Select/Special Topics in Classical Mechanics

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Module No.# 08

Lecture No. # 28

Gauss Law Eq of Continuity (iii)

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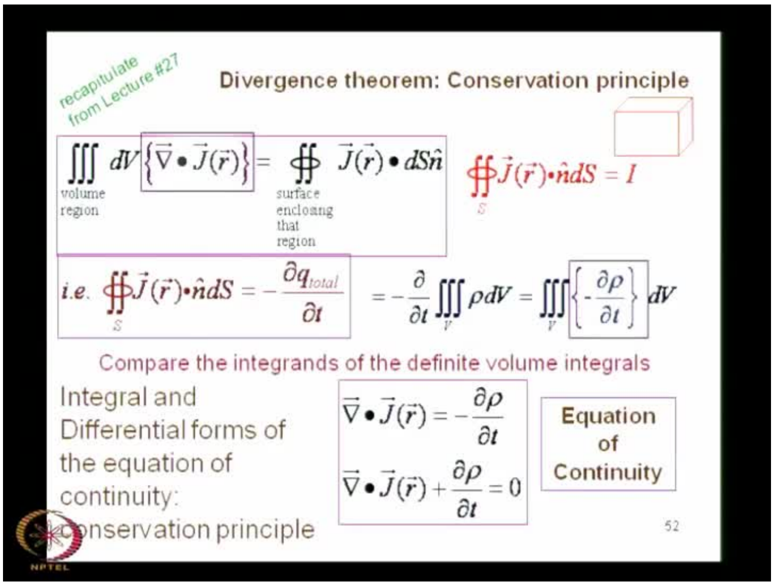

Greetings let us resume our discussion on the Gauss's law. We studied the Gauss's divergence theorem as it is called; we introduced ourselves to what is known as the divergence of a vector field and it let us through a conservation principle which is the physical reality that matter is not created nor destroyed, but we also excepted that this would apply also for a field, as you can certainly go from matter to field and vice versa, but that is the different story all together.

So, within the realm of classical mechanics and classical electrodynamics, we have conservation of matter and conservation of fields. And we established the conservation principle, which express itself as what we called as the equation of continuity.

So, this is just a quick recapitulation of major results, in which we establish the fact, that the Gauss's divergence theorem connects this surface integral to the volume integral, and this surface integral over a close surface gives you the net current, which is whooshing out of the volume which is trapped in that close surface. And this current will of course be the rate of change of charge and the minus sign takes care of the fact that what is coming out, will have to be at the cost of what is inside. The charge itself being the volume integral of the charge density, then we admitted the fact that the operations derivation with respect to time, taking the time derivative and taking the space integration carrying out the space integration.

These are over independent decrease of freedom, these are quite independent processes; so, they can be carried out in any order that you like. So, this time derivative of this volume integral is exactly equal to the volume integral of the time derivative of the integrant row. And now if you notice the fact that, this volume integral is equal to this volume integral; both being definite integrals, the corresponding integrants are the same, which means, that the divergence of J is equal to minus del rho by del t, which is what gives us - the equation of continuity. So, these are respectively the integral and the differential forms of the equation of continuity, which says a conservation principle.

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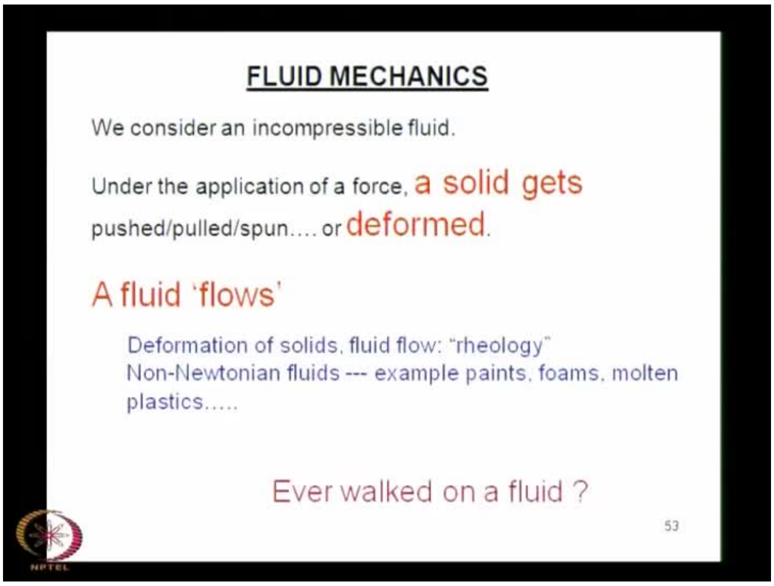

Now, what we are going to do? Today is set up an equation of motion and we know what an equation of motion is - an equation of motion is the mathematical relationship between positions, velocities accelerations; it must connect all of these quantitatively. And we have met the Newton's equation of motion, which is f equal to m a. And we have also seen the corresponding formulations based on the principle of variation expressed as either the Lagrange's equations or the Hamilton's equations.

So, for particles we already figured out how to do it and we have some experience and applying the equation of motion and solving problems for mechanical systems, particles and trace the temporal evolution; the evolution over time as to how the mechanical state of a system, a particle system changes with time.

We will now try to see. If we can set up a similar equation of motion for a liquid and the liquid already brings you know different ideas to our mind, because we know that matter is like solid, liquid, gas. And in general, we will apply to what we will cumulatively refer to as the fluid for both liquids as well as gasses.

The word fluid coming from, you know it relates to flow. So, the characteristic feature of a fluid is that, it flows. And this is the characteristic feature. What happens, when you apply a little bit of force on a liquid, is that they start flowing; this force can be an applied force or it can come from some field like gravity, but we will deal with such fluids which are called as Newtonian fluids, and these are obviously to be separated from what else would you call it as Non-Newtonian fluids. And there are examples of Non-Newtonian fluids like paints, foams, molten, plastics, and so on these are complex systems and we will not deal with those nevertheless, it is nice to know what a Non-Newtonian fluid, is just in case you have not applied your thought to that. The way you came to this class room, you walked up and you walked up the stair case or walked up along the surface and so on and there are some [fl], who claim that they can walk on water and so on, and hopefully you do not get impress by that, but that something which you can do and have you ever considered walking on a fluid.

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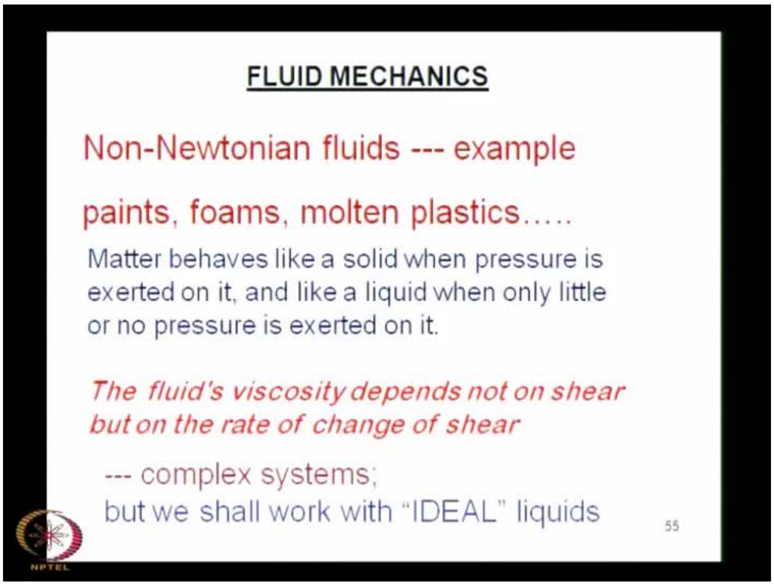

So, what I will do is I will show you a walk on a fluid and this is something, which I have downloaded from this YouTube; you do not have to write down this reference, if you just Google walking on liquid at the YouTube wave, you will easily find about a dozen or even more scores of such links and you can easily find this for yourself, but I have downloaded this, so that you can see it and enjoy this. These are some examples and what I will do, is I will show you this little movie; so, essentially what we have in this film is these boys having lot of fun walking over a fluid and there will be a little bit of element of either fear or at least some sort of uneasiness when you try to walk on fluid, because it is not something do normally, but as you see these boys are having lot of fun; and you know, once they discover that, they can actually do it, then you feel like doing it again and again. And what is interesting about this is that, you need to walk briskly, if you do it a little slowly you are going to sink just the way you would sink in water; so, you need to do this a little quickly.

So, there are many others, and the form of the fluids is very complex together, these are called as complex fluids. And what happens is that, it is the state of matter, which behaves like a solid when you applied pressure on it; so, when you place your foot on it you are applying pressure and it behaves like a solid, so that it would support you. But if the pressure is reduced, so if you do not walk basically enough, then what is going to happen is that, you would sink, because the manual in which the viscosity properties of these fluids is turns out to be, is that, it depends not just on shear but on the rate of change of shear; that is the characteristic feature of some of these fluids; so, these are called Non-Newtonian fluids. And you can make it yourself, means, you can get some cornstarch and mix it with water and if you do it in a certain proportion then you will get these complex fluids.

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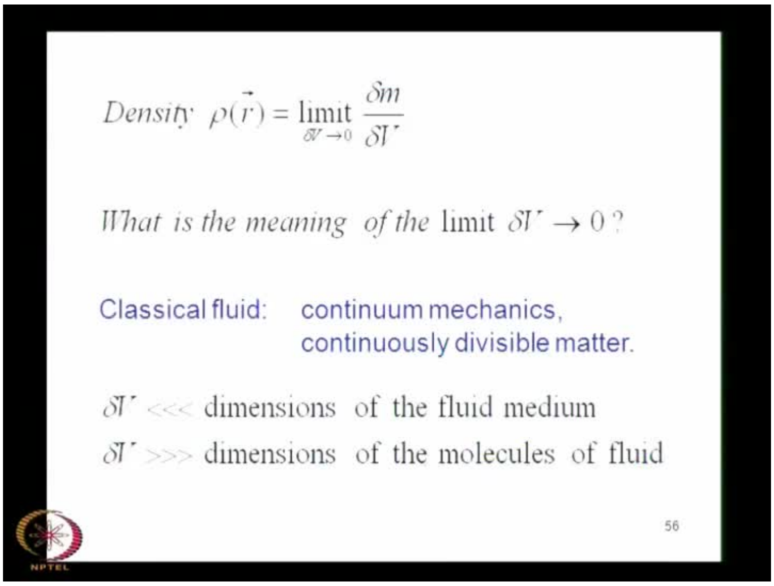

What we are going to discuss or though so called ideal fluid; it is not that water is an ideal fluid, but it is at least pretty close to that; these are the common fluids and we should not venture it to analyze complex fluids, before we learn how to analyze normal fluids or ideal fluids. And ideal fluids, normal fluids, these are characterize by various properties one of which is density, which is mass per unit volume. And this is we defined in the limit delta v going to 0; you take the ratio of delta m by delta v, take the limit delta v going to 0, but we must understand the meaning of this limit delta v going to 0, because 0 is a 0, means, there is nothing, means, it is the volume has to shrink to a point. But if you consider any liquid like water, this water is made up of these two hydrogen atoms and an oxygen atom; there is a certain bond length, there is a certain bond angle.

