with the other. These are not separate postulates; you do not make too many postulates in physics. Physics aims at building the entire logical analysis on the basis of minimal number of principles, absolutely the minimal number of principles.

You do not even propose the first law of inertia. If only, you could interpret it as the special case of the second by putting f equal to 0. You do so, because you have to do it, because it is not contained in the second law of inertia. It is a ground reality of the Galilean principle of equivalence which much be first understood before you talk about change in moment.

So, make the minimal number of postulates, in Newtonian mechanics, you make. Therefore, three fundamental laws of Newton; they provide you the framework. In the special theory of relativity, you make only two fundamental principles, there are only two fundamental principles: one that the laws of physics are the same in every inertial frame of reference which is not new to physics at the time of Newton.

What was new to it that this was applied to light, this was applied to electromagnetic phenomenon. Light is electromagnetic wave. The application of the Galilean principle of equivalence to light, to electromagnetic phenomena, is the new thing but it was there in the Newton’s time also.

So, this is the new thing in the special theory of relativity. That laws of physics must be the same in all inertial frames of references, and that the laws of physics would consider not only mechanics of particles in Newton’s or Hamilton’s or Lagrangian’s time but also electromagnetic phenomena, and the second is the reconciliation that the speed of light must be finite and constant as having the same value in every inertial frame of reference. These are the two pillars of the theory of relativity and amongst the very many consequences of this are the facts that measure of time and measure of distance must undergo a refinement in our thinking and lead us to time dilation and Lorentz contraction or length contraction.

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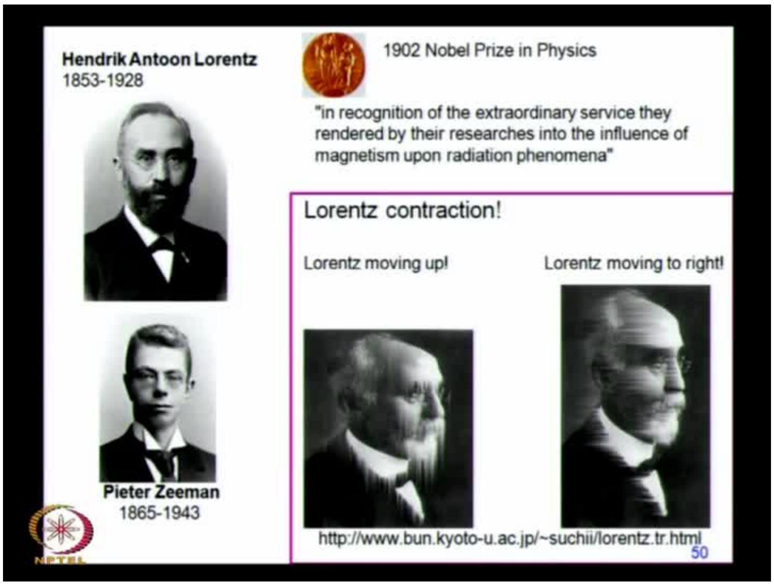

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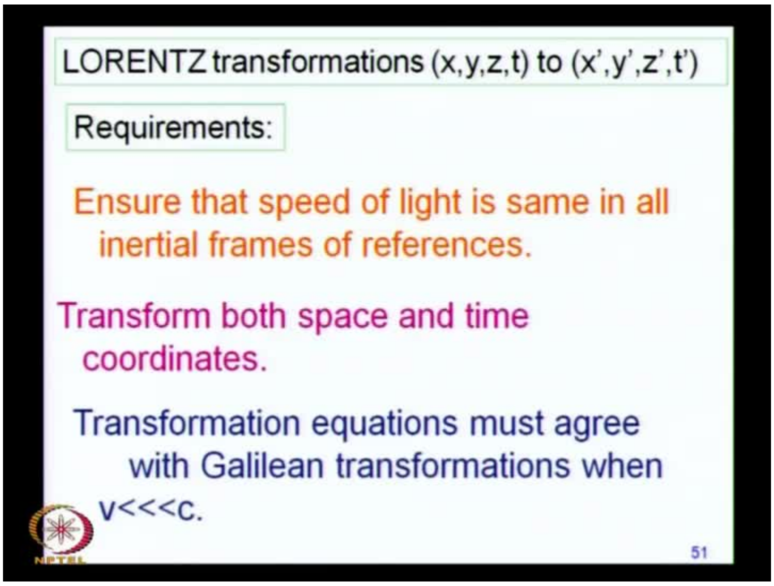

