Select / Special Topics in Classical Mechanics

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Module No. # 07

Lecture No. # 24

Potentials Gradients Fields (ii)

Greetings. We will continue our discussion on the Directional Derivative which we have recognized to be a scalar, having however a directional property. So, we are not going to worry ourselves by asking how come a scalar quantity has a directional property.

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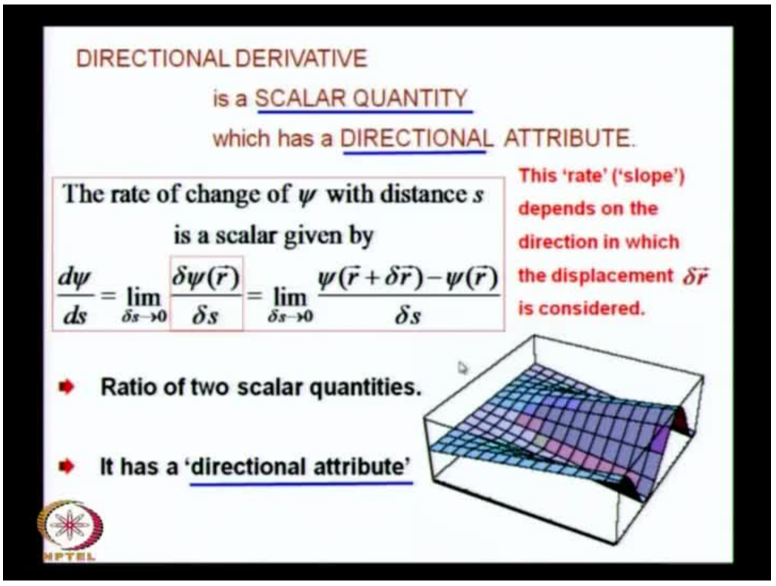

We will not worry about it because we never defined the scalar as a quantity which is defined by magnitude along. We defined it as a quantity which remains in variant with respect to transformation, rotations of a coordinate system, and its values does not change; no matter how you orient the coordinate system. This is our defining criterion of a scalar. So, the directional derivative is a scalar with a directional attribute. We are choosing our words carefully; we do not say it has got a direction; what we say is that it got a directional attribute, property, quality which involves the sense of direction and the reason it has this directional attribute is coming from this argument rather than from the function itself.

The function psi is the scalar, but when you consider delta psi as the difference in the value of this function between 2 points - value of the function at r and value of the function at a neighboring point r plus delta r, then the question is - where is this neighbor? With reference to the first point, this neighbor can be either to the east or to the west. And it is over here, in this argument, that the direction plays a role. So, the directional attribute comes because this delta r displacement that you see over here, this is a vector. It has got a direction and it is this which provides the directional attribute, the directional quality to the directional derivative.

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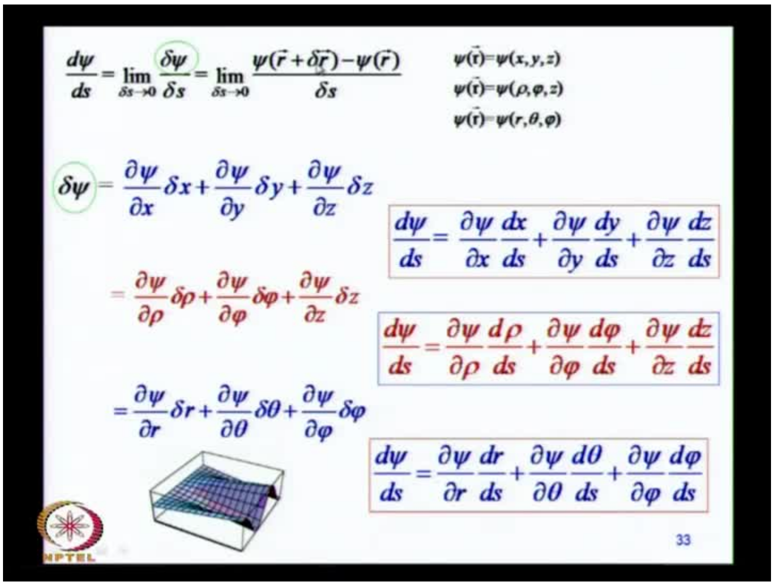

It is the argument of the function, not the function itself; the function itself is a scalar because the direction of this vector delta r is important and we need to consider our formulation regardless of the choice of the coordinate system whether it is Cartesian or polar, cylindrical polar, or spherical polar, no matter what.

We got expressions for the changes in delta psi to neighboring points which are separated comprehensively by delta x delta y and delta z in Cartesian geometry which give the directional derivative to be determined by the rate of change of the function psi with respect to each of these degrees of freedom x, y and z. So, there is a del psi by del x, del psi by del y, del psi by z, and each of these rates is to be scaled by these scaling factors which are dx by ds, because in that particular direction, how much has an exchange though. So, that rate will also come in.

So, let us not to look at it just as mathematics; let us look at it as physics because we are looking at properties of function at a given point in space and how these properties change from one point to the next, and obviously what is important is how you go from one point to the other; What is at the changes when you go from one point to the other. So, if x and y and z are changing, you must take that into consideration. Then rate of visual quantity changes with respect to x, y and z. If what is changing is rho, phi, and z, or r theta and phi, then you must consider the rate at which how these functions change with these independent degrees of freedom.

So, here, in the plane polar coordinate system, you have got rate of change of psi with respect to rho multiplied by change in rho itself which is delta rho, and likewise, similar expressions from the other two independent degrees of freedom. So, the directional derivative of the cylindrical polar coordinates will be - you have to divide each of these quantities by delta s and take the limit delta is going to 0; so, delta rho by delta s in the limit delta is going to 0 gives you this d rho by ds. So, that is how you get the first term del psi by del rho times d rho by ds plus del psi by del phi times del phi by ds. So, you really do not have to look at this as just a problem in calculus; it is a problem in Physics.

And Mathematics provides a rigorous expression, a quantitative expression which comes naturally. And it comes naturally from our understanding of the physical reality that here you are talking about a physical property namely the function psi which could be temperature or anything and how it depends on various points. So, in the spherical polar coordinate system, you have expression for delta psi, two neighboring points, and the directional derivative of psi in the spherical polar coordinate system, depending on the direction in which the displacement delta r, this displacement is considered.

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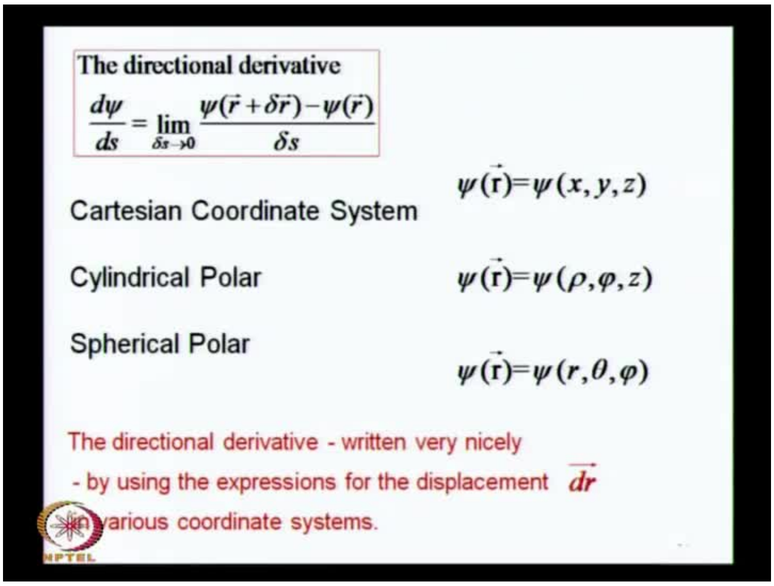

So, we have to develop now rigorous expressions for the directional derivative in another form because to get the value of the directional derivative, either you have to work with this calculus or you can also use vector algebra because you do have directional attributes. So, obviously methods of vector mechanics, vector calculus will be very effective; victor algebra and vector calculus.

