Select/Special Topics in Classical Mechanics
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Module No. # 06
Lecture No. # 21
Special Theory of Relativity (iii)

Greetings everybody, we will resume our discussion on the theory of relativity, specifically on the special theory of relativity and what we will discuss today is, what is famously known as the twin paradox.

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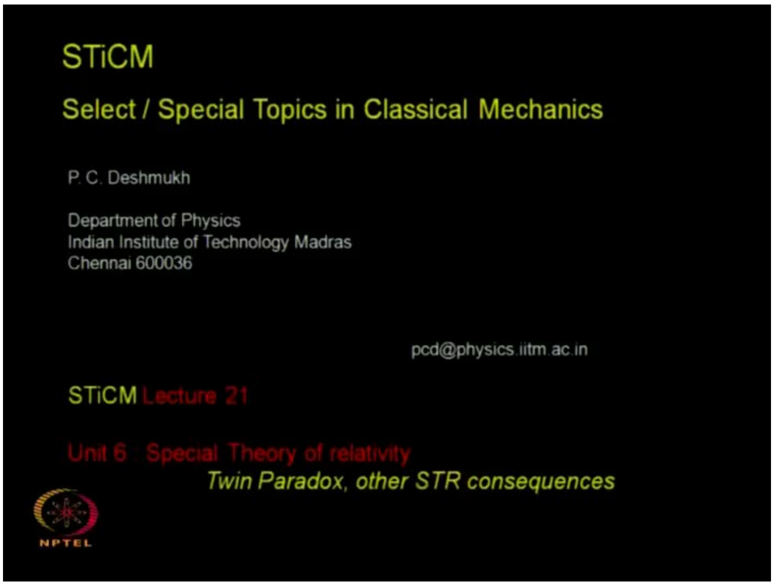

Now, let me very quickly recapitulate the essential considerations that go into this thinking.

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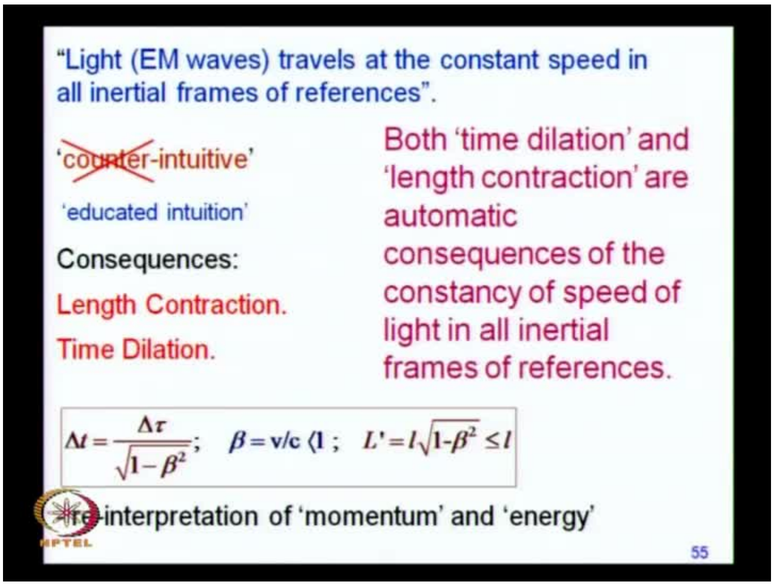

Other than the fact that the laws of physics are the same in all inertial frames of references, we have accepted that the meaning of simultaneity depends on the state of motion of an observer. For two observers, who are moving with respect to each other at a constant velocity both being inertial frames of references, what is simultaneous for one observer is not simultaneous to another, but what is common to both, simultaneity is different, but the speed of light is the same. That for both the observers, if they measure the speed of light in their respective frames of references, they get the same answer, which is 3 into 10 to the 8 meters per second.

So, in a certain sense, this looks like a counter intuitive idea, but as I discussed in our previous class, when you allow your intuition to get educated, then from the point of view of this refined intuition, it does not remain counter intuitive any more. It is a natural consequence of the fact that we understand that simultaneity is different, it is the speed of light which is the same and if the speed of light is the same then what is distance for one is not the same for the other. What is the time interval for one is not the same for the other and we accept it as natural consequences of the fundamental stipulations of the special theory of relativity and it falls very nicely within the framework of what we should now call as an educated intuition and there is no reason anymore to think of this as a paradox; there is no reason to think of this as counter intuitive; there is no reason to think about this as any kind of an anomaly. This is how nature is. At this formalism, the special theory of relativity enables us to describe it correctly and consistently.

The consequences are length contraction and time dilation; both are natural outcomes of this consideration that moving clocks go slow and compared to a clock which is stationary, the time intervals are not the same and the relationship between the proper time and delta t, which is in a stationary observer’s frame of reference; this is in the moving observer’s frame of reference, the relationship between them is given by this first expression over here.

As also, the length intervals in the two frames of references are connected by this relation, which refers to as length contraction or Lorentz contraction. It will also lead us to a reinterpretation of momentum and energy and I will come to that a little later presumably in the next class, but this will also turn out to be an automatic and natural consequence of the fact that the speed of light is the same in all inertial frames of references, even if they are moving with respect to each other. All of these consequences come as a package; they come together; they are not independent postulates. There is only one reconciliation. I hate to call it as a postulate because we can actually reason it out by understanding that simultaneity must be different and if simultaneity is different, then we are automatically led to other consequences.

So, these are the fundamental considerations in the special theory of relativity. They all come as a package and to get a handle on this length contraction and time dilation, we accept it as a natural consequence, all right and we were not really used to it because we use Galilean relativity throughout our school, high school and so on and it seems to work.

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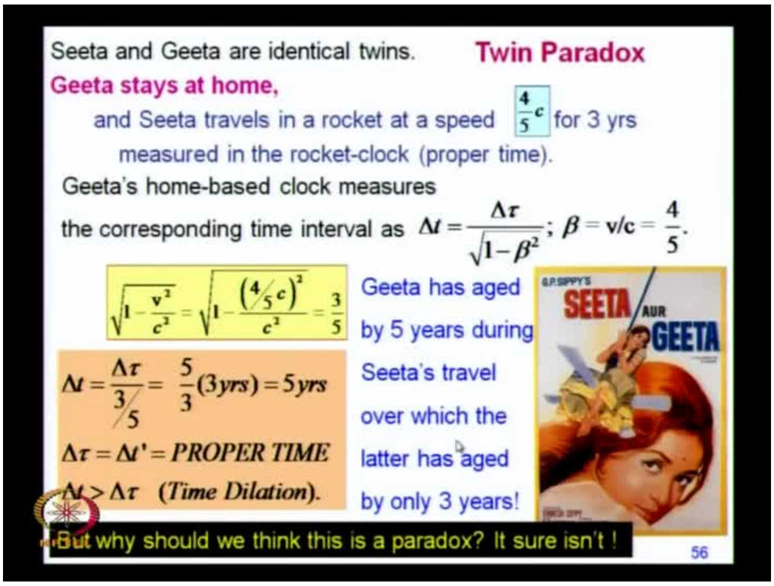

So, I will take up one particular illustration of time dilation and length contraction. One particular example which is rather fascinating which will illustrate these points very clearly and this is what is known as the twin paradox and the story here is that if there are two twins and one stays at home and the other travels, not necessarily hanging on a ceiling fan, but may be in a rocket, then obviously the perceptions of time intervals and length intervals for these two siblings will not be the same.

There is a quantitative relationship between them and if it is going to influence their measures of time and the twins measure time in their own clocks. I cannot measure my time in your clock and you cannot measure your time in my clock. So, they both measure their respective perceptions of length and time in their own measures and the measure of one not being the same of the other sibling, they cannot age at the same rate.

So, let us see how this goes. So, we have these two twins Seeta and Geeta. Let us say Geeta stays at home and Seeta travels in a rocket and she goes at a speed of four-fifth the speed of light, which is 80 percent of the speed of light - 0.8 times c and Seeta travels for 3 years. Whose 3 years? Not your 3 years or my 3 years. I am not going to time this; you are not going to time this. Seeta is going to time it for herself in her own clock. That is what Seeta will be. It will be Seeta's proper time or Eigen time. Proper time is the commonly used term it is the clock on the rocket and that is the only clock that Seeta can really refer to. So, she will refer all her measurements of time to the rocket clock and according to that clock, she travels for let us say, 3 years. What is the consequence of time dilation?

Geeta's clock is home bound. So, delta tau is according to Seeta's clock; this will not be the same as delta t for Geeta, right. The relationship between them is quantitative. We know delta tau and we know beta; beta is the ratio of the rocket speed to the speed of light. So, we can easily determine what this delta t is. What is it?

Delta tau is 3 years according to Seeta's clock; that is the proper time. If you just put in these numbers, determine the square root of 1 minus beta square, divide delta tau by the square root of 1 minus beta square, you get delta tau divided by 3 over 5 and this turns out to be 5 years. Fair enough; it is quite straightforward. What it means is that during the 3 years that Seeta measures in her proper clock, which is the proper time for Seeta, Geeta refers to her own clock which is homebound clock and she finds that she herself has aged through 5 years whereas, Seeta has aged only through 3 years. They are born twins, parents loved both of them equally, no problem there, but one has aged through 3 years and the other has aged through 5 years.

