

Select/Special Topics in Classical Mechanics

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Module No. # 06

Lecture No. # 22

Special Theory of Relativity (iv)

In this class, we will conclude our discussion on the special theory of relativity. We have reconciled with the constancy of the speed of light in all inertial frames of references. We have accepted that the laws of physics are the same in every inertial frame of reference as we expect them to be or we have accepted the consequence which manifest itself in terms of time dilation and length contraction. The important thing that I will like to emphasize over here that whenever you talk about distance, time and measures for this measure is, some unit in which you measure it.

You must use your own measure, that if you are doing this analysis from the point of view of an certain observer, an observer let us say whether it is Seetha or Geetha or Jayalalitha or whoever it does not matter. When using Seetha's perspective use Seetha's measures, her distance scale, her time scale, the division of her distance by her time will give you the velocity in her frame of reference. If you want to relate it to that of any other observer then, the transformations must be done through the Lorentz transformations.

So, if you just keep track of these simple things special theory of relativity is no longer a mystery. One really need not be perturbed by all these discussions that one finds in some book or sometimes even most over the internet and they make it sound as if there is some magic going on. There are only three and a half persons who could solve this and you know that kind of thing, so you do not have to worry about those things just use the correct measures in the correct time scales.

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First few Nobel Prizes in Physics, in reverse chronological order

1922	- Niels Bohr
1921	- Albert Einstein
1920	- Charles Edouard Guillaume
1919	- Johannes Stark
1918	- Max Planck
1917	- Charles Glover Barkla
1916	- The prize money was allocated to the Special Fund of this prize section
1915	- William Bragg, Lawrence Bragg
1914	- Max von Laue
1913	- Heike Kamerlingh Onnes
1912	- Gustaf Dalén
1911	- Wilhelm Wien
1910	- Johannes Diderik van der Waals
1909	- Guglielmo Marconi, Ferdinand Braun
1908	- Gabriel Lippmann
1907	- Albert A. Michelson
1906	- J.J. Thomson
1905	- Philipp Lenard
1904	- Lord Rayleigh
1903	- Henri Becquerel, Pierre Curie, Marie Curie
1902	- Hendrik A. Lorentz, Pieter Zeeman
1901	- Wilhelm Conrad Röntgen

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We will conclude this discussion, we are absolutely not going to get into the general theory of relativity or anything like that, but we will sum up some of the other consequences. This has been a very fascinating field in physics it has made important contributions to the developments in physics. Some other famous names in the theory of relativity are Lorentz, Michelson, Einstein and they are all Nobel laureates amongst many others. Although they did not specifically get Nobel Prize for the work they did in the context of the theory of relativity, it does not matter, but they have done not just one piece of work but much else and it has really revolutionized physics.

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Origins O and O' of the two frames S and S' coincide at $t=0$ and $t'=0$.

$$x' = \gamma(x - vt) \quad x = \gamma(x' + vt')$$

$$y' = y \quad y = y'$$

$$z' = z \quad z = z'$$

$$t' = \gamma \left(t - \frac{vx}{c^2} \right) \quad t = \gamma \left(t' + \frac{vx'}{c^2} \right)$$

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{1}{\sqrt{1 - \beta^2}}$$

Note: $\gamma \rightarrow 1$ as $v \rightarrow 0$.

Lorentz transformations transform the space-time coordinates of ONE EVENT.

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What is the upshot that we do not any more talk about space and time as distinct entities, space and time together constitute the 4 dimensional main course key space, x prime is a mix of x and t , t prime is a mix of t and x , so there is a scrambling that is going on.


Just as a certain arrow has got components along, let us say x axis and a y axis and a z axis and so on. We now talk about events which have got components in 4 dimensions not just 3, it does not automatically mean that the extension from the 3 dimensional world to the 4 dimensional world is of the same kind as the extension from 1 dimensional world to 2 dimensional world or from 2 dimensional to 3 dimensional. You can do geometry on this plane in a 2 dimensional space and you add the third component and you can extend your algebra and your geometry E_x, E_y bases goes to E_x, E_y, E_z . Then, you can develop your algebra and geometry by a straight forward extension, the extension to this 4th dimension is a little messy, but I will not be able to go into those details over here.

It is quite clear from this discussion here that space and time are mixed. We must talk about events described by not x, y, z alone but by x, y, z and t so, there is a 4th component which is called as the time line component **or** you talk about events.

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What is SPACE for one observer, is a mix of space and time for another !

What is TIME for one observer, is a mix of time and space for another !



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What is space for one observer is a mix of space and time for another, what is time for one observer is a mix of time and space for a different observer.

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Now, static charges produce electric fields. **Current (charge in motion) produces a magnetic field.**

$$E'_x = E_x$$

$$E'_y = \gamma_f [E_y - v_f B_z]$$

$$E'_z = \gamma_f [E_z - v_f B_y]$$


STR: What is \vec{E} or \vec{B} for one observer, is a mix of $(\vec{E}$ and $\vec{B})$ for another !

$$B'_x = B_x$$

$$B'_y = \gamma_f \left[B_y + \frac{v_f}{c^2} E_z \right]$$

$$B'_z = \gamma_f \left[B_z + \frac{v_f}{c^2} E_y \right]$$

Faraday-Lenz experiments now make sense!



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So, what you call as space is what I might call as a combination of space and time and vice versa depending on our relative state of motion. This has got very interesting consequences, think of static charges, you have got a certain charge - so many coulombs or whatever - static charge produces an electric field, we are comfortable with this idea. If you have a charge in motion you have a current and the charge in motion generates a

magnetic field, this right hand thumb rule and all these things. We are comfortable with that idea also, but now what is the static charge which is a charge at rest for one observer could be a charge in motion for another.