So, let us see how to work it out. And in fact, we can work it out very easily by expressing this displacement vector dr in different coordinates systems. Once we do that and this is very easy. I want to remind you again and again that you should not try to by heart these expressions, but develop them from first principles. It is very easy because you know how to express the position vector in any coordinate system, whether Cartesian or cylindrical polar or spherical polar. And in any of these coordinate systems if you have developed the expression for the displacement vector, you can automatically get the expression for the directional derivative by using vector algebra and vector calculus. And it is in the use of vector calculus that the vector derivative operators will come in, which is our gradient operator which is what I am now about to introduce.

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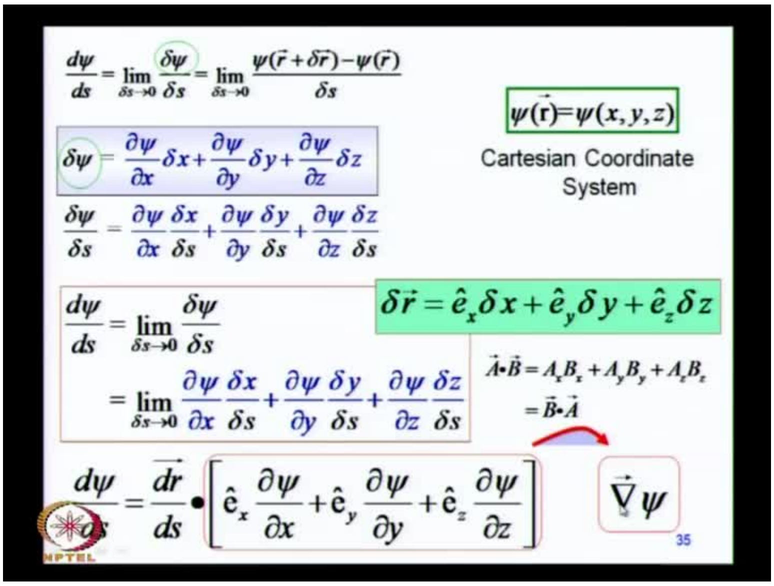

So, let us consider the Cartesian coordinate system. This is the most familiar one for most of you, and in this Cartesian coordinates system, the change delta psi is del psi by del x times del x; similar contribution from the y and z dependents; dependents of psi on y and z because psi depends not only on x, but also on y and z. You divide both quantities by delta s, take the limit delta s going to 0, and you get this particular expression, and this is something that our belief should ring a bell because the form of this expression is something that you have met in vector algebra.

Does it not remind you of this relation that if you take the dot product of the scalar product of two vectors, it is A x B x plus A y B y plus A z B z, and scalar product being commutative, A dot B is the same as B dot A. So, you can write it either as A x B x plus A y B y plus A z B z or B x A x plus B y A y plus B z A z.

Now, the form of this expression here, c does it not remind you of… can you not think of this as A x and this as B x? So, this looks like A x B x plus A y times B y plus A z times B z and it really does not matter what you call as A, what you call as B because A dot B is the same as B dot A, so far, as the scalar product is concerned. So, this form would remind you of the scalar product between two vectors and you can express this directional derivative as a scalar product of two vectors.

The advantage is that in this box, we use only calculus, derivatives, whereas in this box we use vector algebra. So, these are two different mathematical disciplines: one is vector algebra, the other is calculus - deferential calculus, and if you can use the two together to our benefit, we will find that we get a tremendous handle on analyzing the relationships between potentials and fields.

So, let us see how it is done. What you need to do is to identify one of these vectors. You know the combination of this, this, and this (refer Slide Time: 11:04) as components of another vector and the first part del psi by del x and del psi by del y and del psi by del z as components of the another vector. And then, if you look at these as a sum like a x B x plus a y B y plus a z B z, then you can express it as a scalar product of two vectors.

Now, it is not at all difficult to interpret this delta x by delta s as a component of a vector because the components delta x delta y and delta z in the numerator are just components of the displacement vector. So, if you divide the displacement vector by delta s, you get this component over here. So, it comes straight from expression for the displacement vector and you can write the displacement vector delta r in Cartesian coordinate system as we have done, but you can do so in any coordinate system.

So, now, we can see that this directional derivative which is a some of the 3 terms which we interpret as an expansion of the scalar product in terms of its components, in the product of its components, products of the Cartesian components. In the first lecture, we considered it to be this dr by ds because delta r by delta s will give you these components delta x by delta s, and then delta y by delta s, then delta z by delta s.

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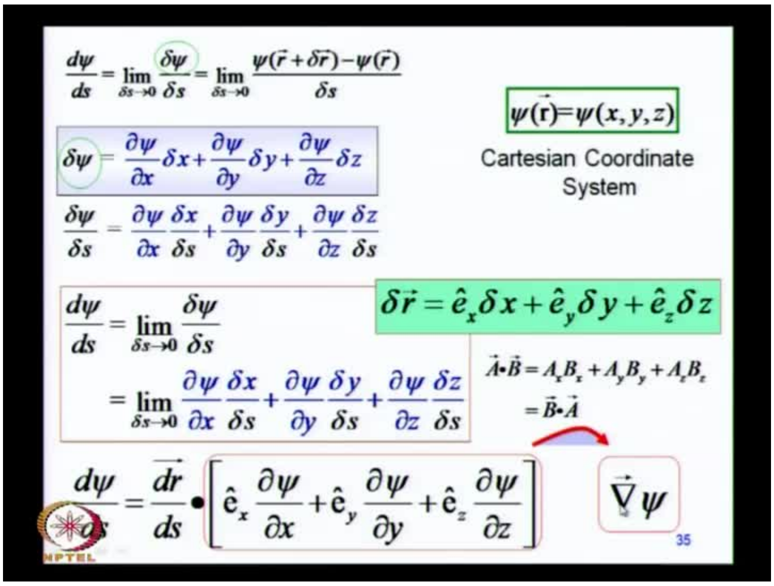

The other component must be - del x by del psi should be the x component, del psi by del y must be the y component, and del psi by del z must be the z component because then if you multiply the corresponding components, you get the dot product A dot B. It is very straight forward now.

So, this quantity that you have in this red box over here this is what we call as the gradient of psi. This is the definition of the gradient of psi. What you have is a Cartesian expression for the gradient of psi, but this is also generating for us a definition of the gradient of psi which we can use independent of any coordinate system and we can use the definition in other coordinate systems as well.

So, let us first see how it works in Cartesian coordinate system. So, let me quickly remind you that you have the directional derivative which you express as a sum of three terms and you can look at the sum of these three terms as the sum of products of components of two vectors generating a scalar product; that is the basic trick. So, you recognize this as a scalar product of two vectors and term by term identify the two vectors A dot B, which will generate the sum of these three terms; quite easy to do.

And this quantity in the red box is what you define as the gradient of psi written as this del with an arrow on the top. The reason to put this arrow on the top is that it must have the vectorial attribute; that does not make it a vector; It is a vector operator. A vector operator is not the same as a vector. A vector is defined by how its components transform when you rotate a coordinate system. A vector operator will seek an operand. It will carry out that operation and at the end of it, you will get a net result whose properties you must then investigate.