It is not a paradox; of course, it is not a paradox; we have already accepted this. This result is coming just by putting numbers in this equation. In our previous class, we actually determined this relationship. We determined that the relationship between delta t and delta tau is given quantitatively by this relationship. Having accepted this relationship, there is no reason for us to be surprised that delta t turns out to be different from delta tau. All we have done is that use this algebraic relationship and plugged in some numbers for beta, right. All we did was to put beta equal to 4 over 5, we put delta tau equal to 3 years; that is all we have done. We have not added any mystery to it. No mumbo-jumbo.

Why should we think this is a paradox? This is not a paradox; we have accepted this relationship. If there was any paradox at all, it was the fact - there was only one shock, that the speed of light is the same in two frames of references. One, which is static and the other which is moving at a constant velocity with respect to the other. This is a completely non Galilean idea. This was shocking from the point of view of Galilean relativity; this would seem to be paradoxical. Once we resolve this paradox through what we now call as an educated intuition, having accepted that simultaneity means different and it leads to the fact that the speed of light is being the same, the measures of length and time turn out to be different, then the consequences cannot be called as a paradox; this is not a paradox, but there is a paradox coming up.

So, when people talk about this twin paradox; this is not the twin paradox. We will accept this. No big worry that one has aged through 5 years; the other has aged through 3 years. This is how nature works; be it so.

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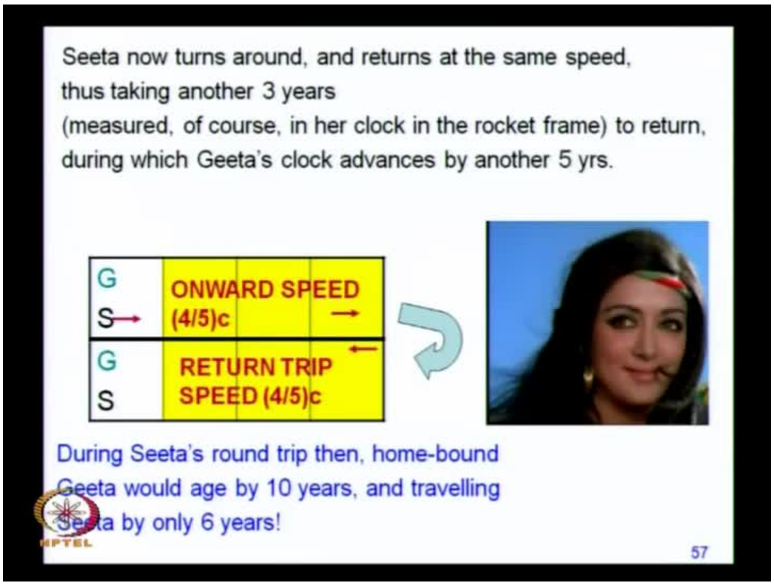

Now, let us consider the following. Seeta has travelled for 3 years. She takes a U-turn and comes back and on her return trip, her rocket comes at the same speed, which is 80 percent the speed of light - four-fifth the speed of light. So, in 3 years she has gone that far, takes a U-turn, comes back at the same speed and will therefore, take as much time to come back and return home. How long will she take? Another 3 years because she has gone that far and she has to return as much; so, she will take another 3 years.

So, she will return in another 3 years and she would have spent a total of years 3 plus 3 - 6 years during which, Geeta would age by 10 years because on her onward journey, Geeta had aged through 5 years according to her own measure of time and on the return trip, Geeta also advances through as much time, which is another 5 years and when the twins meet again one has aged through 6 years and the other through 10 years and the parents loved them both equal. Even this is not a paradox because this is coming straight out of the quantitative relationship between delta t and delta tau; this is not the paradox; it cannot be. It comes simply by putting numbers in an algebraic expression, which we accept.

If we accept xy equal to z, then why should it surprise us that once we put x equal to 2 and y equal to 3, the answer turns out to be 6. So, there is no mystery here.

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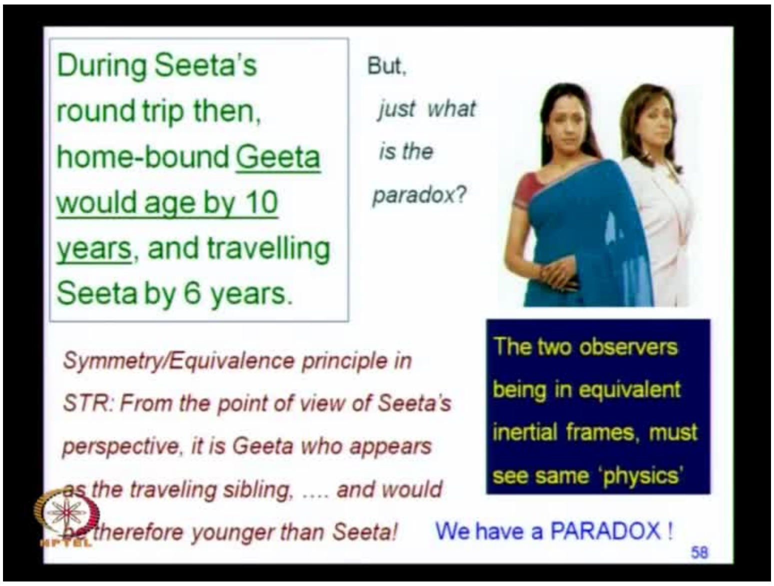

Let us consider a following situation. Now, we really want to know what exactly a paradox is and perhaps, the paradox here is that women do not age at all. So, that could be one, but let us not worry about it. The worry is the following that there is a certain symmetry; there is a certain equivalence principle. We did this analysis from the point of view of Geeta's perspective.

From Geeta's perspective, let me take this half and explain this. I will flash it again from the point of view of Geeta's perspective. Seeta has travelled, comes back and they compare their ages, but now what about Seeta's perspective. Seeta is here. So, in her mindset, she finds that it is the other sibling which is Geeta, who has gone here and this sibling has taken a U-turn and come back.

So according to Seeta, it is Geeta who is the travelling sibling. So, she would think that since Geeta is the traveling sibling, it is the traveling sibling who should age through 6 years and the homebound, which is Seetha now through 10 years. Now, that is a paradox because the perspective of one Geeta's perspective leads you to believe that it is Seeta who should age through 6 and Geeta through 10, but from the point of view of Seeta's perspective, you get an impression that it is Seeta, who should age through 10 and Geeta through 6. How can both be correct? This comes from what we call as the symmetry principle or the equivalence principle; so, this is the paradox.

The perspective of either of the two twins being completely equivalent should not lead us to any contradiction. So, this is really the statement of the paradox, not that they have aged differently.

So, we do have a paradox and we need to resolve it. We have to understand what is really going on because we should not arrive at different conclusions, when we consider the perspective of one twin and then we do the analysis with reference to the perspective of the other twin. This is required by the simple consideration that the two observers who are both in inertial frames of references, they must really see the same physics. So, they cannot see you know, a different outcome of this analysis. So, this is really the statement of the paradox.

Now, the resolution will have to come, if we are able to reason out correctly in such a way that whether we take Geeta's perspective and let Seeta travel for 3 years, come back and this travelling sibling - travelling Seeta undergoes aging through 6 years and Geeta through 10 years and we shift the perspective to Seeta and let her think that it is Geeta who has travelled, we should then still be led to the same conclusion that it is Geeta who has aged through 10 years and Seeta through 6 years. That is the desired goal of a resolution of the paradox. Is that clear?

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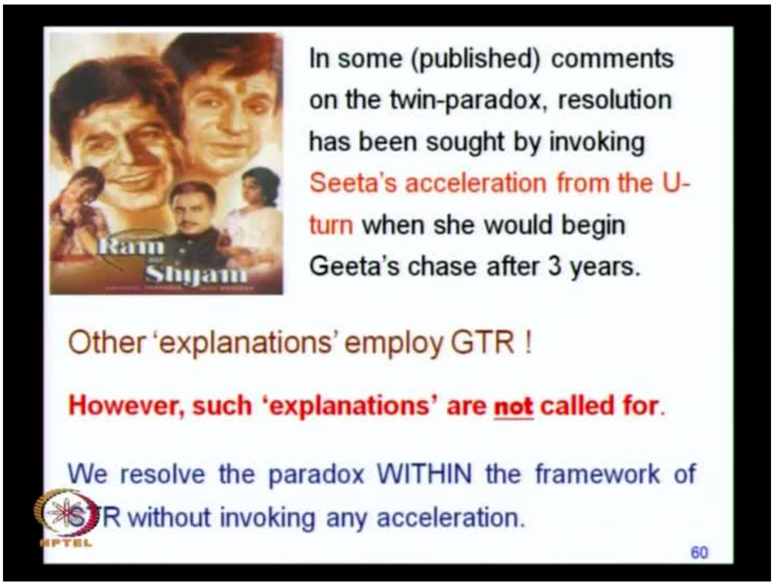

Now, let us see how this is done. I should alert you to the fact that there are lots of you know, there is a lot of literature on the twin paradox in many books. The internet has a lot of websites in which twin paradox is discussed. I mean some, not all. There are certain faulty solutions; there are erroneous solutions. So, I need to alert you to that.