The consequence of this is that if static charges produce electric fields and currents produce magnetic fields what is electric field for one observer can be a combination of electric and magnetic field for another, because what one observer sees as a static charge, the other observer can see it as a charge in motion. So, what is electric field for one observer? For me it may be a purely electric field but if you are moving relative to me, you might see it as a combination of electric field and a magnetic field.

So, electrodynamics is intimately related to the special theory of relativity, it is unseparable these are the equations of transformations, what is an electric field for one observer? E_x , E_y and E_z , this is the electric field for one observer. If the second observer is moving with respect to the first observer at a certain velocity, he will see it as a combination of electric field and a magnetic field.

Here is the superposition of the y th component of the electric field with the z th component of the magnetic field, I am not going to go into these details but let me get this idea across to you. I will get into some of these applications when we discuss electrodynamics in one of the later units. At that point of time we will illustrate these relations, we will spend more time discussing this.

At this point I just want to point out to you that what is an electric field for one observer is the combination of electric and magnetic field for another observer. The transformation from E_x , E_y and E_z , and B_x , B_y and B_z to two new fields E prime and B prime is done according to these transformation equations which I have assembled over here. They have a certain resemblance to the Lorentz transformations which transform space and time and mix them up. These are transformations for electromagnetic field and they show up as combinations superpositions of electric and magnetic fields, but we will illustrate this when we have a unit on electrodynamics alone because Maxwell's equations need to be discussed in terms of the divergence and curl of electric and magnetic fields. We have not even introduced the idea of divergence of a vector, the curl of a vector in this course, so that is what I am going to do in the next unit.

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Faraday's experiments

Loop held fixed; Magnetic field dragged toward left.
NO Lorentz force $q(\vec{v} \times \vec{B})$

Current: identical!

Strength of B *decreased*.
Nothing is moving,
but still, current seen!!!

Decreasing $B \downarrow$

$I \propto \frac{dB}{dt}$

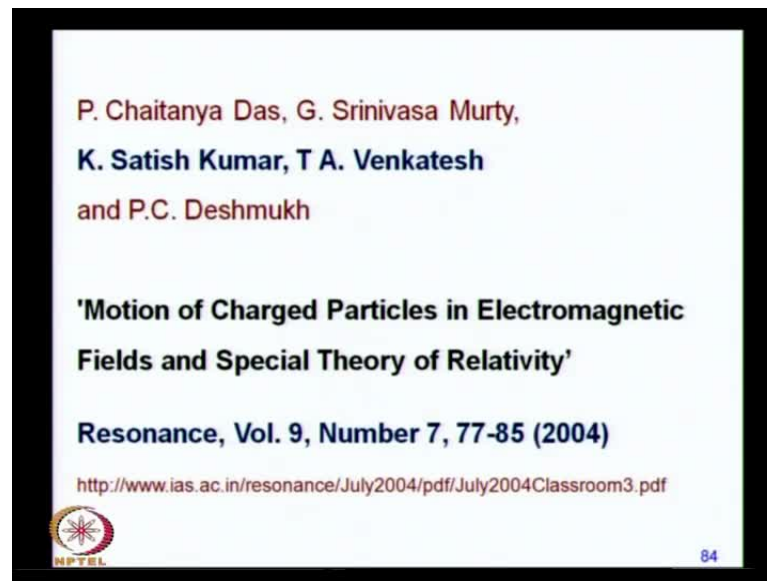
Einstein:
Special Theory of Relativity

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The Faraday-Lenz experiment that we have discussed when we introduced this topic of relativity, now really makes sense. Remind yourself that when you had a magnetic field and you have a circuit which is placed in this magnetic field and you drag the field itself, the Lorentz force is 0, because there is no velocity imparted to any charge in the circuit, but we found that there is a current just as one would get, if the magnetic field is held steady and the electrical circuit is dragged.

We also saw that if you move neither, but just change the magnitude of the magnetic field even then we found that there is a current. So, this Faraday-Lenz experiment which goes into Maxwell's equations, it really makes perfect sense once you understand that space time continuum is what we must use to do this analysis. You cannot separate space from time and then still expect electrodynamics to work. So, electrodynamics and the special theory of relativity go hand in hand. As I mentioned when we introduce this subject, perhaps this more than the measurements of Michelson and Morley were the motivation for Einstein to ponder over these issues and finally, he came up with brilliant answers.

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This particular transformation between electric and magnetic field is very well illustrated in this paper which you can look up. It is nice software which is developed by two of my students Sathish Kumar and Venkatesh and subsequently this software package was improvised by Chaitanya Das and Srinivas Murthy and it is available at this reference in resonance, it is also there at my website you can look it up. When we discuss electrodynamics we will actually run this package, Jobin we can do that means, we will set it up on this laptop and we can run it.

We will do it as when we discuss electrodynamics not at this stage. It really shows how these electromagnetic fields transform from one frame of reference to the other frame of reference and how the observations make perfect sense and the analysis of the implications of the Lorentz force whatever acceleration it is to impart. The physics will turn out to be the same in both the frames of references, because what is electric field for one observer is a combination of electric and magnetic field for the other.