So, if you really go down to delta v going to 0, you do not know, if you are going to look at the hydrogen atom or the oxygen atom or the other, you know hydrogen atom and so on, so that is not what is implied by delta v going to 0; what we do is an approximation to this and this is called as the continuum approximation; so, mechanic studied in the continuum approximation, in which we presume matter to be continuously divisible which it is not, because if you divide molecule liquid and look at smaller and smaller parts, you will certainly hit certain parts, we do not have the properties of water, means, hydrogen, of course it is not water.

So, you do not go to the limit delta v going to 0 in that limit, what it really means is that, the volume under our consideration is much smaller than the dimensions of the fluid medium; so, if you are looking at water flowing in pipe or in a river, then compare to total dimensions of the fluid medium then the volume elements that, you are talking about a really tiny and you could consider them to be infinitesimally small and mathematically you can think of this as a limit delta v going to 0, but it is not really going down to the sub atomic level and seeking the limit delta v going to 0 in a mathematical sense.

So, this is what is called as the continuum limit, matter being studied in continuum limit the volume is much smaller than the dimensions of the fluid but much larger than what the structure of the liquid at a molecular level is really made of, so it has to be much larger than that; so, in this limit, we define the density of the liquid.

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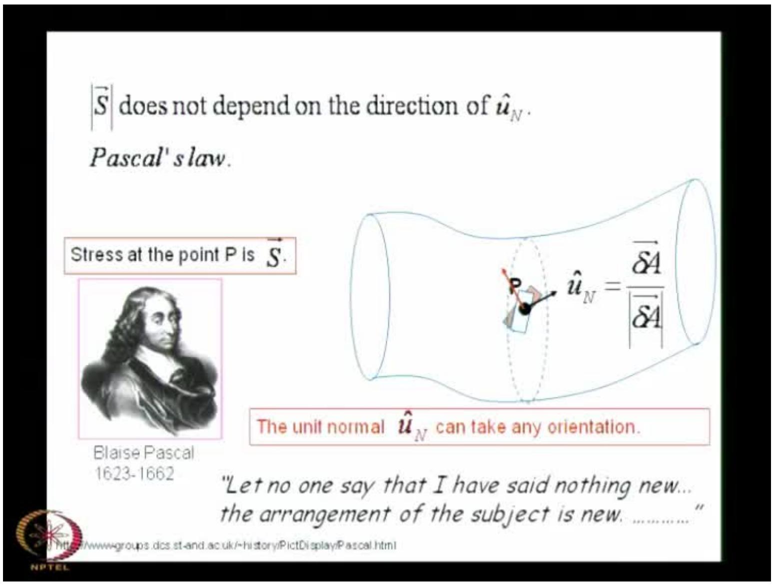

And let us consider a certain hose pipe of some arbitrary shape. And this will have fluid entering one side and exit from the other; in between you are consider a certain cross section as you seen in this figure. And in this cross section you think of a certain point.

Now, construct a very tiny piece of elemental area like a patch of area - a very tiny patch which is passing through that point; so, if the point is here, then this is the patch, but I can through the same point I can pass this plane, this plane, this plane, this plane, through essentially the same point .

So, you can think of any patch which will have a different orientation and if the orientation of that patch is to be indicated by a direction, which is normal to the patch, which is orthogonal to the patch, it is a 90 degrees to the patch, then the orthogonal to this patch is obviouslydifferent from the orthogonal to this patch; but both the patches are passing through the same point.

So, if there is another, means, if this is an orthogonal to the patch which is shown in one, but again to any patch you can think of two orthogonal, one going one way and the other which is opposite to that. And this sense has to be developed using the right hand screw rule, which we have discussed in the previous class. In the previous class, we assigned a certain convention, so that is the direction in which you think of traversing the perimeter of that patch and if you turn a right hand screw along that particular sense, then the forward motion of that would give you the direction of the elemental area; so, this is one particular orientation that is possible. And we will examine what is the stress at the point P; so, stress is like pressure force per unit area.

We all sense it specially when the exams are close and at the point P you can ask, what is the stress, if there is a different patch which is oriented differently; it will have a different orientation shown by a different vector which is orthogonal to that and the direction again will be set by the right hand screw rule and this unit normal to the patch can really take any different orientation, because you can have a patch through a point in a variety of different ways; so, the unit normal can take any orientation, so remember that.

Now, what is interesting is that, if you look at the magnitude of the stress which is indicated by the modulus of S; this magnitude does not depend on the direction of this patch and this is the physical content of Pascal's law, that the magnitude of this stress is independent of the direction, in which, you consider this patch this was recognized by Blaise Pascal in the seventeenth century, wonderful physicist who made somewhere exciting observation, son, the properties of fluids; he is well known for this quotation that, let no one say that I have said nothing new, the arrangement of the subject is new, and this famous law that the modulus of stress does not depend on the direction of the orientation of the patch is known as Pascal's law

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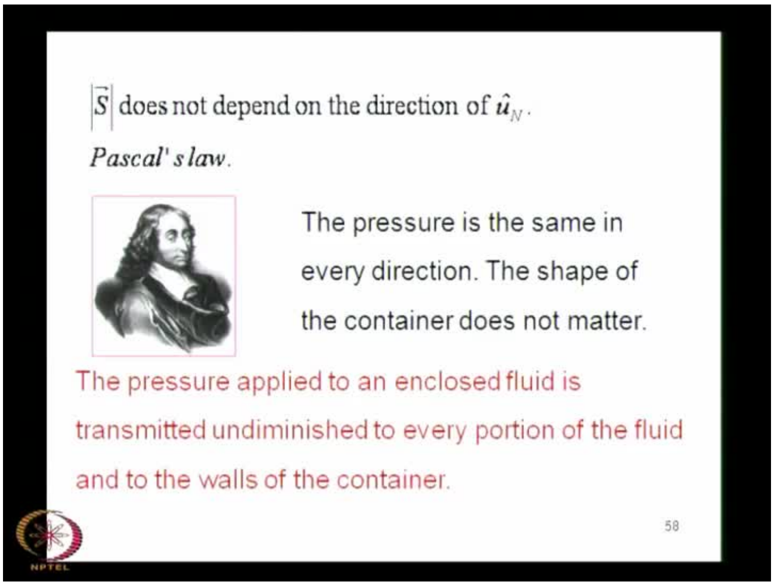

Now, what it, means, is that the pressure is the same in every direction; the pressure of course is force per unit area; force is a vector quantity, so pressure would become a vector quantity, but what is implied over here is a magnitude of the pressure, what is implied over here is the magnitude of the stress; so, there is no inconsistency there that the pressure is the same. So, in this context we will continue to use it as pressure, it is the same in every direction, quite independent of the shape of the container.

In other words, what it, means, is that the pressure applied to any fluid which is enclosed in a certain container, it is transmitted undiminished to every portion of the fluid and also to the walls of the container; now, this is at the heart of Pascal's law, this come directly from this consideration.

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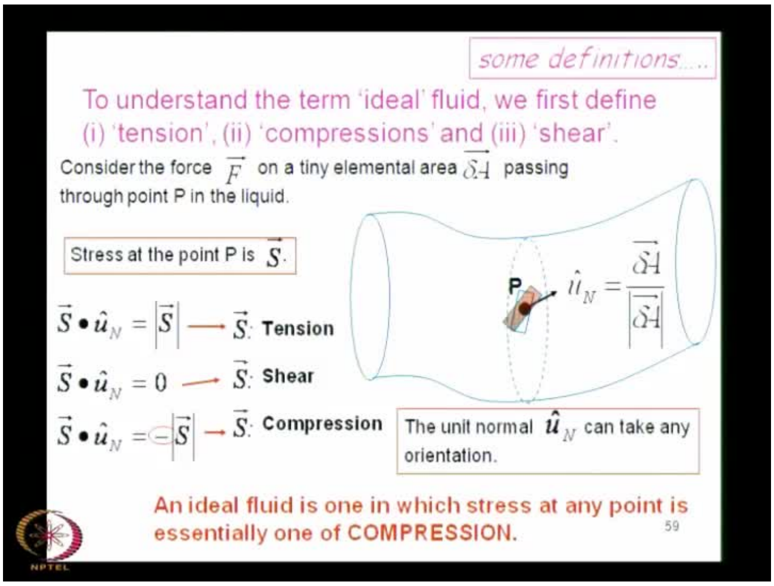

Now, we are pretty much ready to define what an ideal fluid is and to be able to define it, we needed this background. So, let us now introduce certain a few terms: we will introduce tension, we will introduce what is called as compression, and we will introduce what is called as shear.

So, what we do, is we consider the force on a tiny elemental area, this elemental area is considered as a vectorial quantity; it is the cross product of the two sides of the rectangle, so it will have a certain directional attribute this comes from the right hand screw rule as we have already seen. And with reference to this, we can define the force per unit area, which will give us the stress.

Now, let us ask what are the properties of the stress? So, when you think of a certain direction, which is the unit normal to a patch, this u vector - the unit vector u that, which you see in this figure; if the component of stress along this direction is equal to the magnitude of the stress, then the stress is said to be tension; if it is 0, it is set to be shear, and if it is negative of the magnitude of the stress, then it is set to be compression.

So, these are now the three definitions of what we call as tension, shear, and compression; having defined this, we can now define what an ideal fluid? It is an ideal fluid, is, one in which stress at any point is essentially one of compression and we know exactly what compression is, compression is when the component of stress along a direction is exactly equal to the negativeof the modulus of the stress, so that is the criteria. And when the stress is one of compression then the fluid is said to be an ideal fluid and it is fluids of this kind to which we will apply our analysis, in this class, we will not deal with non-ideal fluids, we will not deal with Non-Newtonian fluids.