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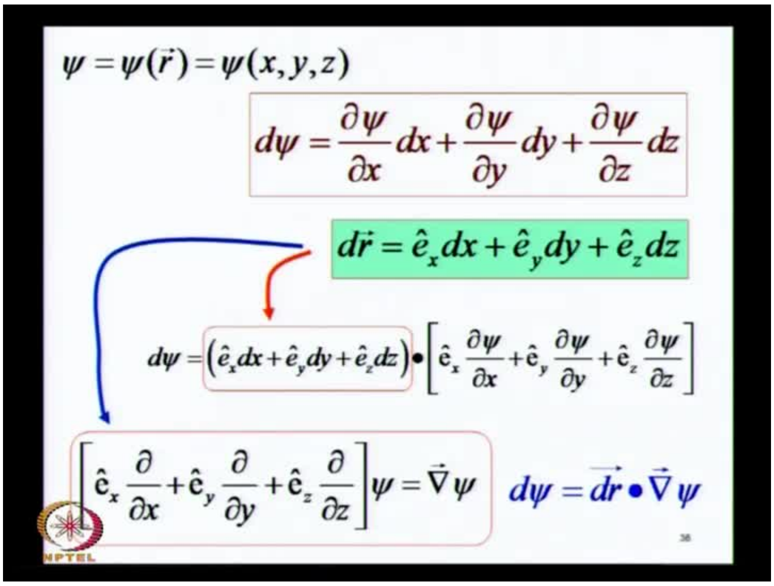

So, these are the main ideas that for any scalar function, you define the increment in that function which is d psi. This increment is because of the rate of change of this function with respect to the independent degrees of freedom times the change in the independent degrees of freedom, for all the three independent degrees of freedom.

And then, you express this displacement vector, this change. Why is this change taking place? Because you are comparing the value of this function between two neighboring points which are connected to each other through displacement vector which is dr, and this recognition of the displacement vector dr is what immediately provides you with the Cartesian expression for the gradient which is in this box.

It is the recognition of this displacement vector as e x dx plus e y dy plus e z dz which is what generates and interpretation of this vector as the gradient of psi. So, this is fundamental. How you write dr in a given coordinate system and we can use dr in any coordinate system and then follow the same trick because now we have got a technique with us. So, the expression of the displacement vector is fundamental to this process and this expression gives us the net result that d psi, the differential increment in psi because of a change in the points and the consideration, when you go from r to r plus dr through the displacement vector dr is then given by this scalar product dr dot del psi.

Now, look at this equation at the bottom right of the screen. This equation which is in a lovely blue color, the royal blue as it is I think; d psi is equal to dr dot grad psi. Now, this is the scalar. What does it mean? It is in variant with respect to changes in the orientation of the coordinate system and its interpretation cannot depend on the choice of the coordinate system. It cannot depend on the Cartesian coordinate system.

We have developed this expression in the Cartesian coordinate system, but the result d psi equal to dr dot del psi must be independent of the coordinate system. So, this result must hold good in every coordinate system, and if you just remember this, you are not going to have to by heart how to write the expression of gradient in spherical polar coordinate system.

I do not want you to remember that. What I need you to remember is that this expression for d psi is independent of any coordinate system, and therefore, you can determine what grad psi must be in any coordinate system so that dr dot del psi turns out to be d psi. So, we will actually work it out and you will see when it comes out very neatly.

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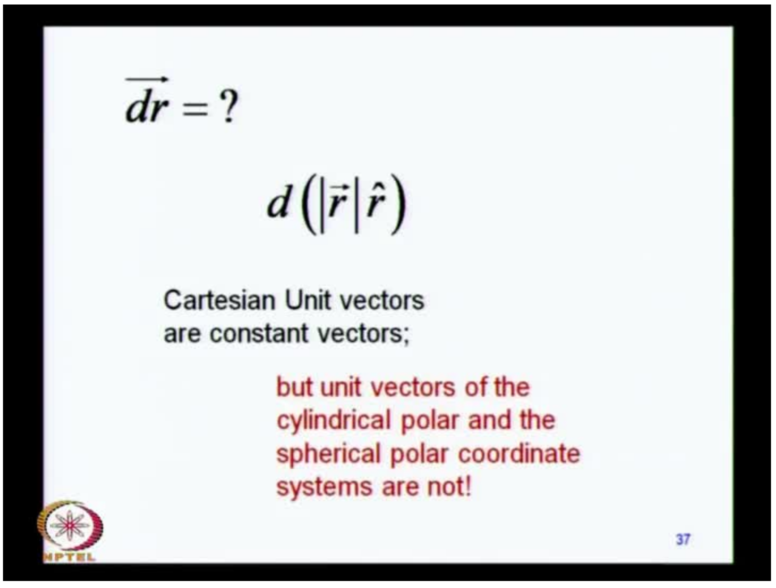

So, we are going to have to write the displacement vector in every coordinate system. We have already done it in the Cartesian coordinate system, but mind you, when you consider this differential increment in the position vector, it is a differential increment in the position vector which has made up its magnitude, which is sitting in this modulus of r and the direction indicted by the unit vector, which I indicate by this caret which looks like a hat.

So, this is a unit vector. This caret is our symbol for unit vector. So, the position vector itself has got two components: one is a magnitude, the other is a direction. And the changes, which is this differential increment may come either because of change in magnitude or else because of changes in the direction, or because of both.

So, whenever you consider the change in a quantity which is made up of a product of quantities, you must consider the differential increments in each one of them keeping the other constant, and then sum them up. It is like taking the deferential of a product of two functions. So, differential of two functions f and g is f times dg plus g times df. So, it will be some of those two and you may therefore, consider changes not only in the magnitude of r, but also in the direction of the unit vector.

Now, what is important is that when you consider the unit vectors in cylindrical polar coordinates or the spherical polar coordinates, they are not constant vectors. This is something that we have done in some details in unit 3, and therefore, changes in these unit vectors must be satisfactorily taken into account.

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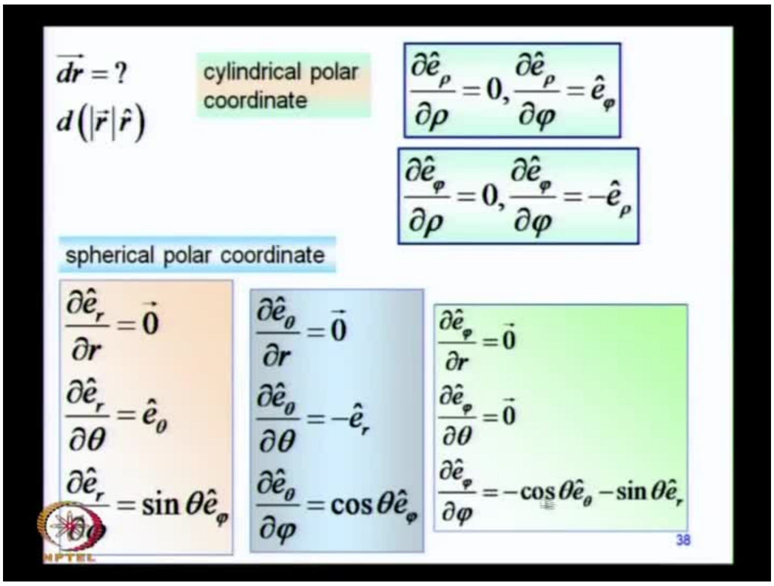

So, let us see how to do that. In cylindrical polar coordinates, the unit vectors e rho does not change with respect to the polar distance rho, but it does change with azimuthal angle phi, and this is the rate at which it changes. Likewise, the other unit vector which is e f, it does not change with a polar distance. So, del e rho by del e phi by del rho goes to 0, but if I does change with respect to the azimuthal angle phi, and the change in the unit vector is always orthogonal to it, so del e rho by del phi will be along e phi and del e phi by del phi will be along e rho, and we have obtained these relations in some details already.