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Other implications of STR: space-time continuum

INVARIANT INTERVALS?

$$\vec{\eta} = \frac{d\vec{r}}{d\tau} = \frac{d\vec{r}}{dt/\gamma} = \gamma \frac{d\vec{r}}{dt} = \gamma \vec{v} \quad \text{'velocity'} \xrightarrow{?} \frac{d\vec{r}}{dt}$$

$$\eta^\mu = \{\gamma c, \gamma \vec{v}\}: \text{"4-velocity"}$$

$$\eta^\mu = \{\eta^0, \eta^1, \eta^2, \eta^3\} = \{\gamma c, \gamma \vec{v}\}$$

$$\eta^0 \eta_0 + \eta^1 \eta_1 + \eta^2 \eta_2 + \eta^3 \eta_3 = c^2 \quad \text{Lorentz Invariant}$$


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Now, there are other implications of the special theory of relativity and again in this class I am only going to sum up some of the essentially consequences but not go into any great details. Let us ask this question what is an invariant interval? Means, if you look at the length interval between this and this, we have already agreed that this length interval between this tip to that tip cannot mean the same to every observer regardless of their relative states of motion.

If you talk about velocity which is a ratio of distance over time, neither the distance nor the time means the same to all observers these do not have absolute meanings they have got meanings in their own frames of reference, because each observer has his own measure of the length scale and he has his own measure of the time scale. He must use his own time and his own length to subsequently do these analyses of the ratios which give velocities subsequent ratios with another time, with which will give him acceleration and then do dynamics.

So, the invariant is not delta r it is not delta t and to come up with the invariant quantities in the 4 dimensional space. We are not expecting invariants in three dimensional spaces anymore, because the 3 dimensional spaces do not have an identity which can be separated from the 4 dimensional worlds.

The invariants of the 4 dimensional world will have some mixed attributes and this is where we end up introducing what is called as the proper velocity. Velocity is dr by dt we define what we call as a proper velocity which is some sort of a hybrid quantity, this is dr by $d\tau$ and actually, if you just look at the expression at the right extreme of this equation it is γ times velocity - you know what this γ is this square root of 1 minus v square by c square.

This η which is γ times v is what is introduced as the proper velocity, the vector v has got 3 components. So, the vector η will also have 3 components because it is only that each component is scaled by the factor γ , but we must look for a velocity which has got 4 components. So, γv which we have now defined as η gives you three out of those 4 components there is an additional component which is γc .

So, γc together with γv is what gives you the 4 components of what is called as a 4 velocity, so this has to be introduced in the theory of relativity. This is very often expressed by using contravariant and covariant notations. I am not going to get into this analysis and use this notation; I will probably use some of it when we discuss electrodynamics in a later unit but not now.

At this juncture, I just want to point out that you must introduce the 4 velocity which is γc and γv , the 3 components sitting in γv and this is the 4th component. The reason to do this is that if you take the metric like the scalar product kind of thing.

I will not go into a detailed definition in terms of covariant and contravariant labels and how to do the contraction and so on, but you can see that these three components look somewhat like the $v \cdot v$ scalar. The scalar product of a vector with itself $a \cdot a$ is a_x^2 plus a_y^2 plus a_z^2 .

It is a_x^2 , a_x^2 plus a_y^2 , a_y^2 plus a_z^2 , a_z^2 to this we must add the $\eta_0 \eta_0$ and if you construct this addition, this addition gives you c^2 regardless of anything else. This is a manifestly invariant quantity because the speed of light is a constant in every inertial frame of reference; therefore its square is also a constant.

So, you get invariant quantities from this 4 component velocity not from the 3 component velocity. So, this is the implication comes as an upshot of the special theory of relativity that velocity has to be defined in terms of these 4 components, with these 4 components it gives you the correct invariant quantity which is c square. What about momentum? Momentum is what we normally define as mass times velocity and if our perception of velocity has been refined to upgrade it from a 3 component velocity to a 4 component velocity we must do for momentum as well.

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$$\eta^0 \eta_0 + \eta^1 \eta_1 + \eta^2 \eta_2 + \eta^3 \eta_3 = c^2$$

$$\vec{\eta} = \frac{d\vec{r}}{d\tau} = \gamma \vec{v} \quad \vec{p} = m \vec{\eta}$$

$$p^\mu = \{p^0, p^1, p^2, p^3\} = \{\gamma mc, \gamma m \vec{v}\} = \left\{ \frac{E}{c}, \gamma m \vec{v} \right\}$$

$$p^0 p_0 + p^1 p_1 + p^2 p_2 + p^3 p_3 = \frac{E^2}{c^2} - \vec{p} \cdot \vec{p} = m^2 c^2 \quad \left[\frac{E}{c} = \gamma mc \right]$$

$$E^2 = p^2 c^2 + m^2 c^4$$

For a photon
 $v = c$
 $E = pc$

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This is the 4 component velocity vector eta, the 3 momentum - the usual 3 dimensional momentum - the momentum in the Euclidian space that we normally talk about is mass times velocity but now, this must go over to the 4 momentum of which m times eta will give you 3 of those 4 components. The 4th component turns out to be gamma mc which is connected to the energy, so the energy is like the 4th component of the momentum.

In the special theory of relativity, energy gets introduced as the 4th component of momentum. In fact, the exact correspondence is here that E over c is this gamma mc which is the 4th component of momentum. The remaining 3 components being gamma times mass times velocity which is gamma times this 3 momentum.

Now, the reason this works is because if you construct the invariant quantity out of this, the quadratic quantity out of this which is $p^0 p_0 + p^1 p_1 + p^2 p_2 + p^3 p_3$.

Just like we constructed the invariant from the velocity, from the 4 velocity we construct the invariant from the momentum, the 4 momentum what we get is E^2 over c^2 minus $\mathbf{p} \cdot \mathbf{p}$, this is what we get.