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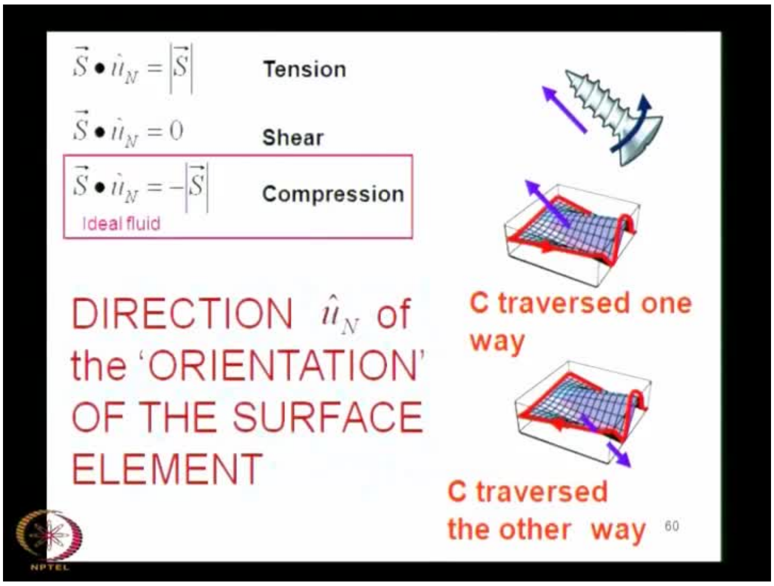

So, now, we know what an ideal fluid is, it is one for which the stress is, one of compression the direction u N could be anything and it is considered in the context of this right hand screw rule, which we have discussed at some length in our last class; so, this is just to remind you of that.

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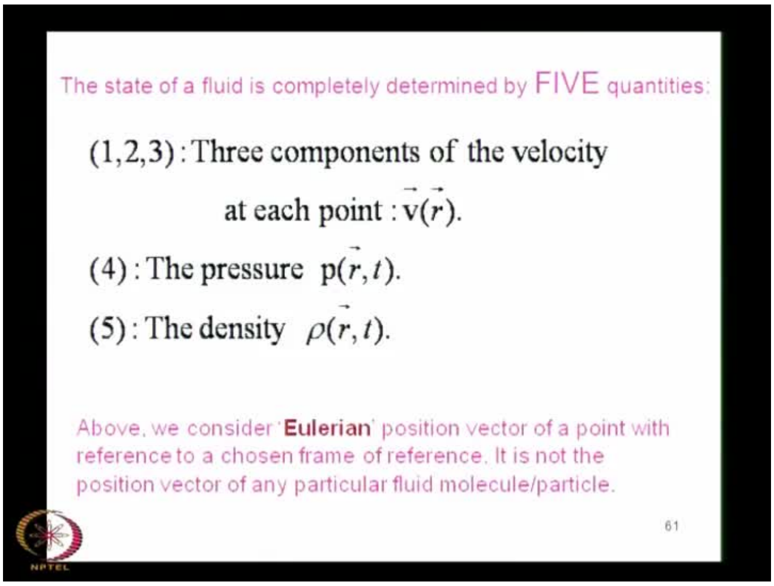

Now, a fluid, I refer to one of the properties of the fluid, namely the density. And we understood how the density is to be interpreted, mass per unit volume, delta m by delta v in the continuum limit of matter, not at the atomic structure level, not at the level of molecular or atomic structure.

So, the fluid is really defined by five quantities, density one of them; along with density you must specify the pressure and you must specify the three components of the velocity of the fluid at each point. Now, you have three components of the velocity; so, there are these three scalars over here, there is another scalar add over here, which is pressure, which is thought of not as the vector force per unit area but the magnitude of the stress which is a vector, scalar quantity. And then the density and all of these quantities are considered in the continuum limit of matter. And these five quantities together describe what a fluid is, what the state of a fluid is, because you have to define… first of all how to describe the mechanical state of the system, and then you have to answer how does this mechanical system evolve a time; this is the central problem of mechanics.

We address this for particles and we are now doing it for fluids; so, you have to define this at every point in space; so, it is the function of special coordinates of the position vector and it could of course change from time to time, because there may be other factors, which influence the pressure and density and velocity and so on; in summe,r if there is no water flowing in a river, is, no velocity, no pressure, no density; so, of course, it depends on time different in summer, different in the monsoon season; I prefer the monsoon.

Now, what do we mean by position vector? Now, there are two alternative perspectives in this context and these are very crucial; for our understanding of the equation of a motion for a fluid, one perception is what is called as the Eulerian position vector.

This is just the position vector of a point in space, whether or not there is a fluid at that point, like the small cannel which passes by the side of your home. And in the monsoon season, there is a lot of water going through it and you think of some point with reference to your house, it is right next to your house, so you can fix its coordinates with reference to some coordinate frame of reference in your leaving room, if you are like. And there is a point in that cannel, this is your Eulerian position vector of that point; this position vector will have its meaning inthe monsoon season when there is lot of fluid and it will have the same meaning in summer when there may be no water running through, it does not matter.

It is a property of that point in space, as oppose to this, there is what is called as the Lagrangian position vector, which is the position vector of a drop of water; a drop of water which is flowing, means, if you let water flow and the drop which was here, now goes here, and now goes here, and now goes here, you are tracing the drop of water as it goes from one part to another; so, the position vector of this drop of water is changing as the fluid would flow. This is not a property; this is not a fix property of a fix point in space, it is a property of a particular drop of water.

Now, I am not going to define what I mean by a drop, because we are not looking at fluid at the molecular level and we will not talk about a certain n number of molecules to define a drop, because at a molecular level, you know there are these hydrogen bonds between different molecules of water; you know at any finite temperature, these molecules are trembling, some of these hydrogen bonds are breaking, new bonds are getting formed and so on; so, we are not talking about this at level at which you talk about the atomic structure or the molecular structure, but any recognizable small piece of element of the water is, what we will call as a drop of water and if you are chasing that particular drop of water, then you have a different idea of a position vector. This is obliviously different, conceptually different from the idea you had of the particular point in your cannel, whether or not there was any fluid passing through it, or what was the density of fluid at that point, what was the pressure of the fluid at that point, what was the velocity of the liquid flowing at that particular point; so, that is called as the Lagrangian description.

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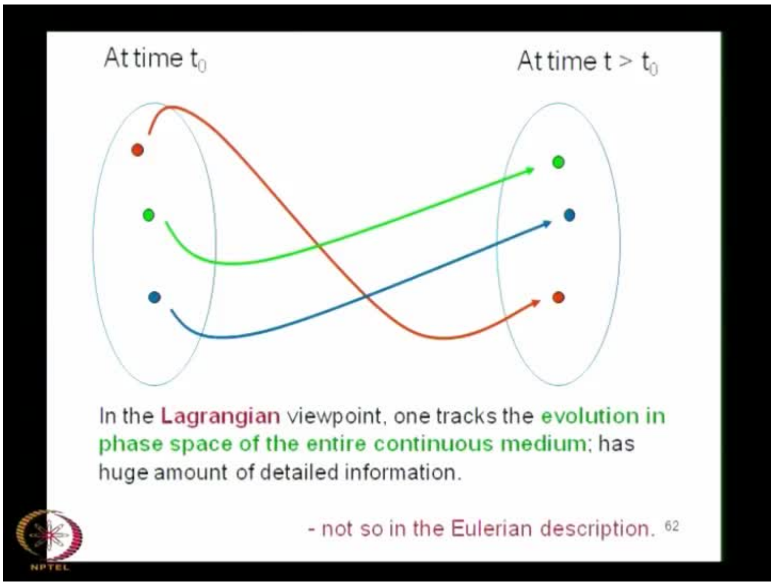

So, let us get a better field for this, so here are two sets and if you follow the trajectory of each drop individually, that, this is the trajectory, this is the position in a configuration space or you can generalize to a it to a phase space if you are like, in which you describe both the position as well as the momentum or position and velocity if you are like. So, if you keep track of the time evolution between time t 0 to time t 0 plus delta t or some later time, and if you follow the trajectories of each particular fluid element, which loosely I will call as a drop.

Then you have with you what is called as a Lagrangian description of the fluid. So, you will have to track every single tiny recognizable fluid element and you can see that, it is quite different from the Eulerian description of positions in space.

So, position vector whenever you talk about a position vector, in the context of fluid mechanics. you must ask yourself are you talking about an Eulerian position vector or aLagrangian position vector because 2 have completely different meanings

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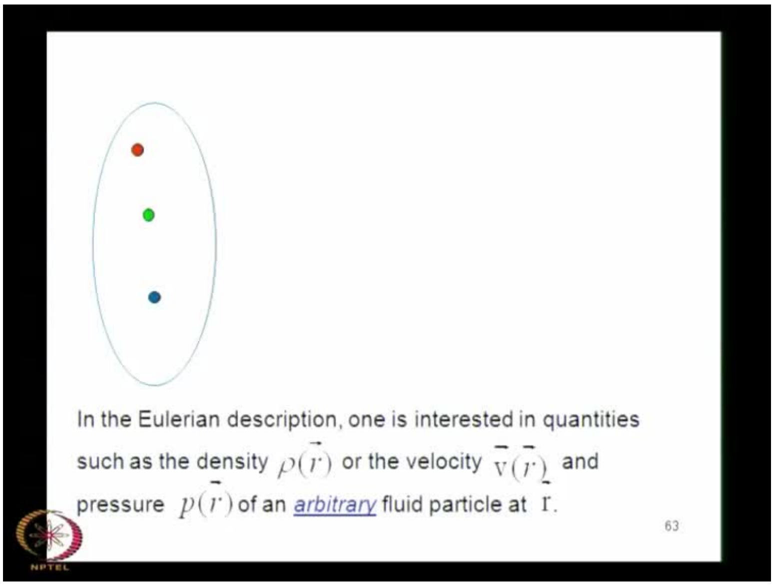

So, in the Eulerian description all you look at is specific points, in the region of the fluid, you do not care, as to what, which particular fluid element is there. So, if you distinguish different drops as drop a, drop b, drop c, drop d, you do not ask which particular drop is at a particular position vector, but you simply ask at that particular position vector what is the velocity of the fluid, no matter which drop is passing through; it is like, all of you have come to this class you have negotiated with the traffic and you had different speeds, you did not come here at a uniform speed, from wherever you have started. And then there are you know some big roads in Chennai, where the traffic moves freely, you may have two lanes, three lanes like the Anna Salai for example, and the traffic could be moving much faster then what it does in a place like Mylapore; so, it does not even move in Mylapore, does it.