In many books, you will find it and I am not going to give you the references. I do like to give reference when I know that the source is correct, not when it is wrong, but you will find it yourself. You will find that there are books and internet sites, where the resolution is sought by invoking Seeta's acceleration. The argument goes like this that you have got the two twins; one goes over here and then takes a U-turn and while taking the U-turn her momentum of course, has to reverse direction, right and when the momentum changes, there must be a force resulting in an acceleration and we must invoke this acceleration and there is no more the two inertial frames of references to compare. So, that is one resolution which is sometimes given.

There are some other explanations which go as far as saying that no you have to invoke the general theory of relativity and only then will you get a correct resolution. These explanations are not called for. The resolution we will find which we are going to discuss now is complete within the framework of the special theory of relativity. You do not have to invoke any acceleration at the U-turn; you do not have to invoke the general theory of relativity and within the framework of the special theory of relativity. All you need to do is to reason it out correctly and you will find that the paradox is resolved, which is what we are going to do now.

So, first of all, let us dispense with this U-turn business that there is some acceleration which is involved over there. We can completely do away with the U-turn at all because even without the U-turn, we could still pose the paradox by saying that Geeta is here, Seeta travels for 3 years, according to her clock whereas, Geeta has aged through 5 years, but from Seeta's perspective, she is stationary; it is Geeta who has travelled through in the opposite direction and she would think that it is the other one, who should have aged through only 3 years and she should have age through 5 years. So, there is still this paradox in just half the trip. So, the paradox is already there and the U-turn is really not required. We can always bring in a third observer.

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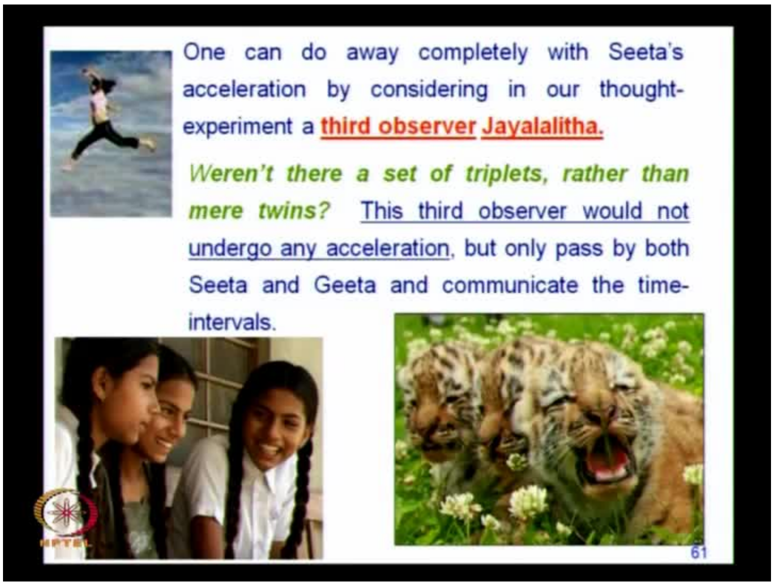

There is this Seeta, Geeta and one more and this third observer because it could be a part of the triplets rather than just twins. What the third observer can do is just fly over both observers and you have this third observer, who flies over Seeta means here our travel between the twin started when Geeta was here and Seeta was here and Seeta started travelling and we could have a third observer, who is flying over both of them and she meets Seeta, clocks the time, then she comes here, clocks the time and communicates the timings through these two. So, they do not really have to meet; nobody has to take a U-turn.

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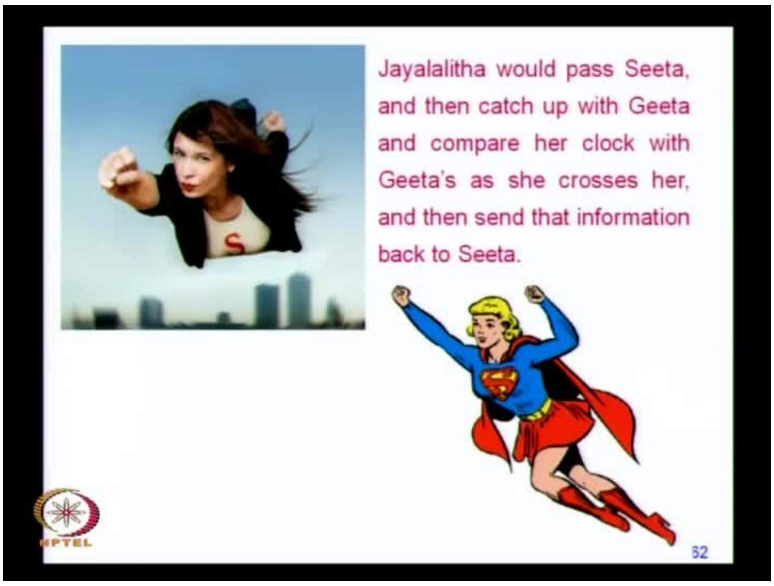

Seeta can continue to travel for another 3 years and this third sibling, who is flying over both of them, she could clock the time when she goes over Seeta and then again when she goes over Geeta, if she is flying fast enough and communicate the information and then this business of acceleration at the U-turn simply disappears. So, that is not required to be invoked at all in this analysis. The first thing we do is, before we discuss the correct solution, we dispense the erroneous solution which involved acceleration at the U-turn because that is not required; that is not a part of the paradox.

The part of the paradox - the only paradox is the asymmetry or what is rather an apparent asymmetry. If we are able to resolve this, if we are able to remove this asymmetry then the paradox gets resolved. So, we will do away with this U-turn business and then frame the paradox so that we do not have to worry about the U-turn. So, first we will pose the paradox or repose the paradox - reframe the paradox in such a manner that the U-turn is eliminated because we do not want it to interfere with our thinking; so, let us do that.

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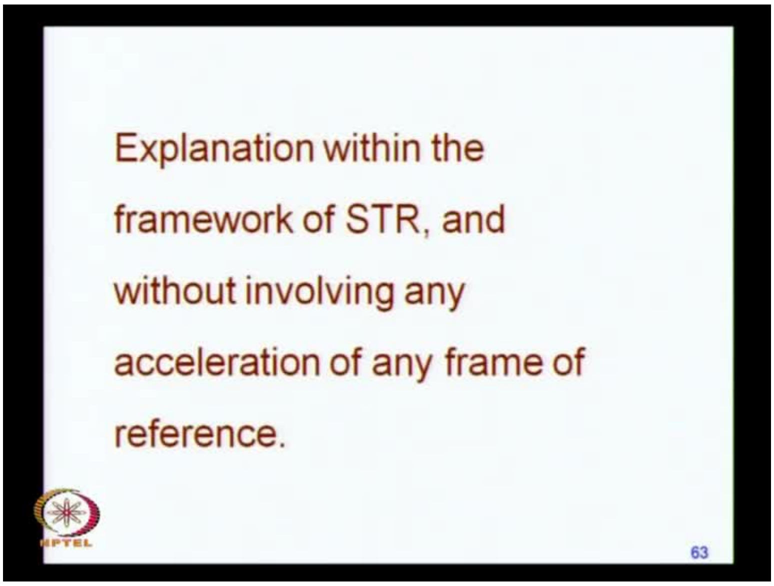

Our discussion will be completely within the framework of the special theory of relativity. It will not invoke any general theory of relativity and it will not invoke any acceleration of any frame of reference.

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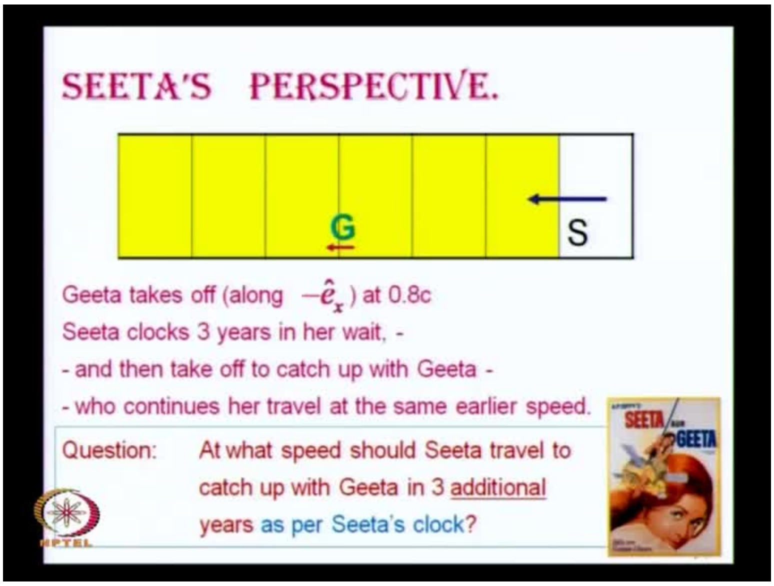

So, let us now take Seeta's perspective. So, Seeta is the travelling twin; she goes, takes a U-turn and comes back. What we will do is we will take Seeta's perspective and from this point of view - from Seeta's point of view, we find that Geeta takes off along minus x, so, if this was plus x, this is minus x right. So, Geeta takes off along minus e x at 80 percent the speed of light - 0.8 c. Seeta who is sitting here, clocks 3 years in her wait and then even as Geeta continues to travel, Seeta takes off and catches her up. Now, there is no U-turn.