This is manifestly invariant because you get it equal to $m^2 c^2$, where m is the rest mass which is again it does not change from 1 frame of reference to another. Here again, I will like to alert you to the fact that we introduce only 1 mass which is the rest mass, some older books also talk about a relativistic mass. You already see the energy mass equivalence in this relation, because if you multiply both sides by c , you get E equal to γ times mc^2 . These two relations give you E^2 equal to $p^2 c^2$ plus $m^2 c^4$.

This relation that you see in this block is the energy mass equivalence which is mc^2 . If the 3 velocity is different then, the only component that will contribute is the rest mass and then the rest mass will give you this energy. There is no point in introducing a relativistic mass, because if energy and mass are equivalent why do you need to introduce. Two quantities when they are connected, so there is only one mass that you talk about which is the rest mass. This energy E equal to mc^2 is best written as E equal to γmc^2 , there is a γ factor which is sitting there. If you look at this equation you can ask yourself what would this be for a particle which is traveling at the speed of light, what if $v = c$? It is equal to the speed of light.

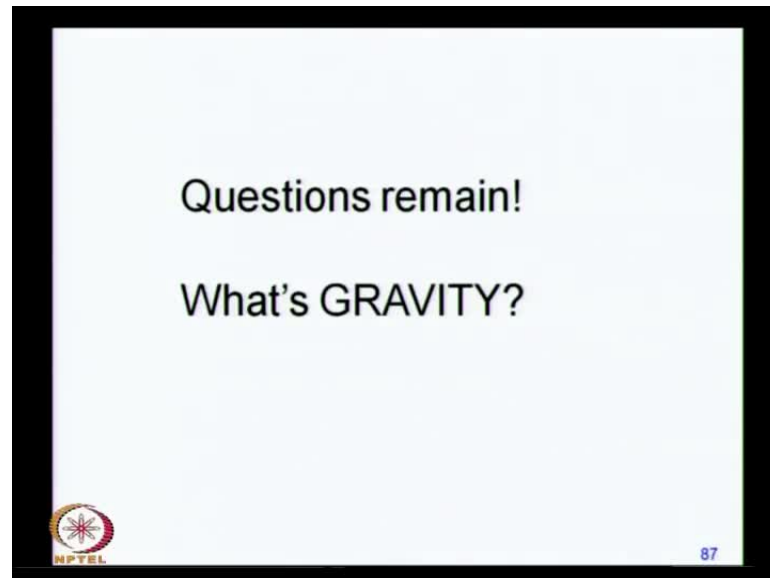
So, put v equal to c here, this becomes c^2 by c^2 which is $1/1 - 1$ gives you a 0 in the denominator and the whole quantity blows up it becomes infinite. If the numerator also goes to 0 then, you get 0 over 0 which is indeterminate and may be it makes sense.

For particles which are traveling at the speed of light which is a photon the rest mass must be 0. So, for a photon v is equal to c and then over here, the rest mass being 0, this quantity vanishes $m^2 c^4$ and the energy becomes E equal to pc for a photon the rest mass being 0.

The most famous equation in physics is perhaps $f = ma$ or $E = mc^2$ one of these two $f = ma$ is of course correct in **the Newtonian limit** in the Galilean limit. $E = mc^2$ must be correctly written as $E = \gamma mc^2$,

because you need to introduce only one new quantity, energy and mass being equivalent, so there is an energy mass equivalence but the equation which I like most is this $E = mc^2$ because it associates energy with my initials.

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We have the mass energy equivalence which comes as an upshot of the special theory of relativity. There are other questions which remain what is gravity? This is something which we learnt from Newton that it is the force between two masses and there is this $G \frac{m_1 m_2}{r^2}$, but let us think about it in terms of an example that we have discussed earlier.

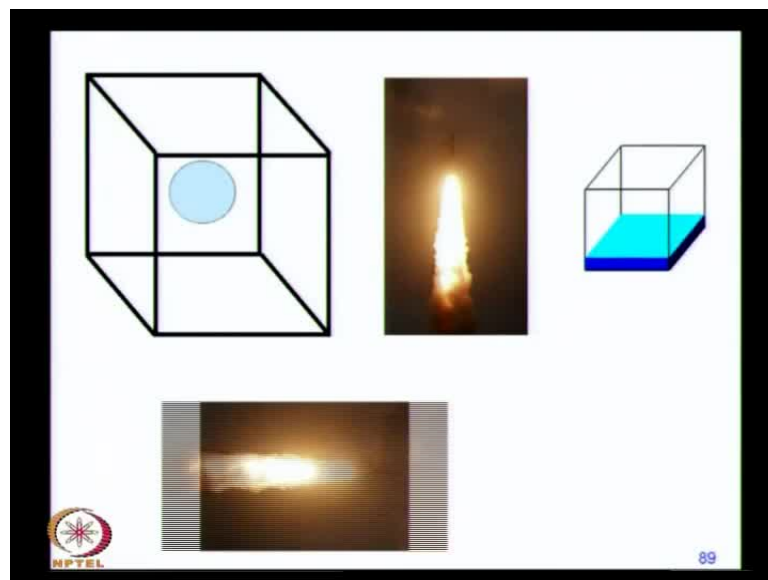
We discussed this example if you remember in the context of how a liquid in a beaker will look if this beaker is kept on a desk in the laboratory, as it is over here, or if you are observing it in a satellite which is in a state of free fall. If we are altogether in a satellite in a state of free fall orbiting the earth, how will this liquid look? We agreed that since everything is in a state of free fall it is not like zero gravity, in the sense that gravity is not present but it is within effective zero gravity state. This is what we discussed, when we discussed the accelerated frames of references.