In the spherical polar coordinate system, if you consider the dependence of the unit vector e r along r theta and phi, then with respect to the polar distance, there is no variation. With respect to the polar angle, the variation will be orthogonal to e r; so it is e theta. And the variation with respect to the azimuthal angle again is orthogonal to e r; it is along e phi and the component is given by the sin of the angle theta - sin of the polar angle.

Likewise, the rate of change of the unit vector e theta in the spherical polar coordinate system - with respect r it is 0; with respect to theta it is minus e r. So, always nice to remember that the changes are always orthogonal to the unit vectors which is why it does not change at all. If it has got a component, how could it be a change? So, the change in e theta with respect to theta is orthogonal to theta; so, this is along e r. Now, that is a necessary information. It is not sufficient because you also have to find what the sine is and not just what the sine is; also, what the scaling factor is.

So, in this case, there is scaling through cosine theta and then there is a change of the unit vector e phi with respect to r theta and phi, and the change in the azimuthal unit vector e phi with respect to phi has got components which are orthogonal to this, but these are components both along e theta and e r both are orthogonal to e phi, but it is in the plane which is orthogonal to e phi, and then they are scaled respectively by minus cos cosine theta and minus sine theta.

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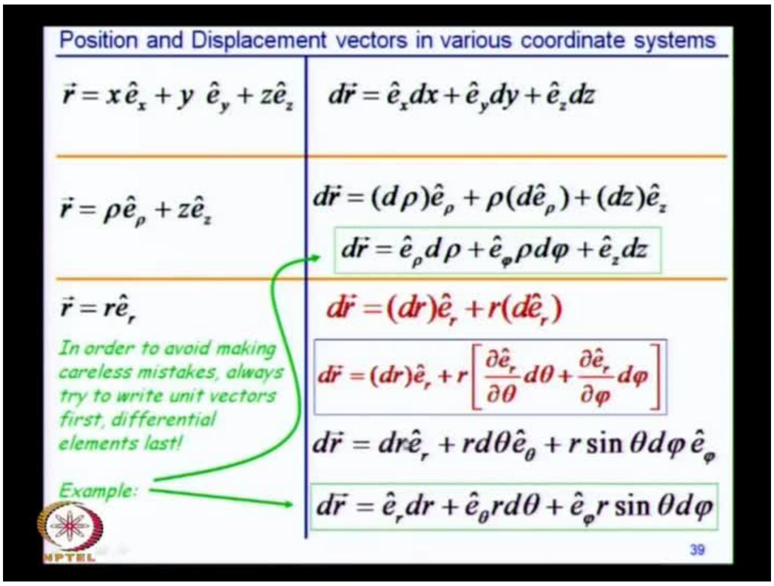

So, with these, you can write the expression for d r in any coordinate system. So, in the Cartesian, you have got a position vector which is xe x plus ye y plus ze z and dr will be e x dx because e x does not change in Cartesian coordinate system. The Cartesian unit vectors are constants, but in plane polar coordinate system, the position vector is a sum of this rho e rho plus the z component. So, this will give you the component of the position vector in the plane x y, but then there is component along the z axis in 3 dimensional space. And therefore, the changes in r which has this displacement vector will come from changes in rho times e rho which is d rho times e rho plus changes in e rho itself. So, it is rho times d e rho and e rho is not a constant vector.

So, this quantity has to be determined, but we just did it in the previous slide and then there is a component in the component to the displacement which will dz times e z, but no component coming from the changes in e z which is the constant vector in the cylindrical polar coordinate system.

So, now we write this. So, e rho times d rho is here. Notice that this is the first term. I have written just as it is with the difference that, I have written the unit vector first and this increment after it. That is the only difference. Here, again this de rho, I get it from the rate of change of e rho with respect to phi; it does not change with respect to rho, and with respect to phi, it will be d phi e phi. So, it is this d phi e phi; this rho (Refer Slide Time: 26:13) comes over here. So, this component is e phi rho d phi and this is the third component here again I have written unit vector first and the increment later. So, this is the expression for the displacement vector in the cylindrical polar coordinates.

And now, you see that, you really do not have to by heart it because you can just get it term by term from first principles. What about the spherical polar coordinates? Here you must take into account which changes in the unit vector e r and these changes will be because of change in e r due to change in theta, and also because of change in e r because of change in phi.

So, it will be a combination of del e r by theta times d theta plus del e r by del phi times d phi because the unit vector can change when you go from one point to another, either because of polar coordinate is changing or because the azimuthal angle is changing or because of both. So, you put everything in it. It does not change just when the polar distance changes because this unit vector changes with respect to it; It does not change with respect to r; the partial derivative of the unit vector e r with respect to r vanishes.

So, now, you have got all of these terms. These partial derivatives you have determined. We just discuss them in previous slide. So, you are plugging in those values and you get this r d theta times e theta coming from this term, and here you have got a scaling factor of r sine thetad phi, and then this has got a component along e phi. And now, again, we will write the unit vectors first and then the scaling factors.

The reason I prefer to do it is because whenever you have a differential operator, means look at this expression over here - dre r. Now, I do have a bracket here, but I have not written a bracket here (Refer Slide Time: 28:19), and if I were to read this expression incorrectly, I could think of this as a differential of r times e r which is not what I really intend to do. So, in the final expressions, I always prefer that the unit vector, as I have written first, because now the differential operator is going to operate only on r, and e r does not appear to the right of this differential operator any more at all. So, it is a safe practice to always write the unit vectors first, which is what I have been doing. So, I strongly recommend that.

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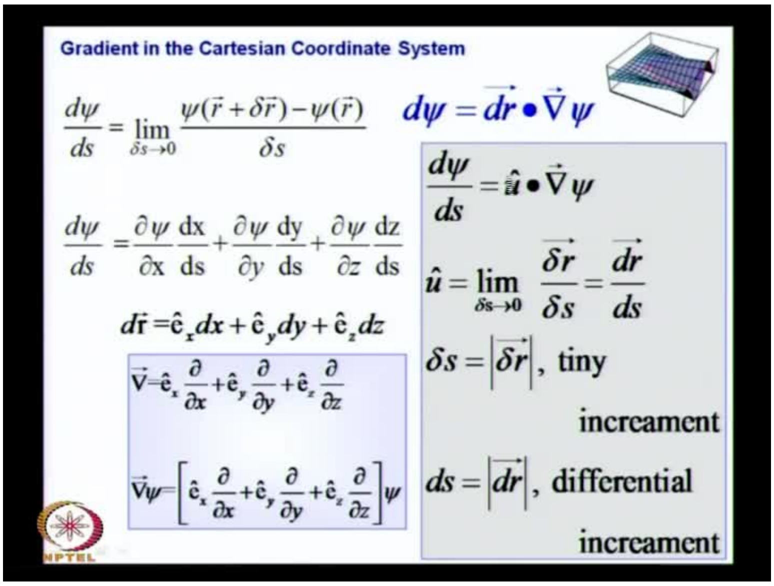

Not that it matters, but then you have keep track of where the brackets are; one can do it, no big deal, but many of us have various levels of proficiency in making careless mistakes; so, this is when we are avoiding some of them. So, this is the expression for the gradient in the Cartesian coordinate system. The result is independent of the coordinate system, but what leads us to the result in the Cartesian coordinate system is the correct interpretation of dr in the Cartesian coordinate system.

So, when we recognized this sum, when we interpreted this sum in terms of a scalar product A dot B, with these coming as components of the displacement vector and these coming as components of the gradient vector (Refer Slide Time: 30:00), we were let to this expression for the gradient operator. This identity of the gradient did not come from any process which required us to by heart what is gradient in the Cartesian coordinate system. It automatically popped out; it popped out from our recognition of d psi as dr dot del psi, and dr has got an obvious expression in Cartesian coordinate system which is this (Refer Slide Time: 30:44), and the moment you do this and you recognize this as a sum coming from A dot B as A x B x plus A y B y plus A z B z, the expression for gradient which you see in this box automatically jumps out.