Lorentz got a noble prize in 1902. The Nobel Prize, I think the first one to be awarded was in 1901 which went to Rontgen. The very second went to Lorentz which he shared with Pieter Zeeman who is very well known for Zeeman spectroscopy, and this brings us to what are known as Lorentz transformations, because we have, now reconciled with the fact that our perceptions of space intervals and time intervals must be refined, and therefore, Galilean relativity cannot hold, it has to be modified.

So, how space and time are to be related when you are carrying out a comparison between events seen by one observer in frame S and another observer in the frame S prime. What is x y z and t in one frame of reference with respect to what it is in another frame of reference. This is normally connected by Galilean relativity must be refined and what should be the requirements of this refinement. We must refine these transformations, and make certain demands on these transformations.

Because the new transformations that we are looking for must be reconciled with the new consequences that we have now come to terms with, namely: time dilation, length contraction that the measure of time cannot be the same, the measure of distance cannot be the same, that the speed of light must be the same.

So, these are the requirements that they must ensure that the speed of light is the same in all inertial frame of reference. The new transformations that we are now going to seek which will connect x, y, z, t from one frame to another. These new transformations must meet this requirement. They must transform both space and time, because if you just transform one and not the other, you cannot get the constancy of the speed of light in the two frames of references. So, they must transform both space and time.

In Galilean relativity, time was not ever transformed to anything else, it always remained itself. No matter in which frame of reference you measure time, t prime was always equal to t in Galilean relativity, it cannot remain so. Both space and time must be transformed. Finally, well, is Galilean transformation totally wrong, we could use it for trucks, cars, bullets, seem to worked alright. So, it was not absurd, and therefore, the new transformation relations must reduce to Galilean transformations in the limit v going to 0 or equivalently c going to infinity.

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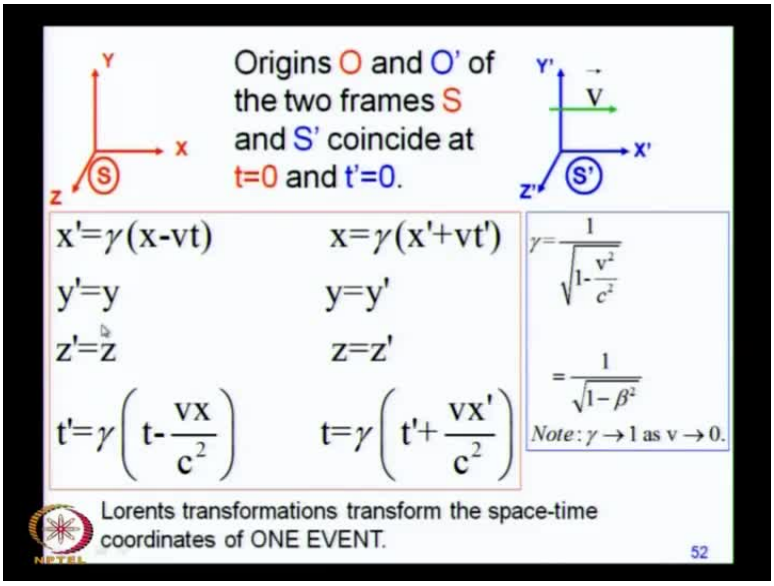

So that the limiting cases should give the Galilean transformation. So, these are the three demands that we make, and the result of these requirements is the following that x, y, z, and t must transform according to these laws. That, if you measure x, y, z and t in one frame of reference, compare it with x prime, y prime, z prime, and t prime. In another frame of reference, in which, this other frame of reference, the blue frame of reference, S prime is moving with respect to this red frame of reference at a velocity V in the red frame of reference. In the blue frame of reference, it is of course still.