So, the meeting was enabled by the U-turn. We do away by that U-turn by enabling the meeting by another process, which is completely equivalent to the earlier situation. The meeting is arranged not by requiring Geeta to take a U-turn, but let Geeta continue to travel, but then Seeta should go at after waiting for 3 years, she should take off at a certain speed and we will find out what the speed must be.

When she takes off at an appropriate speed, she should then catch up in how much time, according to whom - according to Seeta's clock because it is Seeta who is going to who ages through 6 years in the whole process. From their separation to their reunion, 6 years have elapsed in Seeta's clock, of which Seeta spends 3 years in the wait and in subsequent 3 years of her own clock, she must catch up.

So, we will set up the situation like this. So, here is Geeta. This is Seeta's perspective. So, she stays at home. Geeta has taken off; she travels for 3 years according to Seeta's clock. So, these blocks that you see over here do not belong to Geeta; they belong to Seeta's clock. According to Seeta's clock, Geeta has travelled through 3 years - 1, 2 and 3. There is no U-turn. Geeta continues to travel after the third year and at this instant of time, when Geeta has finished her third year, Seeta must take off at an appropriate speed so that she will catch up with Geeta.

Obviously, Seeta must travel at a considerably larger speed; otherwise, she not going to able to catch up. So, we ask this question; we have agreed that Seeta must travel at a larger speed. Let us now ask ourselves at what speed should she travel, so that she will catch up with Geeta. What must be her speed? Any quick answer. Let us work it out. Let us think of a very similar situation here.

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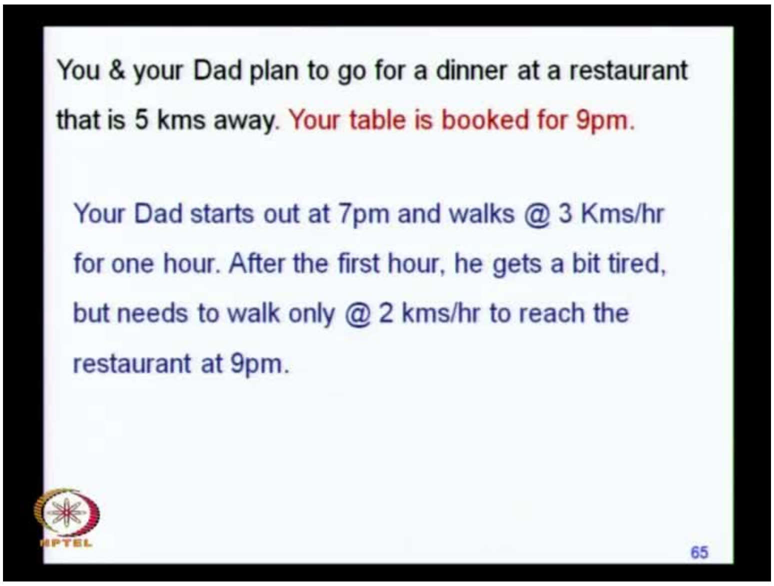

Let us say you and your dad plan to go for a dinner. Good idea. We do not need to discuss anything else, but let us say you plan to go for a dinner. You have booked your table at a restaurant at 9 O' clock in the evening; the restaurant is 5 kilometers away.

Your dad, let us say, he starts off at 7' O clock at the evening. He starts walking and in the first hour, he makes 3 kilometers. So, he walks at a speed of 3 kilometers per hour for 1 hour and in the next hour, so that he can reach the restaurant at 9 O' clock, he is a little tired now and he travels at 2 kilometers per hour, a little leisurely. 3 kilometers is already leisurely for this dad; no matter how old he is.

But in the next subsequent second hour, he must travel at 2 kilometers per hour so that at 9 O' clock he will reach the restaurant. Now, you tell your dad that you have got your homework to finish and you will start from home only at 8 O' clock and you are being young, you will move faster and you will catch up. That, you do not have to do your homework, you are only chatting with your friends on the phone is a different matter.

But then you start off at 8 O' clock and not at 7, 1 hour after this. At what speed, must you go? 5 kilometers per hour, which is the sum of the two speeds. So, your dad's speed during the first hour must be added to your dad speed in the second hour so that you go at the sum of the speeds so that you can meet him at 9 O' clock at the restaurant.

So, that is what you should expect Seeta to be required to do. From Seeta's perspective, Geeta has taken off, in Seeta's clock, 3 years have elapsed, and Geeta would have reached a certain point. In the next 3 years, Geeta would have gone further and now, Seeta takes off after the first 3 years according to her own clock and she must go at a faster speed. So, at what speed must she go?

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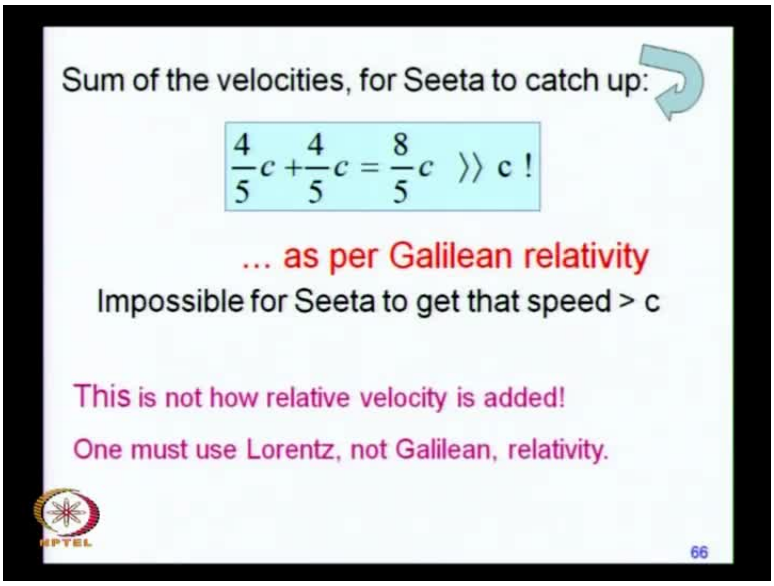

Eight-fifth the speed of light. That is faster than the speed of light. She is never going to make it. Now, you have got eight-fifth the speed of light because what you did was that you added the speeds according to Galilean relativity. Now, that is not how speeds add; you must apply the Lorentz relativity, not Galileo.

At eight-fifth the speed of light, Seeta is never going to make it, but that is not a worry because this is not the correct answer. We have to find out how these speeds should be added so that she can get there in the next 3 years according to Seeta's own clock. So, we will make use of Lorentz transformations now and we already wrote them out in our last class. So, I will use them now.

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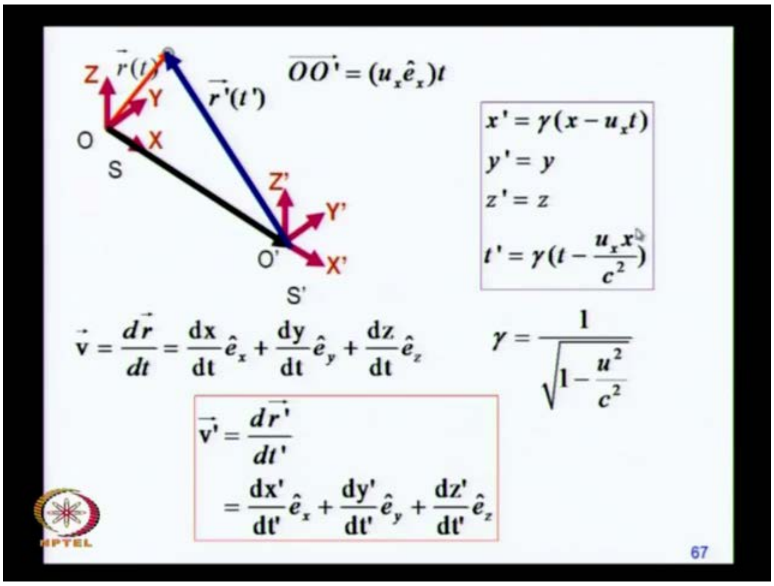

You have got one frame of reference x, y and z. This is the origin of this frame of reference and you observe a certain object, whose position vector is r and the velocity of this object in this frame of reference is delta r by delta t in the limit delta t going to 0, which is the sum of these three components.

So, this is the velocity of this object in the frame of reference s. In another frame of reference s prime, which is moving with respect to s along the x axis and this is along the positive x axis, the displacement vector is given by this because we are assuming that the frame s prime moves at a constant velocity with respect to the frame s and the constant velocity is this u x times e x; it has got no component along y and z.

So, in this frame of reference the velocity of the same object is r prime, which is this blue vector; this is the position vector and its differential with respect to t prime, not with respect to t will be the velocity of that object in the frame s prime because time is different for s prime.