So, any how you can talk about the average speed at the certain point on a traffic at Anna Salai, and at average traffic speed at a point in Mylapore, no matter who is passing through; you could be driving, you could be riding your bicycle or motor bike or car or whatever or somebody else could be driving, but all of your speeds are going to be control by the traffic conditions. So, it does not matter, which particular drop of water, which particular element which is in motion is being talked about, but there is a certain velocity, which is a property of a particular point in the region of the flow.

This is the Eulerian description, that you can talk about the density at a position, you can talk about the velocity at a position, you can talk about the pressure at that position, no matter which fluid particle you are talking about, no matter which drop of fluid you are talking about, but it is a property of that particular point; now, this is the Eulerian description.

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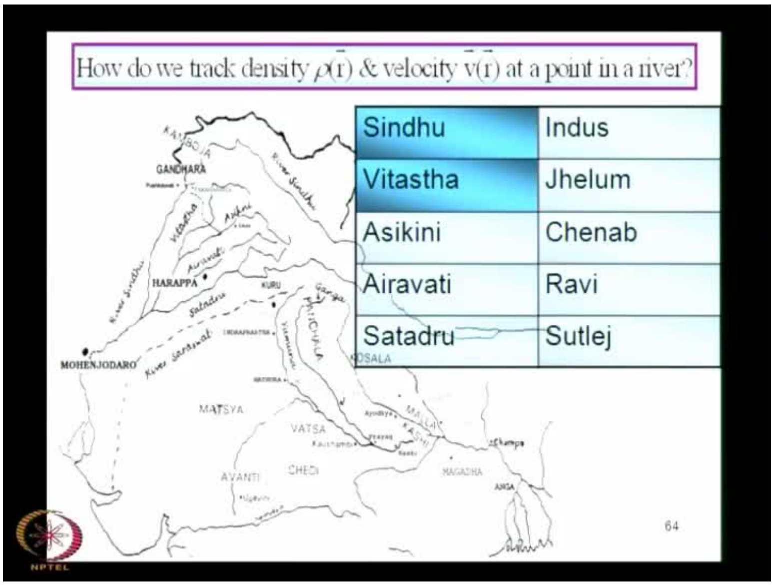

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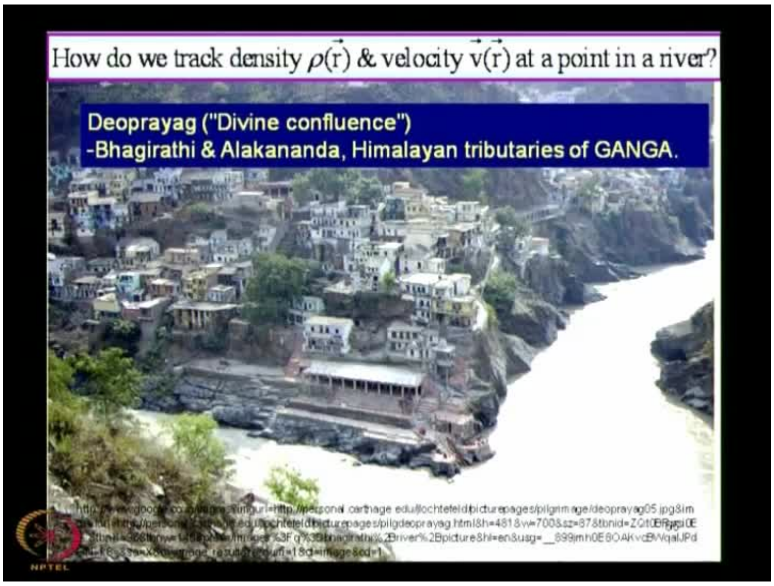

Now, I assume that the description of a fluid, in terms of Eulerian and the Lagrangian parameters is quite clear in your minds. Let us take this idea little further to get a very strong handle on this, because this is absolutely fundamental to setting up the equation of motion for a fluid. And let us think of some reverse in northern India, you have got the Sindhu, you have got the Vitastha, which is now called as a Jhelum; you have got the Asikini, now called as a Chenab lovely rivers, in the beautiful Himalayas; you have got the Satadru, now called as the Sutlej. And you have, you are going to ask the question how do we track the density and velocity at a particular point in a river?

So, I will take up a very famous example, which is of this confluence, this is a Sangam; this is the confluence of two wonderful rivers Bhagirathi and Alakananda, these are tributaries to the ganga, both Himalayan rivers and this is the picture at a town called Devprayag which is called as the Devine confluence. And you have the Alakananda coming from this side, and the Bhagirathi coming from this side and they merge over here, this is the confluence.

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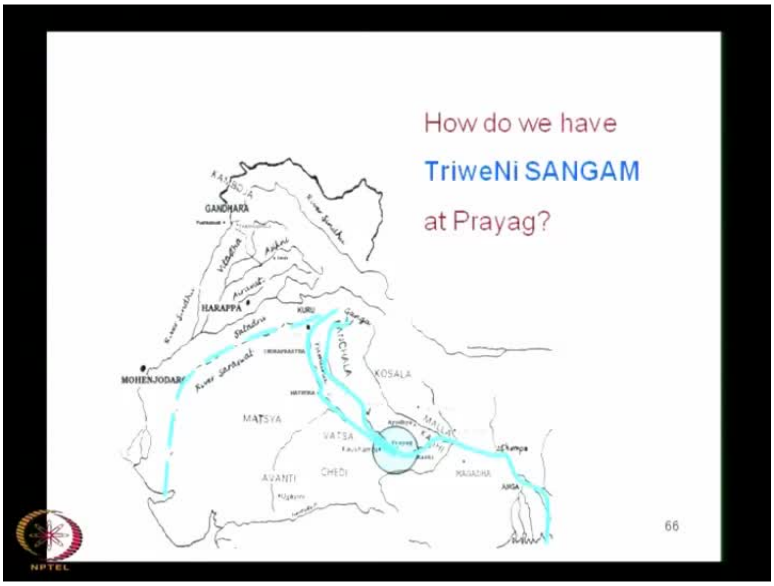

Now, the most famous confluence is of course the Prayag, the TriweNiSangam which everybody knows about and this is the confluence of the Ganga; so, this is the Ganga, this originates at Gangothri and Ganga flows like this. And this is the Yamuna which flows from Yamunotri, it has its origin in the Yamunotri glacier; it flows like this and it meets the Ganga at Prayag.

So, this is the TriweNiSangam; where is the third river, which is the third river and where is it? Saraswathi is the third river and it is supposedly underground, but underground, where under the ground, where you are sitting, where is it?

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Under those areas no it is not there. Saraswathi is under ground to a certain extent not fully parts of it have now being detected, but that is a different story, I will not going into that but most of it is underground that is quite true, but it is not under Prayagit is over here this is Saraswathi; this is the Saraswathi, do you see, it goes through Rajasthan and Gujarat and flows into the bay of Bengal, it never heads east toward Prayag.

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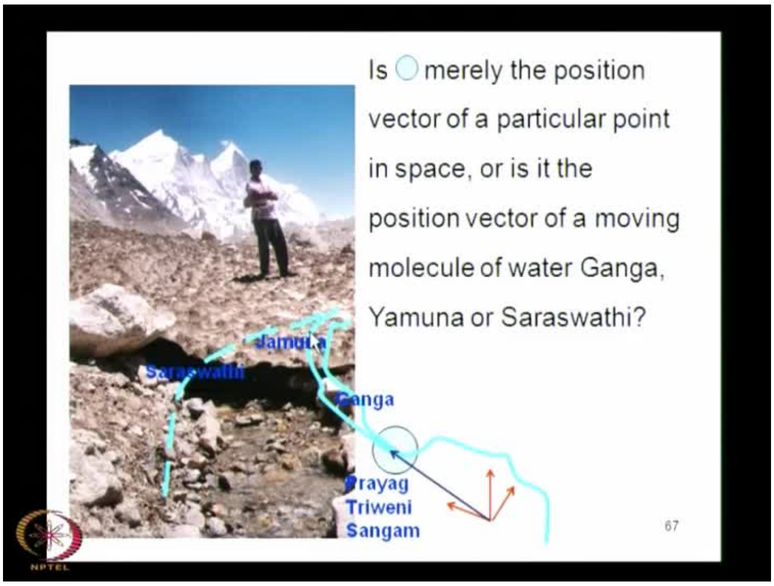

So, now, how is the TriweNiSangam now and even here it is mostly underground which is why I have shown it by a dash line; it is underground that part of the answer is correct except that, it is not underground in the UttarPradesh region or certainly nowhere close to Prayag it goes way of the west in India through Rajasthan and Gujarat or what is really happening is this, that here if you look at a molecule of water or a drop of water; again do not take the word molecule literally, because we have agree to talk about fluids in the limit of what we call as the continuum limit of matter.