Then, what is x, y, z, t to an observer in s, and what is x prime, y prime, z prime, and t prime to an observer in s prime. If the two frames coincided at t equal to 0, if both the clocks are set, when the two frames of references are at 0, and the motion is only along the x axis. This is just for the sake of simplicity, but you can use vectors and do it in any direction, it does not matter. Then, the relation between x, y, z, and t is given by these relations.

So, look at just the first column, that in terms of x, y, z, and t, x prime, y prime, z prime, and t prime are given by this. There is no change in y prime and z prime. So, in transverse directions, the perceptions of length measures remain the same. That the changes take place that the Lorentz contraction takes place only in the direction of motion of the relative motion, which is why I showed you the previous picture here, let me go back to that for a moment. This is Lorentz moving up, and this is Lorentz moving to the right. So, the Lorentz contraction takes place only in the direction of motion.

So, y and z do not change, but x prime is now a mix of x and t, and t prime is a mix of x and t, and what appears in the superposition of x and t are the relative velocities of the two frames of references. So, this v appears over here; the speed of light of course comes, and the measure is in terms of gamma which is this 1 over square root factor which we have already met.

So, these are the Lorentz transformations, and they transform space time coordinates from one event to another, and these are the inverse transformations written in the second column. Here, again, these are no different from the first one; the first gave raise to the second, back and forth.

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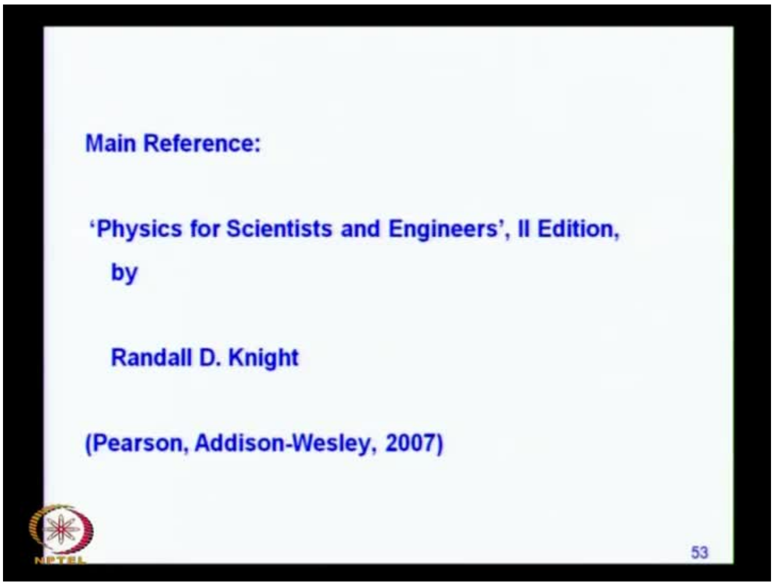

And I think all I want to do at this point is to give you the main reference for this discussion which is a very nice book by Randall Knight called physics for scientists and engineers, and you can read up this discussion from this book if you like.

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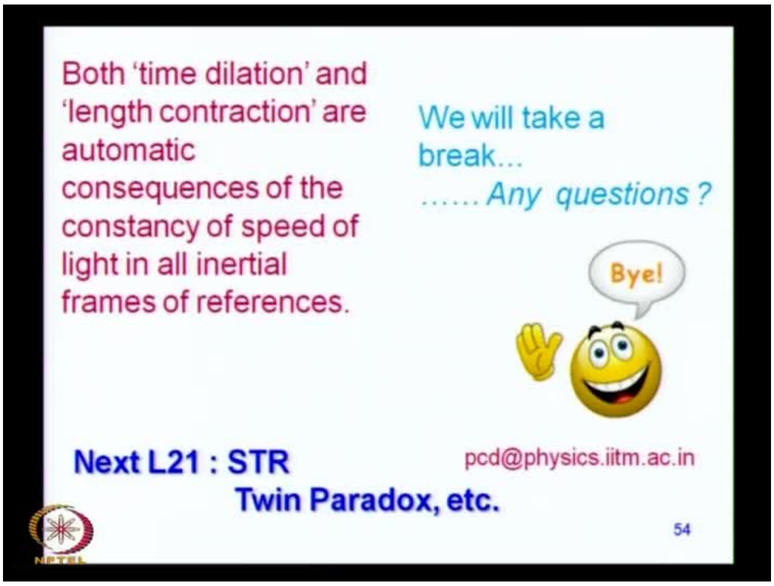