So, you do not have to remember that the gradient is e x times del psi by del x plus e y times del psi by del y; that is correct, but you do not have to remember it. What you do have to remember is that the gradient of a vector is such a vector that its scalar product with a displacement vector will give you the change in the value of the function when you consider a change in the particular direction of the displacement vector dr; that is where the physics lies. Rest of it is elementary vector algebra which automatically comes out. So, this actually gives us a definition of a gradient vector.

If you divide both of these quantities by delta s and take the limit delta is going to 0, dr by ds in the limit delta s going to 0 will give you a unit vector in the direction of the displacement vector. And delta psi by delta s in the limit delta psi going to 0 gives you the directional derivative d psi by ds. Now, this gives us a definition of the gradient. The gradient of a scalar function is such a vector whose component in any direction gives you the directional derivative in that direction and any vector can be defined in terms of its component. So, if you get these components along three mutually orthogonal directions, you get the complete vector itself.

In other words, you can get the directional derivative d psi by ds either by doing calculus; this is calculus - del psi by del x times dx by ds plus del psi by del y times dy by ds plus del psi by dz times dz by ds, or by doing vector algebra by composing this scalar product of two vectors; you can use either calculus or vector algebra. Now, this is not just vector algebra. This is more than vector algebra because the gradient operator contains the derivative operators as well.

Here, you must take the derivative with respect to x; so, it is more than vector algebra; it is vector calculus. So, combination of vector algebra and vector calculus provides you an alternative method of determining the directional derivative, which you can otherwise get from pure calculus alone. But when you use the two techniques in conjunction, you get a very powerful handle on dealing with these ideas.

So, this is now the definition of a gradient that it is such a vector operator, that when it operates on scalar point function psi giving you grad psi, its components along any direction give you the directional derivative in the direction in which the component is measured; this is physics.

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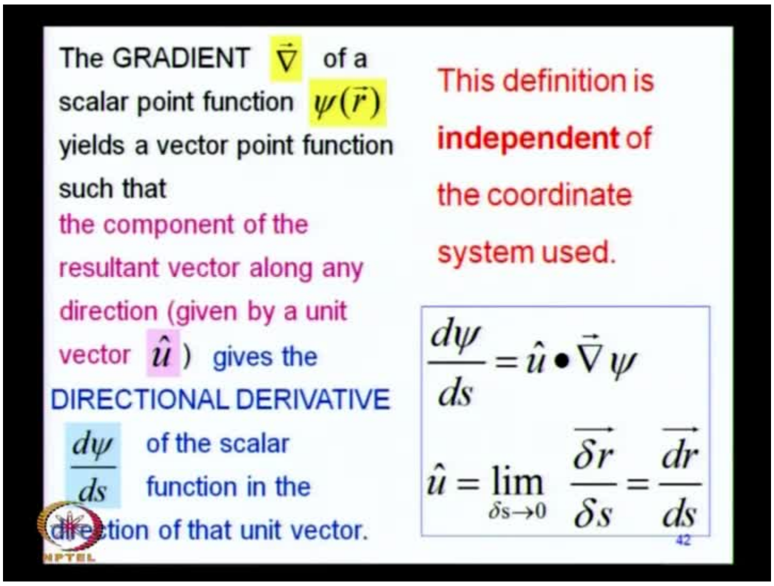

What is at the heart of this is the idea of a directional derivative. This definition is completely independent of the coordinate system. It keeps tracks of the direction attribute. It is coming from this unit vector u because it will be different in different directions, and the unit vector u is what will keep track of the fact that you are dealing with a quantity which is a scalar which however has a directional property.

So, this is our complete definition of gradient of a scalar point function. In this definition, what you see on the screen x, y, z do not appear anywhere, and the definition and the interpretation will be completely independent of the coordinate system. In any coordinate system, this definition must hold.

What will change from one coordinate system to the other is how delta r is expressed in different coordinate systems; the displacement vector dr will have different expressions in different coordinate systems. And because this dr has got a different expression in different coordinate systems, you will have different expressions at different coordinate systems, which will require you to have different expressions for a gradient of psi in different coordinate systems. But the definition the different expressions must be such that the scalar product must give you the left hand side which is independent of any coordinate system. The scalar product itself must be independent of the coordinate system; that is a scalar and that is an Isaac the independence with respect to the coordinate system which is employed.

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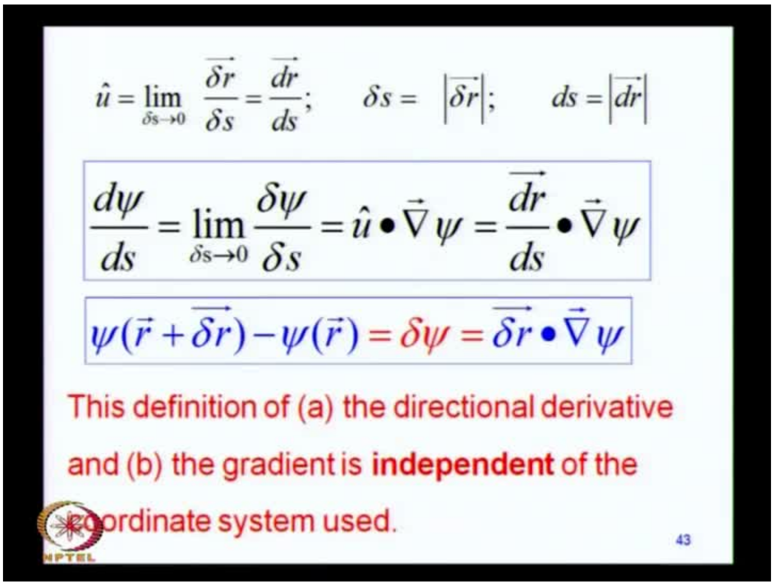

So, let us now proceed to get these expressions for delta r in different coordinate systems which we have just written, and use them to discover for ourselves not out of any memory device, but from this recognition of the change - delta psi as delta r dot grad psi or del psi grad psi. Grad operator is the same as the del operator; this is just a matter of terminology, and in cylindrical polar coordinate, let us try to get this first.

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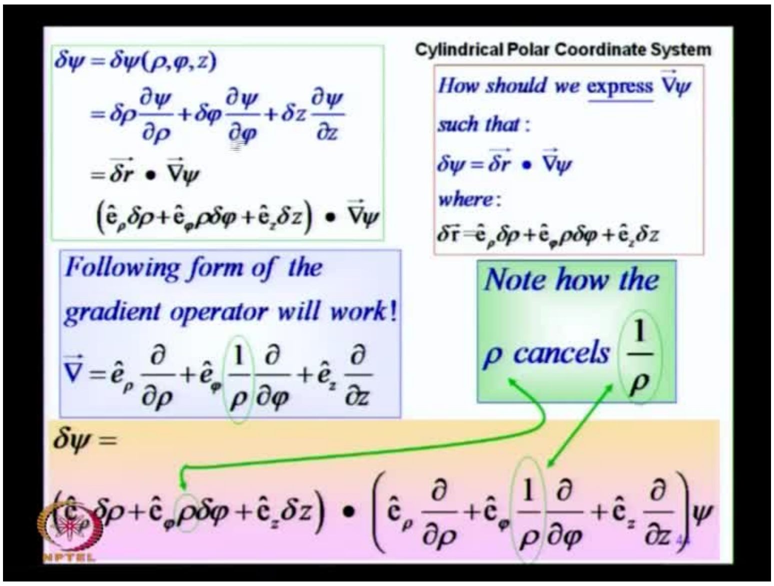

So, the first thing we do is retain our faith that this expression delta r dot delta psi del psi must have the same value in any coordinate system; admit the fact that delta r itself will have different expression in cylindrical polar coordinates. We know what it is. It is e rho delta rho plus e phi rho delta phi plus e z delta z; we just did it.