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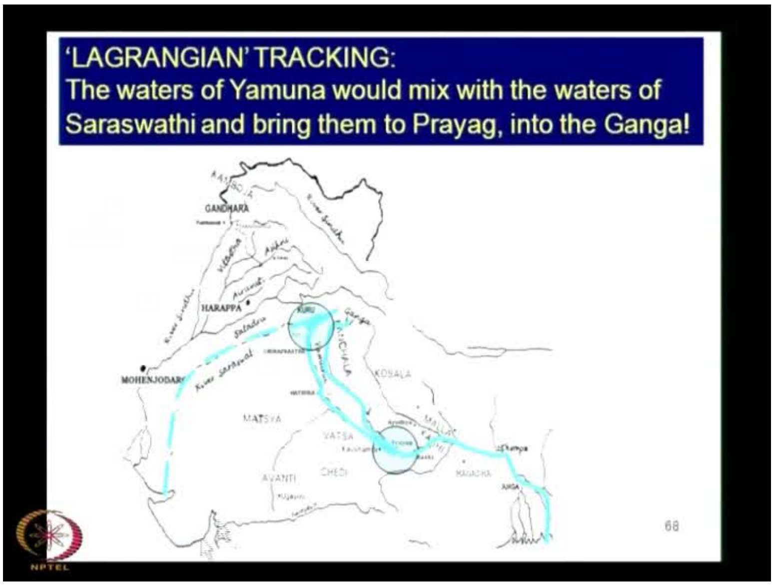

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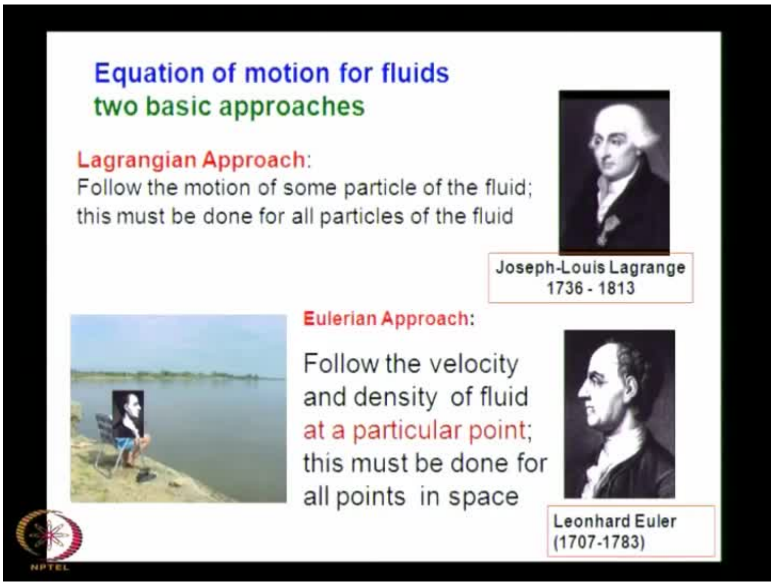

We are going to ask the question, if you look at a drop of water at the Sangam, there is no way of knowing whether this drop has come from Ganga. This is the Gangothri glacier and a drop of water in the Ganga would need to originate at the Gangothri glacier and then run down through its pass and reach Prayag or you would know if that particular drop of water has originated at the origins of the Yamuna and then traversed its pass and met the rivers of Ganga at Prayag, or may be over here the Yamuna mixed a little bit with Saraswathi, it over lapped and it drop to the waters of Saraswathi to Prayag and then what you have at Prayag is the TriweNiSangam. You cannot get this interpretation unless you had the Lagrangian description of the drops of water, because you have to follow the trajectories of particular fluid elements, in the Lagrangian description. In the Eulerian description you would just look at a point in the river flow and ask what is the velocity of the fluid over there? I do not care whether that particular drop of water has come from the Ganga or from the Yamuna or from Saraswathi.

So, this is the primary difference between the Lagrangian description of the fluid and the Eulerian description of the fluid and these are famous you know mathematicians and physicists who have contributed a lot, not just to fluid dynamics but to so many other branches of physics and also mathematics, but that is a big and a very existing story that we will not get into.

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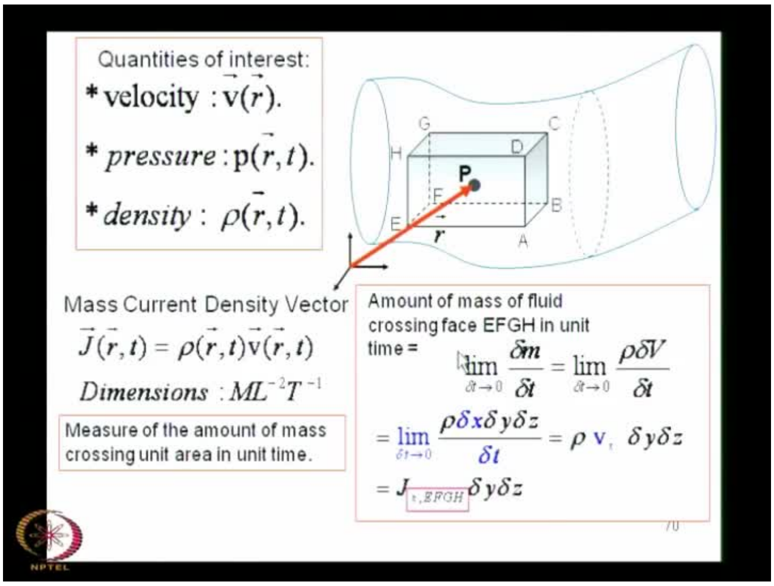

So, now, let us look at a point P, which is situated in the field through which fluid is passing; this is the point P and the quantities of our interest are, what is the velocity of the fluid at that point, what is the pressure of the fluid at that point, and what is the density of fluid at that point; the velocity can also be a function of time, so I will introduce that over here.

Then, I construct as we did in our last class, the mass current density vector which is the product of the density of the fluid and the velocity of the fluid. We figured in our last class that, this much have the dimensions of ML to the minus 2T to the minus, it is a mass per unit area per unit time.

So, this is the mass current density vector; you have got a coordinate say frame of reference for simplicity. I will considered Cartesian coordinate say frame of reference and just for the sake of discussion, I construct a parallelepiped because this is easy to handle in a Cartesian frame of reference but our results will be quite independent of which coordinate system; we are using because we could do this analysis in any other coordinate system and leave the physics in variant which it certainly should be.

So, now what is the mass of the fluid which is crossing the left phase; let us say that fluid is entering from the left phase and exiting from the right phase; what is amount of mass of the fluid which is crossing the left phase? The left phase is marked by these corner E, F, G, H; so, the mass of fluid which is crossing E, F, G, H in unit time; so that is the rate at which mass is flowing which is obliviously delta m by delta t and we will consider this in the limit delta t going to 0; so, at a certain instant of time.

Now, mass is volume into density we have no difficulty defining these quantities delta V going to 0, it is not what we are bother about in our continuum limit of mater, that we are working with. So, our rate of flow of mass which is delta m by delta t or d m by d t in the limit delta going to 0, this is rho times delta V by delta t in the limit delta t going to 0.

Delta V is nothing but the product of the three sides of this parallelepiped, which is delta x into delta y by delta z and the fluid is really passing along the x axis; this is my x axis in this figure, it goes from the left to the right.

So, delta x by delta t will give me the x component of the velocity of the fluid. and this area delta y by delta z sticks out as a multiplicative factor and this rate d m by d t is now rho times v x multiplied by this area, but rho times v x is my current density vector; so this is the x component of the current density vector, but where is this to be determined? It must be determined on the phase E, F, G, H, it may have a different value or different points in the liquid, which is why I place a suffix x on J because it is the x component of that, but I also include in the subscript the phase E, F, G, H, because it is on this phase that I must determine the x component not anywhere else. So, this is the amount of mass of fluid crossing the phase E, F, G, H in unit time.

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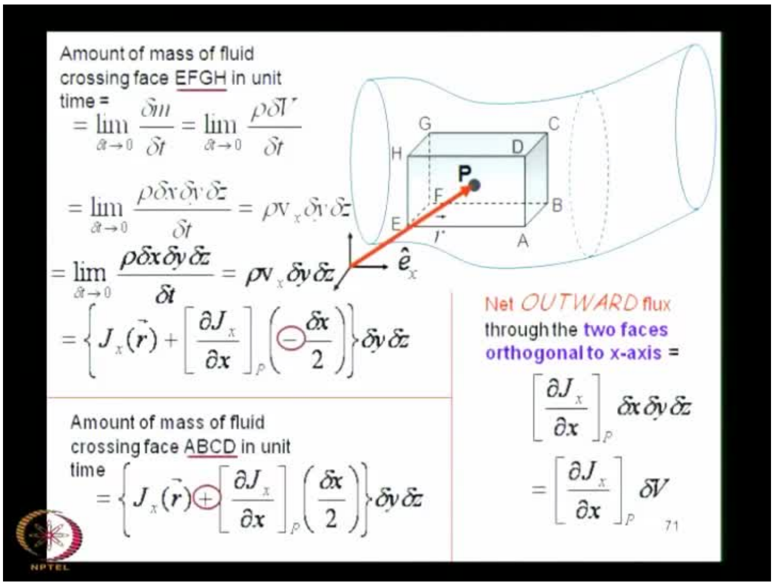

Now, so far so good, we have got this rho times V x delta y delta z and this is the value on the phase E, F, G, H, will it be different from what the value is at P certainly and the difference can be obtain by using a simple derivative, it will be different from the value at the point P by an amount, which is given by the rate of change of J x with x multiplied by the displacement from this point which is half of delta x.

So, I must subtract from the value of the current density vector, the x component of the current density vector at the point P, I must subtract from this, the rate of change of the x component of the current density vector with x multiplied by this difference which is half of delta x; but I have to subtract that because I am going to the left the value of the x parameter is diminishing on the left; it increases when you go from left to right; it diminishes when you go from right to left; so, there is a minus sign over here, are we all together on this.

So, what about the amount of mass of the fluid which is crossing, now the phase ABCD that is where it is exiting from; it will be given by a very similar quantity which is not quite the value of the x component of the current density vector at the point P, but it will be different from it the difference will be given by the rate of change of J x with x multiplied by this displacement, which is delta x by 2. And you should added to the value of J x at this point P; so, that is your result here, that the amount of mass of fluid crossing the phase ABCD in unit time is J x r plus del J x by del x delta x by 2 and then you have got the surface element which is multiplied by it as just as before.

So, now, if you just concentrate on these two phases, there are three pairs of phases, we now focus on these two; so, this is one pair of phase, the net outward flux, from these two phases which are orthogonal to the x axis will be given by the difference of the these two terms and which you take the difference the J x will cancel. And from this, you subtract the previous term which has an inbuilt minus sign; so, the delta x by 2 pieces will add up to give you a net delta x, the delta y, delta z comes as a common multiplier and what sticks out is a product of delta x, delta y and delta z which is nothing but the volume of the parallelepiped; it is a very simple result; are we all together, that is very good.