In the absence of this any effective gravity the liquid will take shape which is not determined by gravity because gravity is missing in the state of free fall. We all agree that it is gravity which is making this water sit at the bottom of bottle and not flying up

within the bottle. In a state of free fall this does not have to happen, so the liquid is inside this bottle will take a shape inside the bottle **and** when the bottle is in a state of free fall depending on whether the cohesive forces of the liquid molecules are stronger or the adhesive forces of - which is the force of attraction between the liquid molecule and the molecules of the beaker - are stronger it could actually stick to the beaker from inside.

If the adhesive forces are stronger leaving a cavity inside and if the cohesive forces are stronger, it could form some sort of a globule just being suspended there, it does not have to sit at the bottom. The reason is, it is sitting at the bottom because of the gravity it experiences over here, but that is not going to happen in the free space - in a state of free fall.

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We consider this liquid in a state of free fall and it is no longer sitting at the bottom, it does not have to, why? It is in a state of free fall and the container itself is falling as freely as the liquid itself. Here the container is not following, the container is held by this table, so it is not in a state of free fall.

This liquid will be suspended somewhere in the middle forming a globule. Now to this beaker you fire rockets from the bottom of the beaker. **Are we all together?** Now, what is going to happen? The rocket is not going to get accelerated, the beaker is getting

accelerated and the liquid will gently come and settle down at the base, just as it would, if it were in a gravitational field, so this is what will happen.

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Gravity - Geometry

The curvature of space-time continuum reproduces the effects that we normally attribute to the gravitational interaction.

The space-time curvature of space-time itself is determined by the presence of matter!

Mass causes the space-time to acquire such a curvature that other matter is attracted toward it... which is what we have referred to as gravitational attraction!

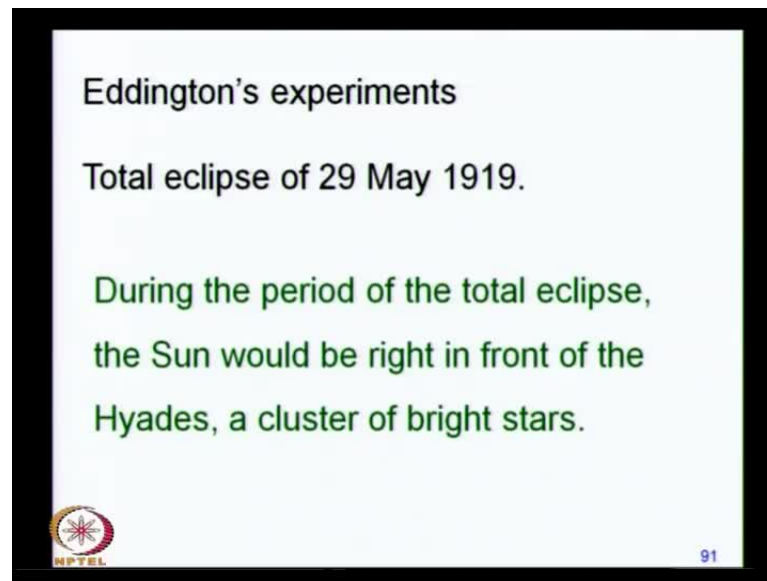
Einstein's General Theory of Relativity
1915 Field Equations of GTR

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On the other hand, if you fire the rockets sideways - why not - instead of firing it like this you fire it like this then, the liquid will settle over here. You start seeing the connections between gravity and accelerations and geometry, you begin to see these connections. These are fundamental to the general theory of relativity; this is what Einstein proposed at the end of 1915 almost 10 years after the special theory of relativity. What he argued is that it is a curvature of space time continuum which reproduces the effect, which we normally attribute to gravitational attraction, because the two situations are completely similar.

Now, the space time curvature itself is determined by what? It is determined by matter. It is the presence of matter which determines the curvature of space and time. Now, all this belongs to the general theory of relativity. I am going not going go in to this details, but these are very fascinating ideas, that, what the presence of a mass does to space time.

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Now, space does not exist by itself, time does not exist by itself what exist is, the 4 dimensional main course key word the space time continuum which has got a certain curvature. What gives the curvature is that it has the presence of matter and this is what generates gravitational attraction. You can see this idea was in fact tested just a few years after Einstein proposed it in 1915, in experiments which were done by Eddington. What this idea suggests is that the presence of mass - the presence of matter - will change the curvature of space time. If the curvature of space time is changed then, it will make the objects swing, so that it goes along a certain path.

If this is to happen then how can you test this prediction that the space time curvature is changed by the presence of a mass. If you want to noticeable affects you really want to look at big events involving big masses, you cannot do it in a laboratory with 1 or 10 kilograms of mass, you need huge parts to do this.

Eddington proposed an experiment that if you look at effects which are generated by the presence of a mass as large as the sun. The sun has got a huge mass and then the mass of sun must influence the space time curvature in the vicinity of the sun and it must make things going past it bend. So, light coming from behind the sun must get bent and we should be able to see this deflection.

Now, you have to look at light rays which are coming from behind the sun, in the vicinity of sun, how do you see that? The sun is so bright, any light coming from anywhere else, coming from some other star which is so much weaker compare to the light that you get from the sun; you cannot really do this experiment easily.

What Eddington proposed is that if you do this experiment during a total eclipse, when the sun is completely eclipsed, so that the light from the sun itself does not reach you. Then, because of the reduced sun intensity see the light which is coming from behind the sun and then ask yourself did you notice any deflection **and if you did**.