Let us use the Lorentz transformations. We have written them up yesterday in our previous class. So, I will use them straight away. These are the Lorentz transformations and according to these transformations we must now determine the velocity of this object in this frame of reference which is delta r prime by delta t prime in the limit delta t prime going to 0.

But this t prime is not the same as t, there is a mixing of time and space, which is what Lorentz transformations do - the mix the space and the time components. So, that you live in what is called as the Merkosky world or the space time continuum, which is the four dimensional space time continuum.

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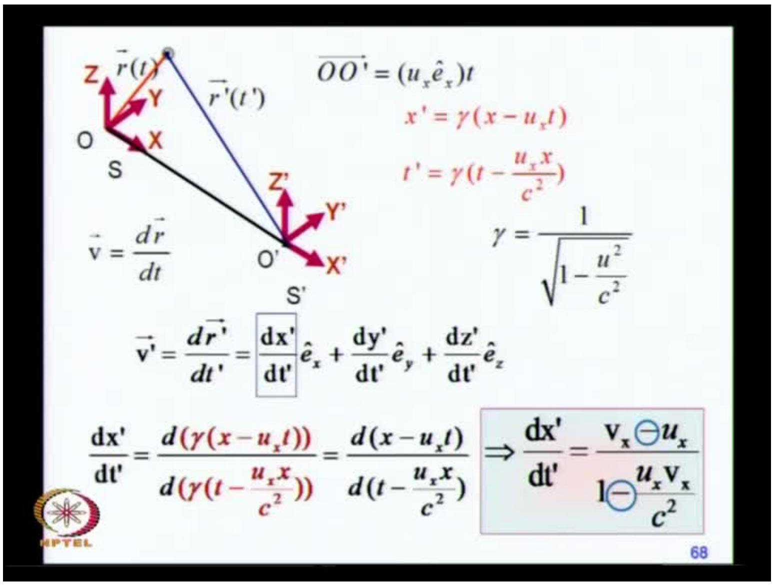

So v prime will be given by this d r prime by dt prime. So, let us do this step by step. dx prime by dt prime, dy prime by dt prime e y, d z prime by dt prime e z. Now, this is the velocity in the other frame. Here, I no more write the y and z components because they remain the same in both the frames of references. So, let us not worry about them.

But x prime is a mix of x and t, t prime is a mix of t and x, gamma is determined by this ratio u over c, which is the relative velocity of the two frames of references and what we need to determine is this dx prime by dt prime. This is the only component which is going to change; the y and z components will not change; this is the only component that must change.

So, we have to find dx prime by dt prime and this is x prime; this is t prime. Now, from these relations straight away we can get dx prime by dt prime. So, let us do that. This is dx prime by dt prime. So, x prime I take from here, which is gamma x minus u x t which comes over here; t prime is here, which is gamma t minus u x x over c square. So, this comes over here, the gammas cancel; you are left with this ratio. You can divide both the numerator and the denominator by the interval delta t and take the limit delta t going to 0 and what you get is this ratio dx prime by dt prime is delta x by delta t in the limit delta t going to 0 will give you v x; this differentiated with respect to time gives you the u x; this differentiated with respect to time gives you 1 and this differentiated with respect to time u x over c square is a constant, dx by dt gives you v x. This is the component of the velocity along the x direction in the frame s prime.

Notice how, the speeds have added. If this was Galilean, c would have been considered to be infinite; this term would have vanished and we would simply have v x minus u x which is the Galilean perspective that, all you have to do is to take the sum of the two velocities, which is what you did. That is how, you got the 8 over 5 c.

Because this motion is you know, In our own analysis we began with Seeta moving in the x direction and from Seeta's perspective, Geeta goes in the minus x direction, we have this negative sign over here. So, instead of sum of the velocities, you get a difference of the velocities, but it is essentially the same; there is no difference between the sum and the difference, except for the minus sign, right.

So, velocities must be added according to Lorentz velocity and your earlier answer of eight over fifth of c was wrong because you missed this factor completely which would not be a terrible error, if the speed of light was infinite, which is the Galilean limit of the Lorentz transformations anyway; so there is no inconsistency in that.

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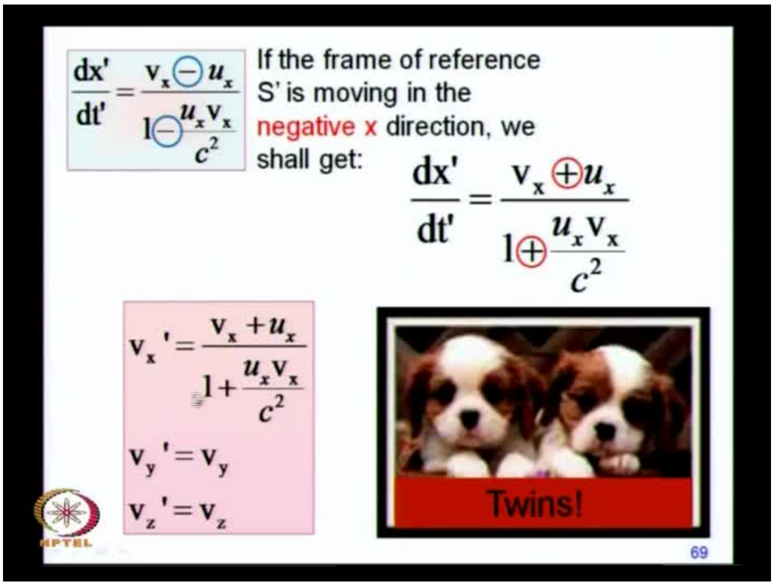

So, this is the minus sign and now that you have the frame of reference s prime moving in the negative x direction, you must plug in a positive sign because Seeta must go at an enhanced speed. She must go at the sum of the two speeds and the two speeds that we are now talking about are four-fifth, the speed of light. In the original example that we had Geeta and Seeta stay together, Seeta takes off at four-fifth the speed of light. So, we must We have got all the numbers now. All we have to do is to calculate this. What about v y prime and v z prime; they remain invariant and all we have to do is to get the x component, which is the sum of the two plus this factor.

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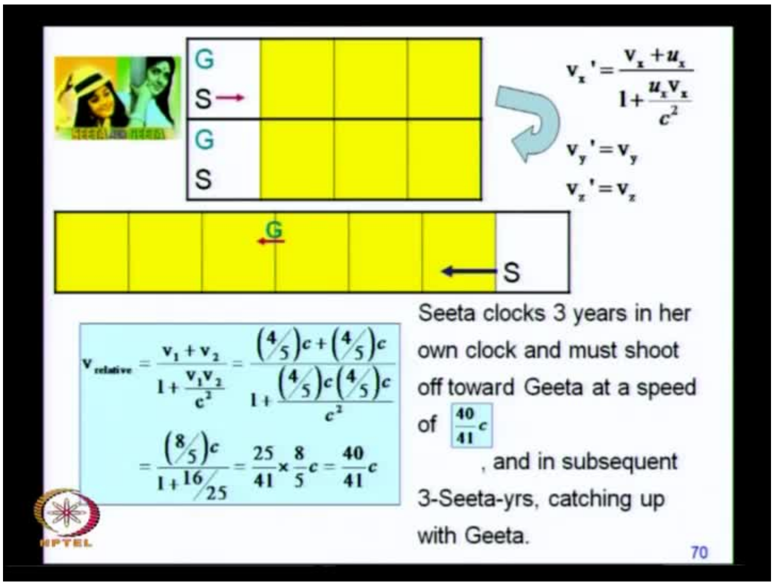

Let us actually evaluate this. So, this is our picture. Seeta clocks 3 years according to her own time and then she takes off after Geeta, she decides to catch up and she must go at this speed and we should find out what the speed is. Now, we have v one plus v two which is coming from this v x plus u x divided by 1 plus v 1 v 2 by c square right. These are four-fifth the speed of light. You just put in these numbers and very simple arithmetic tells you that this relative speed must be 40 over 41 the speed of light. Can you see how this turns out to be 40 over 41? It is a straightforward arithmetic; there is no big difficulty with that.

So, Seeta must take off at 40 over 41 the speed of light and then she will be able to catch up with Geeta in 3 years - in 3 additional years. Now, we need to convince ourselves that in subsequent three years, she will be able to catch up with Geeta.

Do we agree with this? That Seeta must start off, she should clock 3 years in her own watch and then go after Geeta chasing her and go at a speed of 40 over 41, the speed of light and she should catch up with Geeta during which Geeta should age by 10 years according to her own clock; Seeta ought to age by 3 years according to her own clock and if all this works out, the paradox will be resolved.