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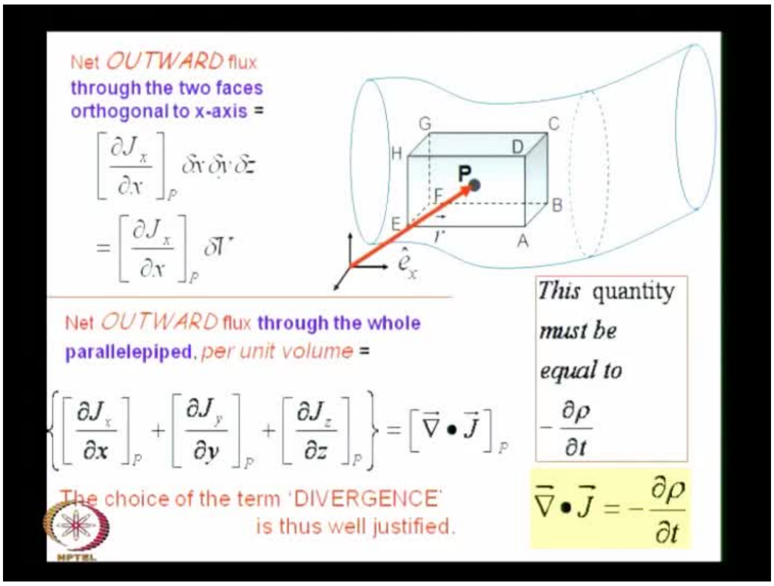

So, this is the net outward flux subject to the consideration, that we have now considered only those two phases which are orthogonal to the x axis, but then there are two other pairs, there are these phases and that these phases; so, let us add them up and that should be very easy because all you have to do, is do an exactly identical analysis for the other two axis. And then each will give delta V for a different reason, the elemental areas will come from delta z delta x in one case, and from delta x delta y in the other. But when you get the delta x and delta y, it is the delta z by 2 which will add up to the other delta z by 2 giving you the delta z; so, in every case, the factor delta V will become common and when you add up all three pieces, you have these three pieces del J x by del x plus del J y by del y plus del J z by del z. And now you ask what will be the corresponding quantity per unit volume, so this delta V will be cancel because to get the corresponding quantity per unit volume you have to divided by that volume element which is delta V.

So, the net outward flux through the whole parallelepiped per unit volume, now dividing it by delta V will be just the some of these three partial derivatives determined at the point P. And we have already seen in our previous class, that, this is nothing but the divergence of the current density vector.

So, the current density vector gives you the net outward flux through the whole parallelepiped, and by conservation of matter, this must be equal to the negative rate of change of density in the region which is bound inside. So, this is the equation of continuity what it essentially, means, that the term divergence is a very well chosen term, because divergence gives you an idea of what is the net flux coming out. And we have seen that this really gives us an exact measure not just a qualitative measure, but a qualitative measure of how much of the fluid is oozing out per unit volume per unit time.

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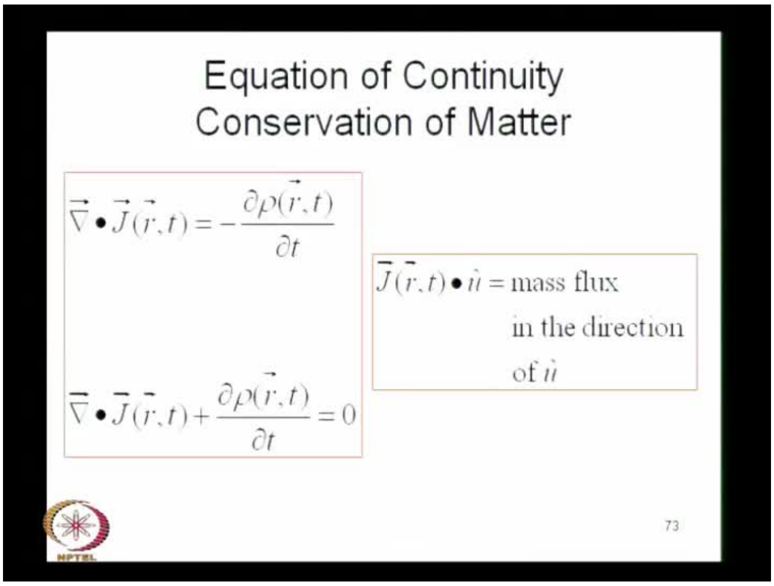

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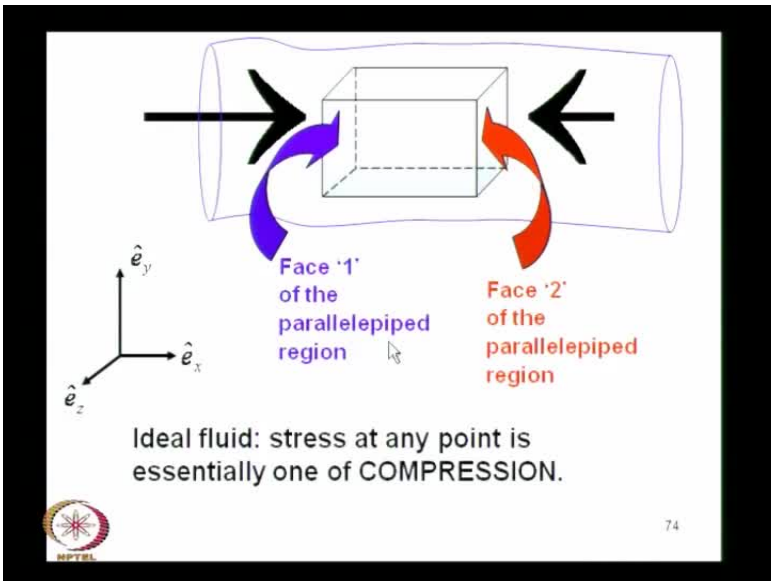

So, the word divergence is very appropriate. This is the equation of continuity as we have seen already. We now remind ourselves that the fluid that we are dealing with is an ideal fluid, which means, that the stress at any point is one of compression and we will now use this to set up the equation of motion for the fluid, which is what we have set ourselves how to do.

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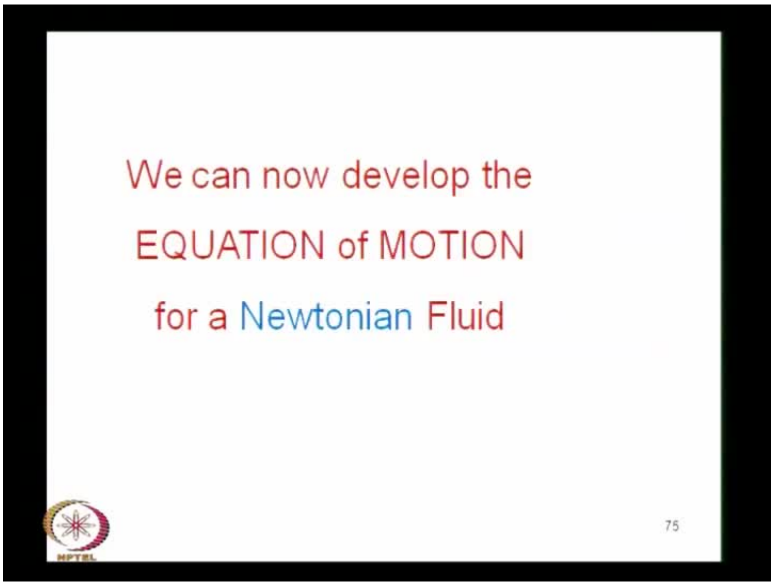

So, let us consider these two phases now; again, face 1 on the left, face 2 on the right and we know that because of fluid we are talking about is an ideal fluid; this stress is one of compression and it is this compression which I indicate by the arrows that you see. And using this we will develop the equation of motion for a Newtonian fluid, we already know what a Newtonian fluid is and we already know what are Non-Newtonian fluid is.

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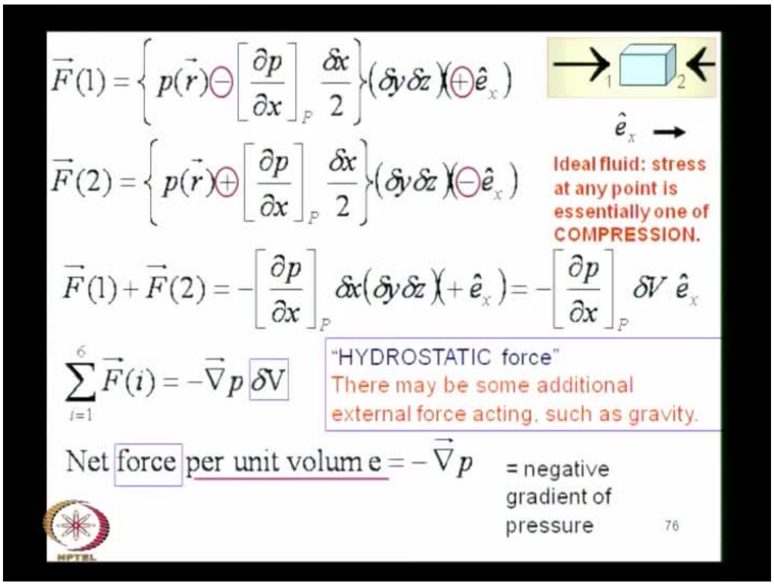

So, now to get the equation of motion, if an equation of motion will give us a relationship between position, velocity, and acceleration in the Newtonian spirit, it must give us… Does it have a camera, dual SIM card, good I need it.

It is confiscated, thank you very much; you do not have to be sorry for that, I have to thank you for that you have lost it; I hope it is recorded there is no reason to cut this, because I want to record my gains of this lecture .

So, let us continue our discussion here, I desperately need cell phone with the dual SIM card. So, the force acting on face 1is pressure multiplied by area; if pressure is force per unit area force better is pressure multiplied by area. But then, where is a pressure to be determined? The pressure is to be determined on face 1, not at the point which is inside why the pressure may be different.

So, it is pressure at the internal point p and then different from it by an amount which is given by the rate of change of the pressure with x multiplied, by the displacement from that point which is delta x by 2, but they must be a minus sign over here and what is the direction of this force it is from left to right; so, it is along plus e x.

So, this sign of plus e x must be kept track off, this minus sign must be kept track off, because you are evaluating the force on the face 1, which is to the left of the point p and when you go to the left the value of x decreases; so, the displacement is minus delta x by 2; so, there is a minus sign over here and the plus sign omega.

What about the force of the face 2; again, it is pressure multiplied by area, so you have got the pressure quantity here multiplied by the area, which is delta y by delta z. The direction of this force is from right to the left why? Because it is an ideal fluid, in which this stress is essentially one of compression, so it has to be inward to the region you are talking about.

So, there is a minus sign over here and this force on the face 2 is not the force at the point p but to the right of it; so, it will be somewhat different. And the difference will be the pressure at the point p plus the rate at which the pressure change is with x multiplied by the displacement, which now is plus delta x by 2 because x is increasing from left to right.

So, now, we know why we have got our minus sign here, a plus sign here, and a plus sign here, and a minus sign here and then we can determine the net force on the parallelepiped first by considering the net force on these 2 faces and then by adding up corresponding term from the other 2 pairs of faces.