Having done this, all we now need to do is interpret grad psi such that its dot product will compose this particular sum. It cannot be difficult. Done? Have you worked it out in your note books already? What must be the expression for grad psi that its dot product with the displacement vector will give you delta psi? Is a question that you are going to ask yourself. And if you answer it for yourself you automatically get a unique answer for grad psi. What is it?

This must be the dot product because now if you multiply term by term, you get del psi by del rho times delta rho which is here, which is the first term; from here you get del psi by del phi divided by 1 over rho times rho delta phi; so the rho will cancel and you will be left with delta phi times del psi by del phi. So, you cannot have anything other than this over here, because only this term will give you the correct delta phi del psi by del phi to come here; nothing else. There is therefore, no reason to memorize that the expression for gradient in the cylindrical polar coordinates must involve this one over rho factor; without that, you cannot get this term here.

The criterion is this that delta psi equal to delta r dot gradient psi must be independent of a coordinate system, but delta r must have different expressions; therefore, this expression for grad psi will also be different. So, we have thus discovered what it ought to be from this simple consideration and this automatically gives you that the gradient must be given by this expression - e rho del by del rho coming from here, e phi 1 over rho times del by del phi plus e z del over del z, and notice how the rho factor over here cancels the 1 over rho here to give you the correct term in the middle (Refer Slide Time: 40:16 to 40:26).

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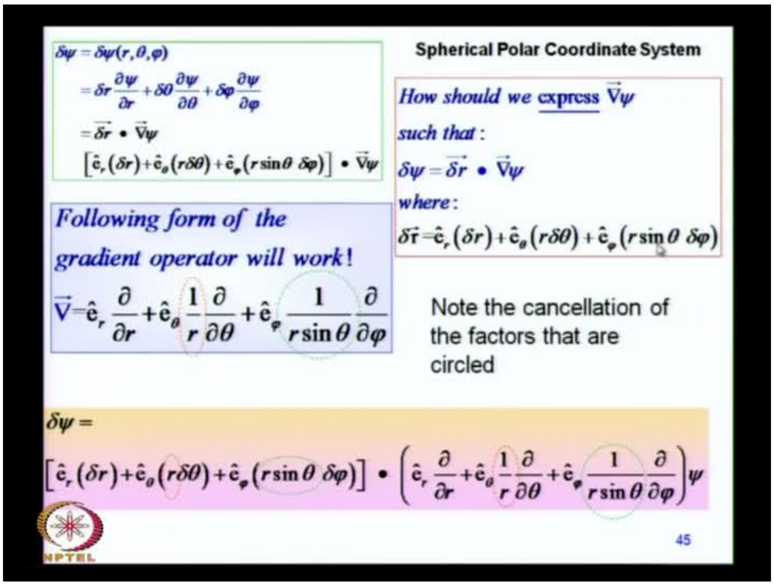

This cancellation of rho is important and what this technique tells us? That it is absolutely unnecessary to remember what the formula for the gradient operator is, in any coordinate system. So, we got it in the Cartesian coordinate system; now, we got it in the cylindrical polar coordinate system; let us do it in the spherical polar coordinate system. In the spherical polar coordinate system, again, we use the same thing. This change in the scalar function psi will be given by the component of the gradient of psi along the displacement vector.

The expression for the displacement vector has to be written for the corresponding coordinate system. So, in spherical polar, it is e r delta r plus e theta r delta theta plus e phi r sine thetadelta phi which we just did. From the chain rule we know what it is and we must generate this equivalence by discovering for ourselves what must be the expression sine theta according to the chain rule. So, we use that as a criterion and we discover that if you write the gradient of psi as what you see over here that it is e r del psi by del r plus e theta 1 over r times del psi by del theta plus e phi 1 over r sine theta del psi by del phi; I have written all of this with psi written outside the bracket because the whole operator operates on the function psi.

Term by term, this scalar product gives us what we know must result from the chain rule of ordinary calculus. Notice that this 1 over r cancels this r this 1 over r sine theta cancels this r sine theta so that this chain rule which you see over here is automatically reproduced; absolutely no need to remember this 1 over r in the definition (Refer Slide Time: 42: 46 to 42:59).

If you forget it, there is no way you can get this cancellation. So, we get the correct expression for the gradient operator in the spherical polar coordinate system, and now we have got the scaling factors 1 over r as well as 1 over r sine theta in the azimuthal part. And these components 1 over r sine theta and 1 over r happily cancel this r over here, and r sine theta over here because they are coming from the expression for the differential increment in the position vector itself; delta r is what has got this extra r factor over here and this r sine theta over here; so these are the ones which you need to handle.

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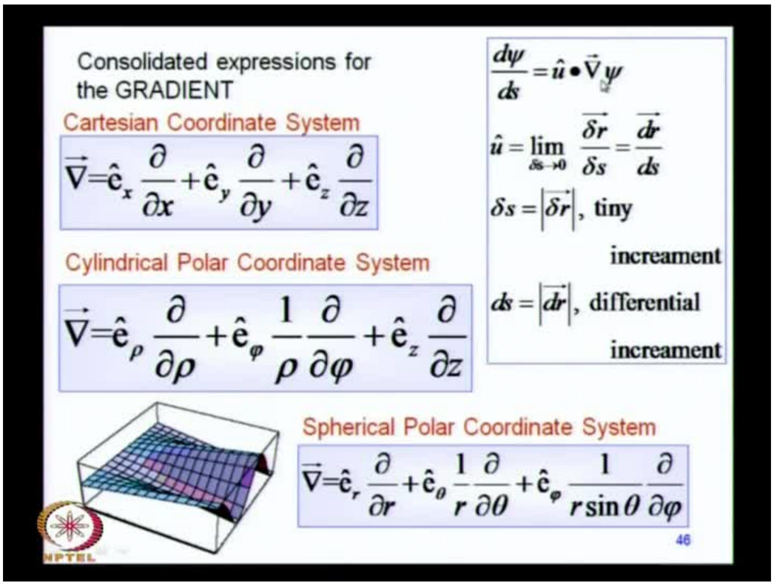

So, we have now got the expression for the gradient operator in all the coordinate systems: in Cartesian, in cylindrical polar, and in spherical polar coordination system. They all come from this basic definition that the gradient of psi is such a vector. It is a vector function of the scalar function psi, that it is component along any unit vector gives you the directional derivative in the direction of the unit vector. This is the criterion, it is the definition, and it gives us the correct expression regardless of any coordinate system.

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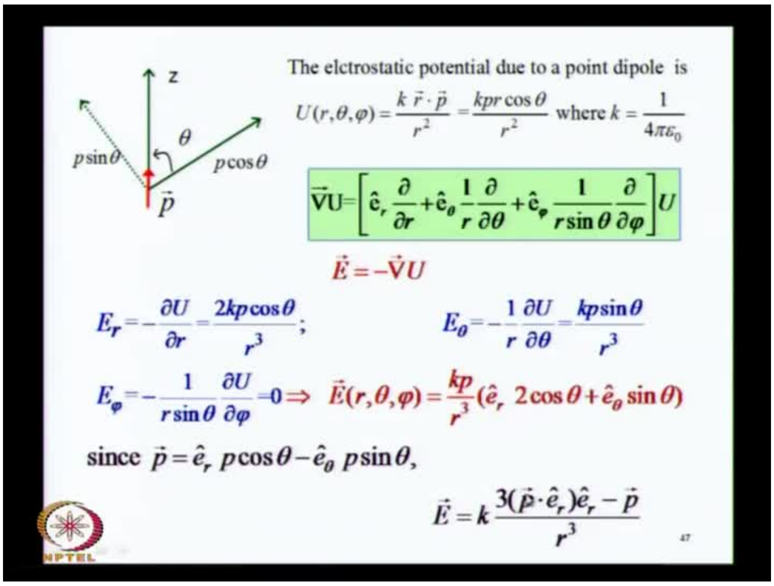

You can do some small exercises; for example, if you write the potential due to a dipole which has got this general form, this is the potential (Refer Slide Time: 44:41) due to an electric dipole and in this geometry in which the angles and the distances are defined as it is. You can get the gradient of this potential and here I make use of the expression for gradient in the spherical polar coordinate system which we have just obtained. And by using this definition, you can get the corresponding components because the electric intensity is the negative radiant of the potential; that is the connection between field and potential which I mentioned earlier.