And I guess, I will take a break here. I will be happy to take some questions, but at the next class we will discuss the twin paradox, and that is a very interesting case, because it will be a very nice application of what we have learnt: the time dilation and the length contraction and the consequences of the special theory of relativity.

So, the both time dilation and the length contraction, these are essential and automatic consequences of the constancy of speed of light; these are not new postulates or anything; these are automatic consequences, essential consequences, and not the only consequences, there are other consequences, but then, those can be discussed may be in another course or in at some other time. Be happy to take some questions, otherwise, good bye for now, and then, we meet again, when we continue our discussion on the twin paradox.

Question? Yes

Now, Hawking says that instead of black hole, time is imaginary. Now, does he mean that v is greater than c?

No, I do not think I am going to get into the hawking problem. There are very complex issues; I will be referring to some of them in the next class, because the issues that we need to factor in our thinking are not restricted to the special theory of relativity alone. There is more to the hawking idea than this, because there is gravity, and now, gravity is omnipresent, and the special theory of relativity does not deal with gravity, the general theory of relativity does, which is actually theory of gravity, and to talk about the Hawking radiation, you really have to get into much more complex discussion than what we can get into in this course, because you have to throw in gravity. Any other question?

So, we really do not know if we have yet a theory of everything. So, there are various conjectures which are being made with reference to ones perception of space time, because here we have only dealt with, we have come to terms with space and time are not independent of each other. When you transform space coordinates, you must also transform time, and you, therefore talk about a space time continuum.

And we are talking about a 4 dimensional universe, but then, when you have to factor in gravity, then you presumably have to go well beyond the 4 dimensional universe, may be 10 dimensional or 10 plus 1 11, there are various theories which are being proposed, I do not believe that any one is established as yet, and I do not know if anyone can be established in the very near future, may be yes, may be no, but that is completely beyond the scope of this course.

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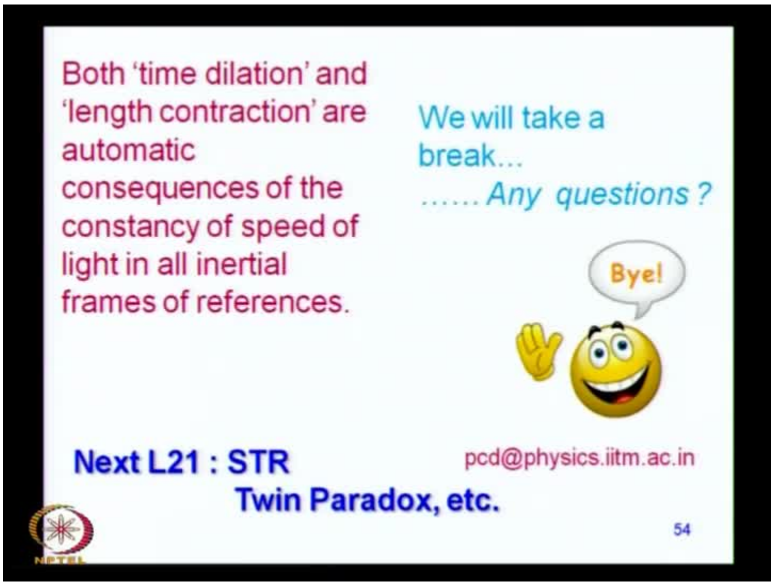

Any other question? But I will certainly make a few remarks on some of these related issues, just to prepare the foundation for this discussion, but that will come in the next class or actually in the class after this because in the next class, I will be discussing the twin paradox.

Any other question? So, thank you very much and good bye, and thank you also for putting up with my bad throat, it was completely beyond my control.