If this deflection turns out to be exactly what is predicted by Einstein's general theory of relatively, you can get a quantitative estimate from what are known as Einstein's field equations. Then, you can conclude that Einstein's theory is correct otherwise, how you can get an experimental proof. This was the experiment which was proposed by Eddington, fortunately there was a total eclipse and at the time of the year when the eclipse took place which was in 29 of May 1919.

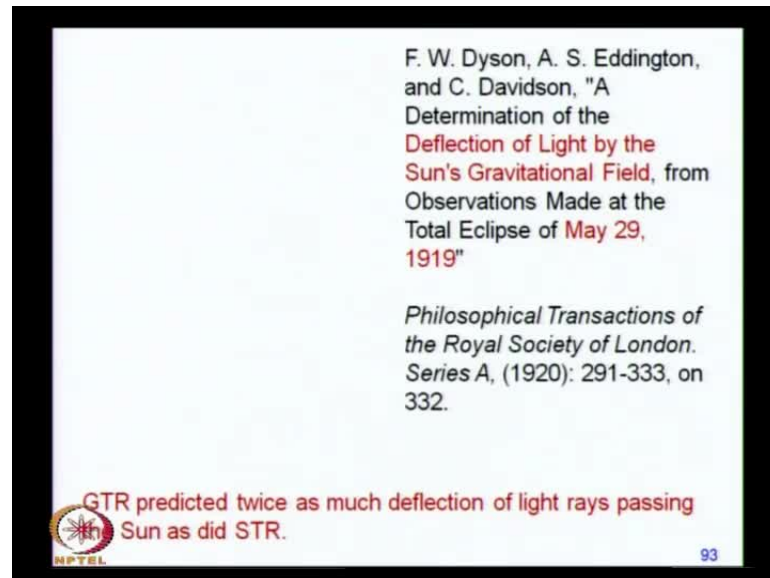
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What would be behind the sun would be light coming from Hyades, which is a cluster of bright stars. If you are familiar with the night sky - this is the picture of the night sky - can you recognize what is what? This part of the sky familiar to you, let me give you a hint, you have got the constellation Orion over here, Sirius star or this is called as a **[FL]**

this is somewhere over here, you have got the Orion do you recognize, are you familiar with the night sky.

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I hope you can at least identify the [FL] and then can you recognize the Taurus, Aldebaran which is the brightest star in Taurus and then, over here you have the theories and this is the region of the [FL] or the Hyades and this is about a 152 light years away. Sun would be in front of this and light from the Hyades would go past sun and reach the earth during the period of that total eclipse. What you would need to do is to measure the deflection of light. This experiment was reported by Dyson, Eddington and Davidson in a paper called a Determination of the deflection of light by sun's gravitational field from observations made at the total eclipse on May 29, 1919 in a paper that was publish in the philosophical transaction of the royal society of London.

While the deflection is to be expected even on the basis of the special theory of relativity - why not - because we have already seen a mass energy equivalence. If electromagnetic energy is going pass the sun, there is a certain mass associated with it even according to the special theory relativity, but the deflection predicted by the special theory of relativity is not the same as the deflection predicted by the general theory of relativity.

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The general theory of relativity predicted the deflection which is almost twice as much as the deflection predicted by the special theory of relativity. So, Eddington recognize the fact that this difference will be measurable and it turns out to be twice as much as is predicted by the special theory of relativity, then Einstein's general theory of relativity must be correct. So, that was the hope with which this experiment was carried out by Arthur Eddington. This is the path of the total solar eclipse or that day which is in May 1919, it went from this place in South America and it went pass this town of Sobral in Brazil, which is where the experiment was carried out. The path of the solar eclipse, the totality went even across the Atlantic Ocean, so a second experiment was carried out on a ship in the Atlantic.

What you will expect that light from this place if it is coming like to this to the earth, it would get bent and come along this line from the earth it would appear as if it has not come from here but from here, so that would be the apparent position of the source. If that apparent position of the source is not the same that you are aware of from your measurements on other days outside the eclipse, then from comparison you can tell if the deflection did take place and was that deflection twice as much as is predicted by the special theory of relativity.

The story of this experiment is a very fascinating one, I will not go through the history of that, but I hope that some of you will read it and enjoy it. Eddington carried out this

experiment and the conclusions were positive. The general theory of relativity was verified, it is in fact a marvelous experiment that was carried out, this could be done because the eclipse was reasonably long lived, it was almost like 7 minutes which is almost the same interval that we had recently in January.

Jobin when we had in a Trivandrum - we had a totality and partial eclipse in Chennai and in Trivandrum there was a totality for almost 7 minutes. I do not know if this experiment was repeated during the recent total eclipse, if any laboratory did it. I will like to hope that somebody did it and if nobody did, I think we missed a great opportunity because, totality does not last too long, in this case it lasted like 7 minutes. The Trivandrum eclipse earlier this year did last for like 7 minutes or there about and the general theory of relativity was established.

I also mentioned that there are other consequences of the special theory of relativity not just length contraction and time dilation there are other things. Again I will not go into details, I will assume that you have heard about atomic wave functions, you have heard of quantum mechanics; you know that electron states are described by spin orbitals.