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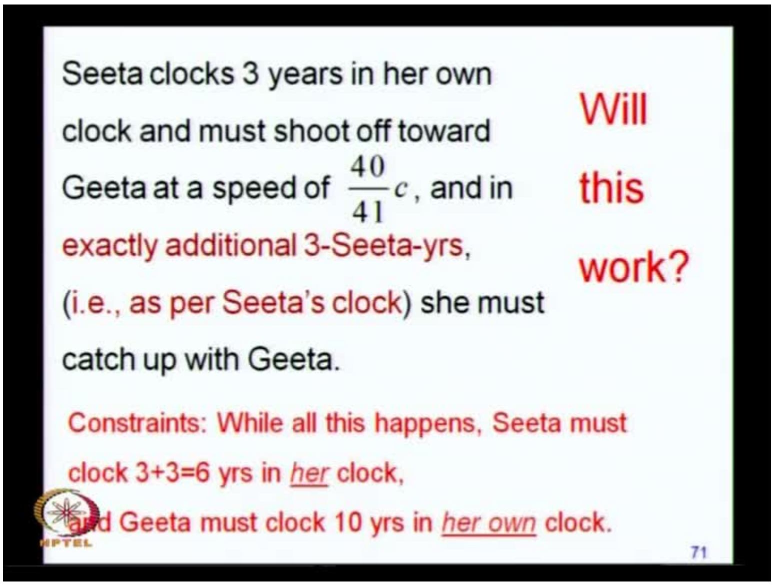

Now, let us see if that happens. That in exactly 3 additional Seeta years, this is what I call as the Seeta year because it is a measure of time in her own clock - in Seeta's clock. Let us find out if this is going to happen because there are certain constraints. The constraints are that Seeta must clock exactly 3 additional years in her own clock.

We have not checked her out and we need to also assure ourselves that during this period, Geeta measures in her own clock, 10 years. We have not verified that either. Now, if that is something that we can verify, then the paradox will be resolved. Let us see how that happens. So, our first requirement is that Geeta would clock 10 years in her own clock, but now from Seeta's perspective, Geeta is a travelling twin. So, Geeta will be referring to her own clock, which is now in a moving frame and what Geeta would call as a proper time; that proper time should have advanced through 10 years.

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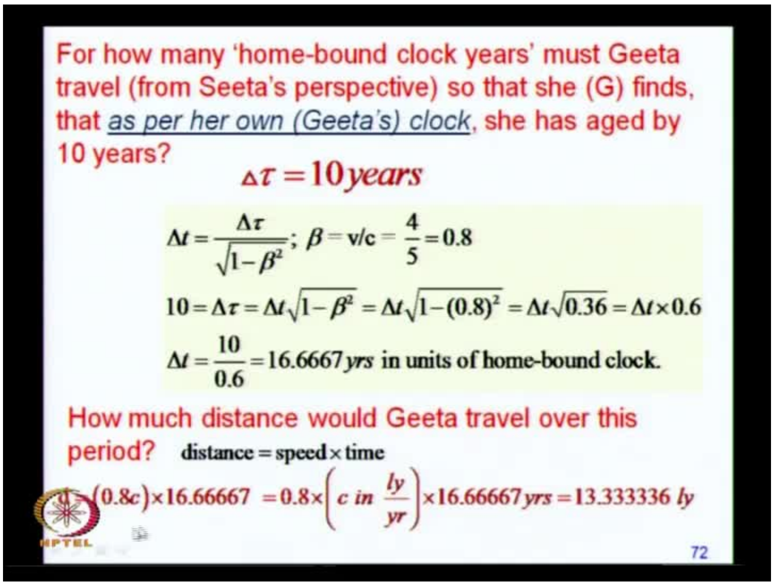

Now, we should ask for how many years of the home bound clock must Geeta be travelling so that she measures 10 years in her own clock. She has to refer to her own clock; everybody uses his own time and his own scale. So, for how many years must Geeta travel, how many which years, the home bound years? According to Geeta's clock, the proper time in her own clock must be 10 years.

But those 10 years may be different for this observer. Let us find out this. delta tau must be 10 years. So, what is corresponding delta t, if beta is equal to 80 percent the speed of light? If beta is equal to 4 over 5, what will be delta t, so that delta tau is 10 years?

This is a very simple arithmetic. You just plug in the numbers and you find that this delta t must be 10 over 0.6. This really does not truncate at the fourth decimal place, but I do not want to keep writing for hour, so I have truncated it after a few decimal places and this is more than 16 years; this is 16.66666 and it goes on and on and on and on. You can very easily see this because beta is equal to 0.8. So, you take the square root of 1 minus beta square which is 1 minus 0.64 which is the square root of 0.36 and that is 0.6.

So, you divide 10 by 0.6 and you get 16.666 and so on, but according to Geeta's clock, she is not going to measure 16.6 years. These are 16.6 home bound years, but according to the proper time, the proper clock which Geeta is referring to in her own frame, she has gone through 10 years; that is what we have put over here delta tau is equal to 10 years.

How much distance would Geeta have travelled during this time? Now let us calculate that. Now, distance is speed into time. We know the speed and we know the time. The speed is 0.8 times the speed of light and the time interval is 16.666 so many years.

We will use the unit of We will measure the light speed in units of light years per year. You know what a light year is. Light year is the distance which light travels through in 1 year. So, you can measure you can record speed of light in meters per second or in units of light years per year. This is a convenient unit over here. So, I am using c in light years per year - ly per year.

So, the distance is 0.8 c which is speed measured in light years per year multiplied by the number of home bound years; so this is 0.8 times 16, so it is 80 percent of this 16.66, somewhat less than 16.66 and so on; it turns out to be 13.333 and so on. So, that is the distance in light years.

It is a lot of distance. She has gone really far and now, Seeta must catch up and at what speed is she going to travel? At 40 over 41 times, the speed of light.

How much distance does she have to cover to catch up?

13.33 light years.

No, it is not 13.3 3 because this distance will be Lorentz contracted. This is the distance as seen from the home bound frame of reference, but in the next 3 years, Seeta herself is going to be travelling. So, she is now in her own rocket. [FL] She is going to have to cover a certain distance to catch up, but that will not be 13.333 light years; it will be Lorentz contracted.

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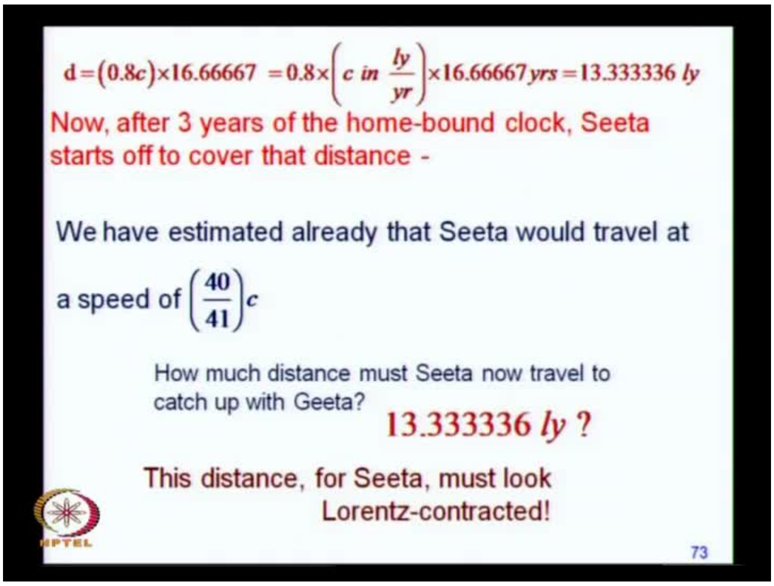

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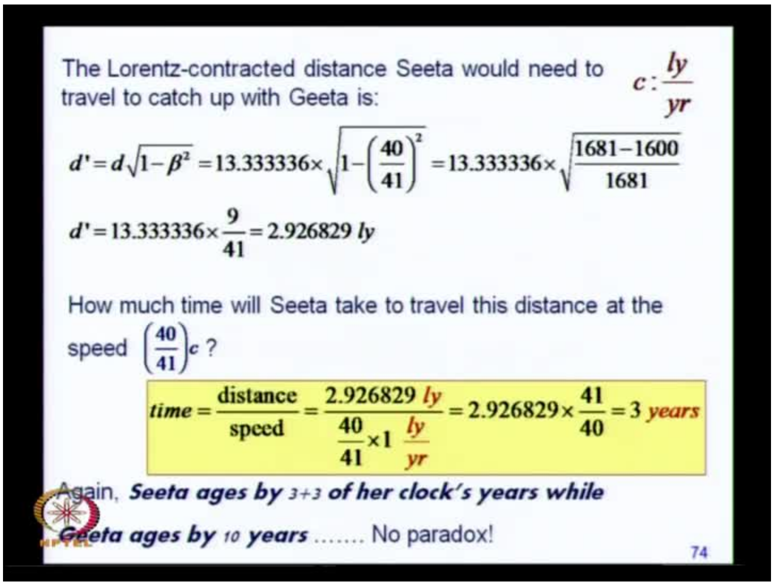

Let us find out what the distance turns out to be. What is the distance that she must cover? The distance that she must cover is not 13.33 light years, but it must be Lorentz contracted and we know the formula. This formula is here. d prime equal to d times square root of 1 minus beta square, where d itself is 13.33. So, you multiply this distance in light years by this factor 1 minus beta square, where beta now is 40 over 41 and this distance, when you calculate this, this will be 41 square minus 40 square.