The field is the negative gradient of the potential. So, you can apply this in gravitational problems; you can apply this in electrostatic problems. Very easily, all you have to do is to get the gradient and recognize a field to be the negative gradient of the potential. So, you can rewrite by doing some simple transformation; you can write this electric intensity in terms of the dipole movement itself, but this is just a matter of rowing simple transformations. The basic expression comes from this and what we have used here is the expression for the gradient operator in the spherical polar coordinate system.

So, one can do exercises very easily using this, and remember, you do not have to remember the formula for the gradient operator in any coordinate system; that is the last thing I will recommend.

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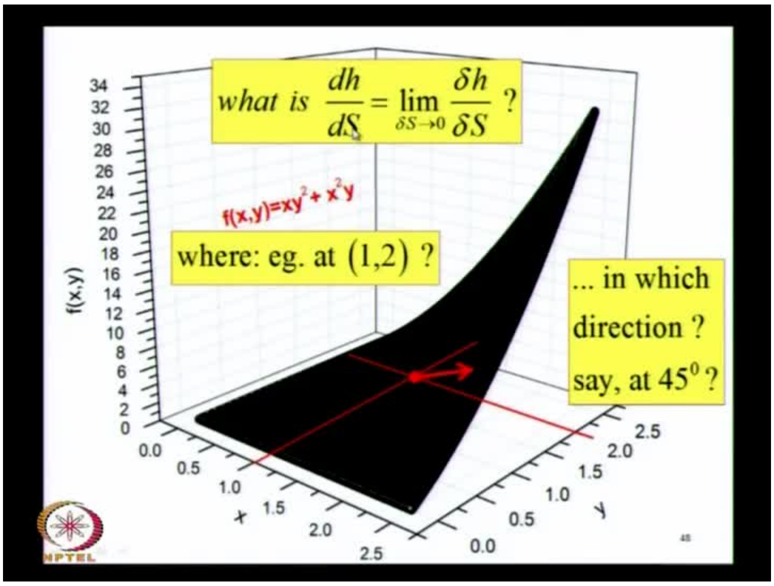

Now, what about this problem which Domain gave us? He generated the surface for us and the question we had is - what is the rate of change of this height with respect to distance at a given point 1 comma 2, in a given direction say 45 degree? Now, if you have to solve this problem using any other techniques, try doing it using calculus, try doing it using cylindrical polar coordinates if you like, or spherical polar coordinates if you like. There are various ways of doing it. They will all give the same answer. You will always get the correct answer, as long as you to use the correct technique. But now, we have learnt another way of doing it which is using a combination of vector algebra and vector calculus that you can get the directional derivatives just by doing calculus or also by using vector algebra and vector calculus.

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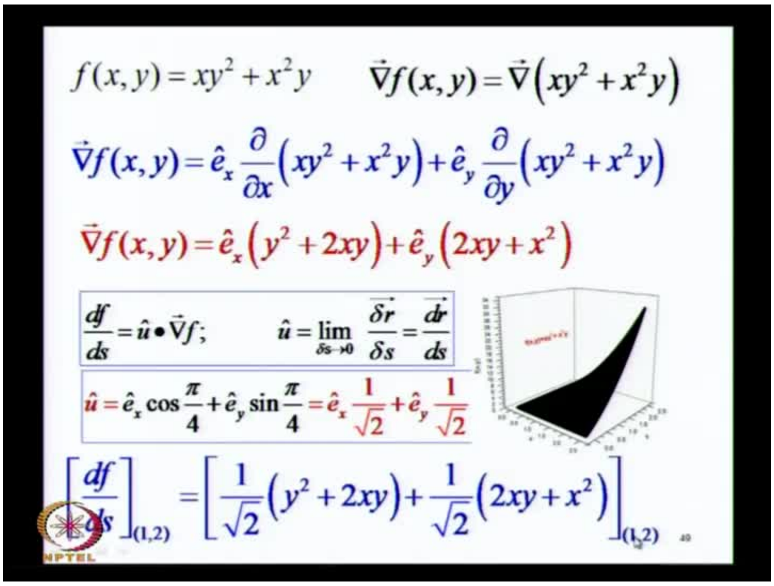

So, one can do it; our problem was this: what is the directional derivative at a given point in a given direction? So, what we do is, we begin with the function which is x y square; this is a function which is plotted; x y square plus x square y plotted in the first coordinate. And what we do is find its gradient. Now, we know that the gradient is e x times del f by del x plus e y times del f by del y. So, we find what the gradient is, and by taking these derivatives, you can get the complete form for the gradient. And all we have to do now is to take the component of this gradient along the unit vector.

Which is this unit vector? It must be a unit vector which is pointing in the direction of 45 degrees. So, you can construct that unit vector because you know the angle at which it should point. That unit vector will obviously be cosine pi by 4 times e x plus sine pi by 4 times e y. We know these quantities which are 1 over rho 2. Both of them, now you know this unit vector, and now you ask, what is the component of this gradient? You know the gradient and you know the unit vector; you can get the dot product.

So, you construct the dot product; you get the directional derivative; you have determined it at a particular point namely x equal to 1 and y equal to 2. So, after you determine the scalar product, put x equal to x and y equal to 2, and you have your answer done; so, very simple.

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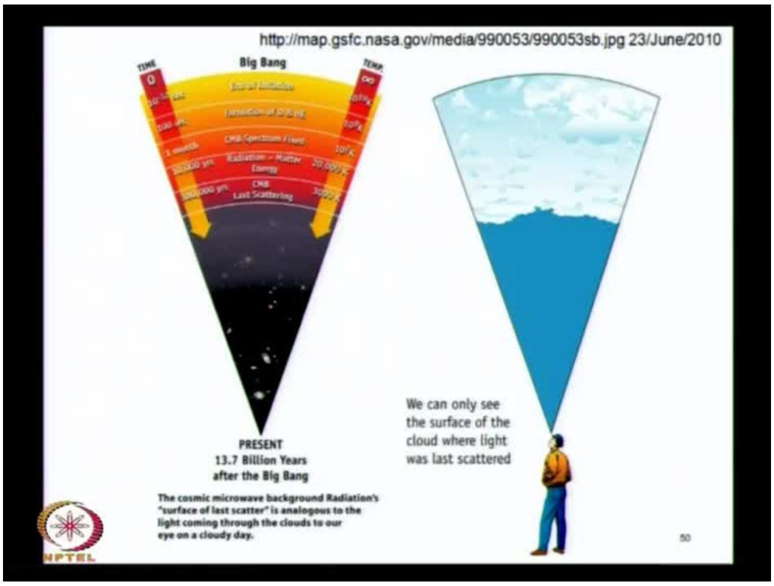

Now, this has got applications in gravitational problems, electromagnetic problems and so on, and we will certainly have a unit on classical electro dynamics. So, we will discuss some of these things. In Maxwell's equation, we will find applications, but I am going to just draw your attention to something which is completely different because we all concerned about.

What is all this about? You, me, universe, what is all this? And people tell us that all this began with the big bang something like 13.7 billion years ago and over this period of time, certain things have happened. So, what is the evidence that this is how the universe evolved?