So, from these 2 pairs of faces, the net force from the face 1 and face 2 is the sum of these two of which this term cancels, because this term comes with this plus sign and this term comes with this minus sign; so, this term cancels and these two will add up but with a negative sign, because there is this minus over here, this is the product of this minus and this plus and here this is the product of this plus with this minus.

So, the net force which is the sum of the all the forces, this is the addition of all the forces will be minus del p by del x, which is the rate at which the pressure changes at the point p multiplied by the area element which is delta y into delta z and then you have these two pieces of half delta x adding up to give you a net delta x; the product of these three delta x delta y and delta z giving you the volume delta V; so, this is the net force on the parallelepiped coming from then two phases 1 and 2.

Now, you add up from the other 2 pieces, it is a very straightforward addition, now you can add the corresponding pieces; so, you will add del p by del x e x plus del p by del y e y plus del p by del z e z, that will give you the radiant of it; we have already defined it earlier so that gives us the gradient of p.

So, the net force per unit volume, when you divided by this volume element will be nothing but the negative gradient of pressure; this is the net force per unit volume on the fluid at the point p, in the continuum limit when delta V goes to 0, which is why you can talk of it as a quantity which is a property of that particular point p, because you have shrunk the volume of the parallelepiped to a point. So, this is the negative gradient of pressure this is what is called as the hydrostatic force.

The term hydro, of course, comes from water butit is always called as hydrostatic force, no matter which fluid you are talking about, as long as you are dealing with ideal fluids. So, it always called as hydrostatic pressure or you can also call it as the fluid force because of the fluid pressure, but in addition to this, there may be other forces the fluid may be in the presences of some external field, maybe we already know this fluid is in the presence of gravity; so, at any point inside, there is a force due to the hydrostatic pressure but then there is also force due to gravity.

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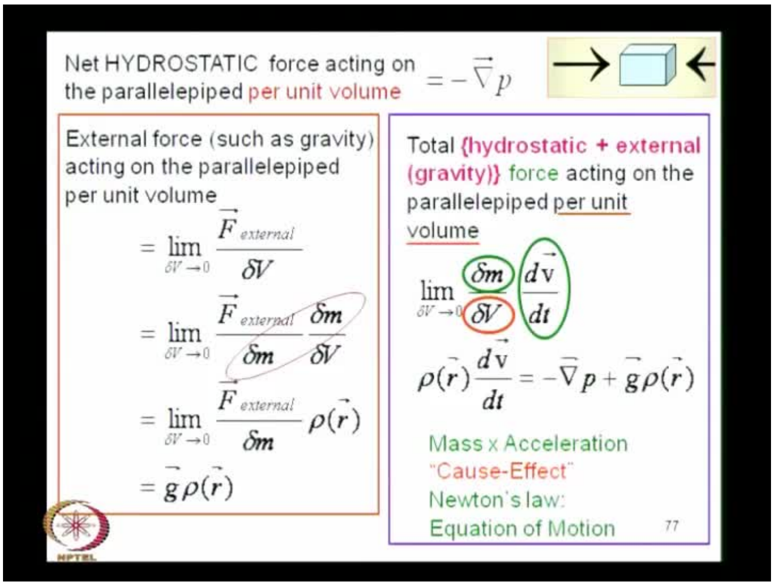

So, that is the external force that we are talking about so let us put that in now. So, if the external force and for the sake of common application, and common consideration I will consider gravity; so, the external force acting on the parallelepiped per unit volume is what we want, so force per unit volume is what we want and to get this what I do, is I multiply and divide by a mass element, because these two masses will cancel each other happily and then I have force per unit mass multiplied by mass per unit volume in the continuum limit delta V going to 0, which gives me the density. And from this quantity I get essentially the acceleration due to gravity. So, what I get is the external force acting on the parallelepiped per unit volume is given by the acceleration due to gravity, for which I have use the common symbol g; this is the g vector multiplied by density; so, this is the external force.

So, now, there are two forces which are acting on the fluid one is the hydrostatic force, which we defined in the previous slide, which is the negative gradient of pressure. And an external force which is the product of the acceleration due to gravity and the density of the fluid. What is this equal to? This is the net force per unit volume or what is force? Forces mass into acceleration; this is the principle of causality we learnt, this is how Newton explained why equilibrium gets disturbed. Equilibrium sustains itself through Galileo's law of inertia, as long as no force acts on the body, as the law of inertia, the first law which we learn from Galileo incorporated in the Newtonian scheme as the first law of Newton. When equilibrium is disturbed, it happens, because there is a cause which results in an effect; the effect is the acceleration that causes the force in the linear responsible relationship of Newtonian mechanics; the acceleration is proportional to the cause. So, mass, times, acceleration is this force, the proportionality is the mass, but here we are talking about the mass per unit volume, because we have defined this external force per unit volume; so, the quantities we are dealing with are defined per unit volume.

So, this is mass times acceleration, which is what you see in the green loops with the difference that the quantities, our, under our consideration are recognized, in the sense that they are scaled per unit volume; so, we must divide by delta V.

So, this is now your result, delta m by delta V is mass per unit volume, which is the density; so, the density times, this acceleration is equal to the total force, which is the sum of the negative gradient of pressure which is a hydrostatic force plus the external force which in our case is due to gravity which is g times density; so, we have got the equation of motion.

F equal to i m a; this equation that we see infront of us is exactly F equal to i m a, except that it has been scaled per unit volume. So, this is the equation of motion for the fluid. This is coming from the cause effect relationship of Newtonian dynamics, this is the equation of motion.

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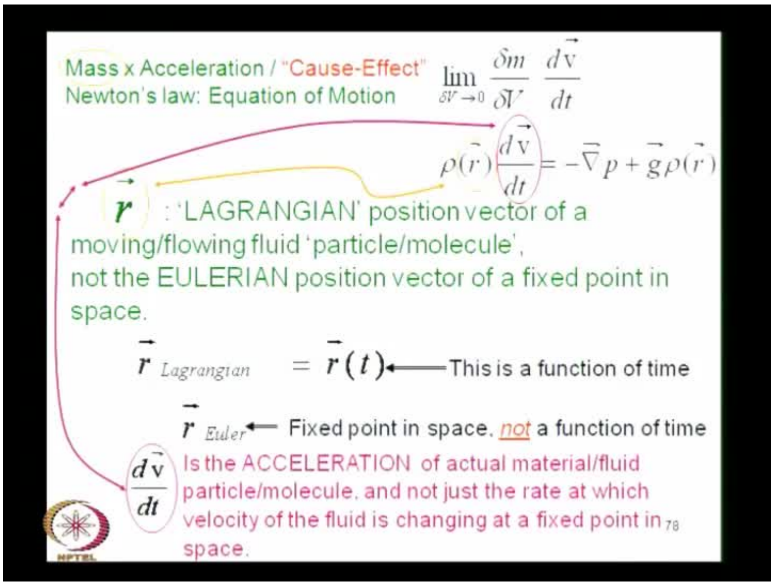

The only thing we have to remember, is that, when we talk about mass times acceleration, this is the acceleration; when you said if you look at this object and drop it under gravity or throw it and fall its trajectory, then you are looking at the acceleration of this particular object, and you must trap this object from time to time, you must have a Lagrangian description of this object Newton's law; the acceleration which is defined in Newton law, must of course refer to the acceleration and the corresponding coordinates being described in the Lagrangian, since it is not the property of a fix Eulerian point in space.

So, this quantity over here is a Lagrangian position vector, this velocity is a velocity rate of change of velocity of a particular fluid particle or molecular drop used in a loose since I am not using the molecule in the sense of H 2 O that is not the level structural detail that we have in our consideration.

We are working with the continuum limit in which delta V is much smaller than the volume of the fluid but much larger than the atomic and molecular structure which makes of the fluid. So, this is not the Eulerian position vector, this is the Lagrangian position vector, because it is entering directly the equation of motion in the Newtonian sense.

So, this Lagrangian position vector, which is the position vector of a particular drop of fluid will change from time to time; an Eulerian position vector stays wherever it is and we will remain so ever after, like a happily married couple.

This is a position vector which is a function of time, it must change with time; it is only the Eulerian position vector which is the fixed point in space it is not a function of time and this derivative is the acceleration of an actual material fluid particle or a drop of fluid, it is not just the rate at which a velocity of fluid is changing at a fixed point, at a fixed Eulerian point in space; so, you have to remember that.

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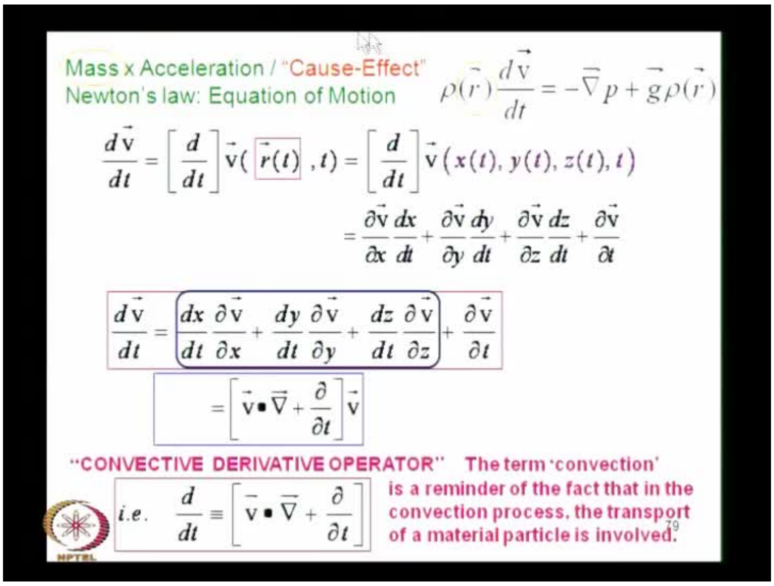

What it means is that, this velocity is a function of this position, because the velocity of this object is one at this instant of time, it is different at later instant of time. And it changes because the position itself changes from time to time.

So, the velocity is a function of position, which in turn is the function of time; so, this is what we call as an explicit dependence on time coming from here and then implicit dependence on time coming from its dependence on the position.

So, the velocity depends on time in two ways, one is through an explicit dependence on time, this is an implicit dependence on time, because it depends on the position vector, which itself is changing from time to time; this is the Lagrangian position vector. And then depend of course explicitly on time as well, because it will be different from yesterday to today to tomorrow, at the same time, at the same position.