So, that will give you 81 in the numerator and the square of 41 in the denominator. You get the square root of 81 over 41 square which is 9 over 41. This is a fairly small fraction 9 over 41. So, 13.33 gets multiplied by a small fraction, it turns out to be a rather small number which is under 3 light years, less than 3 light years; it is about 2.92 light years.

So, now, Seeta has a speed of 40 over 41 the speed of light, which is very nearly the speed of light and she has 2.92 years to catch up. Let us ask how much time she is going to take up. She will not catch up in 2.92 years because her speed is not the speed of light. She will take a little bit more, but the speed is almost that of speed of light. So, she is going to take just a little bit more. Instead of 2.92, she ends up taking 3 years.

You should get these numbers exactly. So, calculate time as distance over speed. Distance now is 2.92 light years divided by the speed which is 40 over 41; plug in the numbers exactly in your calculators and you get exactly 3 years.

Now, there is no paradox. Our conclusion is that whether you do this analysis from Geeta's perspective or from Seeta's perspective, you do not arrive at different physics; that the laws of physics are the same in all inertial frames of references. Time dilation is an admitted consequence of the fact that the speed of light is the same in every inertial frame of reference; there is just one residual paradox that we need to address.

Here, we have already resolved that Seeta ages by 3 plus 3 of her own clock years, while Geeta age is by 10 years of her own clock years. So, it does not matter whose perspective we take, the principle of symmetry and equivalence of the two observers is maintained; that the laws of physics are the same in all inertial frames of references; that the speed of light is constant; that the formula for time dilation and length contraction involve this factor gamma or beta of this v over c and if you put in these numbers correctly, you get exactly the correct resolution of the paradox.

We have not of course, employed general theory of relativity. We have not even mentioned it; we do not even know what it is. We have not invoked any U-turn, we have not invoked any acceleration at the u turn. The entire resolution of the paradox is within the framework of the special theory of relativity and I think this morning Jobin did a few examples, just to convince himself that I am not cheating here and Jobin you tried some other numbers, right. So, what is it that you have tried? means you

Seeta is travelling by 6 by 7 c.

Ok.

For 3 years

So, you had in the first, from Geeta's perspective, you had Seeta travel at six-seventh the speed of light - 6 by 7 times the speed of light, travel for 3 years and then she comes back and she would have aged through 3 plus 3, 6 years, but the time dilation will be different this time. So, how much was the time dilation for Geeta.

5.824 years.

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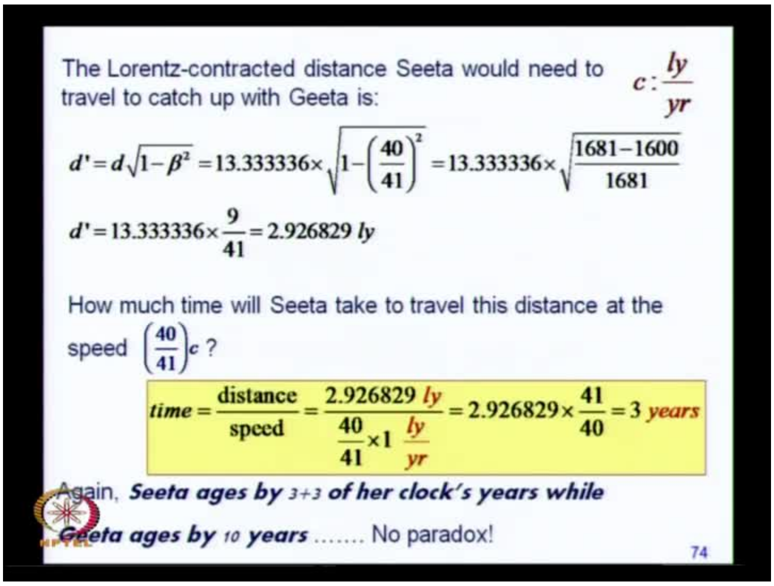

So, Geeta advances through 5.8 and then how did you resolve this?

Sir, the rate of velocity with which Seeta has to travel the second time.

Yes.

So, it is 84 by 85, sir.

84 over 85

So, Jobin finds that from Seeta's perspective, if he lets Seeta stay at home for the first 3 years and Geeta take off and then after 3 years, Seeta shoots off, then she must go at a speed of 84 divided by 85 times the speed of light. Is that right? 84 by 85 c so that in the next 3 years, she catches up and how much distance?

Sir, it is 22.615.

22.615 of the home bound years. 22.615 light years according to the home bound distance scale, but the length contracted distance is much less. How much is that?

That I have not recorded.

So, then if we calculate the time

So, he finds that the length contracted distance is much shorter and it can be covered at the speed of 84 by 85 c in exactly 3 additional years. So, you can do this exercise for yourself and put in different values for, you know, v over c. Let the travelling sibling go for 3 years or 3 and a half years or for 7 years or 8 years or 130 years. Do not make it too long we have human average life is about a 100. So, anyhow, it works.

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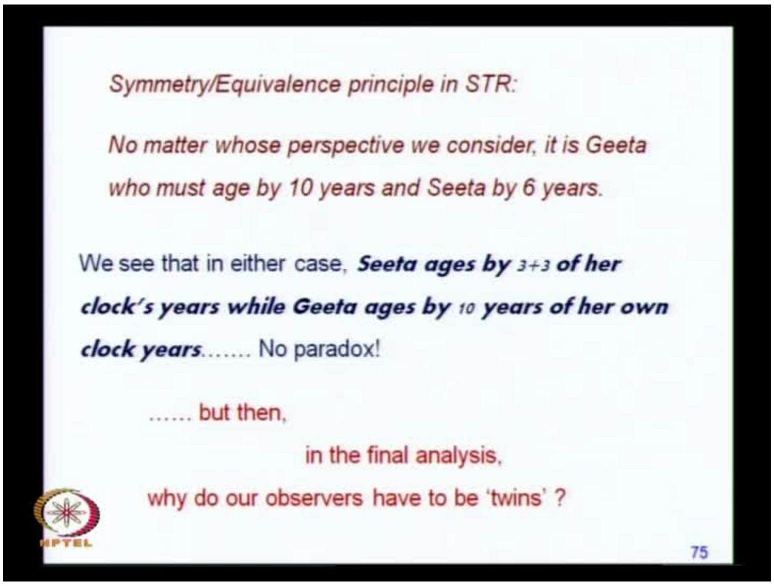

So, this is the complete resolution of the twin paradox. This is within the framework of the special theory of relativity. It does not matter whose perspective we take; it is Geeta who must advance through 10 years and Seeta through 6 years. There is no paradox any more. It is only that the problem is posed so that it looks like a paradox when it really is not.

The only thing is that in the final analysis, the two observers really do not have to be twins. They could be two brothers, two sisters, two friends or even two enemies; it does not matter. It has nothing to do with their relationship, it has to do with their clocks and with their time measures and the distance measures and both must be invoked to get the correct resolution.

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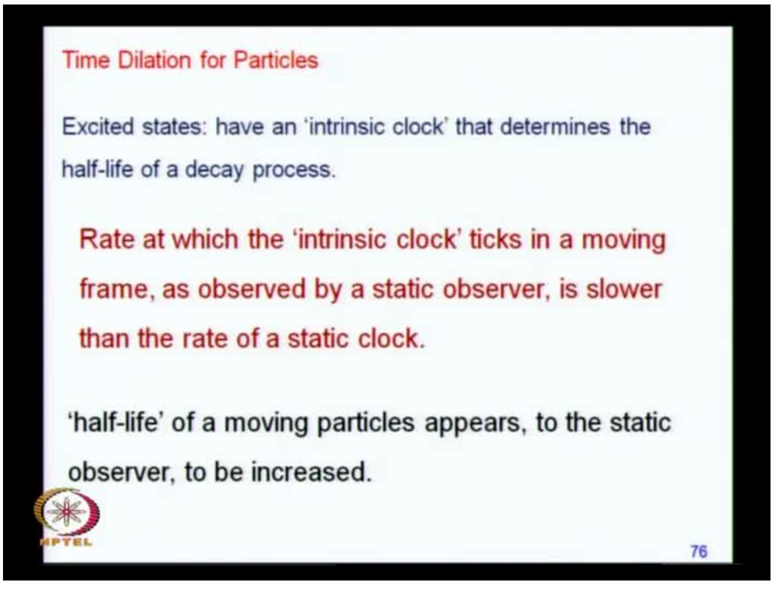

This is seen in particle physics because in an excited state a system can decay after a certain life time. So, excited states have their own intrinsic life time mechanism, but if this is excited state of a resonance which is actually travelling, then its own measure of time will be different. So, in particle physics these things are well studied, well observed, there is consistent experimental verification of this.

I will not go through those applications because our discussion here is not quite in the domain of nuclear physics or particle physics or decays of resonances and so on. It is only confined to the discussion on what the twin paradox is and how it is resolved, but this is well known that half life of moving particles appears to be increased; they live little bit longer.