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SPIN-ORBITALS

$$u_i(q_j) = u_{n_i, l_i, m_{l_i}}(\vec{r}_j) \chi_{m_{s_i}}(\zeta_j)$$

$\vec{s} \times \vec{s} = i\hbar \vec{s}; \quad [s_x, s_y] = i\hbar s_z$
 $s^2 |s, m_s\rangle = \hbar^2 s(s+1) |s, m_s\rangle$
 $s_z |s, m_s\rangle = \hbar m_s |s, m_s\rangle$

$s = \frac{1}{2}$: fixed internal property of an electron
 $m_s = (-s, \dots, s) = -\frac{1}{2}, +\frac{1}{2}$

1928: Dirac STR+QM
 Relativistic Quantum Mechanics
 Provided formal basis for electron's spin

HPTEL

We talk about the spin and the orbital but in quantum mechanics there is really nothing like an orbit and there is nothing like a spin, so these models are wrong, these are misleading. Spin is in fact an intrinsic angle of momentum which is defined by certain

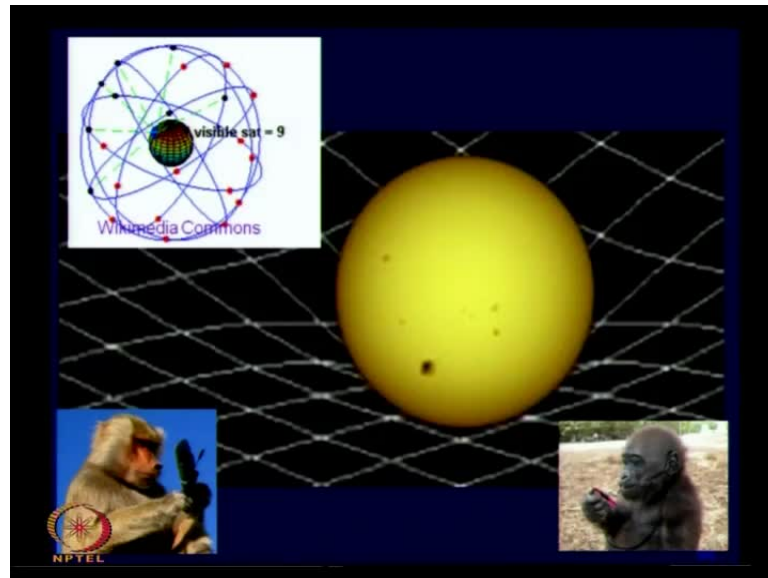
commutation properties and of course, I am not going to get into quantum mechanics to discuss this.

What I want to point out is that there is an intrinsic angle of momentum which elementary particles have and these particles then turn out to be either fermions or bosons. The origin of this electron spin comes from quantum mechanics but not from Schrodinger's quantum mechanics. Schrodinger's quantum mechanics is not Lorentz invariant, it makes use of space and time and so on, it employs potential in which as a function of distance and the position that goes into the Schrodinger equation is the Euclidian distance - the non-relativistic distance, so that is really not relativistic. This is not going to be the discussion on formulation of relativistic quantum mechanics which is the big subject by it needs a full course by itself.

When you put the special theory of relativity and quantum mechanics together you get the Dirac equation for the electron and what comes out of it neatly from the Dirac equation and this is the electron spin. So, electron spin has got no place in non-relativistic mechanics except in an ad hoc manner. You can always make an ad hoc assumption but from the point of view of relativistic quantum mechanics it comes out nicely as a natural consequence.

So, the electron spin is a natural consequence not just of quantum mechanics, but also of the special theory of relativity. It does not come from quantum mechanics alone, it does not come from the special theory of relativity alone either, but it comes from the special theory of relativity without which it has got no place. Amongst the several consequences which come as the package of the special theory of relativity, you must include not only length contraction and time dilation but also quantum properties like electron spin and so on. It should now become clear as to why physicists, chemists, material scientists and engineers everybody must be somewhat familiar with the special theory of relativity.

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This is how a mass affects the curvature of the space time continuum around it. This is suggestive, we are not going to get into the Einstein field equations or anything and do this in any detail but you get an idea.

What you see is that the presence of matter changes the curvature of space and continuum around it and then, if an object - another mass - which is in the vicinity then it will go along this path and get attracted towards this mass. Now you understand gravity, why two masses attract each other, because they find themselves in a space time continuum with such a curvature which encourages their acceleration towards each other. So, what was only postulated by Newton gets explained in this mechanism through the curvature of the space time and what imparts this curvature to the space time is the presence of matter - is the presence of mass - so all these are consequences of the general theory of relativity.

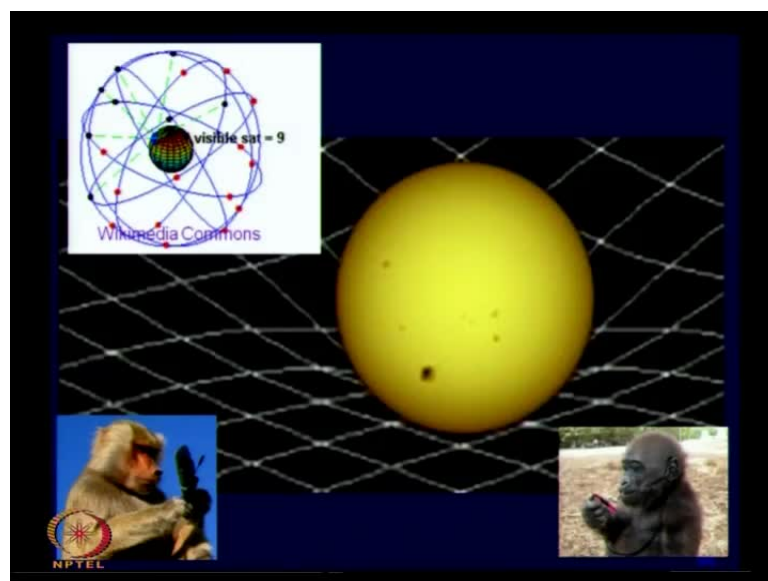
We talked about monkeys talking to each other on the cell phones, every monkey does that and I pointed out that this has something to do with the relativity theory. It has to do not just with the special theory of relativity but also with the general theory of relativity, because the cell phones work because of this global positional system. You have got these 24 odd satellites which are communicating with the receivers and these receivers are in your pockets. They are in the cell phones and any GPS instrument, any navigation instrument, it could be a GPS map in your car or whatever, it tells you how to find your

way when you are driving on a highway and you do not know how to get to your destination.