At a given position in space like the traffic example that we considered you had a certain velocity, at a certain point at Anna Salai, but if you want to go through the same point on a Sunday, you could perhaps go much faster, or at 3'o clock in the morning when everybody else is sleeping you could go much faster; so, it will have an explicit dependence on time, because it will be different from Sunday to Monday and from 2'o clock in the afternoon to 2'o clock in the morning, it will be different. So, that is an explicit dependence on time and it also dependence implicitly on time, because it dependence on the position, which itself is a function of time.

So, when you take the time derivative of the velocity you have to use the chain rule, because the time derivative of velocity which is the acceleration of the fluid particle or the fluid drop. This has to be determine as the rate of change of velocity with respect to time, but this change in velocity comes because x changes of a time and also because the velocity itself depends explicitly on time.

So, you have to use the chain rule; so, it will come as a partial derivative of the velocity with respect to x times, the derivative of x with time plus del v by del y which is the partial derivative of the velocity with respect to y times d y by d t and there is a similar term coming from the dependence on z, and then there is an explicit dependence on time indicated by the partial derivative of the velocity with respect to time; so, there are four terms one, two, three and four which contribute to this acceleration.

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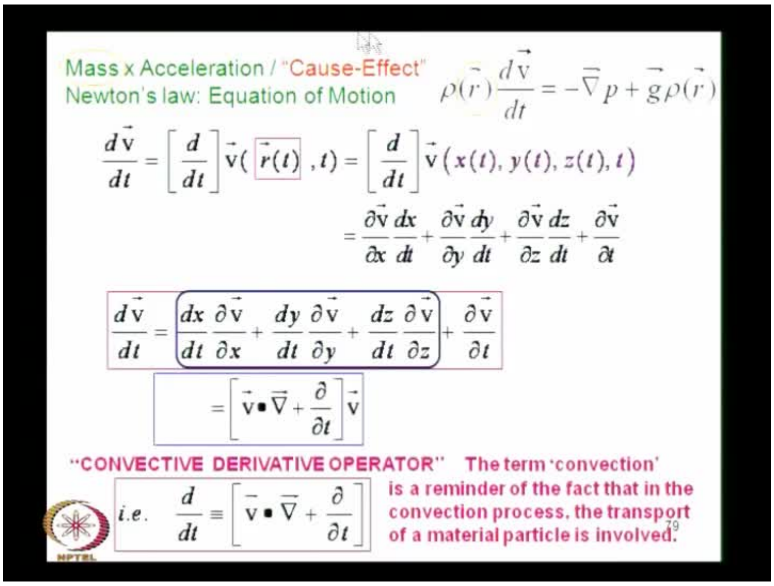

So, let us write out these four terms neatly, all I have done is to written the d x by d t to the left and del v by del x to the right, it does not matter where you write it; these are products, which whose positions I have interchange and it is the same set of four terms, but now you can see that, if you look at the first three terms they look like the scalar product of v and another quantity. Because the scalar product of two vectors a dot b is a x b x plus a y b y plus a z b z, do not you see a relation like that; it has some similarity to that.

There is a big difference this is not a scalar product of two vectors, but it then operational equivalent notation, you can write this quantity in this blue box, this del v by del t I have written separately this del v by del t, I have written over, here this is del v by del t. But what is inside this blue box is what I have written as, v dot del operator operating on v; this is not the scalar product of two vectors, it has got some similarity because it is structurally similar in the certain sense. The reason it is not a scalar product of two vectors because the gradient operator really operates on a scalar function and not on a vector but I have exploited the structural similarities to write this relationship.

So, the gradient operator does not operate on the velocity, it can do so, only through the divergence or curl operations, out of which the divergence we have defined curl, we will define in the next class.

So, this is what is called as the convective derivative; so this gives us an operational in equivalence this d by d t is now an operator which is the time derivative operator, which is completely equivalent to what you see in this box, which is v dot del plus del by del t; this is called as the convective derivative, because it reminds you of convection. In convection, you know that whenever heat is transmitted through convection currents, it is the hot elements which are bubbling out, which will carry that physically; so, there is a physically transport of those hot objects.

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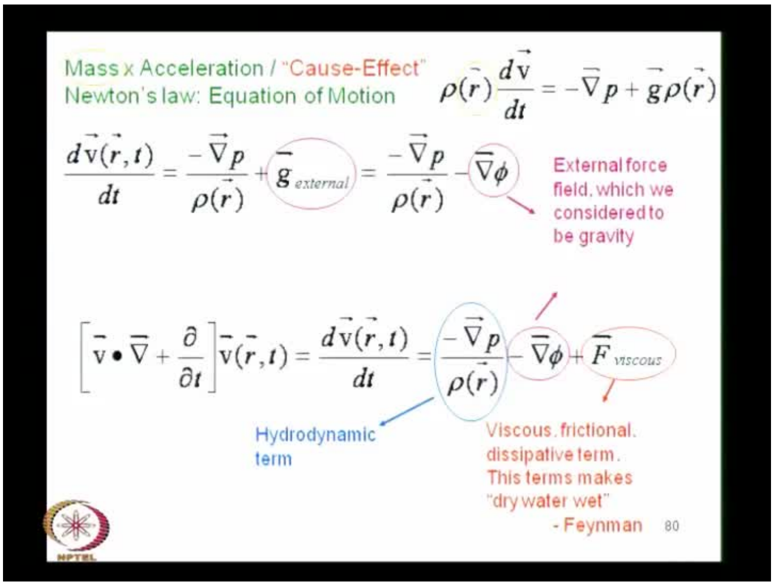

So, that is the Lagrangian description that you must involve it is that idea which is central to this term convective derivative; so, the d v by d t which has now entered your equation of motion is the convective derivative and this d v by d t must be replaced by these four terms, you can write it in anyone of these forms, they are all mathematically completely equal to each other. And what we have set up is the equation of motion, which is the cause effect relationship for the fluid; and the relationship for the acceleration, if you divide both sides by the density which is just a common form in which you meet this equation of motion.

So, it is usually written not for density times the acceleration but for acceleration itself; so, all you do is to divide both the sides by the density, so of the left hand side you have got the acceleration which is the convective derivative of the velocity and from this term you get the ratio of the hydrostatic pressure, or rather, the negative hydrostatic pressure divided by the density plus over here, when you divide this quantity by density it cancels this density and you are left only with the acceleration due to gravity, which is nothing but the negative gradient of the gravitational potential.

So, you can write this both the quantities as gradient of scalars, this is the scalar gradient of the pressure, this is the gradient of the gravitational potential; so that is the advantage in writing it in this form.

As a matter of fact, this really completes the derivation of equation of motion for an ideal fluid. So, this is the external field which we have consider to be gravity, in our case, but very often you see an additional term sticking out over here; so, the convective derivative, if now written in the form in which we met it in the previous slide.

You have got the hydrodynamic term which is this one then you have got this external field which is the negative gradient of the potential which in our case is the gravitational potential. Usually, this term is added when you see the equation of motion this is added on an ad hoc basis because you know that you are really not dealing with an ideal fluid, what is ideal; in this world other than Prabhu Ramachandra may be there is nothing which is really ideal.

So, there are some losses, some factors unspecified degrees of freedom, there is friction there are some losses; so, you add through in that term on an ad hoc basis which you sometimes called as the viscous term. So, usually the equation of motion for a fluid is written in form, in which you see it here, this is the viscous or the frictional deceptive term; this results any departure of the fluid from what is ideal Feynman calls is as the term, which makes dry water wet because water is made up of two hydrogen atoms and oxygen, everything is dry when you compose water, why should it be wet, what is wetness, means, if I pore some water and I think it is wetting my finger, why this finger is dry, this one is wet.

The reason is that some of this water has got in stuck to my finger. and this is because of rather complex and detailed interactions at the molecular level between the molecules of the liquid and the molecules which out on the top of the surface of my skin.

So, you really need to get into those molecular interactions to understand the wetness, because at that level, there is a certain property which has a neighbour some of that water to get struck to this, the rest of the water has fallen through.

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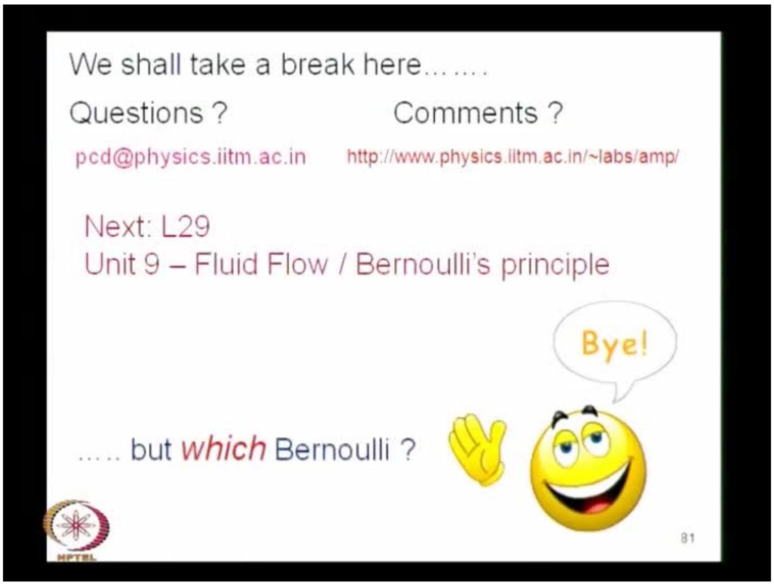

So, this is what Feynman calls, as the term which makes dry water wet and what we really have is the complete equation of motion for an ideal fluid or at least a nearly ideal fluid or at least a fluid, whose property is do not depart significantly from an ideal fluid, not so violently as to make it like a complex fluid, like the one that you saw on which you can walk, what will be a Non-Newtonian fluid.

So, the fluid we are talking about is still in Newtonian fluid very nearly ideal but not exactly and the departure from ideal nature is not very violent, we can sort of deal with it by through in and ad hoc viscous term; so, that is what we have done.

So, I will be happy to take a few questions or comments and then we will meet in the next class, which will be lecture number 29 on unit 9, when we shall discuss the Bernoulli's principle, there are so many Bernoulli's, but we will talk about it next time. If there any questions, I will be happy to take the cell phone, of course, I have to take there, is anything, any other comment or question. I can exchange mine, actually I have one, which is very old and it can take only one SIM card, I need one with two. So, I was hoping that you know the cell phone will ring and it will be a good one; so, thank you for it. Any question.