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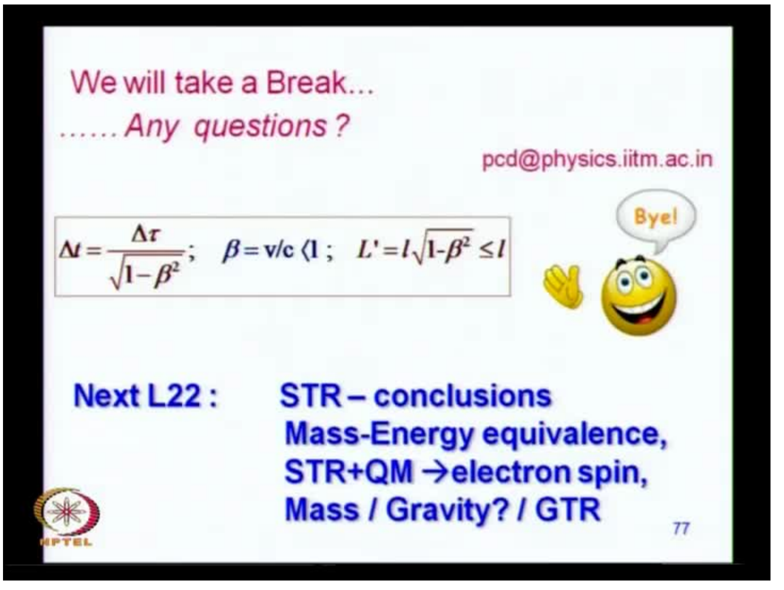

So, there are a large number of applications and with that I will conclude this class and in the next class, I will summarize some of the essential conclusions of the special theory of relativity. I will comment on some other consequences like the equivalence between mass and energy, which we have not discussed as yet.

So, I will comment on that. I will also point out that there are other consequences, not just time dilation and length contraction, but there are other consequences and then of course, this is only the special theory, which is the special case of the general theory of relativity which is clearly beyond the scope of this course, but we will comment on some consequences of gravity. So, that will be for the next class. So, any questions?

According to the travelling sibling, the home one is actually in a state of motion. So, they should be actually you know at a younger age or younger. How do you explain that?

That is what we did.

No, we found out with respect to whether they turn

No, I think this needs a little bit of thinking. Let us go back and ask ourselves, how did we pose the paradox? That from Geeta’s perspective, Seeta is the travelling sibling who ages through less time through 3 years while Geeta ages through 5 and when Seeta ages through 6 years, Geeta has aged through 10 years. Who is older? Geeta. From Seeta’s perspective, Geeta is the travelling kid. So, Seeta should end up with the conclusion that Geeta being the travelling kid, travelling sibling, should stay younger.

This is the paradox that if you analyze this from either Geeta's perspective or Seeta's perspective, you should not reach different conclusions at the end of 6 years of Seeta or if Geeta ages through 10 years. It should remain so whether or not you consider Geeta as the travelling sibling or Seeta as the travelling sibling which is what we just did.

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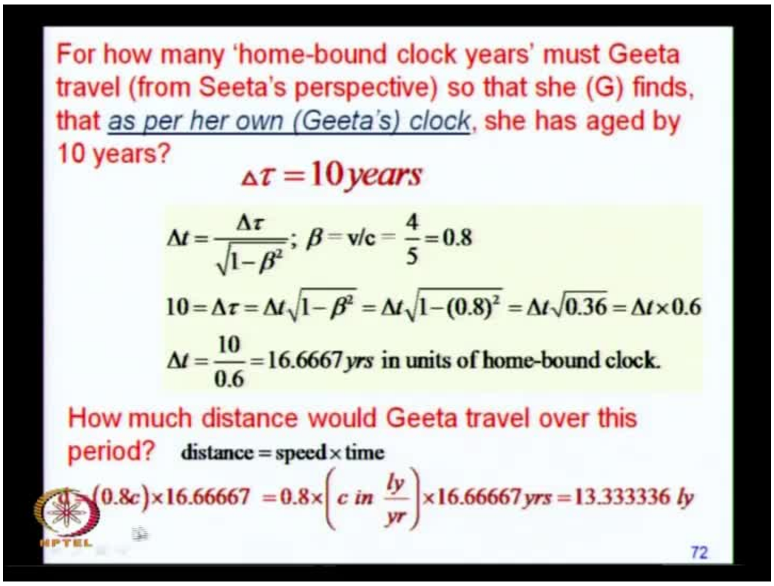

Let us go back and see our solution. That from the point view of Seeta's perspective, Geeta travels through 10 years, according to whose clock? According to Geeta's clock. Geeta is going to make a reference only to her own clock; that is the eigen clock, the proper clock. So, in Geeta's clock, she must measure 10 years, but if this is the proper time delta t is longer; if delta tau is 10 years, delta t is 16.66. We have allowed Geeta to age by 10 years; that has gone into our analysis that Geeta has aged through 10 years. Now, we let Seeta wait for 3 years and in the next 3 years, she catches up according to her own clock.

So, our conclusion is the same that from Seeta's perspective, she waits, she clocks 3 years in her own watch and in the next 3 years, she catches up with Geeta during which Geeta according to her own watch has aged through 10 years. It is the same conclusion; that is exactly what we did.

Yes, our conclusion is that Geeta has aged through 10 years according to the Geeta clock and Seeta has age through 6 years according to the Seeta clock. Does not matter whether you take Geeta's perspective and consider Seeta as the travelling kid or Seeta's perspective and consider Geeta as the travelling kid; either way, you arrive at the same conclusion. That is the resolution law of the paradox. Let it sink in. Do a few examples, take different speeds, do this calculation, it will sink in and the mystery disappears. That is what learning is about. Any other question?

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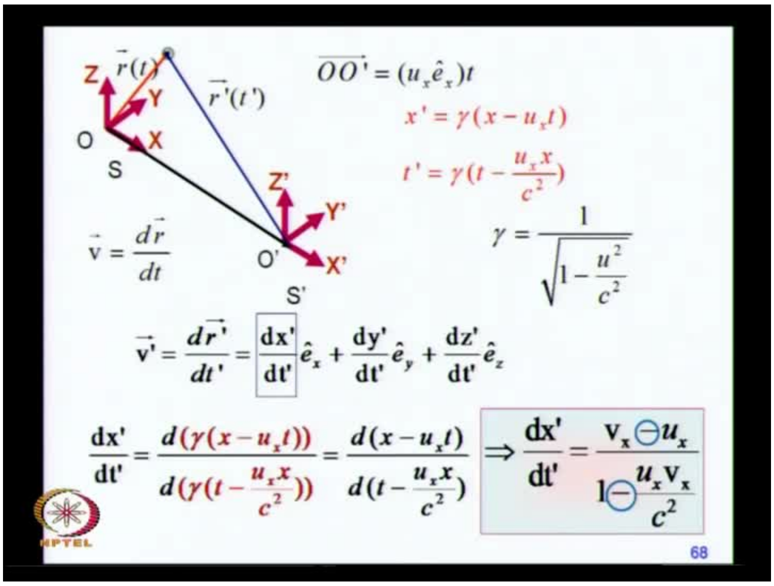

So, the key lies here that when you measure the velocity of an object here, in one frame of reference and this velocity is to be compared with the velocity in another frame of reference, these velocities do not add or subtract according to Galilean relativity, but from the point of view of another observer who is moving at a constant velocity with respect to this observer, this observer must subtract his own velocity with respect to the other observer; that subtraction must be carried out so that he can get the right velocity in his own frame of reference and this must include this factor, u v over c square, which would be 0, if c were infinite.

So, if you add the velocities correctly. So, there are three key points over here. One is how the velocities are to be added correctly according to Lorentz transformations, second is make use of time dilation correctly. Do not stop at that; make use of length contraction correctly and you do all these three things and you get the correct answer.

So, let us take a break and we will continue from here to sum up the discussion on relativity.

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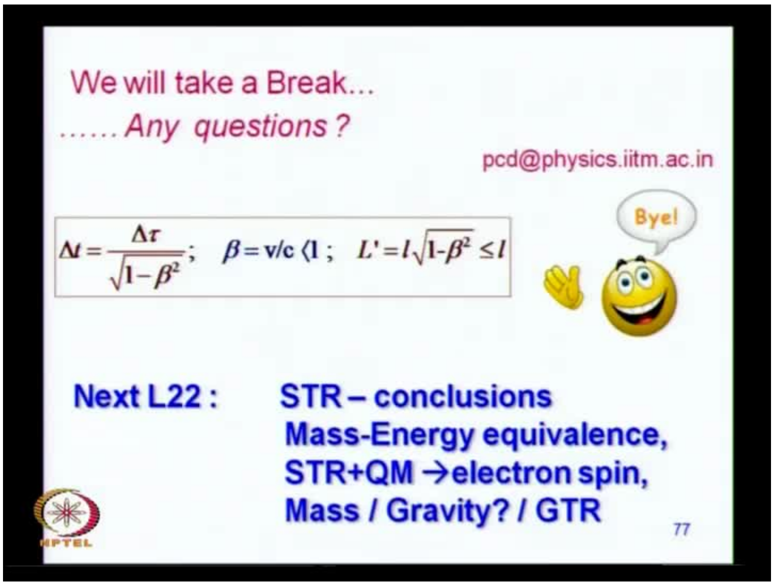

So, that is for the next class. Thank you.