These satellites will tell you they will navigate you through this and all this works because the information sent by the satellite is correctly received by the receiver in your pocket. It should get to your pocket pass through a certain distance over a certain amount of time and if this distance and time is not calculated correctly there will be errors. Distance and time are not absolute quantities they must be determined with reference to the state of motion of the receiver, the satellite and everything else, so to do this correctly you must employ special theory of relativity.

Otherwise, the errors are quite large, the message that you get on your cell phone when your friend calls you, it lands on your cell phone, could easily land if these errors accumulate that if you did not include relativistic effects they could land like 10 kilometers away. Your girlfriend is calling you and the message goes into your father's phone that is terrible. So, these things need to be calculated correctly and you must include the special theory of relativity, so that time and distance is correctly estimated. Not only that the receivers are in your pockets, they are on the surface of the earth, the satellites are like 20000 kilometers away, so look at the curvature or space time.

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This curvature is bent most closely to the mass and least as you go further away. This is not due to special theory of relativity this is due to the general theory of relativity. 20000 kilometers is a distance that is large enough for this factor to influence the time intervals and both special theory of relativity and general theory of relativity must be properly accounted for otherwise, the errors are quite large. I believe they turn out to be something like 7 microseconds per day, if you ignore special theory of relativity and something like 45 microseconds per day if you ignore general theory of relativity, but in the opposite direction. So, the net error turns out to be like 37 microseconds per day or something like I forget the numbers.

It is for this reason that a comfortable acquaintance with the special theory of relativity at least is what I strongly encourage is, at the undergraduate level and I hope that we at least have some sort of an introduction to it.

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Is
Newtonian / Lagrangian / Hamiltonian Mechanics
Wrong?

Is Galilean Relativity Wrong?

$$\frac{v}{c} \ll 1; \quad v \rightarrow 0; \quad \hbar \rightarrow 0$$

NPTEL 97

Finally, we must ask ourselves we have used Newtonian mechanics, Lagrangian mechanics, Hamiltonian mechanics, Galilean relativity with good amount of success and now we are saying that space does not have an identity of itself, which is disjoint from time. So, it is not that absurd, yes it is wrong, because it takes only a part of the story, it is not totally absurd because the speed of light is not infinite but it is very large.

So, in the limit that v over c goes when v over c is much smaller than one, when v goes to 0 for example, it works out. Of course, Newtonian and Lagrangian mechanics has to be corrected not only for relativity but also for quantum mechanics. The reason classical mechanics works is also because the Planck's constant of action what I have written here is h over 2π , this is also a small number and in the limit that h cross goes to 0, you will get classical mechanics in the limiting case not always. Again, these are very certain issues I will go into that at this point.

By and large the v tending to 0, the h tending to 0 is the limit in which Galilean relativity works, but with the special theory of relativity we are able to include in our range of applications also the electrodynamic phenomena which we have already discovered must have a relativistic basis.

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We conclude the unit 6 with a quote from Albert Einstein:

If at first the idea is not absurd, then there is no hope for it
- Albert Einstein

-No guarantee that there is hope for every absurd idea!
- Our experience !!!

Next L23 : Unit 7
Potentials, Gradients, Fields
pcd@physics.iitm.ac.in
http://www.physics.iitm.ac.in/~labs/amp/

Bye!

NPTEL 98

So, with that we conclude this discussion I will just point out that Einstein's contributions go well beyond relativity. Now, I mentioned his contribution to the Brownian motion to the interpretation of the photoelectric effect, it just goes on and on. Then, together with Satyendra Nath Bose, the Bose Einstein statistics which is a very exciting subject especially in the context of laser cooling and Bose Einstein condensation and other things that you know the scientists are working with.

The Einstein has made huge contributions to physics in very many disciplines and we will conclude this discussion on unit 6 with one quote from Albert Einstein that "if an idea is not absurd, if it does not appear to be absurd at first then, there is no hope for it" because you see that this really looks like an absurd idea or the special theory of relativity that the speed of light is the same in every inertial frame of reference. How can that be true means, we talked about intuition then, counter intuition then, educated intuition and then, we came to grips with all that is fine, but when this idea was introduced it was absurd, it looked absurd but our experience is there is no guarantee that there is hope for every absurd idea.

So, thank you very much if there are some questions I will be happy to take, otherwise good bye. Then we will have the next unit which is unit number 7 in which we will deal with potentials fields and gradients, so that will be for the next unit if there is any question I will be happy to take.

Let me just say one thing over here that I have met students who feel shocked when they see that one of the siblings has aged more than the other. Now, this shock is not called for because this shock comes after the student has accepted that the speed of light is the same in all inertial frames of references and the idea of time dilation is accepted.

So, please do not die twice of the same shock, once you accept that the speed of light is the same in all inertial frames of reference then, any consequence which is a natural outcome of this consequence which comes simply by putting numbers in this square root of $1 - \beta^2$ should not shock you. If it shocks you again it will be dying twice by the same shock, so do not let that happen. So, good luck and we will meet for the unit 7.