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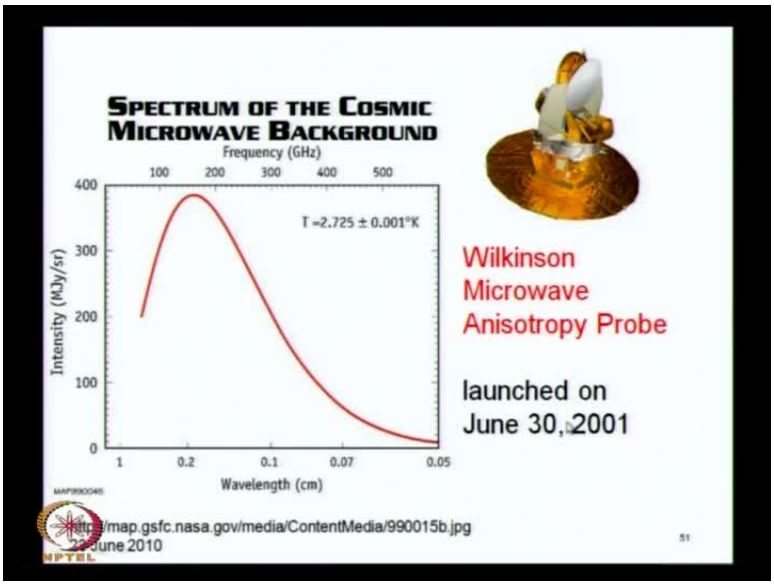

These are fundamental questions for a physicist and it turns out that one of the consequences of the big bang theory is what is called as the cosmic microwave background. One of the predications of the big bang model - that if the big bang model is correct, then this cosmic microwave background would be emitted after the big bang, and if this was time 0, then this cosmic microwave background would originate perhaps some time like in 1 month after the big bang when the universe who was a baby (Refer Slide Time: 50:43).

And then after the last catering, the light from this microwave background would reach us and this is very weak light. It is like if you consider a spectrum as a function of wave length and if you plot this cosmic microwave background as a function of wavelength, it has got a profile of this kind. And this profile would be characteristic of a black body would radiate if it were at temperature of about 3 degrees kelvin as it is called; it is actually less than 3 degrees kelvin; it is like 2.725 plus or minus something.

This is called as the 3 degree background radiation and this should there throughout the universe. So, this k is measurable and this was measured first by a satellite called Cobe and then some further measurements were to be carried out which are being carried out by device called Wilkinson Microwave Anisotropy Probe that W MAP as it is called. And there, I have taken this material from a NASA website; so the references are given at the internet link at the bottom and you can easily find it on the internet.

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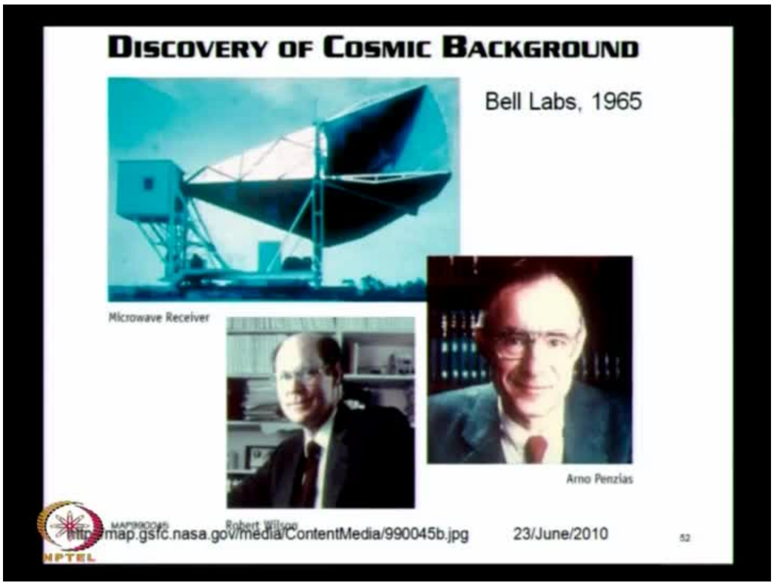

This probe was launched almost exactly 9 years ago, June 30th 2001, and the first time this background was really detected was by Penzias and Wilson in an experiment carried out at the Bell labs. And studies on this microwave background radiation would be a good test on the big bang model. So, that is a reason you want to measure it; very deep question, fundamental questions; exciting questions; not just gravitational potential, not just electromagnetic potential, but you still need a very good handle on these ideas of potentials and fields to study this.

The question I am going to raise here is - you want to measure this; you need a probe which will be sensitive to the microwave radiation. This is the 3 degree kelvin radiation. (Refer Slide Time: 53:35) It will be in this wave length part of the electromagnetic spectrum. So, you need some detectors, some sensors, and you need to keep the sensor somewhere do you want to keep it here on this desk? Where do you want to keep it? You want to keep it in the garden? Where do you want to keep it? Where, the measurements will be most sensitive to the background radiation. Where do you want to keep it?

So, the first experiment was done in space. You know the probe was launched. This was the Cobe which was a satellite orbiting the earth and then you needed more precise measurements because there was a suggestion in the first earliest experiments which were around 1989 or something, which is when the probe was launched I believe. And in the 1990’s, they suspected that the radiation from different parts of the cosmos are not exactly identical; there is a little bit of Anisotropy. Anisotropy means, it is from different directions, it is different. So, really need very sensitive measurements.

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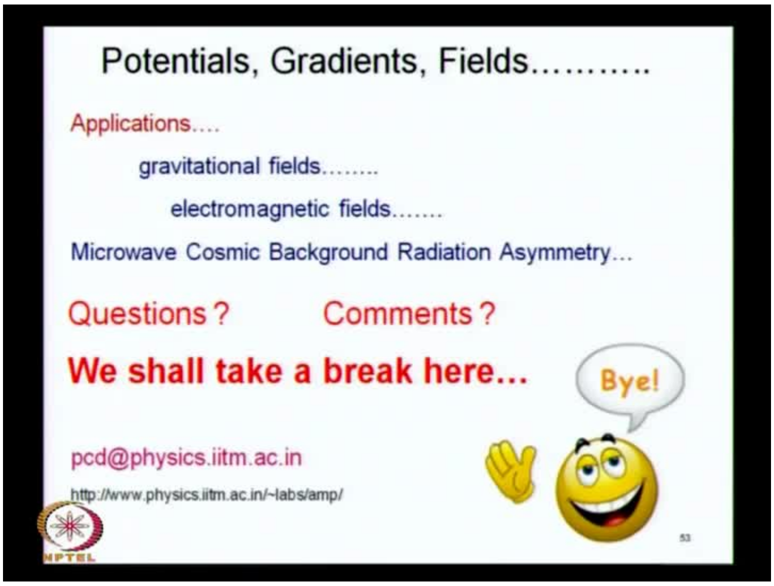

So, a new experiment was device which is this Wilkinson microwave anisotropy probe and the question is - where do you want to keep this probe? Because it needs to measure this microwave background radiation, it should be very sensitive to it. If you keep it in your yard, it will get a lot of light from the sun as well, which was swampert and that effect will be so huge that these tiny variation due to the microwave anisotropy will hardly been detected. So, I will leave you with this question and we will take a break here, and we will continue from this point in our next class; may be in the mean time, you have got these answers.

The answers have to do with the fact that you must understand potentials fields nicely and thoroughly. So, thank you very much. If there is any question, I will be glad to take. So, we will consider in our next class, some applications in the realm of gravitational fields and how the W MAP - the Microwave, The Wilkinson Microwave Anisotropy Probe exploits these things. So, thank you very much and we will take a break here.